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Authors

Goodman, A.L.

Struble, G.L.

Goswami., A.

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C.L.

University of California

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Departments of Nuclear Engineering and Chemistry

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Lawrence Radiation Laboratory
University of California
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ABSTRACT

A generalized quasiparticle transformation is presented which includes n-n, p-p, and n-p ($T=0$ and $T=1$) pairing correlations. The resulting gap equations for $N = Z$ nuclei are solved for Mg^{24} , indicating that n-p ($T=0$) pairing correlations restore axial symmetry to the equilibrium shape.

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A. L. Goodman and G. L. Struble

Departments of Nuclear Engineering and Chemistry

and

A. Goswami[†]Lawrence Radiation Laboratory
University of California
Berkeley, California

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The equilibrium shape of Mg^{24} in its ground state is predicted to be ellipsoidal by Hartree-Fock (H.F.) calculations¹⁾. The SU_3 classification scheme²⁾ also indicates an ellipsoidal shape for Mg^{24} and this is often quoted as a further support of the H. F. result. Both the H. F. and the SU_3 method leave out the very important nuclear pairing correlations which are known to prefer more symmetric shapes. For example, pairing correlations are responsible for the occurrence of a vast number of spherical nuclei³⁾. Kumar and Baranger⁴⁾ have shown that the shapes of the deformed nuclei in the rare earth region are in fact axially symmetric rather than ellipsoidal also because of the pairing correlations.

The H. F. calculations for Mg^{24} give several minima. Of these the lowest one of course corresponds to the axially asymmetric shape. There is also an axially symmetric (prolate) solution which is less bound only by ~ 3.5 MeV. A very important difference between the axially symmetric and the asymmetric solutions is that the latter shows a large energy gap between the

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[†] Present address: Case Western Reserve University, Cleveland, Ohio.

occupied and the unoccupied single particle states, whereas the former exhibits a much smaller single particle gap. As a result of this, pairing correlation effects should be prominent in the axially symmetric state and should lower its energy whereas the axially asymmetric state would remain unaffected by pairing correlations. The purpose of the present note is to show that pairing correlations if properly taken into account actually lower the energy of the axially symmetric state below that of the triaxial state, i.e., restores the axial symmetry of the equilibrium shape of Mg^{24} . To state this slightly differently, we wish to show that configuration mixing of a generalized pairing type actually lowers the energy of the axially symmetric state more than the $K \pm 2$ and $K \pm 4$ configuration mixing implied by the triaxial H. F. solution.

Goswami and Kisslinger⁵⁾ have shown that in light $N = Z$ even-even nuclei one should expect correlated $T = 0$ ($J \neq 0$) independent pairs in the ground state (rather than the usual $J = 0$, $T = 1$ pairing observed in heavy nuclei) in view of the importance of the n - p $T = 0$ force and of isospin conservation. They have also demonstrated by a single j -shell calculation that the $T = 0$ pairing correlations are indeed coherent. Recently Chen and Goswami⁶⁾ have shown that both $T = 0$ and $T = 1$ pairing⁷⁾ can be simultaneously treated by introducing a complex order parameter Δ_{np} corresponding to n - p pairing. They have also derived simple gap equations for $N = Z$ nuclei in a deformed H. F. representation by ensuring that $\langle T \rangle = 0$, i.e., the expectation value of all components of isospin vanish in the ground state. In this paper we shall numerically solve the gap equations for the case of Mg^{24} in the axially symmetric and the asymmetric H. F. representations of ref. 1.

We start with the Hamiltonian

$$H = \sum_{\alpha\mu} \epsilon_{\alpha\mu} C_{\alpha\mu}^+ C_{\alpha\mu} + \frac{1}{2} \sum_{\alpha\mu\beta\nu, \gamma\rho\delta\sigma} V_{\alpha\mu\beta\nu, \gamma\rho\delta\sigma} \times C_{\alpha\mu}^+ C_{\beta\nu}^+ C_{\delta\sigma} C_{\gamma\rho}$$

where $\mu\nu\rho\sigma$ are isospin indices. $C_{\alpha\mu}^+$ denotes the creation operator for the single particle H. F. state $|\alpha\mu\rangle$

$$|\alpha\mu\rangle = \sum_{jm} \phi_{jm}^{\alpha} |jm\mu\rangle \quad (2)$$

expanded in terms of the spherical $|jm\rangle$ representation. The coefficients ϕ_{jm}^{α} are independent of μ for $N = Z$ nuclei and are obtained by solving

