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Critical Infrastructure Systems: Distributed Decision Processes over Network and Uncertainties

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Critical Infrastructure Systems:
Distributed Decision Processes over Network and Uncertainties

By

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Dissertation

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To my family.

Contents

List of Figures

List of Tables

Abstract

Critical Infrastructure Systems:
Distributed Decision Processes over Network and Uncertainties

Critical infrastructure systems (CISs) provide the essential services that are vital for a nation's economy, security, and health, but the analysis of CISs are challenged due to their inherent complexity. This dissertation focuses primarily on the system analysis of critical infrastructure systems, with a particular interest to address the modeling and computational challenges brought by uncertainties, interdependencies and distributed decision making of various components and stakeholders involved in CISs, so that a secure, reliable, efficient and resilient system can be further pursued. Through two examples, the first one is on electric vehicle charging infrastructure planning in a competitive market, and the second one is on power generators planning in a restructured electricity market, we illustrate how our general modeling framework, N-SMOPEC, can be adapted to formulate the specific problems in transportation and energy system. Each example is solved by decomposition based approach with convergence properties developed based on recent theoretical advances of variational convergence. Median size numerical experiments are implemented to study the performance of proposed method and draw practical insights. In addition, we have shown some knowledge from different domains, such as microeconomics, energy and transportation, can be shared to facilitate the formulation and solution process of seemingly unrelated problems of each other, which could possibly foster the communication between different fields and open up new research opportunities from both theoretical and practical perspectives.

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Chapter 1

Introduction

Critical infrastructure systems (CISs), including transportation, energy, telecommunication, banking and finance, water supply, etc., provide the essential services that are vital for a nation's economy, security, and health. Large amount of efforts have been spent in the past 20 years to integrate across the cyber-physical, engineering and social, behavioral and economic sciences [RIPS, 2016] in order to achieve secure, reliable, efficient and resilient critical infrastructure systems. However, due to the inherent complexities of CISs, achieving these goals is still challenging both from modeling and computational aspects. This dissertation aims to provide a general modeling framework, associated with computational techniques, that can capture three main challenges during system analysis of CISs: uncertainties, interdependences, and decentralization.

1.1 Motivations

The significance of CISs has drawn attention from governments for a long time. In 1996, U.S. President Clinton ordered the establishment of President's Commission on Critical Infrastructure Protection (PCCIP), with the goal of: (1) mitigating both physical and cyber threats to CISs; and (2) fostering the cooperation between government and private sector to develop a strategy for improving the reliability and security of CISs [Clinton, 1996]. Similarly, different countries and regions have also founded their CISs organizations/programs, such as the European Program on Critical Infrastructure Protection (EPCIP), the Critical Infrastructure Program for Modeling and Analysis (CIPMA) in Australia, the Critical Infrastructure Protection Implementation Plan (CIPIP) in Germany and the Critical Infrastructure Resilience Program (CIRP) in the UK. However, three main challenges: uncertainties, interdependences, and decentralization, hamper the further analysis of the CISs at a system level.

Planning decisions of CISs typically need to be made for long term and can not be easily or quickly modified in the future. These facts lead to the first CISs modeling challenge that is how to embed various sources of future uncertainties into decision processes. Well known uncertainties include system components failures, natural resource fluctuation, possible technology breakthrough/shutdown, demand variation, unknown policies and regulations in the future, etc. These uncertainties will directly affect the overall cost-effectiveness and service level of CISs. Another type of uncertainties, commonly referred to as risk, could occur with relatively low probability but will cause undesired outcome or significant losses. Some catastrophic events, such as the 9/11 terrorist attack (2001) and Hurricane Katrina (2005) in U.S. , Sichuan earthquake (2008) in China, earthquake and tsunami (1995, 2011) in Japan, forest fire (2016) in Canada, belong to this category. These events can bring damage to the infrastructure systems, human lives, economic development, and eventually threat the stability and security of the whole society.

However, the impacts of uncertainties will not only affect one part or one single infrastructure system, due to the highly interdependency within and between CISs. For example, on August 14, 2003 the loss of a few power stations due to high energy load escalated into the worst blackout in history that affected 50 million people in the Midwest, Northeast and Ontario, Canada, with a combined load of 61.8 gigawatts lost power for up to 4 days [Liscouski and Elliot, 2004]. This effect is well known as a cascading power transmission failure that relatively small number of failures of power stations can possibly lead to a massive chain of transmission line and generating failures across the whole electrical grid. In addition, CISs are becoming increasingly interdependent both physically and through cyber connection. For example, the Californian power disruptions in 2001 affected oil and natural gas production, refinery operations, pipeline transportation within California and to its neighboring states, the movement of water from northern to central and southern regions of the state for crop irrigation, and also the normal operation of telecommunication, which further impacted the productivity and functioning of other industries [Rinaldi et al., 2001].

The problem is becoming even more challenging in a distributed decision making environment. In many countries, some critical infrastructures are provided through a decentralized structure [Bird, 1994] and the current situation makes that trend likely to be more true in the future. As a result, more decision makers will be involved in shaping an infrastructure system. For example, due to the global trend of electricity market deregulation [Lai, 2001], investor owned vertically integrated utilities (IOU) or publicly-owned municipal utilities (POU) are no longer able to vertically control the power supply chain and pass the investment risk to consumers by charging arbitrary electricity prices. Power suppliers are required to decide their own investment and production, facing competition from both renewable and non-renewable sectors. In addition to power suppliers, electricity supply chain typically includes several other stakeholders: such as transmitter(s), power retailers, independent system operator (ISO), and consumers, who make decisions in a decentralized manner. In shaping the transportation infrastructure system, from demand side, there are (many) individual drivers deciding which routes to take and/or which service facilities to use; from supply side, there can be one or several government(s)/investor(s) deciding where and how much to invest. In these examples, each decision entity makes her own decision, but needs to simultaneously account for other decision entities' behaviors given the interdependence among them.

In summary, CISs involve multiple decision makers who are facing interdependencies within/between systems and uncertainties/risks for the future. It is critical to model these close couplings among the systems and gaining a better understanding about these complexities, so that effective planning, maintenance and emergency decisions can be made at a system level. This dissertation pursues along this direction and aims to provide a holistic modeling framework and accompanied computation techniques to deal with the challenges brought by uncertainties, interdependencies, and decentralization during system analysis of CISs.

1.2 Overview of Existing Literature

With the increasing attention from governments on CISs, researchers in critical infrastructure community have applied both qualitative and quantitive methods trying to gain a better understanding of these systems.

For qualitative studies, Bologna and Setola [2005] proposes specific recommendations in terms of how to deal with uncertainties that exist in CISs, such as using information/data to improve the local capability to autonomously react to anomalies, preparing for the worst scenario, and identifying common failure events; Rinaldi et al. [2001] classified infrastructure interdependencies into four types: physical, cyber, geographic and logical. While physical, cyber and geographic emphasize the interdependence due to the functioning of infrastructures, logical interdependence emphasizes the predominant role of human decisions. Briere [2011] recommends a fusion center to facilitate a community stakeholder-driven rapid restoration of CISs after extreme events. These qualitative studies echo the importance and challenges of dealing with uncertainties, interdependencies and decentralization in CISs, and provide some helpful conceptual frameworks that pave the way for better quantitative modeling and analysis.

The quantitative studies over the past decade on CISs have been emphasizing the importance of taking a system(s) approach in disaster impact analyses and mitigation planning.

For disaster impact analyses, several studies aim to quantify the social economic impact of critical infrastructure failures due to extreme events. For example, Rose et al. [1997] quantify the regional economic impacts of electricity lifeline disruptions due to earthquake based on input-output and linear programming models. But in this study, the physical network structure and spatial relation are omitted, which can lead to bias of the interdependence representation between different system(s) components. Chang and Nojima [2001] incorporate the transportation network structure and develop post-disaster measures for transportation system performance. Kerivin and Mahjoub [2005] review various network measures to quantify the survivability of a spatially distributed telecommunication infrastructure system, based on the concept of node-disjoint and edge-disjoint. Other important measurements for system resilience are also proposed. Conceptually, resilience is usually characterized as the ability to recover after a major disturbance in the context of civil and industrial engineering system [Reed et al., 2009]. Ip and Wang [2009] define network resilience as a function of the number of reliable paths between all node pairs, which borrow the similar concept from system redundancies. Zhang et al. [2009] measure resilience as a function of change in system mobility, accessibility and reliability from pre-disruption levels in transportation system.

System-level mitigation planning have been conducted for various critical infrastructure systems such as transportation [Liu et al., 2009, Miller-Hooks et al., 2012], power [Chen et al., 2014], communication [Kerivin and Mahjoub, 2005, Sterbenz et al., 2010], etc. Typically, the problem is formulated as a two-stage optimization model, with the first stage consists of a central planner who can control the whole system and make the best pre-disaster decisions, and the second stage represents the post-disaster interactions of different system components, including possible recourse decisions being made to adjust the system to the new operating conditions. The first stage decisions can also take into account some forms of equilibrium conditions or different objectives/reactions from the second stage, in which case the problem is typically referred to as bi-level. In either case, one drawback of these studies is that they are limited to single decision maker and single critical infrastructure system.

There are also quite significant amounts of quantitative studies that investigate the interdependence between multiple CISs and between multiple decision makers. Several national laboratories develop bottom-up agent-based simulation models to study CISs interdependencies and identify the optimum or ranking of asset to protect from extreme events, such as Aspen-EE [Barton et al., 2000] and NABLE [Schoenwald et al.] by Sandia, SMART II++ [North, 2001] by Argonne, CIMS [Becker et al., 2011] by Idaho. Agent-based approach is praised for its ability to capture detail interdependencies among CISs and provide scenario-based what-if analysis. But the main drawbacks are the highly sensitivity to the assumptions on agents' interacting rules and the difficulties to calibrate parameters due to data scarcity [1]. On the other extreme, researchers develop top-down methods to simplify the detailed interaction assumptions. CIP/DSS (Critical Infrstructure Protection/Decision Support System) [Min et al., 2007], developed jointly by Los Alamos, Sandia and Argonne National Laboratories, use system dynamics based approach, which describe the interdependence of CISs at an ag-

[1] Data on CISs may be harder to obtain due to the security concerns.

gregate level through causal-loop diagram and stock-and-flow diagram. However, the main disadvantages of this approach are the somewhat arbitrary and over-simplified assumptions on the causal relationships between different systems and lack of analysis ability at component level. Other studies lie between these two extremes, such as approaches based on input-output model [Haimes and Jiang, 2001], computable general equilibrium (CGE) [Zhang and Peeta, 2011], and network topology [Buldyrev et al., 2010] and flows [Lee et al., 2007]. Although these approaches well capture the interdependences between CISs and some of them (e.g. CGE models) can incorporate the behaviors of different decision makers, they are generally deterministic and based on individual scenario analysis.

1.3 Contributions

This dissertation focuses primarily on the system analysis of critical infrastructure systems, with a particular interest to address the modeling and computational challenges brought by uncertainties, interdependencies and distributed decision making of various components and stakeholders involved in complex infrastructure systems. The general methodological contribution of my dissertation is on establishing network-based multi-agent optimization modeling framework and computing methods to facilitate planning and analysis of interdependent CISs that are shaped by collective actions of multiple decision entities who share global uncertainties but do not necessarily coordinate with each other. Note that the dissertation is not limited to system analysis under extreme scenarios. Instead, our general modeling framework is flexible enough to incorporate general uncertainties/risks into decision making processes.

Some specific contributions are summarized as follows:

1. The main contribution of Chapter 3 is on the establishment of a theoretical foundation, from both modeling and computational aspects, for business-

driven EV charging infrastructure investment problem. We demonstrate how the original multi-agent problem can be reformulated to a problem of finding a maxinf-point of certain bifuncation, which is inspired by the solution approach for Walrasian Equilibrium, followed by convergence analysis and algorithm design based on the recent theoretical advances of maxinf-points convergence.

2. The main contribution of Chapter 4 is on the development of modeling and solution methods to address challenges brought by uncertainties and oligopolistic competition among energy producers over a complex network structure. To overcome the computational difficulty, we have combined two ideas. The first is using stochastic decomposition techniques to convert a large-scale stochastic problem to many smaller scenario-dependent problems that are more easily solvable or can be solved in parallel. The second is using variational inequalities to prove the equivalence of our scenario dependent multi-agent optimization problem with a single traffic equilibrium problem, which have been formulated as a convex optimization problem. This allows exploitation of efficient solution techniques that can typically outperform general-purpose solvers.

1.4 Organization

The remainder of this dissertation is organized as follows:

In Chapter 2, we first summarize key methodological elements for our dissertation, including stochastic programming, network modeling, and multi-agent optimization problems with equilibrium constraints (MOPEC). And then we present our general modeling framework: Network-based Stochastic MOPEC (N-SMOPEC) to explicitly capture uncertainties, interdependences, and decentralization in system analysis.

In Chapter 3 and Chapter 4, we apply our general methodology in the context of transportation and energy infrastructures, respectively, and design effective solution approaches for each specific example. Chapter 3 establishes a theoretical foundation for business-driven EV charging infrastructure investment planning problem, which is then solved by a decomposition method rooted in the most recent theoretical development of variational convergence. In Chapter 4, we formulate the power generators planning problem in a restructured electricity market and related this problem to a classic traffic assignment problem through decomposition techniques and variational inequality reformulation.

The last chapter concludes the dissertation with discussions, and future extensions.

Chapter 2

Methodology

The main research question of this dissertation is stated as: how will uncertainties, interdependencies and distributed decision making influence the planning and analyses of an infrastructure system? To capture the interplay of these three key challenges during system analysis, we develop a holistic methodology, Network-based Stochastic Multi-agent Optimization Problem with Equilibrium Constraints (N-SMOPEC), integrating knowledge from three fields: operations research, microeconomics and network science. In this chapter, we first summarize some fundamental building blocks, including stochastic programming [Louveaux, 1986, Birge and Louveaux, 2011], network modeling [Bertsekas, 1998, Sheffi, 1985], and multi-agent optimization problems with equilibrium constraints (MOPEC) [Ferris and Wets, 2012], and then present the general modeling framework of N-SMOPEC in the end. The engineering aspects of applying N-SMOPEC will be discussed with applications from transportation and energy systems in Chapter 3 and 4, respectively.

2.1 Stochastic Programming

Stochastic programming, which was first introduced by Dantzig [1955] and further developed both in theory and computation by Wets [1966], Van Slyke and Wets

[1969b], Wets [1974], is an approach for modeling (single player) optimization problems that involve uncertainty. A classic form of stochastic programming, also known as expected value model, can be depicted as following:

$$\underset{\boldsymbol{x} \in X}{\text{minimize}} \;\; \mathbb{E}[f(\boldsymbol{x}, \boldsymbol{\xi})] \tag{2.1}$$

where $\boldsymbol{\xi}$ represents an uncertain vector [1]; and $\boldsymbol{x}$ is the decision variables that are measurable for their current available information. We typically assume $\boldsymbol{\xi}$ follows a known probability distribution, which can be either provided by domain experts or estimated by statisticians using historical data [2]. In this section, we will limit our discussion on some variants of stochastic programming in the most widely applied and studied setting: two-stage stochastic programming. For multi-stage stochastic programming, which is a natural extension in terms of modeling but with much larger computational challenges, one can refer to [Pflug and Pichler, 2014].

2.1.1 Two-stage Stochastic Programming with Recourse

The basic idea of two-stage stochastic programming (with recourse) is to distinguish two types of decisions based on whether the uncertain parameters $\boldsymbol{\xi}$ is known or not at the time of decision making. The first-stage decisions, such as long-term system planning decisions, are usually made before future uncertainties $\boldsymbol{\xi}$ is revealed and are difficult to readjust once implemented; the second-stage decisions, such as real-time system operational decisions, can be adjusted based on the actual realization of $\boldsymbol{\xi}$.

[1] Note that throughout the entire dissertation, we use lowercase bold font to emphasize the vector parameters/variables.

[2] Recently, there are some studies trying to relax this assumption by looking into how to make decisions taking into account the uncertainty of the distribution (also known as "uncertainty of uncertainty"). For readers interested in this topic, one can refer to [Royset and Wets, 2016]. If the parameters are known only within bounds, one approach to tackling the problem is called robust optimization [Ben-Tal et al., 2009], which could be too conservative in practice.

The classic two-stage stochastic programming, in the simplest form, may be presented as follows:

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad f(\boldsymbol{x}) + \mathbb{E}_{\boldsymbol{\xi}}\left[Q(\boldsymbol{x}, \boldsymbol{\xi})\right] \tag{2.2a}$$

$$\text{subject to} \quad \boldsymbol{x} \in X \tag{2.2b}$$

$$Q(\boldsymbol{x}, \boldsymbol{\xi}) = \inf_{\boldsymbol{y}} \left\{ g(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\xi}) | \boldsymbol{y} \in Y(\boldsymbol{x}, \boldsymbol{\xi}) \right\}, \tag{2.2c}$$

where $\boldsymbol{x}$ represents the first-stage decisions, and $\boldsymbol{y}$ is the second-stage decisions, which depends on the choice of first-stage decisions and the actual realization of the uncertain parameters $\boldsymbol{\xi}$. The objective is to minimize the first-stage cost, $f(\boldsymbol{x})$, plus the expected value of the second-stage cost, $Q(\boldsymbol{x}, \boldsymbol{\xi})$, subject to the feasibility constraints of $\boldsymbol{x}$ and $\boldsymbol{y}$. Adopting expected value in the objective function can be well justified by Law of Large Numbers. That is to say our "optimal" decisions are optimal in the sense that we can conduct the experiment infinite number of times and we value individual experiment linearly with respect to its quantitative cost. However, sometimes we may not be able to repeat the experiment as many times as we want and we may worry about the bad outcomes more than we favor the good ones. This limitation gives rise to another popular branch of stochastic programming, named as risk averse optimization.

