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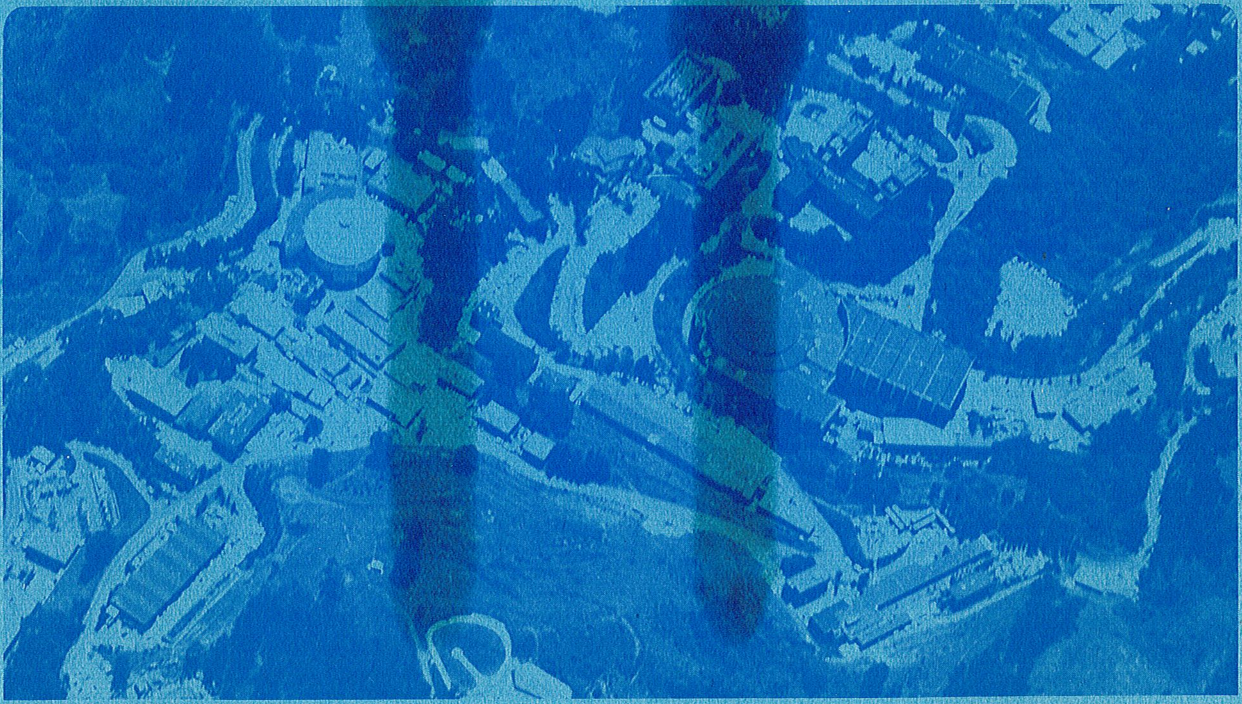

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PETITE UNIFICATION: AN ALTERNATIVE VIEWPOINT

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Grand unification ideas[2] were proposed long before the experimental confirmation of the standard $SU(2)_L \times U(1)_Y$ model[1] as the most viable candidate describing "low-energy" electroweak phenomena. The most popular and, certainly, most economical grand unified theories are SU(5)[3] and SO(10).[4] There are good reasons, such as the approximate agreement of the measured value of $\sin^2\theta_W$ with the theoretical expectations and the economical assignment of known fermions to SU(5) representations, to take the grand-unification idea very seriously. However, there are also well-known difficulties, in particular the large number of arbitrary parameters and especially the lack of a credible scenario of spontaneous symmetry breaking. Furthermore, it requires a rather bold extrapolation of thirteen orders of magnitude, from $M_W \sim 80$ GeV to $\sim 10^{15}$ GeV with relatively little new phenomena in between.