$$H_{H.F.} |\alpha\mu\rangle = \epsilon_{\alpha\mu} |\alpha\mu\rangle \quad (3)$$

where $H_{H.F.}$ is given in the spherical representation by

$$\langle j_1 m_1 \mu | H_{H.F.} | j_2 m_2 \mu \rangle = \epsilon_{j_1 \mu} \delta_{j_1 j_2} + \sum_{j_3 m_3 j_4 m_4 \nu} \langle j_1 m_1 \mu j_3 m_3 \nu | V | j_2 m_2 \mu j_4 m_4 \nu \rangle \langle C_{j_3 m_3 \nu}^+ C_{j_4 m_4 \nu} \rangle \quad (4)$$

and $\epsilon_{\alpha\mu}$ are the single particle H. F. energies. The H. F. solutions satisfy

(i) the time reversal symmetry, i.e., the time reversed state

$$|\bar{\alpha}\rangle = \sum_{jm} \phi_{jm}^{\alpha} (-1)^{j-m} |j-m\rangle \quad (5)$$

is degenerate with $|\alpha\rangle$ and

(ii) reflection symmetry.

This restricts $H_{H.F.}$ to a much simpler form connecting only (jm) states with $\Delta m = 0, \pm 2, \pm 4$.

The axially symmetric and the triaxial solutions of the H. F. equations for Mg^{24} with V having a Rosenfeld exchange mixture, i.e.,

$$V = V_0 \frac{\tau_1 \cdot \tau_2}{3} (0.3 + 0.7 \sigma_1 \cdot \sigma_2) \frac{e^{-r/a}}{r/a} \quad (6)$$

where $V_0 = 50$ MeV, $a = 1.37$ fm, and the oscillator parameter $b = 1.65$ fm have been given in ref. 1.

With the prescription of Chen and Goswami that (i) Δ_{np} be allowed to be complex with real part corresponding to $T = 1$ pairing and imaginary part corresponding to $T = 0$ pairing and (ii) $\langle T \rangle = 0$, the matrix of the BCS-Bogoliubov linearization becomes*

$$\begin{pmatrix} \epsilon_{\alpha} - \lambda & 0 & \Delta_p(\alpha) & \Delta_{np}^*(\alpha) \\ 0 & \epsilon_{\alpha} - \lambda & \Delta_{np}(\alpha) & -\Delta_p(\alpha) \\ \Delta_p(\alpha) & \Delta_{np}^*(\alpha) & \lambda - \epsilon_{\alpha} & 0 \\ \Delta_{np}(\alpha) & -\Delta_p(\alpha) & 0 & \lambda - \epsilon_{\alpha} \end{pmatrix} \quad (7)$$

* neglecting the small change of ϵ_{α} due to pairing

where $\Delta_{np} = \Delta_{np}^{T=1} + \Delta_{np}^{T=0}$, Δ_p is the order parameter corresponding to pairing of protons, and λ is the chemical potential.

The matrix (7) can be diagonalized by introducing the following generalized Bogoliubov canonical transformation

$$\begin{pmatrix} c_{\alpha 1}^+ \\ c_{\alpha 2}^+ \\ c_{\alpha 1}^- \\ c_{\alpha 2}^- \end{pmatrix} = \begin{pmatrix} u_{\alpha} & 0 & -v_{\alpha} & -v_{\alpha}' \\ 0 & u_{\alpha} & -v_{\alpha}'^* & v_{\alpha} \\ v_{\alpha} & v_{\alpha}' & u_{\alpha} & 0 \\ v_{\alpha}'^* & -v_{\alpha} & 0 & u_{\alpha} \end{pmatrix} \begin{pmatrix} c_{\alpha p}^+ \\ c_{\alpha n}^+ \\ c_{\alpha p}^- \\ c_{\alpha n}^- \end{pmatrix} \quad (8)$$

where u_{α} , v_{α} are real and v_{α}' is complex. Here the subscripts 1 and 2 denote the two (degenerate) types of quasiparticles obtained by mixing the neutron and the proton. The quasiparticle energy is given by

$$E_{\alpha} = \sqrt{(\epsilon_{\alpha} - \lambda)^2 + \Delta_p^2(\alpha) + |\Delta_{np}(\alpha)|^2} \quad (9)$$