2.1.2 Risk Averse Optimization

Risk averse optimization is helpful when there are some (significantly) undesirable outcomes that we hope to avoid, even though these outcomes are associated with low probability. Different disciplines develop different methods to capture the risk averse behaviors observed in their own domains, such as expected utility theory, chance-constrained optimization, and mean-risk models. Interestingly, these methods turn out to be related with each other.

2.1.2.1 Expected Utility Theory

In economics literature, expected utility models [Von Neumann and Morgenstern, 1944], as well as some of its alternatives/generalizations (e.g. prospect theory [Kahneman and Tversky, 1979], rank-dependent expected utility [Quiggin, 1982] and regret theory [Loomes and Sugden, 1982]), play an important role to deal with risks that exist in systems. The basic idea of expected utility models is we try to minimize the expected disutility instead of minimizing the expected cost, see (2.3).

$$\underset{\boldsymbol{x}\in X}{\text{minimize}} \;\; \mathbb{E}[u(f)] \tag{2.3}$$

where $u : I\!R \to I\!R$ is a nondecreasing disutility function. Although the existence of utility functions can be guaranteed by a system of axioms [3], in practice they are difficult to elicit.

2.1.2.2 Chance Constraints

Operations researchers also propose multiple alternative approaches to capture the impact of risk in decision making. Miller and Wagner [1965] suggests to impose additional constraints to optimization problem (2.2) to explicitly bound the probability of certain undesirable outcome. These constraints are named as chance constraints, and take the following form: $Pr[Z_{\boldsymbol{x}} \le b] \ge 1 - \alpha$, where $Z_{\boldsymbol{x}}$ is the random outcome vector depends on $\boldsymbol{x}$; b is the fixed target and α is the probability threshold. Chance constraints is later generalized by Dentcheva and Ruszczynski [2003] as stochastic dominance constraint, $Z_{\boldsymbol{x}} \preceq_{SD} B$, where B is the benchmark random outcome, e.g. $B(\boldsymbol{\xi}) = Z_{\overline{\boldsymbol{x}}}(\boldsymbol{\xi})$ for some $\overline{\boldsymbol{x}} \in X$. Some examples of stochastic dominance constraints include $Pr[Z_{\boldsymbol{x}} \ge b] \le Pr[B \ge b]$ and $\mathbb{E}[(Z_{\boldsymbol{x}} - b)_+] \le \mathbb{E}[(B - b)_+]$, $\forall b \in [b_{min}, b_{max}]$. Mathematically, it can be

[3] The axioms guaranteed the existence of utility function in expected utility models are completeness, transitivity, independence and continuity. Other alternative models may require different system of axioms.

shown that the dual variable of the stochastic dominance constraints are related to the utility function used by Von Neumann and Morgenstern [1944] [Dentcheva and Ruszczynski, 2003]. The main drawback of optimization models with chance constraints is that they will lead to non-convex feasible region in general.

2.1.2.3 Mean-risk Models

The third approach, which is widely used in finance and engineering, is called mean-risk models, with the basic idea of trying to minimize the weighted sum of the mean, $\mathbb{E}[Z_{\boldsymbol{x}}]$, and the risk, $r[Z_{\boldsymbol{x}}]$. The general formulation of mean-risk model is shown in (2.4).

$$\underset{\boldsymbol{x}\in X}{\text{minimize}} \;\; \mathbb{E}[Z_{\boldsymbol{x}}] + \kappa r[Z_{\boldsymbol{x}}] \tag{2.4}$$

where κ is the tradeoff parameter between mean and risk, and can be interpreted as the dual variable of the stochastic programming with risk constraints. Notice that $r[Z_{\boldsymbol{x}}]$ may still possibly be non-convex in $\boldsymbol{x}$. For example, Value at Risk (VaR), which is a risk measure derived from chance constraints and defined as:

$$VaR_{1-\alpha}(Z_{\boldsymbol{x}}) = F_{Z_{\boldsymbol{x}}}^{-1}(1-\alpha) = \inf\{t : F_{Z_{\boldsymbol{x}}}(t) \geq 1-\alpha\}, \tag{2.5}$$

where $F_{Z_{\boldsymbol{x}}}^{-1}(\cdot)$ is the (left-side) quantile of $Z_{\boldsymbol{x}}$, is not convex in $\boldsymbol{x}$.

Later Artzner et al. [1999] define a set of properties that investors expect to hold for a risk measure, named as coherent risk measure, including:

1. Normalized: $r[0] = 0$ (the risk of holding no assets is zero);

2. Monotonicity: if $Z_1 \geq Z_2$, then $r[Z_1] \geq r[Z_2]$ (higher portfolio cost, higher risk);

3. Convexity: if $\lambda \in [0, 1]$, $r[\lambda Z_1 + (1-\lambda)Z_2] \leq \lambda r[Z_1] + (1-\lambda)r[Z_2]$ (diversification reduces risk);

4. Translation invariance: if A is a deterministic portfolio with deterministic cost a, then $r[Z_{\boldsymbol{x}} + A] = r[Z_{\boldsymbol{x}}] + a$.

Any coherent risk measures, with the assumption that $Z_{\boldsymbol{x}}$ is convex in $\boldsymbol{x}$, can be shown to be convex in x. The conditional value-at-risk (CVaR) (sometimes called expected shortfall or average value at risk) [Rockafellar and Uryasev, 2000] is one of the widely applied coherent risk measure. The definition of CVaR is shown in (2.6):

$$\mathrm{CVaR}_{1-\alpha}(Z_{\boldsymbol{x}}) = \inf_{t \in R}\{t + \alpha^{-1}\mathbb{E}[Z_{\boldsymbol{x}} - t]_+\}, \tag{2.6}$$

CVaR is constructed to be a conservative and convex approximation to VaR by the additional amount of $\alpha^{-1}\mathbb{E}[Z_{\boldsymbol{x}} - VaR_{1-\alpha}(Z_{\boldsymbol{x}})]_+$ [Shapiro and Philpott, 2007].

2.1.3 Solution Approaches

Small scale two-stage stochastic programming problems can be solved as their deterministic equivalent form under the assumption of finite discrete distributions of the uncertain parameters. However, in reality, two-stage stochastic programming problems typically have large dimension and characteristic structures. Therefore, decomposition-based methods are typically used to solve these problems. In this section, we discuss two widely applied decomposition techniques: cutting plane methods and dual decomposition methods.

2.1.3.1 Cutting Plane Methods

Notice that once $\boldsymbol{x}$ is fixed in (2.2), the second stage optimization (2.2c) can be solved (in parallel) for each $\boldsymbol{\xi}$ separately and possibly efficiently, this provides the incentive to vertically decompose the first and second stage optimization. The idea of cutting plane methods is to gradually approximate $\boldsymbol{x}^*$ using some facets of the domain of $Q(\boldsymbol{x}, \boldsymbol{\xi})$ (feasibility cuts) and the known pieces of the functions $Q(\cdot, \boldsymbol{\xi})$, $\boldsymbol{\xi} \in \Xi$ (optimality cuts). To illustrate one of the most widely applied cutting plane methods in Stochastic linear programming, L-shaped method [4][Van Slyke and Wets, 1969a], we use one specific form of two-stage stochastic programming problems, as shown in (2.7).

$$\underset{\boldsymbol{x}}{\text{minimize}} \qquad f(\boldsymbol{x}) + \sum_{\boldsymbol{\xi}\in\Xi} Pr(\boldsymbol{\xi})Q(\boldsymbol{x},\boldsymbol{\xi}) \tag{2.7a}$$

$$\text{subject to} \qquad \boldsymbol{x} \in X \tag{2.7b}$$

$$Q(\boldsymbol{x},\boldsymbol{\xi}) = \inf_{\boldsymbol{y}} \left\{ g(\boldsymbol{y},\boldsymbol{\xi}) | A_y(\boldsymbol{\xi})\boldsymbol{y} = \boldsymbol{b}(\boldsymbol{\xi}) - A_x(\boldsymbol{\xi})\boldsymbol{x} \right\}, \tag{2.7c}$$

In (2.7), we assume that: 1. the uncertain parameters $\boldsymbol{\xi}$ have finite discrete distributions; 2. $f(\cdot)$ and $g(\cdot)$ are convex and polyhedral with respect to $\boldsymbol{x}$ and $\boldsymbol{y}$; 3. $\boldsymbol{x}$ and $\boldsymbol{y}$ are linked by a linear constraint; 4. no constraints directly relate any second stage decisions in different scenarios. The L-shaped method is summarized in **Algorithm 1**.

Notice that $\mathbb{E}Q(\cdot,\boldsymbol{\xi})$ and the domain of $\mathbb{E}Q(\cdot,\boldsymbol{\xi})$ are convex polyhedral, they can be represented by a finite number of cuts. So L-shaped method can either find an optimal solution or detect infeasibility in finite steps. However, the number of cuts in the master problem may grow extremely large for some problems and there is no easy way to keep it bounded [Ruszczynski, 1997], which slows down the solving process. Another difficulty is that each generated cut may cut only a very small piece of the region. Therefore, the performance of L-shaped algorithm may not be impressive in practice.

2.1.3.2 Dual Decomposition Methods

Compared to cutting plane methods, several alternatives based on duality and augmented Lagrangian were proposed. Consider the following alternative formulation of two-stage stochastic programming (2.11):

[4]L-shaped method is a special case of Benders decomposition method [Benders, 1962].

Algorithm 1 L-shaped method

Step 1: Initialization

$\tau = r = s = 0$

Step 2: Master Problem

$\tau \leftarrow \tau + 1$

Solve master problem (If $s = 0, \overline{q}^{\tau} \leftarrow -\infty$ and excluded the objective (2.8a)):

$$\underset{\boldsymbol{x},\overline{q}}{\text{minimize}} \quad f(\boldsymbol{x}) + \overline{q} \tag{2.8a}$$

$$\text{subject to} \quad \boldsymbol{x} \in X \tag{2.8b}$$

$$< E_k, \boldsymbol{x} > -e_k \geq 0, k = 1, \ldots, r, \text{(feasibility cuts)}, \tag{2.8c}$$

$$< F_k, \boldsymbol{x} > +\overline{q} - f_k \geq 0, k = 1, \ldots, s, \text{(optimality cuts)}. \tag{2.8d}$$

$(\boldsymbol{x}^{\tau}, \overline{q}^{\tau}) \leftarrow$ optimal solutions of master problem.

Step 3: Feasibility Cuts

for each $\boldsymbol{\xi} \in \Xi$ **do**

Solve the linear program:

$$\underset{\boldsymbol{y},\boldsymbol{v}^+,\boldsymbol{v}^-}{\text{minimize}} \quad \omega = \boldsymbol{e}^T \boldsymbol{v}^+ + \boldsymbol{e}^T \boldsymbol{v}^- \tag{2.9a}$$

$$\text{subject to} \quad A_y(\boldsymbol{\xi})\boldsymbol{y} + I\boldsymbol{v}^+ - I\boldsymbol{v}^- = \boldsymbol{b}(\boldsymbol{\xi}) - A_x(\boldsymbol{\xi})\boldsymbol{x}^{\tau} \tag{2.9b}$$

$$\boldsymbol{v}^+ \geq 0, \boldsymbol{v}^- \geq 0. \tag{2.9c}$$

if $\omega > 0$ **then**

$\boldsymbol{\lambda} \leftarrow$ dual variables of constraints (2.9b)

$r \leftarrow r + 1 \qquad E_r \leftarrow A_x^T(\boldsymbol{\xi})\boldsymbol{\lambda} \qquad e_r \leftarrow \boldsymbol{b}^T(\boldsymbol{\xi})\boldsymbol{\lambda}$

Add feasibility cut $< E_r, \boldsymbol{x} > -e_r \geq 0$ to the constraints (2.8c).

go to Step 2.

end if

end for

go to Step 4.

Step 4: Optimality Cuts

for each $\boldsymbol{\xi} \in \Xi$ **do**

Solve the linear program:

$$\underset{\boldsymbol{y}}{\text{minimize}} \qquad \omega = g(\boldsymbol{y}, \boldsymbol{\xi}) \tag{2.10a}$$

$$\text{subject to} \qquad A_y(\boldsymbol{\xi})\boldsymbol{y} = \boldsymbol{b}(\boldsymbol{\xi}) - A_x(\boldsymbol{\xi})\boldsymbol{x}^{\tau}. \tag{2.10b}$$

$\boldsymbol{\gamma}(\boldsymbol{\xi}) \leftarrow$ dual variables of constraints (2.10b).

end for

$F_{s+1} \leftarrow \sum_{\boldsymbol{\xi}\in\Xi} Pr(\boldsymbol{\xi}) A_x^T(\boldsymbol{\xi})\boldsymbol{\gamma}(\boldsymbol{\xi}) \qquad f_{s+1} \leftarrow \sum_{\boldsymbol{\xi}\in\Xi} Pr(\boldsymbol{\xi})\boldsymbol{b}^T(\boldsymbol{\xi})\boldsymbol{\gamma}(\boldsymbol{\xi})$

if $\overline{q}^{\tau} < f_{s+1} - F_{s+1}^T \boldsymbol{x}^{\tau}$ **then**

$s \leftarrow s + 1$

Add optimality cut $< F_{s+1}, \boldsymbol{x} > + \overline{q} - f_{s+1} \geq 0$ to the constraints (2.8d).

go to Step 2.

end if

return $\boldsymbol{x}^{\tau}$

$$\underset{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}}{\text{minimize}} \qquad \sum_{\boldsymbol{\xi}\in\Xi} Pr(\boldsymbol{\xi}) f(\boldsymbol{x}(\boldsymbol{\xi}), \boldsymbol{y}(\boldsymbol{\xi})) \tag{2.11a}$$

$$\text{subject to} \qquad (\boldsymbol{x}(\boldsymbol{\xi}), \boldsymbol{y}(\boldsymbol{\xi})) \in G(\boldsymbol{\xi}), \forall \boldsymbol{\xi} \in \Xi \tag{2.11b}$$

$$(\boldsymbol{\lambda}(\boldsymbol{\xi})) \qquad \boldsymbol{x}(\boldsymbol{\xi}) - \boldsymbol{z} = 0, \forall \boldsymbol{\xi} \in \Xi, \tag{2.11c}$$

where $\lambda(\boldsymbol{\xi})$ is the dual variables of constraint (2.11c). The idea behind formulation (2.11) is that we firstly relax the non-anticipativity of the first-stage decision variables $\boldsymbol{x}$ and make them scenario dependent, i.e. $\boldsymbol{x}(\boldsymbol{\xi})$. However, these decisions may not be admissible in reality. That is why additional constraints (2.11c), named as non-anticipativity constraints, are imposed to the optimization problem.

One can write out the Lagrangian of (2.11):

$$\mathscr{L}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\lambda}) = \sum_{\boldsymbol{\xi}\in\Xi} Pr(\boldsymbol{\xi}) f(\boldsymbol{x}(\boldsymbol{\xi}),\boldsymbol{y}(\boldsymbol{\xi})) + \sum_{\boldsymbol{\xi}\in\Xi} < \boldsymbol{\lambda}(\boldsymbol{\xi}), \boldsymbol{x}(\boldsymbol{\xi}) - \boldsymbol{z} > \tag{2.12}$$

and the associated (scenario decomposable) dual function:

$$\Psi(\boldsymbol{\lambda}) = \inf_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \left\{ \sum_{\boldsymbol{\xi}\in\Xi} Pr(\boldsymbol{\xi}) f(\boldsymbol{x}(\boldsymbol{\xi}),\boldsymbol{y}(\boldsymbol{\xi})) + \sum_{\boldsymbol{\xi}\in\Xi} < \boldsymbol{\lambda}(\boldsymbol{\xi}), \boldsymbol{x}(\boldsymbol{\xi}) - \boldsymbol{z} > \right\} = \sum_{\boldsymbol{\xi}\in\Xi} \psi(\boldsymbol{\lambda},\boldsymbol{\xi}) \tag{2.13}$$

and solve (2.11) by its dual problem. However, because the dimension of $\boldsymbol{\lambda}$ grows with the number of scenarios, a simple scheme of updating dual vectors $\boldsymbol{\lambda}$ is more desired. This is one of the reasons we introduce augmented Lagrangian method. We define the augmented Lagrangian for (2.11) as follows:

$$\mathscr{L}_\gamma(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\lambda}) = \sum_{\boldsymbol{\xi}\in\Xi} Pr(\boldsymbol{\xi}) f(\boldsymbol{x}(\boldsymbol{\xi}),\boldsymbol{y}(\boldsymbol{\xi})) + \sum_{\boldsymbol{\xi}\in\Xi} < \boldsymbol{\lambda}(\boldsymbol{\xi}), \boldsymbol{x}(\boldsymbol{\xi}) - \boldsymbol{z} > + \frac{\gamma}{2}\|\boldsymbol{x}(\boldsymbol{\xi}) - \boldsymbol{z}\|^2 \tag{2.14}$$

where γ is the penalty parameter. The solving procedures of augmented Lagrangian method can be briefly summarized as follow: for iteration $k = 1, 2, \ldots,$

Step 1: given $\boldsymbol{\lambda}^k$, find $(\boldsymbol{x}^k, \boldsymbol{y}^k, \boldsymbol{z}^k) \in \arg\min_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \mathscr{L}_\gamma(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\lambda}^k)$;

Step 2: $\forall \boldsymbol{\xi} \in \Xi$, update multipliers $\boldsymbol{\lambda}^{k+1}(\boldsymbol{\xi})$: $\boldsymbol{\lambda}^{k+1}(\boldsymbol{\xi}) = \boldsymbol{\lambda}^k(\boldsymbol{\xi}) + \gamma(\boldsymbol{x}^k(\boldsymbol{\xi}) - \boldsymbol{z}^k)$

The advantage of augmented Lagrangian method is the simplicity of the multipliers update (**Step 2**), but the minimizing step (**Step 1**) may not be easy to decompose into scenario-dependent subproblems because of the non-anticipativity variable $\boldsymbol{z}$. Rockafellar and Wets [1991] propose a decomposition method, named as progressive hedging algorithm, which gradually approximates $\boldsymbol{z}$ by information on $\boldsymbol{x}$ from each iteration. The procedure of progressive hedging algorithm is summarized in **Algorithm 2**.