Given this situation, it may be of importance to carefully examine less ambitious alternatives. I shall describe here a recent work done in collaboration with J.D. Bjorken and A. Buras[5] related to such alternatives. We assume that at some distance scale, not too many orders of magnitude less than the compton wavelength of intermediate bosons $W^{\pm}$ and Z°, the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory, characterized by three coupling constants, becomes embedded in a gauge theory $G_S \times G_W$ characterized by only two coupling constants, g_S and g_W. The strong group G_S and weak group G_W are assumed each to be either simple or pseudo-simple i.e. a direct product of simple groups with identical coupling strengths. We call such a possibility <u>petite unification</u>. Any subsequent unification of the strong force with the weak at still shorter distances we shall leave unconsidered.

We adopt here a "building-up" procedure, that is to say we use the available inputs from the "low-energy" theory

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$SU(3)_c \times SU(2)_L \times U(1)_Y$ to restrict the choices of G_S and G_W. The inputs we use are the experimental value of $\sin^2\theta_W$ and the known fermion representations. We found that the choices of G_W are quite restricted. The smallest acceptable G_W turns out to be $[SU(2)]^4$. We also found that the most efficient choice of a strong group is SU(4) built <u>à la</u> Pati and Salam,[6] which is the simplest case for which the electroweak $U(1)_Y$ generator is a linear combination of both G_S and G_W generators. Furthermore, leptons provide the fourth color degree of freedom achieving thus an early quark-lepton unification.

We then examine in some details the phenomenology of the minimal petite unification model $SU(4) \times [SU(2)]^4$. In particular we look into rare transitions such as $K_L \to \mu e$ which are induced by the $SU(4)/SU(3)_c \times U(1)_S$ leptoquark gauge bosons ($U(1)_S$ is a subgroup of SU(4)). Interesting bounds (upper and lower) on leptoquark masses are found by combining rare decays with $\sin^2\theta_W$. The model also predicts the existence of heavy mirror fermions which are singlets under the first two SU(2)'s identified as $SU(2)_L \times SU(2)_R$ for ordinary fermions.

1. Choices of G_W

In this section, we derive a formula for $\sin^2\theta_W(M_W^2)_{\text{Theo.}}$ in the general framework of $G_S(g_S) \times G_W(g_W)$ and compare it to the experimental value $\sin^2\theta_W(M_W^2)_{\text{exp.}}$. This comparison serves to restrict the choices of possible weak groups G_W.

We assume the following symmetry breaking pattern: $G_S(g_S) \times G_W(g_W) \xrightarrow{M} SU(3)_c(g_3) \times \tilde{G}_S(\tilde{g}_S) \times G_W(g_W) \xrightarrow{\tilde{M}}$ $SU(3)_c(g_S) \times SU(2)_L(g_2) \times U(1)_Y(g') \xrightarrow{M_W} SU(3)_c \times U(1)_{\text{e.m.}}(e)$. To simplify the discussion, we specialize to the so-called "unlocked" case where the electroweak $SU(2)_L$ is one of the unbroken subgroups of G_W. The generators of $U(1)_{\text{e.m.}}$ and $U(1)_Y$, Q and T_o respectively, are given by

$$Q = T_{3L} + T_o \quad , \tag{1.a}$$

$$T_o = \sum_\alpha C_{\alpha W} T^\circ_{\alpha W} + \sum_i C_{iS} \tilde{T}^\circ_{iS} \quad , \tag{1.b}$$

where $T^\circ_{\alpha W} \in G_W$, $\tilde{T}^\circ_{iS} \in \tilde{G}_S$. For an adjoint representation of G_W (the gauge bosons of G_W) or any representation which is singlet under G_S, the second term on the right-hand side of Eq. 1.b is inoperative. It is easy to obtain

$$[1 + \sum_\alpha C^2_{\alpha W}]^{-1} = [\mathrm{Tr}T^2_{3L}/\mathrm{Tr}Q^2]_{\mathrm{adjoint}} \quad . \tag{2}$$

Equation 2 is used below in the search for the appropriate G_W groups.