The order parameters Δ are obtained from the solution of the following gap equations

$$\Delta_p(\alpha) = \sum_{\alpha' > 0} \langle \alpha \bar{\alpha} T = 1 | V | \alpha' \bar{\alpha}' T = 1 \rangle_a u_{\alpha} v_{\alpha'} \quad (10)$$

$$\Delta_{np}^{T=1}(\alpha) = \sum_{\alpha' > 0} \langle \alpha \bar{\alpha} T = 1 | V | \alpha' \bar{\alpha}' T = 1 \rangle_a u_{\alpha} \text{Re } v_{\alpha}'$$

$$\Delta_{np}^{T=0}(\alpha) = \sum_{\alpha > 0} \langle \alpha \bar{\alpha} T = 0 | V | \alpha' \bar{\alpha}' T = 0 \rangle_a u_{\alpha} \text{Im } v_{\alpha}'$$

$$\begin{aligned}
 u_{\alpha}^2 &= \frac{1}{2} \left(1 + \frac{\epsilon_{\alpha}^{-\lambda}}{E_{\alpha}} \right) \\
 v_{\alpha}^2 &= \frac{1}{2} \left(1 - \frac{\epsilon_{\alpha}^{-\lambda}}{E_{\alpha}} \right) \left(\frac{\Delta_p(\alpha)}{\Delta(\alpha)} \right)^2 \\
 (\text{Re } v'_{\alpha})^2 &= \frac{1}{2} \left(1 - \frac{\epsilon_{\alpha}^{-\lambda}}{E_{\alpha}} \right) \left(\frac{\Delta_{np}^{T=1}(\alpha)}{\Delta(\alpha)} \right)^2 \\
 (\text{Im } v'_{\alpha})^2 &= \frac{1}{2} \left(1 - \frac{\epsilon_{\alpha}^{-\lambda}}{E_{\alpha}} \right) \left(\frac{\Delta_{np}^{T=0}(\alpha)}{\Delta(\alpha)} \right)^2
 \end{aligned} \tag{11}$$

$$\Delta(\alpha) = \sqrt{\Delta_p^2(\alpha) + |\Delta_{np}(\alpha)|^2}$$

Substituting (11) into (10) one can get the gap equations first derived in ref. 6. λ is to be determined from the number equation:

$$Z = \sum_{\alpha > 0} \left(1 - \frac{\epsilon_{\alpha}^{-\lambda}}{E_{\alpha}} \right) \tag{12}$$

The pairing energy is given by

$$\begin{aligned}
 E_{\text{pair}} &= 2 \sum_{\alpha > 0} \{ \Delta_p(\alpha) u_{\alpha} v_{\alpha} + \Delta_{np}^{T=1}(\alpha) u_{\alpha} \text{Re } v'_{\alpha} \\
 &\quad + \Delta_{np}^{T=0}(\alpha) u_{\alpha} \text{Im } v'_{\alpha} \}
 \end{aligned} \tag{13}$$

The gap and number equations (10) and (12) are solved for Mg^{24} using both the axially symmetric and the asymmetric Hartree-Fock solutions of ref. 1. In the axially asymmetric state there is no lowering of energy due to pairing.

The pairing solution for the axially symmetric H. F. solution is shown in Table 1. The energy lowering due to pairing is 4.3 MeV so that the paired ground state is ~0.8 MeV lower than the axially asymmetric H. F. state. Since the zero point fluctuation energy for Mg^{24} is expected to be smaller than this (there being no prominent vibrational structure in Mg^{24}), we conclude that pairing restores the axial symmetry of the equilibrium shape of Mg^{24} .

It is interesting to note that the BCS solution obtained here corresponds to $\Delta^{T=1} = 0$. It seems that the two kinds of pairing are mutually exclusive in this case. It is understandable that one would gain more energy by $T = 0$ pairing because the $T = 0$ force is stronger than the $T = 1$ force. For $T = 0$ pairing, isospin remains a good quantum number.

The moment of inertia corresponding to the paired state can be calculated from the cranking formula⁸⁾.

$$\mathfrak{I} = 2\hbar^2 \sum_{\alpha' > 0} \frac{|\langle \alpha | j_x | \alpha' \rangle|^2}{E_{\alpha} + E_{\alpha'}} (u_{\alpha} |v_{\alpha}| - u_{\alpha'} |v_{\alpha'}|)^2$$

all α

$$\text{where } v_{\alpha}^2 = v_{\alpha}^2 + (v'_{\alpha})^2 \quad (14)$$

The result is $\hbar^2/2\mathfrak{I} = 0.25$ as compared to the triaxial value of 0.23. Since the cranking model is well known to overestimate the value of $\hbar^2/2\mathfrak{I}$, the present value may be in better agreement with the experimental moment of inertia of $\hbar^2/2\mathfrak{I}_{\text{exp}} = 0.23$.