Algorithm 2 Progressive Hedging Algorithm

Step 1: Initialization

for $\boldsymbol{\xi} \in \Xi$ **do**

Solve for each scenario dependent sub-problem:

$$\underset{\boldsymbol{x},\boldsymbol{y}}{\text{minimize}} \quad f(\boldsymbol{x}(\boldsymbol{\xi}), \boldsymbol{y}(\boldsymbol{\xi})) \tag{2.15a}$$

$$\text{subject to} \quad (\boldsymbol{x}(\boldsymbol{\xi}), \boldsymbol{y}(\boldsymbol{\xi})) \in G(\boldsymbol{\xi}) \tag{2.15b}$$

$(\boldsymbol{x}^0_{\boldsymbol{\xi}}, \boldsymbol{y}^0_{\boldsymbol{\xi}}) \leftarrow$ optimal solutions

end for

$\boldsymbol{z}^0 \leftarrow \sum_{\boldsymbol{\xi}\in\Xi} Pr(\boldsymbol{\xi})\boldsymbol{x}^0_{\boldsymbol{\xi}} \qquad \boldsymbol{\lambda}^0_{\boldsymbol{\xi}} \leftarrow \gamma(\boldsymbol{x}^0_{\boldsymbol{\xi}} - \boldsymbol{z}^0), \forall \boldsymbol{\xi} \in \Xi \qquad \epsilon \leftarrow \sum_{\boldsymbol{\xi}\in\Xi} \|\boldsymbol{x}^0_{\boldsymbol{\xi}} - \boldsymbol{z}^0\|$

Step 2: PH-iteration

$\tau \leftarrow 0$

while $\epsilon \geq \epsilon_0$ **do** ▷ Check converge criteria

for $\boldsymbol{\xi} \in \Xi$ **do**

$\tau \leftarrow \tau + 1$

Find $(\boldsymbol{x}^\tau_{\boldsymbol{\xi}}, \boldsymbol{y}^\tau_{\boldsymbol{\xi}})$ such that

$(\boldsymbol{x}^\tau_{\boldsymbol{\xi}}, \boldsymbol{y}^\tau_{\boldsymbol{\xi}}) \in \arg\min_{\boldsymbol{x},\boldsymbol{y}} \mathscr{L}_\gamma(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}^k, \boldsymbol{\lambda}^k)$ over all feasible $\boldsymbol{x}_{\boldsymbol{\xi}}$ and $\boldsymbol{y}_{\boldsymbol{\xi}}$

end for

$\boldsymbol{z}^\tau \leftarrow \sum_{\boldsymbol{\xi}\in\Xi} Pr(\boldsymbol{\xi})\boldsymbol{x}^\tau_{\boldsymbol{\xi}} \qquad \boldsymbol{\lambda}^\tau_{\boldsymbol{\xi}} \leftarrow \boldsymbol{\lambda}^{\tau-1}_{\boldsymbol{\xi}} + \gamma(\boldsymbol{x}^\tau_{\boldsymbol{\xi}} - \boldsymbol{z}^\tau), \forall \boldsymbol{\xi} \in \Xi$

$\epsilon \leftarrow \sum_{\boldsymbol{\xi}\in\Xi} \|\boldsymbol{x}^\tau_{\boldsymbol{\xi}} - \boldsymbol{z}^\tau\| + \sum_{\boldsymbol{\xi}\in\Xi} \|\boldsymbol{x}^\tau_{\boldsymbol{\xi}} - \boldsymbol{x}^{\tau-1}_{\boldsymbol{\xi}}\|$

end while

return $(\boldsymbol{x}^\tau, \boldsymbol{y}^\tau, \boldsymbol{z}^\tau, \boldsymbol{\lambda}^\tau)$

2.2 Network Modeling

2.2.1 Concepts

Network modeling is deeply rooted in graph theory so that network flow problems are typically mathematically modeled with graph-related notions [Bertsekas, 1998]. Given a directed graph, $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the set of nodes (indexed by n) and $\mathcal{A}$ is the set of links/arcs (indexed by a_{ij}) that connect two nodes (n_i, n_j) [5]. We would like to begin with an introduction of two key concepts in graph theory, which are going to be used in later chapters: path and flow. A path p in a directed graph can be described as a sequence of nodes $n_1, n_2, ..., n_k$ with $k \geq 2$ and a corresponding sequence of $k-1$ arcs connecting each pair of consecutive nodes in that node sequence. Typically n_1 can be referred to as the start node or origin, and n_k as the end node or destination. A pair of origin and destination (r, s) is referred to as an OD pair. In many applications of network modeling, such as transportation, communication, energy, it is useful to introduce a variable that measures the quantity passing through each arc or path. We refer to this variable as link flow, v_a, or path flow, x_p.

2.2.2 Modeling

The specific formulation of network flow problems can be varied case by case [6]. But different problems may share similar characteristics.

The objective function of a network flow problem typically takes either a linear or convex form of the link flow vector, $f(\boldsymbol{v})$, which may not necessarily be the actual total network cost. Take traffic assignment as an example, there are two alternative criteria suggested by Wardrop [1956]:

1. Drivers individually choose to follow their shortest time routes from their

[5] In a directed graph, the order of nodes matters. (n_i, n_j) means an arc starting from node n_i and pointing to node n_j.

[6] see [Bertsekas, 1998] for some specific examples including the shortest path, the max-flow, the assignment, and the transportation problems.

origins to destinations. In equilibrium, the journey times in all routes actually used are equal and less than those that would be experienced by a single vehicle on any unused route;

2. Drivers are centrally directed to choose routes. In equilibrium, the average journey time is at a minimum.

Assume that the link cost function, $c_a(\cdot)$, only depends on its own link flow, v_a [7]. The objective function of the second criterion, also known as System Optimial (SO), can be formulated as minimizing the total system cost: $f(\boldsymbol{v}) = \sum_{a\in\mathcal{A}} v_a c_a(v_a)$. However, the first criterion, also known as User Equilibrium (UE), was later mathematically formulated by Beckmann et al. [1956], as minimizing a manufactured cost function $f(\boldsymbol{v}) = \sum_{a\in\mathcal{A}} \int_0^{v_a} c_a(u)\mathrm{d}u$, whose interpretation is less intuitive especially in high dimension. We will go back to the traffic equilibrium problem in **Section 2.2.3.3**.

In terms of constraint set for network modeling, some commonly used constraints are summarized in (2.16).

$$A\boldsymbol{v} = \boldsymbol{s}, \tag{2.16a}$$

$$B\boldsymbol{x} = \boldsymbol{v}, \tag{2.16b}$$

$$C\boldsymbol{x} = \boldsymbol{d}. \tag{2.16c}$$

where A is node-link incidence matrix of network, with 1 at starting node and -1 at ending node; B is link-path incidence matrix, with element equals to 1 if a path p consists of link a, and 0 otherwise; C is OD-path incidence matrix, with element equals to 1 if a path p connect a OD pair (o, d), and 0 otherwise; $\boldsymbol{s}$ and $\boldsymbol{d}$ are given vectors, representing the nodal flow injection (can be negative) at each node and the OD demand for each OD pair, respectively.

[7] This assumption is made just for simplification of illustration and can be relaxed.

Constraint (2.16a) guarantees flow conservation at each node; Constraint (2.16b) transforms path flow to link flow [8]; Constraint (2.16c) requires the summation of different paths connecting the same OD pair should equal to the OD demand. Notice that not all of the constraints in (2.16) need to be imposed in one network problem. For example, for link based problems, we typically use Constraint (2.16a) only, while for path based problems, we use Constraint (2.16b & 2.16c).

2.2.3 Solution Approaches

Network problems are special cases of linear/integer/non-linear programming, therefore, any algorithms for linear/integer/non-linear programming can be directly applied. However, the special structure of network problems typically can lead to substantial simplification of the general methods. Theoretically, there are two main categories of algorithms available for network flow problems: primal methods and dual ascent methods [Bertsimas and Tsitsiklis, 1997]. Primal methods, e.g. primal simplex method, maintain and keep improving a primal feasible solution; while dual ascent methods, e.g. dual simplex method, maintain and update a dual feasible solution and an auxiliary primal solution, which satisfy complementary conditions, to increase the objective function and reduce the infeasibility of primal solution. In this section, we will only present three algorithms, with increasing complexity, that are widely used in solving network problems: search algorithms, Dijkstra's algorithm, and Frank-Wolf algorithm.

2.2.3.1 Search Algorithms

Search algorithms are lie at the heart of many network algorithms and aim to find all nodes in a network that satisfy a particular property [Ahuja et al., 1988]. Assuming that we want to find (marked) all the nodes that are reachable through directed paths from a source node, the search algorithm can be summarized in

[8]Sometimes, different path flows solutions can yield the same link flow pattern. So path flow solutions may not be unique.

Algorithm 3.

Algorithm 3 Search Algorithm

Step 1: Initialization

Unmark all nodes in $\mathcal{N}$

Mark the source node s

LIST := $\{s\}$

Step 2: Search

while LIST $\neq \emptyset$ **do**

  Select a node i from LIST ▷ e.g. breadth-first or depth-first

  for $a \in \mathcal{A}$ **do**

    if $a = (i, j)$ is admissible, i.e. j is unmarked **then**

      Mark node j

      pred(j) := i ▷ record the precedent node

      Add node j to LIST

    end if

  end for

  delete node i from LIST

end while

return all the marked nodes and ***pred***

Algorithm 3 does not specify the order for selecting and adding nodes to the LIST. There are two main rules: the first one is that the LIST is maintained as queue, i.e. first in first out; the second one is that the LIST is maintained as stack, i.e. last in first out. Following these two data structures, we will have breadth-first and depth-first search, respectively.

2.2.3.2 Dijkstra's Algorithm

The shortest path problem is an important problem appearing in different applications, e.g. transportation, communication and optimal control. The one source node, one sink node (one-to-one) shortest path problem can be formulated as linear programming, as shown in (2.17).

$$\underset{\boldsymbol{v}}{\text{minimize}} \quad \sum_{a \in \mathcal{A}} w_a v_a \tag{2.17a}$$

$$\text{subject to} \quad A\boldsymbol{v} = \boldsymbol{s} \tag{2.17b}$$

$$\boldsymbol{v} \geq 0, \tag{2.17c}$$

where w_a is the cost of traveling through link a; $\boldsymbol{s}$ is flow injection at each node, with the source node equal to 1 and the sink node equal to -1; A is the node-link incident matrix.

The intuition of linear programming formulation of shortest path is that $\boldsymbol{v}$ are the indicator variables (0/1) for whether each link is part of the shortest path or not. Note that we do not need to force variables $\boldsymbol{v}$ to be integer because every basic optimal solution of (2.17) will always be integral.

The shortest path problem can be solved using standard linear programming techniques (e.g. simplex method) for each OD pair. But when we interested to know the shortest paths from one source node to multiple/all nodes, and assume that all the link costs are non-negative, Dijkstra's algorithm, as shown in **Algorithm 4**, is proven to be more efficient. When link costs can be negative, Bellman-Ford algorithm [Shimbel, 1954] is more appropriate.

2.2.3.3 Frank-Wolfe Algorithm

Equilibrium over urban transportation network is one of the more advance network modeling examples, which consist of shortest path as a subproblem. Two notion of traffic equilibria was mentioned in **Section 2.2.2**, with one denoted as user equilibrium (UE) and the other denoted as system optimal (SO). In this section, we use the UE problem to illustrate one of the important algorithms in network problems: Frank-Wolf Algorithm.

Algorithm 4 **Dijkstra's Algorithm**

Step 1: Initialization

LIST := $\mathcal{N}$

for $n \in \mathcal{N}$ **do**

  if $n ==$ source **then**

    $dist[n] \leftarrow 0$

  else

    $dist[n] \leftarrow \infty$ ▷ currently known shortest distance from source node

    $prev[n] \leftarrow$ unknown ▷ previous node achieving $dist[n]$

  end if

end for

Step 2: Update

while LIST $\neq \emptyset$ **do**

  $u :=$ node in LIST with smallest $dist[\cdot]$

  Remove u from LIST

  for neighbor v of u **do** ▷ neighbor is defined as directly connected by link

    $temp \leftarrow dist[u] + w(u, v)$

    if $temp < dist[v]$ **then**

      $dist[v] \leftarrow temp$ ▷ update currently known shortest distance

      $prev[v] \leftarrow u$ ▷ update previous node

    end if

  end for

end while

return $\boldsymbol{dist}$,$\boldsymbol{prev}$

The traffic assignment problem associated with the notion of UE can be formulated as the following mathematical program (2.18):

$$\begin{aligned}
\underset{\boldsymbol{v}}{\text{minimize}} \quad & \sum_{a\in\mathcal{A}} \int_0^{v_a} c_a(u)\mathrm{d}u && \text{(2.18a)}\\
\text{subject to} \quad & \sum_{k\in K^{rs}} x_k^{r,s} = d^{rs}, \qquad \forall r \in R, s \in S && \text{(2.18b)}\\
& v_a = \sum_{rs} \sum_{k\in K^{rs}} x_k^{rs}\delta_{a,k} \geq 0, \qquad \forall a \in \mathcal{A}, && \text{(2.18c)}\\
& x_k^{rs} \geq 0, \qquad \forall r \in R, s \in S. && \text{(2.18d)}
\end{aligned}$$

where R $(/S)$ is the original (/destination) set, indexed by r $(/s)$; K^{rs} is the set of paths connecting r and s; $\delta_{a,k}$ is the indicator if link a belongs to path k. The Frank-Wolfe algorithm for problem (2.18) can be summarized in **Algorithm 5**.

2.3 Multi-agent Optimization with Equilibrium Constraints

Single-agent optimization helps find the best feasible decisions; while multi-agent optimization helps understand how collective best feasible decisions can influence a system. A general MOPEC modeling framework was proposed in [Ferris and Wets, 2012], which includes a wide variety of variational problems as special cases: variational inequalities, complementarity problems, fixed points problems, etc. Formulate these problems as MOPEC allows the structure of the problem to be exploited fruitfully in a computational environment, and sometimes would lead to a decomposition of the original problem into smaller tasks, where efficient numerical procedures are readily available [Ferris and Wets, 2012]. MOPEC has wide applications in equilibrium problems such as non-cooperative games (e.g. Nash Equilibrium [Nash, 1950, 1951]) and Walras equilibrium problems [Ferris and Wets, 2012].

Algorithm 5 Frank-Wolfe Algorithm

Step 1: Initialization: find initial feasible solutions

$\tau \leftarrow 0$

Find the shortest path for each OD pair (r, s) assuming $c_a(v_a) = c_a(0), \forall a, v_a$

Assign d^{rs} to the shortest path using all-or-nothing (AON)

Get link flows $\boldsymbol{v}^{\boldsymbol{\tau}}$

Step 2: Update travel time

$t_a^\tau = c_a(v_a^\tau)$

Step 3: Find a moving direction, d^τ, for link flow $\boldsymbol{v}$

Find the shortest path for each OD pair (r, s) assuming $c_a(v_a) = t_a^\tau, \forall a, v_a$

Assign d^{rs} to the shortest path using all-or-nothing (AON)

Get the auxilary link flow $\boldsymbol{w}^{\boldsymbol{\tau}}$

Set link flow moving direction $\boldsymbol{d}^\tau = \boldsymbol{v}^{\boldsymbol{\tau}} - \boldsymbol{w}^{\boldsymbol{\tau}}$

Step 4: Find a moving step $\alpha^{\tau *}$ for link flow $\boldsymbol{v}$ by line search

Find $\alpha^{\tau *}$ by solving: ▷ any α^τ will satisfy flow conservation

$$\underset{0 \leq \alpha^\tau \leq 1}{\text{minimize}} \sum_{a \in \mathcal{A}} \int_0^{x_a^\tau + \alpha^\tau d_a^\tau} c_a(u) \mathrm{d}u \tag{2.19}$$

Step 5: Update link flow

$\boldsymbol{v}^{\boldsymbol{\tau+1}} = \boldsymbol{v}^{\boldsymbol{\tau}} + \alpha^{\tau *} \boldsymbol{d}^\tau$

Step 6: Check convergence (optimality)

if $\sqrt{\sum_a (v_a^{\tau+1} - v_a^\tau)^2} / \sum_a v_a^\tau \leq \epsilon$ **then** ▷ ϵ is the convergence threshold

  return $\boldsymbol{v}^{\tau+1}$

else

  $\tau \leftarrow \tau + 1$

  go to Step 2

end if

2.3.1 Formulation

Consider a collection of agents A whose decisions are denoted as $\boldsymbol{x}_A = (\boldsymbol{x}_a, a \in A)$. A MOPEC model, in its simplest form, can be expressed as:

$$\boldsymbol{x}_a \in \operatorname{argmax}_{\boldsymbol{x} \in X_{\boldsymbol{p},\boldsymbol{x}_{-a}} \subset R^{n_a}} f_a(\boldsymbol{p}, \boldsymbol{x}, \boldsymbol{x}_{-a}),\ a \in A, \tag{2.20}$$

where f_a is their optimization functions, $\boldsymbol{x}_{-a}$ represents the decisions of the other agents $A \backslash a$ and $\boldsymbol{p}$ are the system parameters [9]. Parameters $\boldsymbol{p}$ and the decisions $\boldsymbol{x}_A$ resulting from the multi-optimization problem typically need to satisfy global equilibrium constraints, which can be formulated as a functional variational inequality (2.21):

$$D(\boldsymbol{p}, \boldsymbol{x}_A) \in \partial g(\boldsymbol{p}), \tag{2.21}$$

where $g : I\!R^d \to \overline{I\!R}$ is a proper, lower semicontinuous and convex function and D is a set-valued mapping from $I\!R^d \times I\!R^{\sum_{a \in A} n_a}$ to $I\!R^d$.