Using the definition $\sin^2\theta_W(M^2_W) = e^2(M^2_W)/g^2_2(M^2_W)$ and the evolution equations for g_2, g', g_3 and $\tilde{g}_S$, we can derive

$$\sin^2\theta_W(M^2_W) = \sin^2\theta^\circ_W \left\{ 1 - C^2_S \frac{\alpha(M^2_W)}{\alpha_S(M^2_W)} - 8\pi\alpha(M^2_W) \left[K \ln \frac{\tilde{M}}{M_W} + K' \ln \frac{M}{\tilde{M}}\right] \right\} \tag{3}$$

where $\sin^2\theta^\circ_W = [1 + \sum_\alpha C^2_{\alpha W}]^{-1}$, $C^2_S \equiv \sum_i C^2_{iS}$, $\alpha(M^2_W) \equiv e^2(M^2_W)/4\pi$, $\alpha_S(M^2_W) \equiv g^2_3(M^2_W)/4\pi$, $K = b_1 - (\sum_\alpha C^2_{\alpha W})b_2 - C^2_S b_3$, $K' = C^2_S(\tilde{b} - b_3)$, with b_i and $\tilde{b}$ being coefficients of g^3_i in the relevant renormalization group β_i function. Equation 3 is very reminescent of the Georgi-Quinn-Weinberg[7] formula with two diferences. The first one is the fact that $\sin^2\theta^\circ_W \equiv \mathrm{Tr}T^2_{3L}/\mathrm{Tr}Q^2$ only applies to representations which are singlets under G_S. The second one is the presence of the term $C^2_S\alpha(M^2_W)/\alpha_S(M^2_W)$ which represents the possibility that the electroweak $U(1)_Y$ resides partly in the strong group G_S.

Since we are interested in the mass scale range $M \approx 10^{5\pm1}$ GeV and $\tilde{M} \approx 300$ GeV - a few TeV, the quantity inside the curly brackets of Eq. 3 is more sensitive to the term $C_S\alpha(M^2_W)/\alpha_S(M^2_W)$. Denoting that quantity by R (i.e. $\sin^2\theta_W(M^2_W) = R \sin^2\theta^\circ_W$), we found

$R \approx 0.95, 0.85, 0.65$ for $C_S^2 = 1/6, 2/3, 8/3$ respectively. We notice in passing that $C_S^2 = 1/6, 8/3$ corresponds to the case in which there are higher charged fermions while $C_S^2 = 2/3$ accomodates conventionally charged quarks and leptons. Experimentally, $\sin^2\theta_W(M_W^2) \approx 0.22 \pm 0.014$. Consequently, only gauge groups which satisfy

$$0.23 < \sin^2\theta_W^\circ < 0.30 \qquad \text{for } C_S^2 = 1/6,\ 2/3 \quad , \tag{4.a}$$

$$0.30 < \sin^2\theta_W^\circ < 0.4 \qquad \text{for } C_S^2 = 8/3 \quad , \tag{4.b}$$

will be considered.

We now derive $\sin^2\theta_W^\circ \equiv (\mathrm{Tr}T_{3L}^2/\mathrm{Tr}Q^2)_{\mathrm{adj.}}$ within the framework of $G_W = [SU(N)]^k$. We obtain the following results.

i.) Gauge bosons with charges ± 3 or higher are not allowed. Indeed, for $k = 1$, $|Q_{\mathrm{adj}}^{\mathrm{max}}| = 3$, $N \geqslant 4$, $\sin^2\theta_W^\circ \leqslant [12 - (8/N)]^{-1} \leqslant 1/10$. $Q_{\mathrm{adj}}^{\mathrm{max}}$ is the highest charge of the gauge bosons.