The energy of the lowest two quasiparticle state is found to be 4.8 MeV. The first intrinsic excited state is seen to be at 4.97 MeV which is

consistent with the predicted gap and also shows that there is no prominent vibrational structure in Mg^{24} .

Let us discuss some of the criticisms of the present calculation.

(1) The usual criticism of any n-p pairing theory is that n-p pairing assumes an average property for even-even and odd-odd ($N = Z$) nuclei whereas in reality the odd-odd nuclei look very different. That this is not a serious defect has already been pointed out in ref. 5. Here we note that the main reason that the even-even and the odd-odd nuclei look so different is that the H. F. fields are so much different. For example, the H. F. solutions for odd-odd nuclei⁹⁾ do not have time reversal invariance so that the structure of pairing in actual odd-odd nuclei would be entirely different. The mixing of the odd-odd wave function into that of the even-even wave function as implied here is entirely that of a hypothetical odd-odd nucleus introduced by non-conservation of the number of particles and can be largely remedied by projection method.

(2) We have not carried out the treatment of the pair-field self-consistently with the Hartree-Fock-Bogoliubov field. This however seems to be unimportant firstly because Hartree-Fock-Bogoliubov is warranted only for transitional nuclei where the energy of the paired state is within the ground state fluctuation energy of the lowest H. F. state. This is seen not to be the case.

Secondly, one can directly verify from our calculations that the pairing matrix is approximately canonical in the axially symmetric H. F. representation as calculated here.

Finally, we conclude that isospin pairing of neutrons and protons is very important for Mg^{24} and actually restores the axial symmetry of the equilibrium shape of the Mg^{24} ground state¹⁰⁾. Experimentally the shape is a very

difficult property to determine particularly because the triaxiality given by the H. F. solution is but a small deviation from the axial symmetry. However, the result of the present calculation is very satisfactory to many of us who believe that nature prefers axial symmetry to triaxial shapes!!

Table 1

Best axially symmetric solution for $Mg^{24}(E_\alpha$ and $\Delta(\alpha)$ in MeV)

E_α	-19.38	-15.05	-11.99	-10.16	-6.17	-4.05
Ω_α	1/2	3/2	1/2	5/2	1/2	3/2
u_α^2	0.01	0.22	0.79	0.99	1.00	1.00
v_α^2	0.00	0.00	0.00	0.00	0.00	0.00
$(\text{Re } v'_\alpha)^2$	0.00	0.00	0.00	0.00	0.00	0.00
$(\text{Im } v'_\alpha)^2$	0.99	0.78	0.21	0.01	0.00	0.00
$\Delta_p(\alpha)$	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta_{np}^{T=1}(\alpha)$	0.00	0.00	0.00	0.00	0.00	0.00
$\Delta_{np}^{T=0}(\alpha)$	1.05	-2.01	-2.45	0.81	0.62	1.38

References

- 1) J. Bartouv and I. Kelson, Phys. Rev. 138 (1965) B1035
- 2) M. K. Banerjee, C. A. Levinson, and S. Meshkov, Phys. Rev. 130 (1963) 1064
- 3) A. Bohr and B. R. Mottelson in numerous communications
- 4) K. Kumar and M. Baranger, "Nuclear Deformations in the Pairing-Plus-Quadrupole Model (III) Static Nuclear Shapes in the Rare Earth Region, MIT report, to be published
- 5) A. Goswami and L. Kisslinger, Phys. Rev. 140 (1965) B26
- 6) H. T. Chen and A. Goswami, Phys. Letters 24B (1967) 257
- 7) A. Goswami, Nucl. Phys. 60 (1964) 228; P. Camiz, A. Covello, M. Jean, Nuovo Cimento (10) 42B (1966) 199
- 8) D. Inglis, Phys. Rev. 96 (1954) 1059
- 9) G. Ripka, "The Hartree-Fock Theory and Nuclear Deformations," Theoretical Physics Institute, University of Colorado, 1965
- 10) There is some experimental indication that the band mixing of $K = 1/2+$ and $K = 5/2$ bands in Mg^{24} is small Phys. Letters 25B no. 2 (1967)109. Also, explicit analysis of α -scattering experiments by Harvey et al., shows a good fit with an axial symmetric shape. Their results are not conclusive yet. We thank Drs. B. G. Harvey and N. K. Glendenning for discussing their results before publication

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