As a special case of MOPEC, when function g is the indicator function ι_C of a non-empty, closed convex set C, i.e. $\iota_C(\boldsymbol{p}) \equiv 0$ if $\boldsymbol{p} \in C$, and $\iota_C(\boldsymbol{p}) \equiv \infty$ otherwise, functional variational inequality (2.21) will become more conventional geometric variational inequality (2.22):

$$D(\boldsymbol{p}, \boldsymbol{x}_A) \in N_C(\boldsymbol{p}), \tag{2.22}$$

where $N_C(\boldsymbol{p})$ is the normal cone to C at $\boldsymbol{p}$.

Although MOPEC is developed under the context of single-level equilibrium problem, one can extend the same framework to describe the multi-level equilibrium problem considering the possibly anticipation of the upper-level agents on the reaction of the lower-level agents by moving the equilibrium constraints into

[9] Parameters $\boldsymbol{p}$ may be endogenously determined by the system, such as prices.

the optimization problems. For example, consider Stackelberg game [Von Stackelberg, 1952], where there is one leader and multiple followers. Leader will optimize its decisions considering the anticipated followers' behaviors in response to its decision; the behaviors of the followers can be described by either (2.21) or (2.22). The resulting formulation is typically referred to as mathematical program with equilibrium constraints (MPEC), or equilibrium problem with equilibrium constraints (EPEC) if we combined multiple Stackelberg games together.

To illustrate how this general modeling framework can be applied, we take the Nash equilibrium in non-cooperative games as an example. A non-cooperative game consists of a set of players A whose individual reward $r_a(\boldsymbol{x}_a, \boldsymbol{x}_{-a})$ depends both on one's own decision $\boldsymbol{x}_a$ and the others' decisions $\boldsymbol{x}_{-a}$. A Nash equilibrium point is a collection of decisions $\boldsymbol{x}_A$, where $\forall a \in A$, $\boldsymbol{x}_a^* \in \arg\max_{\boldsymbol{x}_a \in C_a} r_a(\boldsymbol{x}_a, \boldsymbol{x}_{-a})$.

To formulate this Nash game using the general MOPEC framework, we introduce a global parameters $\boldsymbol{p} = \boldsymbol{y}_A \in C_A$, which can be interpreted as the previous decisions each player made. So,

$$\boldsymbol{x}_a^* \in \arg\max_{\boldsymbol{x}_a \in C_a} r_a(\boldsymbol{x}_a, \boldsymbol{y}_{-a}), \forall a \in A \tag{2.23}$$

Then we enforce the global equilibrium, which is simply $\boldsymbol{y}_A - \boldsymbol{x}_A = 0$ or equivalently $D(\boldsymbol{y}_A, \boldsymbol{x}_A) \in N_{R^{n \times |A|}}(\boldsymbol{y}_A)$, where $D(\boldsymbol{y}_A, \boldsymbol{x}_A) = \boldsymbol{y}_A - \boldsymbol{x}_A$. Notice that this formulation considers an "iteration" process in obtaining the solution and provides a structure that can facilitate the design of a decomposition solution algorithm due to the isolation of other agents' decision variables in one's optimization problems.

2.3.2 Solution Approaches

For some special cases, MOPEC can be solved as a single convex optimization problem. We leave the discussion of this approach to Chapter 4, where we prove the Nash equilibrium of power generators planning problem can be cast as iteratively finding traffic UE solutions of (2.18). In the remaining of this section, we focus our discussion on the solution approaches for the single-level equilibrium problem and briefly summarize the main research progress in solving bi-level equilibrium problems in a more general setting.

2.3.2.1 Single-level Equilibrium

One possible solution approach to solve single-level MOPEC is to formulate the problem as mixed complementarity problem (MCP) using the first order optimality conditions for the (convex) optimization problems and pass the problem to some existing MCP solvers, e.g. PATH [Dirkse and Ferris, 1995]. Assuming the optimization problems for all the agents are convex, the sufficient and necessary first-order conditions for optimality can be expressed as :

$$0 \in -\partial[f_a(\boldsymbol{p}, \boldsymbol{x}_a, \boldsymbol{x}_{-a}) + \iota_{X_{\boldsymbol{p},\boldsymbol{x}_{-a}}}(\boldsymbol{x}_a)] = -\partial f_a(\boldsymbol{p}, \boldsymbol{x}_a, \boldsymbol{x}_{-a}) + N_{X_{\boldsymbol{p},\boldsymbol{x}_{-a}}}(\boldsymbol{x}_a), \forall a \in A \tag{2.24}$$

where the subgradient is taken with repect to $\boldsymbol{x}_a$ only. The overall problem can then be rewritten as an MCP (2.25):

$$\partial f_a(\boldsymbol{p}, \boldsymbol{x}_a, \boldsymbol{x}_{-a}) \in N_{X_{\boldsymbol{p},\boldsymbol{x}_{-a}}}(\boldsymbol{x}_a), \forall a \in A \tag{2.25a}$$

$$D(\boldsymbol{p}, \boldsymbol{x}_A) \in N_C(\boldsymbol{p}) \tag{2.25b}$$

But MOPEC has more structure than a general MCP formulation and existing solvers can capture, and this structure can sometimes lead to decomposition of the whole problem into smaller pieces, for which efficient computation approaches might be readily available [Ferris and Wets, 2012]. Notice that the decomposition strategies will not be universal for all the problems. In this section, we will use a classical equilibrium model in economics, the Walras barter model (Pure Exchange model), to illustrate how some recent theoretical developments of variational convergence theory can help to solve a MOPEC.

- Walras Barter Model Description

  In a pure exchange economy, there is a finite number of agents $a \in A$ who try to maximize individual utility function $u_a : I\!R^L \to [-\infty, \infty), a \in A$ by deciding acquisitions of L goods $\boldsymbol{x}_a \in I\!R^L$ given their own finite initial endowments (goods) $\boldsymbol{e}_a \in I\!R^L, a \in A$. Trading will take place at a per-unit market price $\boldsymbol{p}$. So the optimization problem for each agent $a \in A$ can be formulated as (2.26):

$$\boldsymbol{x}_a(\boldsymbol{p}) \in \text{argmax}_{\boldsymbol{x} \in X_a} \{u_a(\boldsymbol{x}) | \langle \boldsymbol{p}, \boldsymbol{x} \rangle \leq \langle \boldsymbol{p}, \boldsymbol{e_a} \rangle\} \tag{2.26}$$

  where $X_a \subset I\!R^L_+$ is non-empty, convex set representing agent a's survival set (i.e. $u_a > \infty$). Notice that $\boldsymbol{x}_a$ is homogeneous of degree 0 with respect to prices, i.e. $\boldsymbol{x}_a(\boldsymbol{p}) = \boldsymbol{x}_a(\boldsymbol{\alpha p})$, we may restrict $\boldsymbol{p} \in \Delta \doteq \{\boldsymbol{p} \in I\!R^L_+ | \sum_{j=1^L} p_j = 1\}$. The equilibrium conditions (i.e. the market is operational) for Walras barter model is (1) total supply exceeds total demand, i.e.

$$\boldsymbol{s}(\boldsymbol{p}) \doteq \sum_{a \in A} (\boldsymbol{e}_a - \boldsymbol{x}_a(\boldsymbol{p})) \geq \boldsymbol{0} \tag{2.27}$$

  and (2) local insatiability, i.e.

$$\langle \boldsymbol{p}, \boldsymbol{s}(\boldsymbol{p}) \rangle = 0 \tag{2.28}$$

- Maxinf Point of Bifunctions

  Jofré and Wets [2014] show that that the equilibrium prices $\boldsymbol{p}^*$ can be characterized as the maxinf point of a bifunction (referred to as the Walrasian) $W(\boldsymbol{p}, \boldsymbol{q}) : \Delta \times \Delta \to I\!R$, where $W(\boldsymbol{p}, \boldsymbol{q}) \doteq \langle \boldsymbol{q}, s(\boldsymbol{p}) \rangle$, by **Proposition 1**.

  Proposition 1 *(Walras equilibrium prices and maxinf-points) Every maxinf-point $\overline{\boldsymbol{p}}$ of the Walrasian such that $W(\overline{\boldsymbol{p}}, \cdot) \geq 0$ on Δ is an equilibrium point.*

Moreover, under local insatiability, every equilibrium point $\overline{\boldsymbol{p}}$ *is a maxinf-point of the Walrasian such that* $W(\overline{\boldsymbol{p}}, \cdot) \geq 0$ *on* Δ.

Proof. See [Jofré and Wets, 2014].

- Approximation and Decomposition

  The transformation of equilibrium prices to maxinf points of Walrasian opens up opportunity for approximation and decomposition. One can use a sequence of bifunctions, e.g. $W^\nu(\boldsymbol{p}, \boldsymbol{q}) = \langle \boldsymbol{q}, s^\nu(\boldsymbol{p}) \rangle$ on $\Delta \times \Delta$, where $s^\nu(\boldsymbol{p}) = \sum_{a \in A}(\boldsymbol{e} - \boldsymbol{x}_a^\nu(\boldsymbol{p}))$, to approximate $W(\boldsymbol{p}, \boldsymbol{q})$. To guarantee the convergence of maxinf points, a new notion of bifunction convergence is proposed by [Jofré and Wets, 2014], see **Definition 1**.

Definition 1 *(lop-convergence of bifunction) A sequence in finite-valued bi-function (fv-biv)* $fv\text{-}biv(I\!R^{n+m})$, $\{W^\nu : C^\nu \times D^\nu \to I\!R\}_{\nu \in N}$ *lop-converges to a function* $W : C \times D \to I\!R$, *if*

(a) for all $\boldsymbol{y} \in D$ *and all* $(\boldsymbol{x}^\nu \in C^\nu) \to \boldsymbol{x} \in C$, *there exists* $(\boldsymbol{y}^\nu \in D^\nu) \to \boldsymbol{y}$ *such that* $\limsup_\nu W^\nu(\boldsymbol{x}^\nu, \boldsymbol{y}^\nu) \leq W(\boldsymbol{x}, \boldsymbol{y})$

(b) for all $\boldsymbol{x} \in C$, *there exists* $(\boldsymbol{x}^\nu \in C^\nu) \to \boldsymbol{x}$, *such that given any* $(\boldsymbol{y}^\nu \in D^\nu) \to \boldsymbol{y}$, $\liminf_\nu W^\nu(\boldsymbol{x}^\nu, \boldsymbol{y}^\nu) \geq W(\boldsymbol{x}, \boldsymbol{y})$ *when* $\boldsymbol{y} \in D$, *and* $W^\nu(\boldsymbol{x}^\nu, \boldsymbol{y}^\nu) \to \infty$ *when* $\boldsymbol{y} \notin D$

Lop-convergence is ancillary tight when (b) is strengthened to:

(b-t) (b) holds and for any $\varepsilon > 0$ *one can find a copact set* B_ε, *such that for all* ν *sufficiently large,*

$$\inf_{D^\nu \cap B_\epsilon} W^\nu(\boldsymbol{x}^\nu, \cdot) \leq \inf_{D^\nu} W^\nu(\boldsymbol{x}^\nu, \cdot) + \varepsilon \tag{2.29}$$

With this ancillary tight lop-convergence property of bifunctions, one can prove the convergence of maxinf-points, which is given in **Theorem 1** [Jofré and Wets, 2014].

Theorem 1 *(convergence of maxinf points) When the bifunctions* $\{W^{\nu}_{\nu \in N}\}$ *lop-converge ancillary tightly to* W*, all in fv-biv(*$\mathbb{R}^{n+m}$*) with* $\sup\inf W$ *finite, and* $\varepsilon^{\nu} \searrow \varepsilon \geq 0$*, then every clcuster point* $\overline{\boldsymbol{x}} \in C$ *of a sequence of* ε_{ν}*-maxinf-points of the bifunctions* W^{ν} *is a* ε*-maxinf-points of the limit function* W*, where* ε*-maxinf-points* $\boldsymbol{x}_{\varepsilon}$ *of* W *is defined as* $W(\boldsymbol{x}_{\varepsilon}, \cdot) \geq \sup\inf W - \varepsilon$*.*

Proof. See [Jofré and Wets, 2014].

Based on **Theorem 1**, one can solve the Walras barter model by iteratively solving a sequence of approximated Walrasian, W^{nu}, which can be constructed to have nicer properties we desired (such as convexity). The iteration procedure also enables us to decompose agent's optimization problems by solving them individually. We will employ this approach in Chapter 3 to solve a network-based MOPEC problem related to electric vehicle charging infrastructure planning.

2.3.2.2 Bi-level Equilibrium

For general bi-level equilibrium, such as MPEC and EPEC, a global and large scale solution algorithm is still lacking. The problem of applying standard NLP algorithms for MPEC is problematic because the constraint qualifications (CQ), such as the Mangasarian-Fromovitz Constraint Qualification (MFCQ) and Linear Independence Constraint Qualification (LICQ) assumed to prove convergence of standard algorithms typically fail to hold for MPECs [Pieper, 2001]. But there are several ways people have been trying and yield promising results. For example, the piecewise sequential quadratic programming (PSQP) iteratively solve a quadratic programming to determine the moving direction and is shown to be local convergence under mild conditions [Luo et al., 1996]; the penalty interior-point algorithm (PIPA) replace the complementarity conditions of lower level by Hadamard product to allow small perturbation for each row, and gradually tighten this relaxation [Luo et al., 1996]. The solution approaches of EPEC typically will depend on the solution of MPEC. Diagonalization methods are widely used in engineering literature, but the absence of convergence results is one of their main drawbacks [Su, 2005].

2.4 N-SMOPEC Modeling Framework

To capture the interplay of uncertainties, interdependencies, and distributed decision making processes in systems, we develop a holistic methodology, Network-based Stochastic Multi-agent Optimization Problem with Equilibrium Constraints (N-SMOPEC).

Assume that each agent $a \in A$ have access to the same information at each stage t, $\boldsymbol{\xi}_t \in \Xi_t, t \in T$, where T is the set of stages. We further denoted the total information revealed to agents at stage t as $\boldsymbol{\xi}_{[t]}$, i.e. $\boldsymbol{\xi}_{[t]} \doteq \{\boldsymbol{\xi}_\tau, \tau = 1, 2, \ldots, t\}$. For agent $a \in A$, a multistage mean-risk optimization will be solved to determine its optimal decisions for each stage:

$$\{\boldsymbol{x}_a^t(\boldsymbol{\xi}_{[t]}), \forall t \in T\} \in \underset{\boldsymbol{x} \in X_{\boldsymbol{p}, \boldsymbol{x}_{-a}}}{\arg\max} (\mathbb{E} f_a + \kappa r_a)[\boldsymbol{\xi}_{[t]}; \boldsymbol{p}^t(\boldsymbol{\xi}_{[t]}), \boldsymbol{x}^t(\boldsymbol{\xi}_{[t]}), \boldsymbol{x}_{-a}^t(\boldsymbol{\xi}_{[t]}), \forall t \in T] \tag{2.30}$$

In optimization problem (2.30), each agent will try to make his/her own decisions at each stage t given the available information $\boldsymbol{\xi}_{[t]}$. Note that the $\boldsymbol{x}^t$ need to be measurable on $\boldsymbol{\xi}_{[t]}$ and should not depend on future unknown events. Alternatively, one can formulate the individual's stochastic optimization problem (2.30) by relaxing the decision variables $\boldsymbol{x}^t$ to be $\boldsymbol{\xi}_{[T]}$ dependent and imposing non-anticipativity constraints for each stage. The risk measures, $(r_a(\cdot), \forall a \in A)$, will be the aggregation of the risks faced at each stage.

The decisions of individual agents will be connected through functional variational inequality,

$$D[\boldsymbol{p}^t(\boldsymbol{\xi}_{[t]}), \boldsymbol{x}_A^t(\boldsymbol{\xi}_{[t]}), \forall t \in T)] \in \partial g[\boldsymbol{p}^t(\boldsymbol{\xi}_{[t]}), \forall t \in T)] \tag{2.31}$$

Notice that if there is only one decision maker involved, e.g. central planner case, our modeling framework degenerate to network-based stochastic programming. Furthermore, when $\kappa = 0, T = 2, |A| = 1$, the above formulation is the classic two-stage stochastic programming discussed in **Section 2.1.1**; when $\kappa > 0, T = 2, |A| = 1$, the N-MOPEC degenerates to the mean-risk model in **Section 2.1.2**; and when $\kappa = 0, T = 1, |A| > 1$, the above formulation is equivalent to the standard MOPEC model introduced in **Section 2.3**.

The network structure can underlie both the agents' optimization problem (2.30) and the global equilibrium constraints (2.31). In the former case, individual agent's optimization problem can simply be some network flow problems, such as traffic UE equilibrium or electricity economic dispatch model; the latter case is due to the fact that decisions of different agents are interdependent both spatially & temporally, and sometimes can be easier linked together in a network structure using, for example, flow conservation, link capacity, network equilibrium constraints.

Chapter 3

Application I: Planning of Fast Charging Stations in a Competitive Market

In this chapter, we focus on the modeling and computation of an application in transportation infrastructure system, considering distributed decision making and interdependencies between multiple decision makers and between infrastructures. More specifically, we look at the planning of electric vehicles fast charging stations in a competitive market and illustrate how our general modeling framework N-SMPOEC can be adapted to formulate this problem. Uncertainty is temporarily omitted in this chapter. But as we will demonstrate in Chapter 4, formulating and solving stochastic problems can be closely related to their deterministic counterpart.