ii.) There can only be at most two doubly charged gauge bosons. In particular, for $k = 1$, $|Q_{\mathrm{adj}}^{\mathrm{max}}| = 2$, $N \geqslant 3$, $\sin^2\theta_W^\circ = 0.25$. With four or more doubly charged gauge bosons, $\sin^2\theta_W^\circ < 0.20$.

iii.) For $k = 1$, $|Q_{\mathrm{adj}}^{\mathrm{max}}| = 1$, $N \geqslant 2$, we derive

$$2/N \leqslant \sin^2\theta_W^\circ \leqslant [2(1 - (1/N)]^{-1} \qquad N \text{ even} \quad , \tag{5.a}$$

$$2N/(N^2 - 1) \leqslant \sin^2\theta_W^\circ \leqslant [2(1 - (1/N)]^{-1} \qquad N \text{ odd} \quad . \tag{5.b}$$

Notice that for $k = 1$, $N = 2$, i.e. $G_W = SU(2)$, $\sin^2\theta_W^\circ = 1$. Also for $G_W = [SU(N)]^k$ with permutation symmetry assumed, one obtains

$$\sin^2\theta_W^\circ = (1/k)\sin^2\theta_W^\circ\big|_{k=1} \quad . \tag{6}$$

More general cases can be found in Ref. 5. We see that in the case of

SU(2) we need k = 4 i.e. $G_W = [SU(2)]^4$ in order to obtain a reasonable value for $\sin^2\theta_W^\circ$ which is here equal to 1/4. Since the upper bound of $\sin^2\theta_W^\circ$ is always less than or equal to one, we can see that for $G_W = [SU(N)]^k$ with $|Q_{adj}^{max}| = 1$, the maximum number of factors allowed is k = 4. Notice that within our framework of petite unification, it is hard to accomodate the case $G_W = [SU(2)]^2$ since here $\sin^2\theta_W^\circ = 1/2$ and we would need a partial unification mass of the order $\sim 10^{12}$ GeV in order to have the correct value for $\sin^2\theta_W^\circ(M_W^2)$.

The G_W groups which give $\sin^2\theta_W^\circ = 1/4$ are $[SU(2)]^4$, $[SU(3)]^3$, $[SU(4)]^2$ and SU(8) for $N \leqslant 8$. Other groups which satisfy the constraints (4.a,b) can be found in Ref. 5. It turns out that when we include fermions, many of these groups are themselves ruled out. In what follows, we shall discuss in more details the most minimal one (in a sense of gauge groups) among them, namely $G_W = [SU(2)]^4$.

2. Choices of G_S

What could G_S be? If G_S were just $SU(3)_c$, $\sin^2\theta_W^\circ \equiv TrT_{3L}^2/TrQ^2$ would apply to <u>all</u> representations of G_W, fermions <u>and</u> gauge bosons. Since fermions and gauge bosons should give the same $\sin^2\theta_W^\circ$ in this case, we would need extra fermions with or without unconventional charges. To illustrate this point, let us take, as an example, $G_S = SU(3)_c$ and $G_W = SU(3)_W \supset SU(2)_L \times U(1)_Y$. There are two cases.

i.) $|Q_{adj}^{max}| = 1$. In this case, one obtains $\sin^2\theta_W^\circ|_{adj} = 3/4$. Furthermore quarks and leptons belong to $\underline{3}$ and $\underline{8}$ of $SU(3)_W$ respectively. There are <u>extra</u> charge - 1/3 quarks and charge -1 leptons. This case is of no interest since $\sin^2\theta_W^\circ$ is too big.

ii.) $|Q_{adj}^{max}| = 2$. As discussed earlier, this case, containing two doubly charged bosons, gives $\sin^2\theta_W^\circ|_{adj} = 1/4$. Again, quarks $\in \underline{3}$ and leptons $\in \underline{8}$ of $SU(3)_W$. In addition to quarks and leptons with conventional charges, there are charge - 4/3 quarks and charge -2 leptons.

The moral of the above example is that it is nicer to generalize $SU(3)_c$ in order to have quark-lepton unification. Furthermore, our point of view is that, in the absence of strong interactions, quarks and leptons are indistinguishable with respect to the weak group G_W.

It turns out that the simplest extension of $SU(3)_c$ is SU(4) à la Pati and Salam[6] where leptons play the role of the fourth color. Therefore, we take $G_S = SU(4)$ and the petite unification group to be $SU(4) \times [SU(N)]^k$. The strong group SU(4) can be arranged to break down to $SU(3)_c \times U(1)_S$. The generator of $U(1)_S$ which is the 15th generator of SU(4), T_{15}, is proportional to B - L. In fact for a fermion representation which transforms as (4; N, 1,..., 1) under $SU(4) \times [SU(N)]^k$, one has

$$1/2\ (B - L) = \sqrt{2/3}\ T_{15} = \begin{pmatrix} 1/6 & & & 0 \\ & 1/6 & & \\ & & 1/6 & \\ 0 & & & -1/2 \end{pmatrix} , \qquad (7)$$

where $\mathrm{Tr}T_{15}^2 = 1/2$. The generator of $U(1)_Y$, T_o, is then given by

$$T_o = \sqrt{2/3}\ T_{15} + \sum_\alpha C_{\alpha W}\, T^\circ_{\alpha W} \quad . \qquad (8)$$

The quantity $C_S^2 = 2/3$ for this case. The fermion representation (4; N, $\overline{N}$, 1,..., 1) gives $C_S^2 = 8/3$. It turns out that for weak groups which give $\sin^2\theta^\circ_W = 1/4$, $C_S^2 = 8/3$ is unacceptable because it gives too small a value for $\sin^2\theta_W(M_W^2)$. Furthermore this case gives rises to higher charge fermions. It also turns out that a more detailed study of the constraints on fermions reveals that many weak groups which satisfy the constraints (4.a,b) are ruled out.

3. Minimal Petite Unification Model

The minimal model[5] which we study in more details is $G_S \times G_W = SU(4) \times SU(2)_L \times SU(2)_R \times SU(2)_{L'} \times SU(2)_{R'}$. This model gives $\sin^2\theta^\circ_W = 1/4$ which upon using Eq. 3 predicts a value of $\sin^2\theta_W(M_W^2)$ consistent with experiment for the petite unification mass $< 10^8$ GeV. Fermions transform according to:

$\Psi = (4;\ 2,\ 1;\ 1,\ 1)_L$; $\Psi^* = (\bar{4};\ 1,\ 2;\ 1,\ 1)_L$; $\Psi' = (4;\ 1,\ 1;\ 2,\ 1)_L$; $\Psi'^* = (\bar{4};\ 1,\ 1;\ 1,\ 2)_L$. Symbolically,

$$\Psi = \begin{pmatrix} u_1 & d_1 \\ u_2 & d_2 \\ u_3 & d_3 \\ \nu & e \end{pmatrix}_L \updownarrow SU(4) \ , \quad \Psi' = \begin{pmatrix} U_1 & D_1 \\ U_2 & D_2 \\ U_3 & D_3 \\ N & E \end{pmatrix}_L \updownarrow SU(4) \ , \tag{9}$$
$$\leftarrow SU(2)_L \rightarrow \qquad\qquad \leftarrow SU(2)_{L'} \rightarrow$$

and similarly for Ψ^* and Ψ'^* corresponding to $SU(2)_R$ and $SU(2)_{R'}$, respectively. $\Psi^{(*)}$ are ordinary fermions transforming non-trivially under the left-right symmetric $SU(2)_L \times SU(2)_R$. A new feature emerges. It is the existence of a "mirror" left-right symmetric gauge group $SU(2)_{L'} \times SU(2)_{R'}$ and its "mirror" fermions $\Psi'^{(*)}$.