3.1 Introduction

The emergence of electric vehicles (EVs) has brought great opportunities to both transportation and energy sectors. For the transportation sector, EVs are considered as a promising alternative vehicle technology for GHG emission reduction. For the power sector, EVs provide potential in accommodating high levels of intermittent renewable generation via vehicle-to-grid (V2G) technologies. However, several obstacles impede immediate large EV adoption, one of which is scarcity of charging infrastructure and long charging duration.

The topic of EV charging has attracted attention from both transportation and power sectors. Different charging types (i.e., home, workplace, or public charging) have called for different research emphases. For home and workplace charging, since charging locations are fixed and allowable charging duration is long, most existing studies focus on the control of electricity resource allocation on charging. For example, Huang and Zhou [2015] developed an optimization framework for workplace charging strategies considering different charging technologies and employees' demographic distributions; Lopes et al. [2009] studied smart charging strategies to enhance grid performance and maximize renewable energy resource integration. There are also studies assessing the impacts of EV charging on existing power grid operation [Putrus et al., 2009], emissions [Jansen et al., 2010], and both [Sohnen et al., 2015]. For public charging, since the infrastructure is yet to be developed, most studies focus on identifying the best facility deployment strategies. There are two main schools of thought: from either transportation/location science or power system viewpoint. For example, with a focus on power side, Sadeghi-Barzani et al. [2014] considered the impact of EV charging on grid reliability and proposed a method to minimize the facility development cost as well as charging cost. EV charging infrastructure planning studies in the transportation literature focus more on capturing the interaction between charging and travel (destination and route choice) behaviors. These studies can be further categorized into node-based [Hakimi, 1964] and flow-based [Hodgson, 1990] approaches. Node-based approaches, with a strong root in classic facility location models (e.g. p-median, center, and max-coverage), consider charging demands (typically assumed exogenous) happening at given nodes [Goodchild and Noronha, 1987, Frade et al., 2011]. Dong and Lin [2012] and Dong et al. [2014] combined node-based infrastructure deployment approaches with activity-based travel demand modeling to identify charging location, quantity, and duration based on real travel activities. He et al. [2013] developed an integrated model to capture the interaction between power grid and traffic network. In contrast, flow-based infrastructure deployment approaches, such as Flow Intercepting Location Model (FILM) [Hodgson, 1990, Berman et al., 1992], allocate charging resources to support travelers' preferred routes (such as shortest paths). Different objectives, such as budget-constrained maximum flow coverage [Kuby and Lim, 2005, Kuby et al., 2009] and set-covering minimum cost problem [Wang and Lin, 2013], have been investigated. Building upon [Berman et al., 1995], deviated paths were considered in [Kim and Kuby, 2012, Li and Huang, 2014]. All of the above studies take a central planner's point of view.

Our vision is that as EV demand grows, more investors from the private sector are likely to enter EV charging business. In this context, the future charging infrastructure system will be shaped by collective investment actions of many individual decision entities, who are selfish and competitive by nature. How would such business-driven investment decisions influence the layout of future public charging infrastructure? A good understanding of this question is critical to support effective energy planning at a system level. However, as listed above, most existing studies on EV charging take a central planner's perspective, assuming that investment decision can be fully controlled by a single decision entity. Business-driven investment behaviors of EV charging facilities have been recognized in Schroeder and Traber [2012], but models capturing selfish and competitive investment behaviors are lacking. To our knowledge, only two studies [Bernardo et al., 2015, Yu et al., 2015] have addressed the market side of charging station allocation. Bernardo et al. [2015] studied fast charging stations planning with free entry. Discrete-choice structural models were developed for the travelers as well as the investors' decision processes. Yu et al. [2015] considered the market dynamics of electric vehicle diffusion using a sequential game model. However, both studies ignored the network effect of the underlying transportation and charging infrastructure, which is a critical component that directly influences travel and charging behaviors. For example, Bernardo et al. [2015] is built based on a simple transportation model where origins and destinations are directly connected with known link costs. This treatment simplifies the problem, but is inadequate to capture the congestion effect of the underlying traffic network.

In the broader community of location science, there is a rich body of literature on competitive location problems [Smith et al., 2009, Kress and Pesch, 2012, Hakimi, 1983]. However, consideration of congestion is typically at the facility level. For example, [Brandeau and Chiu, 1994, Drezner and Weolowsky, 1996, Lee and Cohen, 1985] characterized the equilibrium in disaggregate facility choice systems subject to congestion-elastic demand at each facility. Congestion on the transportation network, which is important for our study because it directly influences drivers' accessibility to potential charging facilities, has not attracted much attention. Yang and Wong [2000] proposed a mathematical model for assessing market share among given facilities, considering network congestion and elastic demand of the customers. Even though that paper does not address facility location decision, it sheds light on integrating network analysis with facility location problems.

The goal of this chapter is to establish a mathematical model to support EV charging facility planning in a competitive market environment. To this end, several modeling challenges need to be addressed. First of all, the system involves multiple decision entities with different objectives: investors make infrastructure deployment decisions to maximize their individual profits, while travelers decide where and how to fulfill their travel and charging needs to maximize their own utilities. These decisions are interconnected and must be modeled simultaneously as a whole. Secondly, the physical infrastructure of charging facilities, power grid and urban traffic network are interdependent in terms of physical, spatial, and functional relations, which naturally brings a complex network structure into the problem. To address these modeling challenges, we use our network-based multi-agent optimization model, which reflects the selfish nature of each decision entity while simultaneously capturing the interactions among all over a complex network structure. To overcome the computational difficulty imposed by non-convexity of the problem, we exploit recent theoretical development on variational convergence of bivariate functions to design a solution algorithm with analysis on its convergence properties. To our knowledge, this study is the first in the EV charging literature that provides a theoretical foundation, from both modeling and computing perspectives, for analyzing business-driven EV charging infrastructure investment planning while considering the traveler-infrastructure interactions in a transportation network.

The remainder of this chapter is organized as follows. In Section 3.2, we present the general modeling framework with specific problem formulation for each decision entity involved in the fast charging infrastructure system. In Section 3.3, we demonstrate how the original multi-agent problem may be reformulated to a problem of finding a maxinf-point of certain bifurcation, followed by convergence analysis and algorithm design. In Section 3.4, we present numerical results of a widely used benchmark case study in the transportation literature and draw planning and policy implications. The last section concludes the chapter with insights, discussions, and future extensions.

3.2 Mathematical Modeling

3.2.1 Modeling Assumption

We are interested in studying whether there exists a profitable and self-sustainable fast charging network to support EV demand in the long run. Let us consider multiple investors, each making a facility deployment decision to maximize its own profit. Our main research question is: how to identify an equilibrium state of EV fast charging infrastructure network in such a competitive decision environment? Considering the potentially large number of EV charging service providers and the easiness of entering the market, we model the market structure as perfect competition. Based on this assumption, no participant has the market power to set the price of a homogeneous product; locational EV charging prices are determined by the market in an equilibrium condition[1]. Locational EV charging demand is assumed to depend on traffic conditions and charging services. Locational charging prices signal the market situation to both investors and travelers, so that market could be cleared at equilibrium point. Note that the charging prices may differ across locations - due to network congestion and accessibility, charging services at different locations should no longer be considered as homogeneous product.

3.2.2 Conceptual Framework

This problem falls into the general framework of Network-based MOPEC model, which reflects the "selfish" nature of each decision entity while simultaneously capturing the interactions among all over a complex network structure. The fundamental concept of this modeling framework is illustrated in Figure 3.1.

[1]We acknowledge that such economic theory is a simplification of real markets. Using retail gasoline market as an example, a few empirical studies have shown that market competition may be influenced by vertical integration of supply chains and locational effects [Hastings, 2004, Borenstein and Bushnell, 2005, Houde, 2012]. On the other hand, there are still many independent gasoline retailers (unbranded, and leasers or contractors of branded gasoline) in the market, forming a more competitive market than an oligopoly.

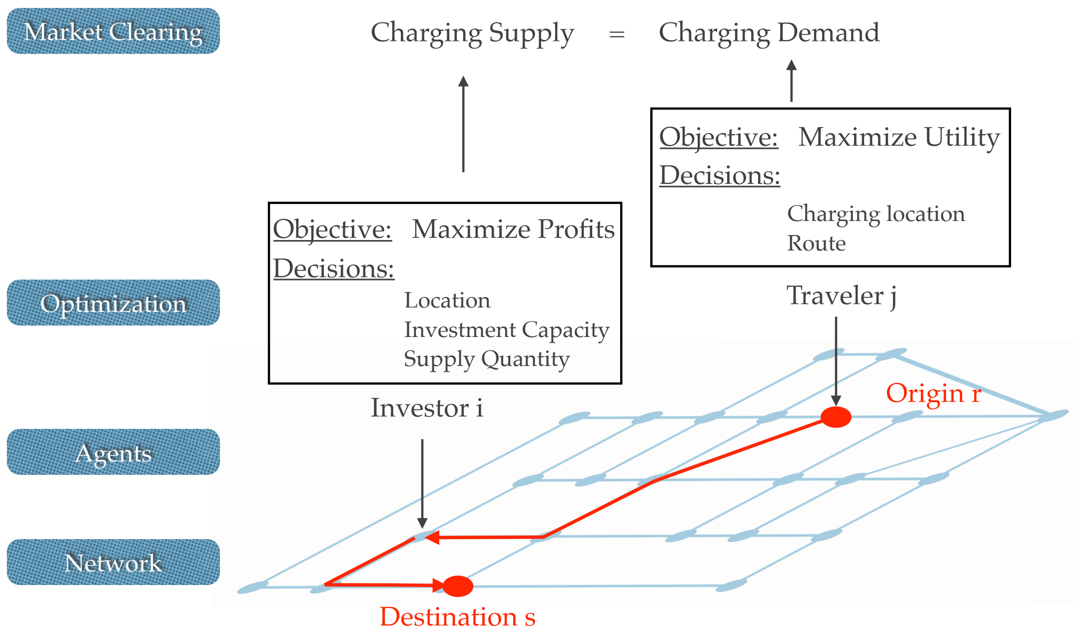

Figure 3.1: Illustration of Network-based MOPEC

In Figure 3.1, there are two types of decision makers (agents), who are making decisions over transportation network: investor i is deciding the location, charging capacity and supply quantity in order to maximize his own profits; traveler j , departing from one node, will decide destination and route in order to maximize his/her own utility. Their decisions are made individually but will be interdependent. In equilibrium state, market clearing conditions, i.e. charging supply equals to charging demand for all nodes, also need to be satisfied.

3.2.3 Recap of Basic MOPEC Modeling Framework

A general MOPEC modeling framework was proposed in [Ferris and Wets, 2012], which has wide applications in such as non-cooperative Nash-equilibrium games and Walras equilibrium problems. Consider a collection of agents A whose decisions are denoted as $x_A = (x_a, a \in A)$. A MOPEC model, in its simplest form, may be expressed as:

$$x_a \in \text{argmax}_x \; f_a(p, x, x_{-a}), \; a \in A, \tag{3.1}$$

where f_a is their criterion function, x_{-a} represents the decisions of the other agents $A \backslash a$ and p is a parameter such as price. The relationship between p and the decisions x_A resulting from the multi-optimization problem satisfies a global equilibrium constraint.

3.2.4 Detailed formulation for each decision entity

3.2.4.1 Modeling the Decisions of Fast Charging Investors

A new fast charging supplier entering the system has two types of decisions to make. During the planning stage, each new supplier decides the locations and the capacities of fast chargers in which to invest. At the operational stage, each supplier will choose its supply quantities based on market charging price. All these decisions should be made while taking into account the decisions of other competitors in the system. For each firm $\forall i \in I$:

$$\begin{aligned}
\underset{c_i^k, g_i^k}{\text{maximize}} \quad & \sum_{k \in K_i} \left[\rho^k g_i^k - \phi_g(g_i^k)\right] - \sum_{k \in K_i} \phi_c(c_i^k) && \text{(3.2a)}\\
\text{subject to} \quad & g_i^k - c_i^k \leq 0, \; \forall k \in K_i; && \text{(3.2b)}\\
& g_i^k \geq 0, \; \forall k \in K_i; && \text{(3.2c)}\\
& c_i^k \geq 0, \; \forall k \in K_i. && \text{(3.2d)}
\end{aligned}$$

where:

K_i : set of candidate investment locations of firm i, indexed by k;
c_i^k : charging capacity allocated at location k by firm i;
g_i^k : total charging supply at location k by firm i;
ρ^k : unit charging price at location k determined by the market;

$\phi_c(\cdot)$: total capital cost function with respect to charging capacity;
$\phi_g(\cdot)$: total operational cost function with respect to supply quantity;
I : set of investors.

The decision variables of each investor include the investment capacity c_i^k and suppling amount g_i^k at each candidate location. The objective function (3.2a) maximizes the net benefits during operational stage minus the total investment cost $\sum_{k \in K_i} \phi_c(c_i^k)$, which may include costs associated with land acquisition and equipment purchase. The net operational benefit is calculated as the total revenues $\sum_{k \in K_i} \rho^k g_i^k$ minus the operating cost $\sum_{k \in K_i} \phi_g(\boldsymbol{g}^k)$. Note that throughout the entire chapter, we denote vectors in lowercase bold font. For suppliers who have co-located business that could benefit from attracting more EV drivers, one may include additional terms to quantify the value of attracted trips. We assume that electricity will be sold at a uniform price at each candidate location, which is derived from the market clearing conditions shown in Section 3.2.4.3. Constraint (3.2b) ensures that the peak-hour electricity supplied at each location by each firm does not exceed its total capacity. Notice that we only consider the deterministic case, charging supply g will always equal to charging capacity c in equilibrium (otherwise investor can reduce their investment on charging capacity and be strictly better off). But we still want to separate c and g so that this formulation can be extended more naturally to the stochastic case, in which one can consider different charging demand scenarios due to peak/off-peak hour, uncertain traffic conditions, future EV penetration rate and battery capacity, etc. The remaining constraints are non-negative restrictions.

To ensure that the objective function is concave, $\phi_c(\cdot)$ and $\phi_g(\cdot)$ must satisfy certain properties. In this chapter, we restrict the summation of $\phi_c(\cdot)$ and $\phi_g(\cdot)$ to be a quadratic function, with positive coefficients. For example, the fixed cost may follow a linear function, and the production cost may be a quadratic function. Besides mathematical convenience, a quadratic production cost function also reflects two facts: (1) as electricity demand at a location increases, it causes more congestion in the transmission lines, which may lead to higher electricity price at that location; (2) as demand increases, higher-cost supplies start to enter the system as electricity is typically dispatched based on cost ranking.

3.2.4.2 Modeling the decisions of travelers in a congested transportation network

The demand for charging is typically derived from EV ownership and the travel patterns expected for these vehicles [Nicholas et al., 2013]. In this chapter, we treat EV ownership as exogenous variable. We allow travel behaviors (in terms of charging facility choice and route choice) to be affected by charging facilities in additions to the underlying transportation network. We extend the Combined Distribution and Assignment (CDA) model [Sheffi, 1985, Lam and Huang, 1992], named as Generalized Combined Distribution and Assignment (GCDA), to explicitly model the interaction between drivers and infrastructure, considering the intermediate facility choice and path deviation.

A multinomial logic model is used to describe the choice of different destination from origins r, with the deterministic component of utility function as follows:

$$U^{rsk} = \beta_0^k - \beta_1 t^{rsk} + \beta_2 \sum_{i \in I_k} c_i^k - \beta_3 \frac{\rho^k e^{rs}}{inc^{rs}} \tag{3.3}$$

where:

I_k	: set of investors who consider k as a candidate location, indexed by i;
U^{rsk}	: utility measure of a user to go from r to s and receive service at k;
β	: utility function parameters (model input);
t^{rsk}	: equilibrium travel time from r to s, with detour to service location k;
e^{rs}	: average charging demand from r to s (model input);
inc^{rs}	: average income in zone r, who travel to zone s (model input).

The utility function of traveler from node r to node s is assumed to be the summation of four parts: locational specific attractiveness factor, travel time, total charging capacities, and charging cost. This type of utility function has been adopted in other studies in the EV travel modeling literature [He et al., 2013, Bernardo et al., 2015, Yu et al., 2015]. Note that commuting trips, which typically have fixed destinations, should be distinguished from those non-commuting trips (such as shopping), which tend to have more flexible destination choices. Therefore, the utility function including all the parameters involved should be trip-type specific. The utility setting adopted here can be adjusted based on trip types. For example, for commuting trips, β_0 can be set to a relatively high value, which means the destination attractiveness is the dominant factor; while for recreational trips, several destinations may have similar attractiveness, therefore the destination choice could be affected by travel time and charging services.