Why do we need the "mirror" fermions $\Psi'^{(*)}$? It is because we want to have a correct $\sin^2\theta_W^\circ$. In fact, from Eqs. (1.a,b) it is easy to derive

$$\sin^2\theta_W^\circ = [1 + \sum_\alpha C_{\alpha W}^2]^{-1} = \mathrm{Tr}T_{3L}^2/(\mathrm{Tr}Q^2 - C_S^2\mathrm{Tr}T_{15}^2)_{\text{fermions}} . \tag{10}$$

Notice, in passing, that Eq. 10 reduces to Eq. 2 for the weak gauge bosons. Using Eq. 10, it is easy to see that if we had only the ordinary fermions $\Psi^{(*)}$, $\sin^2\theta_W^\circ = 1/2$ in contradiction with the value obtained using the adjoint representation of G_W.

How heavy could the mirror fermions be? They must be heavy enough to escape detection and light enough — i.e. $m_{\Psi'} < \tilde{M}$ with $\tilde{M}$ being the scale of $[SU(2)]^4$ breaking — to ensure the permutation symmetry among the SU(2)'s when $[SU(2)]^4$ is a good symmetry. It means that all the fermions (ordinary and "mirror") would have to contribute to the relevant renormalization group functions for $Q^2 > \tilde{M}^2$. It is then natural to expect the mirror fermions to populate the mass range between 20 GeV to a few TeV's (see below for more details on mass scale range).

Even with this minimal model some new interesting physics can occur. The first one is rare transitions induced by leptoquark gauge bosons which connect quarks to leptons. They are triplets under $SU(3)_c$ and carry charges ±2/3. In the breakdown of SU(4) to $SU(3)_c \times U(1)_S$, they gain masses of the order M. Of particular interest is the muon-number changing effective interactions for $Q^2 \ll M^2$. Making some kind of "kinship" hypothesis whereby one has (d_i, e^-), (s_i, μ^-), ... with i = 1, 2, 3, the effective Lagrangian describing $d\mu \to es$ is given by

$$\mathcal{L}_{eff}(d\mu \to es) = \sqrt{2}\, G_S \sum_{i=1}^{3} (\bar{d}_i \gamma_\mu e \bar{\mu} \gamma^\mu s_i + h.c.) \quad , \tag{11}$$

where $\sqrt{2}\, G_S = g_S^2/2M_G^2$ with g_S and M_G ($\approx M$) being the gauge coupling constant of SU(4) and mass of the leptoquark gauge bosons respectively. A Fierz-Michel rearrangement of Eq. 11 given an effective Lagrangian describing $ds \to \mu e$. We then use it to compute the branching ratio $B(K_L \to \mu e)$.

We compare $B(K_L \to \mu e)$ with $B(K_L \to \mu\bar{\mu})$. Using $B(K_L \to \mu e) < 2 \times 10^{-9}$ and $B(K_L \to \mu\bar{\mu}) = (9.1 \pm 1.8) \times 10^{-9}$, we obtain the following bound

$$\alpha_S^2(M_G)/M_G^4 \lesssim 10^{-24}\ \mathrm{GeV}^{-4} \quad , \tag{12}$$

where $\alpha_S(M_G) = g_S^2(M_G)/4\pi$. According to our "minimal" petite unification scheme, $\alpha_S(M_G) = \alpha_3(M_G)$ where $\alpha_3(M_G)$ is the $SU(3)_c$ coupling constant. Using the evolution equation for α_3 taking into account the effect of "mirror" fermion threshold, the bound (12) is translated into

$$M_G > 300\ \mathrm{TeV} \quad , \tag{13}$$

Not only do we have a lower bound on M_G ($\approx M$) from rare decays, but we also have an upper bound as well which comes from the experimental value of $\sin^2\theta_W(M_W^2)$. For M or M_G too large, $\sin^2\theta_W(M_W^2)_{Theor.}$ will not agree with $\sin^2\theta_W(M_W^2)_{exp.}$. The upper bound on M_G depends on