Denote the transportation network by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N}$ is the set of nodes (indexed by n) and $\mathcal{A}$ is the set of links (indexed by a). In a traffic network, a node may represent a community (source/sink of travel demand), an intersection, or a freeway interchange. A link may represent a road section that connects two nodes. The GCDA model is formulated as follows [2]:

$$\underset{\hat{\boldsymbol{x}},\check{\boldsymbol{x}},\boldsymbol{x},\boldsymbol{q}}{\text{minimize}} \quad \sum_{a\in\mathcal{A}}\int_0^{v_a} t_a(u)\mathrm{d}u + \frac{1}{\beta_1}\sum_{r\in R}\sum_{s\in S}\sum_{k\in K^{rs}} q^{rsk}\left(\ln q^{rsk} - 1 + \beta_3\frac{\rho^k e^{rs}}{inc^{rs}} - \beta_2\sum_{i\in I_k} c_i^s - \beta_0^k\right) \tag{3.4a}$$

$$\text{subject to} \quad v_a = \sum_{r\in R}\sum_{s\in S}\sum_{k\in K^{rs}}(\hat{x}_a^{rsk} + \check{x}_a^{rsk}), \forall a\in\mathcal{A} \tag{3.4b}$$

$$(\boldsymbol{\gamma}) \quad \hat{\boldsymbol{x}}^{rsk} + \check{\boldsymbol{x}}^{rsk} = \sum_{p\in P^{rsk}}(B_{\hat{p}} + B_{\check{p}})x_p,\ \forall r\in R, s\in S, k\in K^{rs} \tag{3.4c}$$

$$(\hat{\boldsymbol{\lambda}}) \quad A\hat{\boldsymbol{x}}^{rsk} = q^{rsk}E^{rk},\ \forall r\in R, s\in S, k\in K^{rs} \tag{3.4d}$$

$$(\check{\boldsymbol{\lambda}}) \quad A\check{\boldsymbol{x}}^{rsk} = q^{rsk}E^{ks},\ \forall r\in R, s\in S, k\in K^{rs} \tag{3.4e}$$

$$(\mu^{rs}) \quad \sum_{k\in K^{rs}} q^{rsk} = d^{rs}, \forall r\in R, s\in S \tag{3.4f}$$

$$\hat{x}_a^{rsk}, \check{x}_a^{rsk} \geq 0,\ \forall a\in\mathcal{A}, r\in R, s\in S, k\in K^{rs} \tag{3.4g}$$

$$x_p^{rsk} \geq 0,\ \forall p\in P^{rsk}, r\in R, s\in S, k\in K^{rs} \tag{3.4h}$$

$$v_a \geq 0,\ \forall a\in\mathcal{A} \tag{3.4i}$$

$$q^{rsk} \geq 0,\ \forall r\in R, s\in S, k\in K^{rs}. \tag{3.4j}$$

where:

v_a : traffic flow on link a;

$t_a(\cdot)$: travel time function of link a, e.g. the Bureau of Public Roads (BPR) function;

d^{rs} : travel demand from r to s (model input);

[2]Note that when multiple types of trips are considered, the variables ($\boldsymbol{x}$ and $\boldsymbol{q}$) and all the parameters involved in the utility function should carry an additional subscript corresponding to a specific trip type, which is omitted here for brevity.

q^{rsk} :traffic flow from r to s and service at k;

x_p^{rsk} : traffic flow on path p that connects r, k, s;

$\hat{x}_a^{rsk}$: traffic flow on link a that belongs to the travel from r to k associated with Origin-Service-Destination triple rks;

$\check{x}_a^{rsk}$: traffic flow on link a that belongs to the travel from k to s associated with Origin-Service-Destination triple rks;

A : node-link incidence matrix of network, with 1 at starting node and -1 at ending node;

$\hat{p}$: sub-path of path $p \in P^{rsk}$ that connect r to k;

$\check{p}$: sub-path of path $p \in P^{rsk}$ that connect k to s;

B_p : link-path p incidence vector, with ith row equals to 1 if path p includes link i and 0 otherwise;

E^{ij} : O-D incidence vector of O-D pair ij with 1 at origin i, -1 at destination j;

$\gamma, \check{\lambda}, \hat{\lambda}, \mu$: dual variables of corresponding constraints.

Constraint (3.4b) calculates the aggregate link flow v_a from the isolated link flow associated with rsk: $\hat{x}_a^{rsk}$ and $\check{x}_a^{rsk}$; Constraint (3.4c) guarantees there is always a feasible path flow solution $x_p (p \in P)$ that can yield a given link flow pattern. Constraint (3.4d, 3.4e) ensures the flow conservation at each node, including the origin, intermediate stop, and destination nodes; Constraint (3.4f) further restricts the total trips originated from node r should be equal to the total travel demand at that location. The rest of the constraints set non-negative restrictions on path flow and O-D demand. In the objective function (3.4a), the first term corresponds to the total user cost as modeled in a conventional static traffic equilibrium model, the second term involving $q \ln q$ corresponds to the entropy of trip distribution, and the remaining terms correspond to the utility measure of the travelers. This objective function does not have a physical interpretation, but it guarantees the first Wardrop principle [Wardrop, 1956] and the multinomial logit facility choice assumption being satisfied. This can be seen from **Lemma 1**. For the original CDA model, one can refer to [Sheffi, 1985]. Notice that the formulation proposed here include CDA as a special case. For some numerical examples to illustrate some special cases of our model, please see Section 3.4.2.

Lemma 1 *(Generalized Combined Distribution and Assignment) The optimal solutions* $(\hat{\boldsymbol{x}}^*, \check{\boldsymbol{x}}^*, \boldsymbol{x}^*, \boldsymbol{q}^*)$ *of problem (3.4) are the equilibrium solutions for the service location choice with logit facility demand functions and Wardrop user equilibrium.*

Proof. See Appendix B. □

3.2.4.3 Market clearing conditions

To calculate the equilibrium market charging prices, we have a market clearing condition that specifies the total demand equals the total supply at every supply location. If only trips demanding fast charging are considered, we can simply express the following:

$$(\rho^k) \quad \sum_{i \in I_k} g_i^k - \sum_{r \in R} \sum_{s \in S} e^{rs} q^{rsk} = 0, \ \forall k \in K. \tag{3.5}$$

Otherwise, when heterogeneous trip types are considered, variable $\boldsymbol{q}$ in (3.5) should be type-specific to include only trips demanding fast charging. We shall also point out a major limitation of trip-based assignment models as the one adopted here: it is unable to capture tour-based charging activities, for which an activity based network model would be more appropriate.

As stated before, the decisions of all participants in this system are interdependent and should be modeled simultaneously as a whole system. Taking the same idea as Nash equilibrium, at system equilibrium, there is no incentive for each agent to alter their strategies, given the market clearing price $\boldsymbol{\rho}$ and the rest of agents decisions. We state the system equilibrium more formally by the following definition.

Definition 2 *(System Equilibrium).* The equilibrium state of the system is that all investors achieve their own optimality of problem (3.2) (defined in 3.2a-3.2d) and traffic achieves its optimality of problem (3.4) (defined in 3.4a-4.3g) given market clearing price $\boldsymbol{\rho}$ and the other agents' decisions. In addition, the EV charging market is cleared by condition (3.5).

3.3 Solution Methods

Solving the proposed MOPEC model in a complex network presents great theoretical and computational challenges. One may choose to derive the first-order optimality conditions for each agent's optimization problem, and then combine them all to form a mixed complementarity problem (MCP) [Ferris and Pang, 1997]. Recently, there are more theoretical developments by Jofré and Wets [2009], rooted in the foundation of variational analysis, on variational convergence of bivariate functions. The authors showed that a broad family of equilibrium problems can be studied under the framework of finding a maxinf-point of certain bifunction (bivariate function), and established convergence theorems for this category of problems by relying on the notion of *lopsided convergence*. Their theoretical contribution opens up opportunities for in-depth analyses on stability and convergence properties of equilibrium solutions. From an algorithmic viewpoint, the flexibility of constructing a bifunction allows one to choose a sequence of bifunction with desired properties (including convexity and continuity), so that the complexity of an original problem can be reduced to a sequence of easier problems.

In this study, we formulate the problem of finding an equilibrium market clearing electricity price, as a problem of finding a maxinf-point for an appropriate function. Next, we will first discuss the construction of a bifunction and the corresponding convergence theorems, which then leads to a detailed description of the solution algorithm.

3.3.1 Algorithm Design

The construction of a bifunction follows the scheme presented in [Deride et al., 2015], with modifications to adapt to the problem of fast charging infrastructure planning. For a price $\boldsymbol{\rho} \in I\!R_+^K$, denote $(c_i^k(\boldsymbol{\rho}), g_i^k(\boldsymbol{\rho}))_{k\in K_i}$ the solution to the optimzation problem solved by each firm $i \in I$, defined in problem (3.2a-3.2d). Additionally, define the total capacity $c^k(\boldsymbol{\rho}) = \sum_{i\in I} c_i^k(\boldsymbol{\rho})$, $k \in K$. On the other hand, given a total capacity vector $\boldsymbol{c}(\boldsymbol{\rho})$, and a price $\boldsymbol{\rho}$, define $(\boldsymbol{x}(\boldsymbol{\rho}, \boldsymbol{c}(\boldsymbol{\rho})), \boldsymbol{q}(\boldsymbol{\rho}, \boldsymbol{c}(\boldsymbol{\rho}))$ as the solution for the traffic assignment problem defined in problem (3.4a-4.3g).

Defining the *excess supply function* as

$$\mathrm{ES}_k(\boldsymbol{\rho}) = \sum_{i\in I_k} g_i^k - \sum_{r\in R}\sum_{s\in S} e^{rs} q^{rsk},\ \forall k \in K.,$$

the equilibrium condition described in equation (3.5) can be re-stated as the existence for an equilibrium price $\boldsymbol{\rho}^* \in I\!R_+^K$ such that

$$\mathrm{ES}(\boldsymbol{\rho}^*) = 0.$$

In order to get the maxinf-characterization of the equilibrium problem, let us introduce the *Walrasian* function associated with this equilibrium problem, defined as

$$W(\boldsymbol{\rho}, \boldsymbol{\varphi}) = -\sum_{s\in S} \varphi_k (\mathrm{ES}_k(\boldsymbol{\rho}))^2, \quad \text{on } I\!R_+^K \times \Delta_K,$$

where Δ_K corresponds to the K-dimensional unit simplex. The following lemma provides the maxinf intepretation of equilibrium prices.

Lemma 2 *(Walras equilibrium prices and maxinf-points) Every maxinf-point $\boldsymbol{\rho} \in I\!R_+^K$ of the Walrasian function W such that $W(\boldsymbol{\rho}, \cdot) \geq 0$, on Δ_K is an equilibrium point.*

Proof. See Appendix B. □

Additionally, for $\varepsilon \geq 0$, we say that $\boldsymbol{\rho}_\varepsilon$ is an *approximating equilibrium point*, if associated excess supply function $\mathrm{ES}(\boldsymbol{\rho}_\varepsilon)$ is close to satisfy the equilibrium condition. More precisely, in terms of the Walrasian function, this is if the following inequality holds

$$|\inf W(\boldsymbol{\rho}_\epsilon, \cdot) - \operatorname{supinf} W| \leq \varepsilon.$$

Let us denote the sets of ε-approximating equilibrium points as $\varepsilon - \operatorname{argmaxinf} W$.

Usually, the problem of maximizing the function $\boldsymbol{\rho} \mapsto \inf W(\boldsymbol{\rho}, \cdot)$ lacks concavity properties, which do not allow direct application of a traditional duality scheme. One can embed this problem into a perturbed family, and apply an *augmented Lagrangian* for this non-concave formulation; for the general description and further details, consult [Deride et al., 2015]. Considering the self-dual augmenting function $\sigma = \frac{1}{2}|\cdot|^2$, and sequences of nonegative, nondecreasing scalars $\{r^\nu\}$, $\{M^\nu\}$, one can define the sequence of *augmented Walrasian* functions for this problem as

$$W^\nu(\boldsymbol{\rho}, \boldsymbol{\varphi}) = \inf_{\boldsymbol{z}} \left\{ W(\boldsymbol{\rho}, \boldsymbol{z}) + \frac{1}{2r^\nu}|\boldsymbol{z} - \boldsymbol{\varphi}|^2 \middle| \boldsymbol{z} \in \Delta_S \right\}, \text{on } [0, M^\nu] \times \Delta_S$$

Using this procedure, the idea is to approximate the problem of finding maxinf-points of the original Walrasian function W, by computation of approximate maxinf-points given by the sequence of augmented Walrasians W^ν. The appropriate notion of convergence is given by *Lopsided convergence, ancillary tight.* The convergence theorem of the proposed approximation scheme is provided below.

Theorem 2 *(convergence of approximating maxinf-points) Suppose that* $\operatorname{supinf} W$ *is finite. Consider non-negative sequences* $\{r^\nu\}$, $\{M^\nu\}$, *and* $\{\varepsilon^\nu\}$ *such that* $r^\nu \nearrow \infty$, $M^\nu \nearrow \infty$, $\varepsilon^\nu \searrow 0$. *Let* $\{W^\nu\}$ *be a family of augmented Walrasian functions associated with each parameters* r^ν *and* M^ν. *Let* $\boldsymbol{\rho}^\nu \in \varepsilon^\nu - \operatorname{argmaxinf} W^\nu$ *and* $\boldsymbol{\rho}^*$ *be any cluster point of* $\{\boldsymbol{\rho}^\nu\}$. *Then* $\boldsymbol{\rho}^* \in \operatorname{argmaxinf} W$, *i.e.,* $\boldsymbol{\rho}^*$ *is an equilibrium point.*

Proof. See Appendix B.

This convergence result provides the theoretic foundations for the design of an algorithm for finding equilibrium points, based on replacing the original problem of finding (local near) maxinf-points of W by finding (local near) approximated maxinf-points for the approximating sequence $\{W^\nu\}$. Start with an initial approximated price ($\boldsymbol{\rho}$) at the beginning of the procedure, at iteration $\nu+1$, given $(\boldsymbol{\rho}^\nu, \boldsymbol{\varphi}^\nu)$, and $r^{\nu+1}\,(\geq r^\nu)$, $M^{\nu+1}\,(\geq M^\nu)$, the approach taken can be described in two phases:

- **Phase I**: solve the minimization problem

$$\boldsymbol{\varphi}^{\nu+1} \in \operatorname{argmin} W^{\nu+1}(\boldsymbol{\rho}^\nu, \cdot).$$

- **Phase II**: solve the maximization problem

$$\boldsymbol{\rho}^{\nu+1} \in \operatorname{argmax} W^{\nu+1}(\cdot, \boldsymbol{\varphi}^{\nu+1}).$$

As $r^\nu \nearrow \infty$, and $M^\nu \nearrow \infty$, in virtue of theorem 2, $\boldsymbol{\rho}^\nu \to \boldsymbol{\rho}^*$, a maxinf-point of W, i.e., an equilibrium price for the fast charging infrastructure planning problem.

3.3.2 Numerical Implementation

The proposed algorithm was implemented in Pyomo (Python Optimization Modeling Objects, Hart et al. [2012.]), a mathematical programming language based on Python. The problems that we solve come with the following features:

- For the investors' problem, the objective function considered is strongly concave, over a linearly constrained set. This problem is solved using Gurobi [Gurobi Optimization, 2014], a state-of-the-art and efficient algorithm.

- For the traffic assignment problem, the objective function is convex over a linearly constrained set determined by traffic equilibrium network constraints. This problem is solved using the interior point method, Ipopt, implemented by Wächter and Biegler [2006].

- Phase I consists of the minimization of a quadratic objective function over the K-dimensional simplex. This is solved using Gurobi solver [Gurobi Optimization, 2014].

- Phase II is the critical step of the entire augmented Walrasian algorithmic framework. We need to overcome the (typical) lack of concavity of the objective function. Thus, the maximization is done without considering first order information and relying on BOBYQA algorithm [Powell, 2009], which performs a sequentially local quadratic fit of the objective functions, over box constraints, and solves it using a trust-region method.

All the examples were run on a 3.30 GHz Intel Core i3-3220 processor with 4 GB of RAM memory, under Ubuntu 12.04 operating system.

3.4 Numerical Examples

3.4.1 Base Case

3.4.1.1 Data Description

We use Sioux Falls network, a widely used benchmark network as shown in Figure 3.8 [3], to test the numerical performance of our solution method and draw some insights. Sioux Falls network consists of 24 nodes and 76 directed links. The number on each node/link is the node/link index.

The green, red, yellow nodes in Figure 3.8 represent the set of origins, destinations and candidate investment locations, respectively. The link travel costs, t_a, follow a 4^{th}-order Bureau of Public Roads (BPR) function: $t_a = t_a^0[1 + 0.15 * (v_a/c_a)^4]$, where t_a^0 is the free flow travel time (FFT) and c_a is the link capacity parameter [4].

[3] Figure credited to Hai Yang and Meng Qiang, Hong Kong University of Science and Technology

[4] Note that c_a is the "capacity" parameter used in BPR rather than the true link capacity

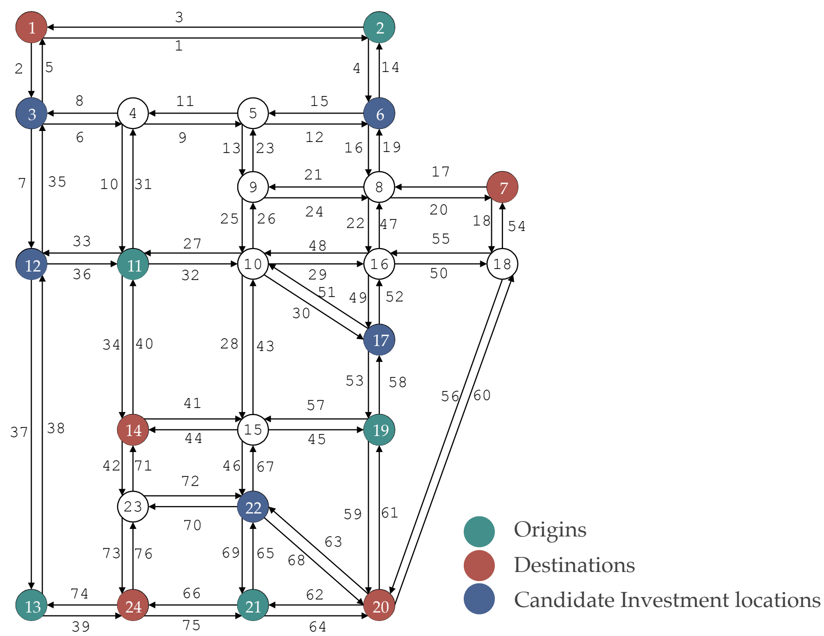

Figure 3.2: Base Case Sioux Falls Test Network

We consider 25 o-d pairs, each has 100 travel demand. We assume that in order to finish the trip, each vehicle needs to charge exactly once, with the same charging demand, somewhere in the middle of the trip [5]. The parameters in the travelers' utility function are set to be $\beta_0 = 0$, $\beta_1 = 1$, $\beta_2 = 0$, $\beta_3 = 0.06$, $e = 1$, $inc = 1$. In addition, we select quadratic form for the investors' costs: $\phi_c(c) = 0.1c^2 + 170c$ and $\phi_g(g) = 0.1g^2 + 130g$, where c and g are in unit of kW and kWh, respectively. The data we use for the base case is documented in Table A.1, A.2 and A.3 in Appendix A. Notice that although the network is the same as Sioux Falls, we scale up the link distance and scale down link capacity and origin-destination (OD) demand accordingly, in order to reflect the long distance travel (inter-city) and road congestion. All of the associated parameters are manufactured for illustration

[5] Notice that: 1) for those EVs that don't need to charge, we can simply treat them as conventional vehicle and model them as background traffic. 2) we will relax this assumption to consider destination charging in Section 3.4.2. 3) One can also easily consider heterogeneous charging demand for each o-d pair depends on their travel distance.

purpose only.

3.4.1.2 Computational Performance

Figure 3.3 shows the convergence performance of the solution algorithm. In base case, the total number of variables is 19216 and the number of constraints is 6111. We can see that the prices and excess supplies at all nodes converge within 23 iterations. The total computing time that our algorithm took to solve the base case is 12068 seconds [6].

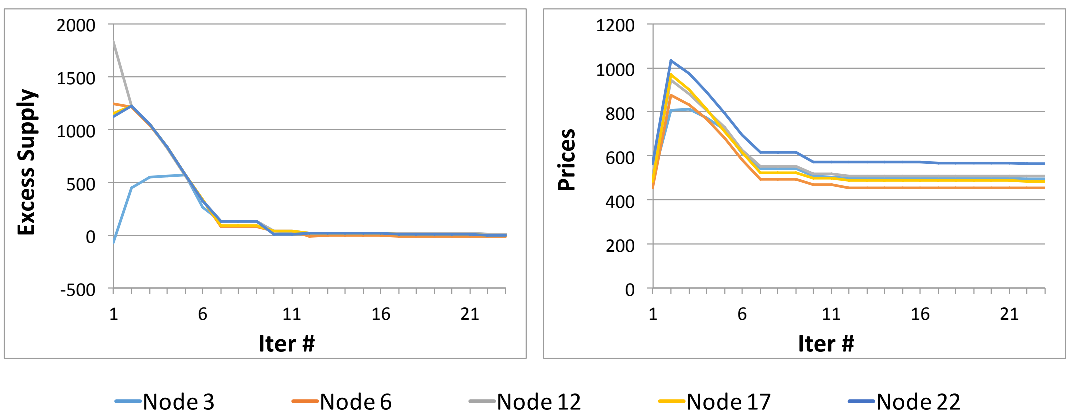

Figure 3.3: Base Case Convergence of Prices and Excess Supply

3.4.1.3 Impacts of Transportation Congestion

We investigate the impacts of transportation congestion on the equilibrium solution of charging capacities and prices, in addition to the actual transportation congestion. The results are presented in Figure 3.4: Case (a) corresponds to the base case; Case (b) represents the case if we ignore transportation congestion during modeling process, and evaluate congestion level using the equilibrium traffic flow solutions.

The equilibrium prices in Figure 3.4 can be loosely interpreted as the "popularity" of each charging station: for example, node 22 has the highest price for

[6] We were not able to solve these examples directly using PATH solver or using diagonal method

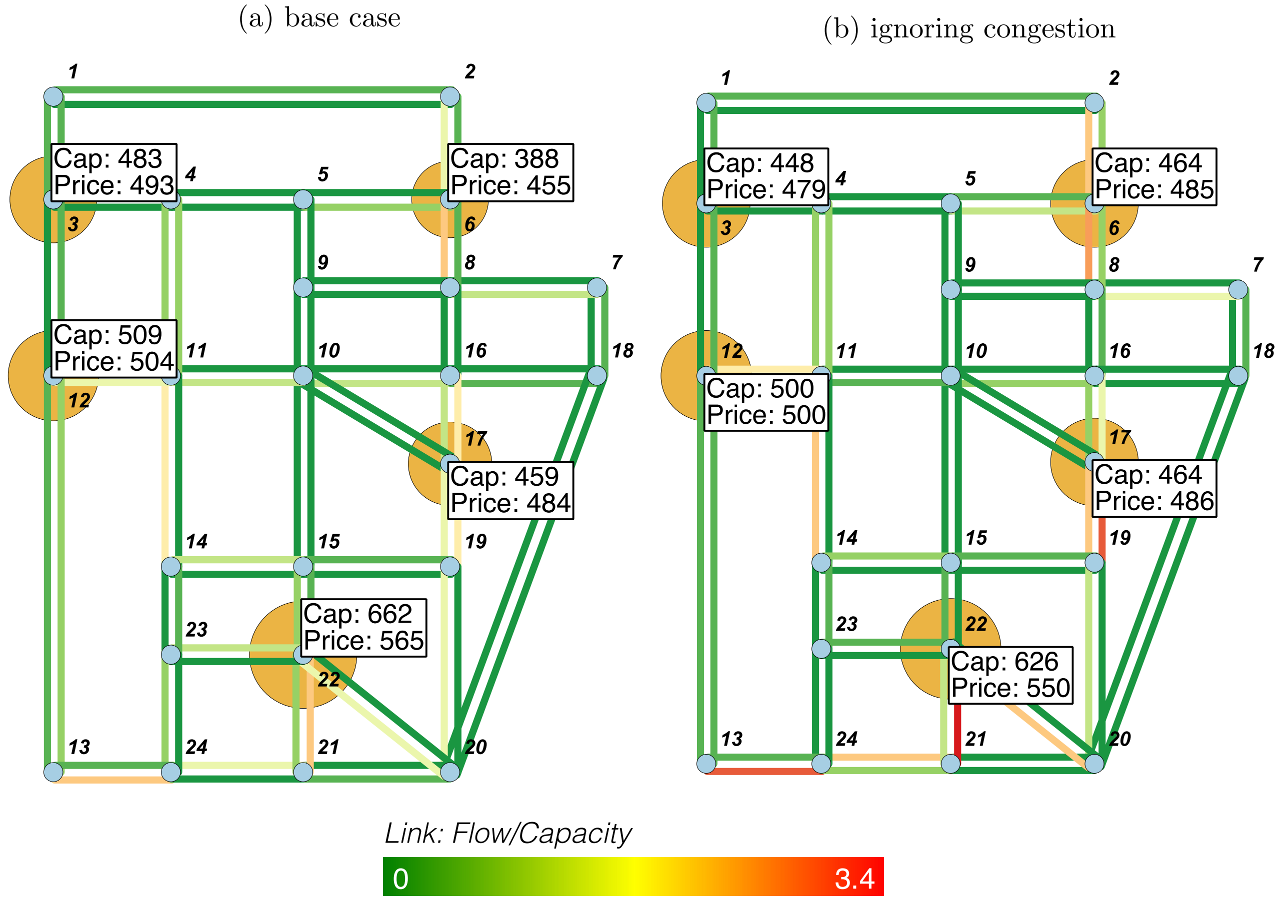

Figure 3.4: Impacts of Modeling Network Congestion

both cases because it is located on/near the path of larger OD travel demand [7] and it has larger connectivity [8] compared to other candidate nodes. Ignoring transportation congestion makes these two advantages of node 22 less significant because: on one hand, people may detour to farther and cheaper charging stations without adding too much travel cost; on the other hand, drivers will just take the shortest path and do not value the alternative accessing options. In Figure 3.4, ignoring transportation congestion in system modeling reduces the difference of locational prices between "popular" (e.g. node 22) and "unpopular" (e.g. node

[7] This can be seen from the sparse charging locations and dense origins and destinations in the lower half of the network (Figure 3.8).

[8] This can be seen from the larger node degree of node 22.

6) charging locations. However, we would like to point out that this observation depends on specific network setting. Conceptually, the changes from Figure 3.4a to Figure 3.4b should related to the relative improvement of "path accessibility" of each charging stations, e.g. how close is one charging station away from the shortest path or how congested to access a charging station, etc. Investigation on the rigorous definition and measurement of "path accessibility" will be left for future work.

3.4.1.4 Impacts of drivers' preferences on charging locations and prices

In this section, we test the impacts of travelers' utility parameters on model solution and numerical convergence of the algorithm. Figure 3.5 summarizes the sensitivity analysis results. The converged solutions of charging prices at various nodes tell us the locations and prices of deployed charging services (with a positive price corresponding to a service deployment). Case (a) corresponds to the base case described in Section 3.4.1.1. Case (b) represents a situation where EV drivers still do not consider station capacity ($\beta_2 = 0$), but are highly sensitive to charging price ($\beta_3 = 0.2$). In this case, EV charging services are provided at all potential locations with closer charging prices. Case (c) represents a situation where EV drivers consider station capacity ($\beta_2 = 0.05$). Note that while network congestion and service accessibility are taken into consideration by drivers, the value of station capacity leads to a more concentrated investment pattern, i.e. node 22 attracts much more investment than the other candidate nodes.

3.4.2 Special Cases

Our model includes two special cases: the first special case, named *Destination Choice Case*, is that drivers can choose their destinations and at the same time charging at their destinations; the second special case, named *Round Trip Case*, is that drivers consider round trip of facility service. Both of these two special cases correspond to the trip type where people travel in order to get services. The

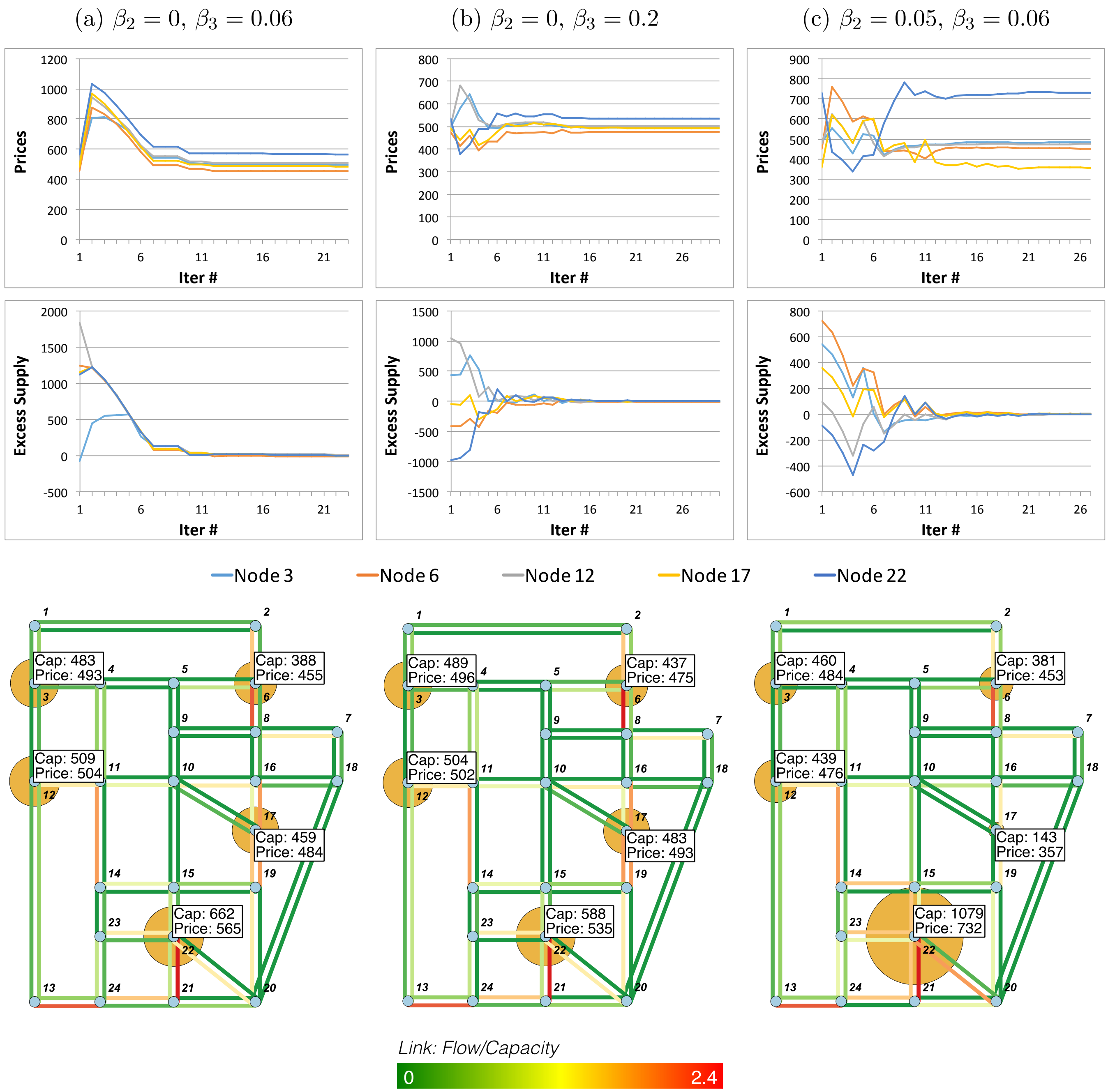

Figure 3.5: Sensitivity Analysis on Drivers' Preferences

only difference is whether travelers incorporate one-way or round-trip travel time into their decision process.

3.4.2.1 Examples

Round Trip Case can be easily incorporated in our model by specifying $r = s$. We constructed an example (see Figure 3.6), which has 5 origins and 5 service locations. The travelers need to come back to their origins after receiving some services, e.g. shopping, recreations, etc. Each O-D pair has 500 travel demand and all the other parameters are identical with base case. To adapt our model to *Destination Choice Case*, we need to add a dummy destination node to all the service locations in Figure 3.6, with zero link cost. The equilibrium solutions of these two special cases are shown in Figure 3.7, from which different investment and travel patterns are generated due to one-way and round-trip travels.

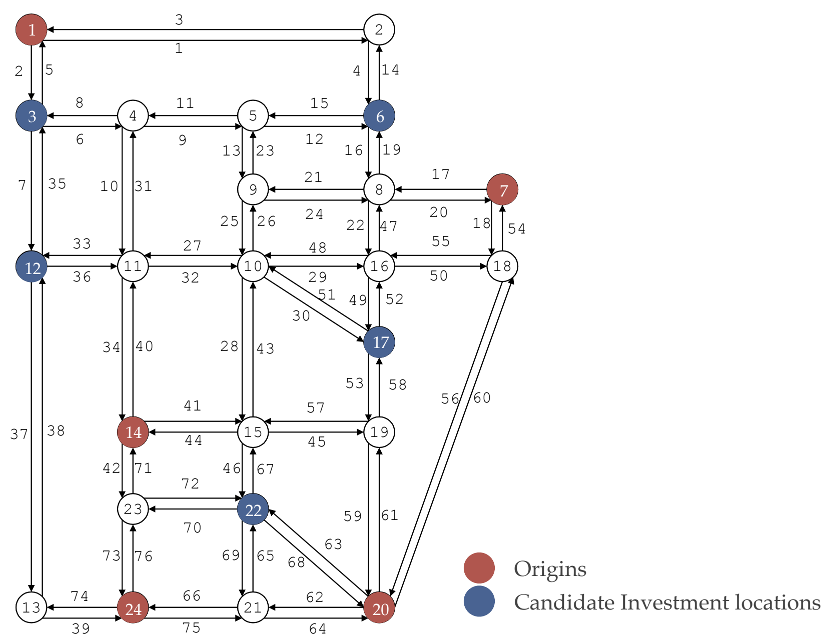

Figure 3.6: Round Trip Case Sioux Falls Test Network

3.4.2.2 Comparison with Central Planner Case

In order to compare with existing literatures on EV charging infrastructure planning, in this section, we present non-commuting trips demanding fast charging at

(a) Round Trip Special Case (b) Destination Choice Special Case

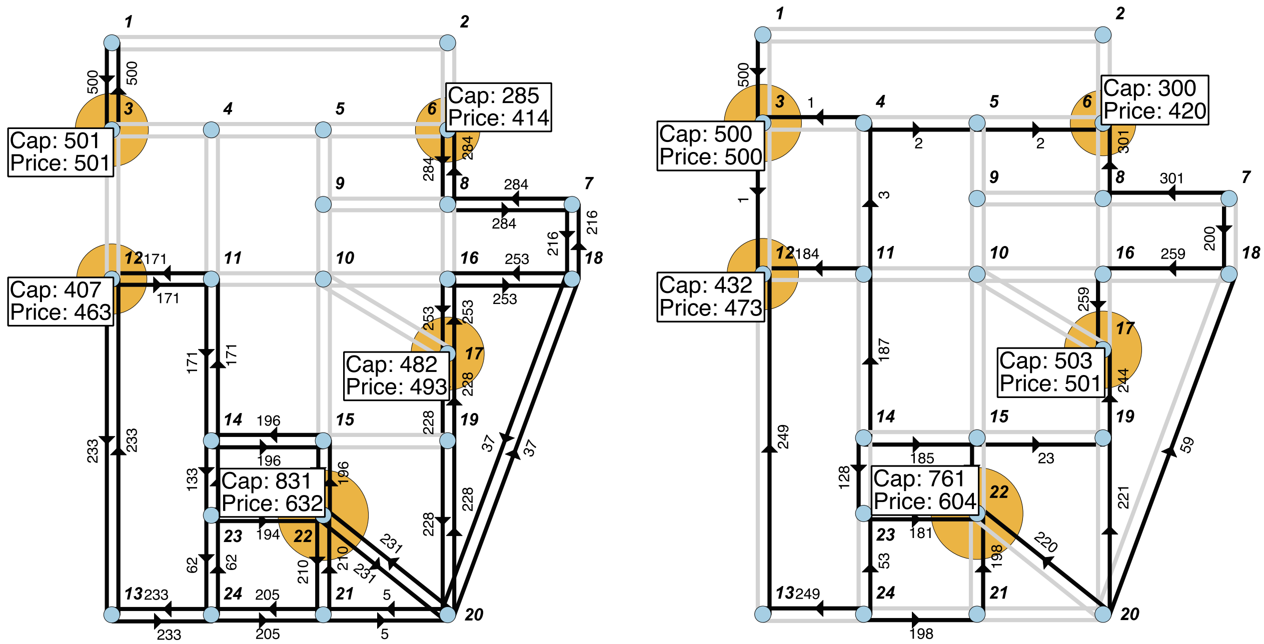

Figure 3.7: Equilibrium Solutions of Two Special Cases

the destinations (typically those fast charging facilities co-located with other commercial/recreational activities) and explore the impact of business-driven charging facility investment behaviors in a competitive market. The detailed computation performance and sensitivity analyses of this special case can be found in [Guo et al., 2016].