$\tilde{M}$ ($\approx$ mass of W_R, $W_{L'}$, $W_{R'}$). In fact,

$$M_G < 10^8 \text{ GeV} \qquad \text{for } \tilde{M} \approx 300 \text{ GeV} \qquad , \tag{14.a}$$

$$M_G < 1000 \text{ TeV} \qquad \text{for } \tilde{M} \approx 1 \text{ TeV} - 10 \text{ TeV} \qquad . \tag{14.b}$$

Any value of M_G exceeding the bounds (14.a,b) is forbidden by $\sin^2\theta_W(M_W^2)_{exp.}$. An interesting feature emerges. The upper bounds on M_G imply lower bounds on $B(K_L \to \mu e)$. The bound (14.b) is of considerable interest since 300 TeV $< M_G <$ 1000 TeV implies

$$10^{-11} < B(K_L \to \mu e) < 10^{-9} \qquad . \tag{15}$$

This range of $B(K_L \to \mu e)$ may be feasible experimentally. We make a plea to experimentalists to give some thoughts on how to improve the precision of these experiments.

In our model as it stands, the proton is stable. However by complicating the Higgs system, it is in principle possible to generate proton decay in higher orders.

Finally, I wish to say a few words about the "mirror" fermions. These "mirror" fermions are the exact duplicates of the ordinary fermions — same electric charges, same SU(4) interactions — except that they only have $SU(2)_{L'} \times SU(2)_{R'}$ weak gauge interactions. They do not couple directly to the electroweak $W_L^\pm$ bosons. We have mentioned earlier that they may be expected to populate the mass range between 20 GeV to a few TeV's. How do we distinguish the lightest among the charged "mirror" fermions from its ordinary counterparts? Let us take for example $E^\pm$ to be the lightest charged "mirror" leptons. They could be produced in reactions like $e^+ + e^- \to E^+ + E^-$. What distinguishes $E^\pm$ from just an ordinary heavy sequential lepton is the possibility that $E^\pm$ could live longer than one would expect from an ordinary weak decay. This could occur if there is no significant mixing between $W_L^\pm$ and $W_{L'}^\pm$, which could be induced by a Higgs representation which transforms as a (2,2) under $SU(2)_L \times SU(2)_{L'}$. Other induced decays e.g. through a (2,2) Higgs exchange could be expected to be small if the Yukawa coupling is itself small. It would be interesting to see if the next heavy lepton (if any) is something new or just another sequential lepton. Similar considerations could be applied to "mirror" quarks as well.

In summary, I have described here an attempt to fill up the desert, at least a few orders of magnitude beyond M_W. It remains to be seen if such a step is necessary but it is certainly worthwhile to contemplate effective theories describing various mass scales.

Because of the lack of space I apologize for not being able to give a complete list of references which could be found in Ref. 5.

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References

[1] S.L. Glashow, Nucl.Phys. 22 (1961) 579; A. Salam and J.C. Ward, Phys. Lett. 13 (1964) 168; S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264; A. Salam, in Elementary Particle Theory, ed. N. Svartholm (Almquist and Wiksells, Stockholm, 1969) p. 367.

[2] For a complete list of contributors to these ideas, the reader is urged to consult the review paper of P. Langacker, SLAC-PUB-2544 (1980), to be published in Physics Reports.

[3] H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438.

[4] H. Georgi, Particles and Fields 1974, ed. C.E. Carlson (AIP, NY, 1975) p. 575; H. Fritzsch and P. Minkowski, Ann. Phys. 93 (1975) 193.

[5] P.Q. Hung, A.J. Buras and J.D. Bjorken, FERMILAB-Pub-81/22-THY (1981).

[6] J.C. Pati and A. Salam, Phys. Rev. D10 (1974) 275.

[7] H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.