We adopt the same manufactured network specifications, link travel costs and user preference data as in [He et al., 2013]. The green nodes in Figure 3.8 represent the set of origins, destinations and candidate investment location for all the firms, i.e. $R = S = S_i, \forall i$. The number on each link is the link index.

The case study in [He et al., 2013] is used as a benchmark, where the power generators and the charging infrastructure investors are all controlled by a central planner to maximize the total social welfare. While in our model, we allow the charging infrastructure investors to make their individual decisions in a competi-

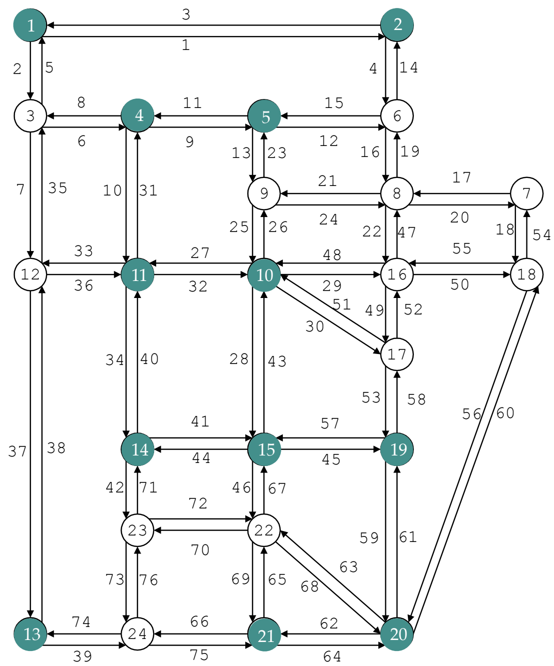

Figure 3.8: Sioux Falls Test Network with 12 candidate investment locations (green)

tive market. Note that the locational marginal electricity prices, which are model output in [He et al., 2013], are taken as exogenous parameters in our model.

Table 3.1 compares the results in terms of the total allocation of charging capacity at each candidate node. There are two equilibrium cases: the first case allows investment on all candidate nodes; the second case assumes that nodes 11-21 already have existing charging facilities and are excluded from the candidate location set, which is consistent with the setting in [He et al., 2013]. We can see that the total investment, which is 126.17 MW, is identical across all three cases. This is straightforward because the setting of this example assumes inelastic total charging demand. However, the charging capacities allocated to different nodes are very different: the result in [He et al., 2013] tends to concentrate more investment on nodes 4, 5 and 10, while our model leads to a more diffused investment outcome in both equilibrium cases. The difference is not surprising. The objective of [He et al., 2013] is to maximize the social welfare, which includes surplus for consumers, investors and generators, while in our model each investor makes decision to maximize individual profits. Even though the value of locational marginal price $\boldsymbol{\rho}^s$ sends signal of the market charging demand to the investors, it cannot capture the externality of traffic and charging congestion. Therefore, a business-driven competitive market generally cannot yield a social optimal solution.

Table 3.1: Comparison between Central Planner Allocation and Market Equilibrium Outcome

Node	Central Planner	Equilibrium	Equilibrium (fixing results at Nodes 11-21)
1	1.77	10.22	13.52
2	1.52	10.06	11.85
4	34.01	10.86	18.85
5	24.63	10.58	16.51
10	37.53	10.55	38.74
11	3.51	11.27	3.51
13	2.04	9.02	2.04
14	3.08	10.65	3.08
15	5.99	12.10	5.99
19	4.60	11.43	4.60
20	3.94	9.66	3.94
21	3.55	9.78	3.55
Total	126.17	126.18	126.18

3.5 Discussion

In this chapter, we domonstrate how our general modeling framework can be adapated to formulate the EV fast charging infrastructure planning in a competitive market. The main contribution of this chapter is the establishment of a theoretical foundation, from both modeling and computational aspects, for business-driven EV charging infrastructure investment planning problem. We have gained encouraging numerical results from a medium-size testing network, but our work is still at a relatively preliminary stage and further efforts are needed to fully understand how market competition might influence the layout of future public charging infrastructure. From a computational perspective, there are further improvements to be made. For example, specialized solution methods for solving the combined destination and assignment mode developed in the transportation literature may be adopted. From a practical perspective, an immediate next step is to implement our model in a real-world case study, where one may investigate relevant planning and policy questions, such as how to design economic/pricing mechanism to guide private investment towards a social-optimal outcome and how travelers' utility might influence business-driven investment strategies. Lastly, from a modeling perspective, one may extend the proposed method to the stochastic case, which is important for long-term planning as knowledge of future parameters is often poor at the time of planning stage. In this case, one may construct a stochastic MOPEC problem, for which the solution method provided in this chapter may be combined with scenario decomposition methods, e.g. Progressive Hedging method [Rockafellar and Wets, 1991]. Another extension is to have a fully integrated power and transportation infrastructure system, which requires explicit modeling of both power grid and transportation network. This extension will add more decision entities to the system thus increasing the problem complexity, but we expect that the general MOPEC modeling framework and variational convergence theories can still apply. Lastly, we shall point out that the MOPEC model presented here is only suitable for identifying an equilibrium condition of the system in the long term. It would not be a suitable modeling choice if the emphasis were on learning effects and system dynamics during a transition state.

Chapter 4

Application II: Power Generators Planning in a Restructured Electricity Market

The second application we are going to present is on power system, in which we incorporate uncertainties, interdependencies and decentralization simultaneously using our general modeling framework N-SMOPEC. The goal is to establish a mathematical model and corresponding computation techniques for analyzing long-term infrastructure investment decisions in a deregulated electricity market. In order to keep a concrete ground for discussion, we focus on the United States as a special case of deregulated market.

4.1 Introduction

First of all, a power supply system often involves non-cooperative behaviors of multiple decision entities. For example, in the United States there are multiple generation companies supplying electricity to a region's electricity grid, which is often operated by a separate non-profit Independent System Operator (ISO). ISO is in charge of coordinating, controlling and monitoring the operation of the electrical system in order to keep stability and efficiency of the network and instantaneously balance supply and demand [CalISO, 2013]. Capturing the interactive behaviors of different system players simultaneously requires modeling techniques beyond conventional optimization approaches based on single decision entity. Secondly, the physical infrastructure for producing and delivering energy are interdependent due to their spatial and functional correlations [Rinaldi et al., 2001], which requires a network-based modeling framework to capture the spatiality of supplies/demands and the transmission network connecting them. As demonstrated by Hobbs et al. [2008], inclusion of transmission network constraints may result in significantly different predictions on generators' behaviors in a competitive market. Coping with uncertainty is another major challenge in long term planning, especially considering the evolvement of technologies and demand in the future. Despite of the importance of addressing uncertainties in energy system planning as identified in [IEA, 2006], very few stochastic models exist in the literature of energy infrastructure planning.

The electricity market in the United States is generally considered as an oligopoly market, even though the levels of market competitiveness vary by regions [Bushnell et al., 2007]. Depending on the decision variables and anticipation of rivals' reaction [Day et al., 2002], an US electricity market is often modeled based on one of the following: in a Cournot competition [Cournot and Fisher, 1897], each generator submits a fixed supply quantity; and in a Supply Function Equilibrium (SFE), each generator submits a production function (i.e. available production quantity as a function of price). It can be shown that if firms know exactly the market realization, SFE and Cournot models yield the same solution [Willems et al., 2009]. The advantage of Cournot models is their simplicity and therefore can be integrated with more complex market and system settings [Willems et al., 2009, Hu et al., 2004]. However, Cournot model is known to be sensitive to demand parameters. SFE models on the other hand provide more flexibility in addressing varying demand conditions [Day et al., 2002]. Based on one of these assumptions, researchers have provided in-depth analyses on the impacts of market competition [Hobbs et al., 2000, Hobbs and Pang, 2007] and transmission network constraints [Hobbs et al., 2008] on power markets. Stochastic oligopolistic models have also been developed to analyze the operation of power markets under demand and cost uncertainties [Genc et al., 2007, Pineau and Murto, 2003]. All these studies focus on the operational aspects of a power market with fixed infrastructure and do not consider investment decisions on the physical infrastructure.

With wholesale electricity market deregulated, traditional capacity expansion models developed for regulated firms, such as [Murphy et al., 1982], became inadequate, and studies dealing with both investment and operations in an oligopolistic electricity market were critically needed [Kagiannas et al., 2004]. A series of studies have been developed based on game theoretic models and multi-agent based simulation [Wogrin et al., 2011]. One of the main benefits of game-theoretic models is their capability to capture strategic behaviors of each player when making long-term investment decision [Ventosa et al., 2002, Murphy and Smeers, 2005]. In a closed-loop model, investment decisions and market operation decisions are assumed to happen in separate stages. Typically a bi-level model is used, with upper level focusing on investment decision and the lower level on daily operation (i.e. generation decision) under given capacities. This type of models can also be categorized as an Equilibrium Problem with Equilibrium Constraints (EPEC) or Mathematical Programming with Equilibrium Constraints (MPEC), for which existence and uniqueness of equilibrium solutions are not always guaranteed [Ralph and Smeers, 2006]. In an open-loop model, investment and generation decisions are assumed to be made simultaneously. This simplification significantly reduces computation difficulty, and has a real implication: forward contract [Ventosa et al., 2002, Murphy and Smeers, 2005], even though it weakens the ability to capture possible market power of players' first-stage investment decisions on short-term markets. In this chapter, we adopt an open-loop approach. Readers may refer to [Wogrin et al., 2012] for more discussion on open vs. closed loop models.

Considering non-cooperative games on a network structure adds more complexity. In the literature on power generation capacity expansion in oligopolistic markets, only a few studies explicitly model the transmission and location effects. In [Kazempour and Conejo, 2012], a stochastic MPEC model is developed with the upper level focusing on a strategic player's investment decision and the lower level capturing the ISO's electricity dispatch problem. In that model, the rivals do not participate in competition on generation capacities as their investment decisions are treated as model input instead of decision variables. In [Kazempour et al., 2013], a deterministic EPEC model is developed for modeling capacity expansion decisions of rival investors considering transmission network constraints. There are also studies approaching from an energy supply chain perspective, in which a detailed transmission network is replaced by direct links between generators and demands. For example, Liu and Nagurney [2011] proposed an analytical model for energy firm merging and acquisition through supply chain network integration. In addition, there are also supply-chain-based studies focusing on the operational issues of power systems without considering the strategic planning of infrastructure, such as integration of renewables with other fuel markets [Matsypura et al., 2007, Nagurney and Matsypura, 2007, Liu and Nagurney, 2009].

In summary, little work has been reported in the literature for modeling generation infrastructure planning considering all three challenges: uncertainty, infrastructure interdependency, and oligopoly competition. In this chapter, we model using our N-SMOPEC framework to support strategic infrastructure planning in an oligopolistic power market. In the proposed model, uncertain parameters are described by a discrete set of scenarios and their associated probabilities. Each investor aims to maximize the expected total profit by choosing the best first-stage investment decision and the second-stage scenario-dependent generation decisions. The ISO dispatches electricity from the generators to the demands to maximize consumer surplus while satisfying the transmission network constraints and the market clearing conditions. A system equilibrium is achieved when all agents solve their problems optimally.

The remaining part of this chapter is organized as follows. In Section 4.2, we first recap the general stochastic multi-agent modeling framework, then describe the behavior of each party involved in the power grid, and give specific assumptions and formulation of the proposed model. In Section 4.3, we demonstrate how the original energy problem may be reformulated and converted to multiple user-equilibrium traffic network assignment problems. In Section 4.4, we present numerical results and draw planning and policy implications. The last section concludes the chapter with insights, discussions, and future extensions.

4.2 Mathematical Model and Analyses

4.2.1 Recap of Stochastic Programming and N-SMOPEC

The research question is stated as: How should energy investors strategically plan their production infrastructure (where and at what capacities to build their production facilities), to ensure long-term economic benefit while integrating with the existing power grid?

Even though our emphasis is on the strategic planning of production infrastructure, the cost-effectiveness of a planning decision depends on how the system is likely to be operated afterwards. To model the planning and operational stages in an integrated framework, one should recognize the very distinguishable natures of the two types of decisions against uncertainty, which may be related to demand, supply, and technology. At this point, let us use a general notation $\boldsymbol{\xi}$ to represent the uncertain vector. We assume $\boldsymbol{\xi}$ follows a discrete probability distribution, described by a set of discrete scenarios and associated probabilities. Planning decisions, such as infrastructure setup, are usually made before future uncertainty is revealed and are difficult to readjust once implemented. On the other hand, operational decisions such as electricity production and dispatching quantities can be adjusted based on the actual realization of uncertain parameters (for example, the actual demand or a more accurate hour-ahead demand forecast). This feature fits well in a stochastic programming framework [Louveaux, 1986, Birge and Louveaux, 2011], which recognizes the non-anticipativity of planning decisions while allowing recourse for operational decisions.

The classic two-stage stochastic program for a single decision maker, in the simplest form, may be presented as follows [Birge and Louveaux, 2011]:

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad f(\boldsymbol{x}) + \mathbb{E}_{\boldsymbol{\xi}}\left[Q(\boldsymbol{x},\boldsymbol{\xi})\right] \tag{4.1a}$$

$$\text{subject to} \quad \boldsymbol{x} \in F \tag{4.1b}$$

$$Q(\boldsymbol{x},\boldsymbol{\xi}) = \inf_{\boldsymbol{y}} \left\{ g(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\xi}) | \boldsymbol{y} \in G(\boldsymbol{x},\boldsymbol{\xi}) \right\}, \tag{4.1c}$$

where $\boldsymbol{x}$ represents the planning-stage decision, and $\boldsymbol{y}$ the operational decision, which depends on the choice of planning decision and the actual realization of the uncertain parameters $\boldsymbol{\xi}$. The objective is to minimize the first-stage planning cost, $f(\boldsymbol{x})$, plus the expected value of the second-stage operational cost, $Q(\boldsymbol{x},\boldsymbol{\xi})$, subject to the feasibility constraints of $\boldsymbol{x}$ and $\boldsymbol{y}$.

In our problem, each decision entity makes her own decision, but needs to simultaneously account for other decision entities' behaviors given the interdependence among them. For example, too much electricity generation at a local point may increase transmission congestion, which could affect all parties in the power system. This problem fit in well with our general modeling framework N-SMOPEC. Using a two-player problem as an example, the above formulation

(4.1a ∼ 4.1c) may be extended to the following:

$$
\begin{aligned}
(\boldsymbol{x}_1, \boldsymbol{y}_1) &= \underset{\boldsymbol{x}_1, \boldsymbol{y}_1}{\arg\min} \left\{ f_1(\boldsymbol{x}_1, \boldsymbol{x}_2) + \mathbb{E}_{\boldsymbol{\xi}} \left[g_1(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{y}_1, \boldsymbol{y}_2, \boldsymbol{\xi}) \right] \right\} \\
(\boldsymbol{x}_2, \boldsymbol{y}_2) &= \underset{\boldsymbol{x}_2, \boldsymbol{y}_2}{\arg\min} \left\{ f_2(\boldsymbol{x}_1, \boldsymbol{x}_2) + \mathbb{E}_{\boldsymbol{\xi}} \left[g_2(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{y}_1, \boldsymbol{y}_2, \boldsymbol{\xi}) \right] \right\} \\
\text{s.t.} \quad & (\boldsymbol{x}_1, \boldsymbol{x}_2) \in F \qquad \text{and} \qquad (\boldsymbol{y}_1, \boldsymbol{y}_2) \in G(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{\xi}),
\end{aligned}
$$

where $\boldsymbol{x}_i$ and $\boldsymbol{y}_i(\boldsymbol{\xi})$ represent the planning decision and the operational decision of player i ($i = 1, 2$), respectively; and f_i and g_i are the first-stage and second-stage costs of player i, respectively. Each player aims at minimizing her own total expected cost, i.e. the planning plus the expected operating cost. Note that $\boldsymbol{y}_i(\boldsymbol{\xi})$ is $\boldsymbol{\xi}$-specific, but for brevity, we write it as $\boldsymbol{y}_i$. Also, for generality, we specify the objective functions and constraints in a form relating both players decisions, which may be decomposable to individual player in many cases. For those agents only make second stage decisions, their optimization problem degenerate to deterministic optimization problem.

4.2.2 Detailed Formulation for Each Decision Entity

The conceptual modeling framework is illustrated in Figure 4.1. ISO and individual generators make their own decisions in order to fulfill their own objectives. Their decisions determine the power flows of the supply and demand side, which endogenously determines the locational electricity prices. We will explain our modeling assumptions in more details and present the formulation for each decision entity in the following sections.

4.2.2.1 Modeling the decision of electricity generation companies

In this study, we assume the generators follow an open-loop Cournot competition. A new energy generator that is entering the system has two types of decisions to make. During the planning stage, it decides where and at what capacity to invest its production facilities. At the operational stage, it chooses its best production

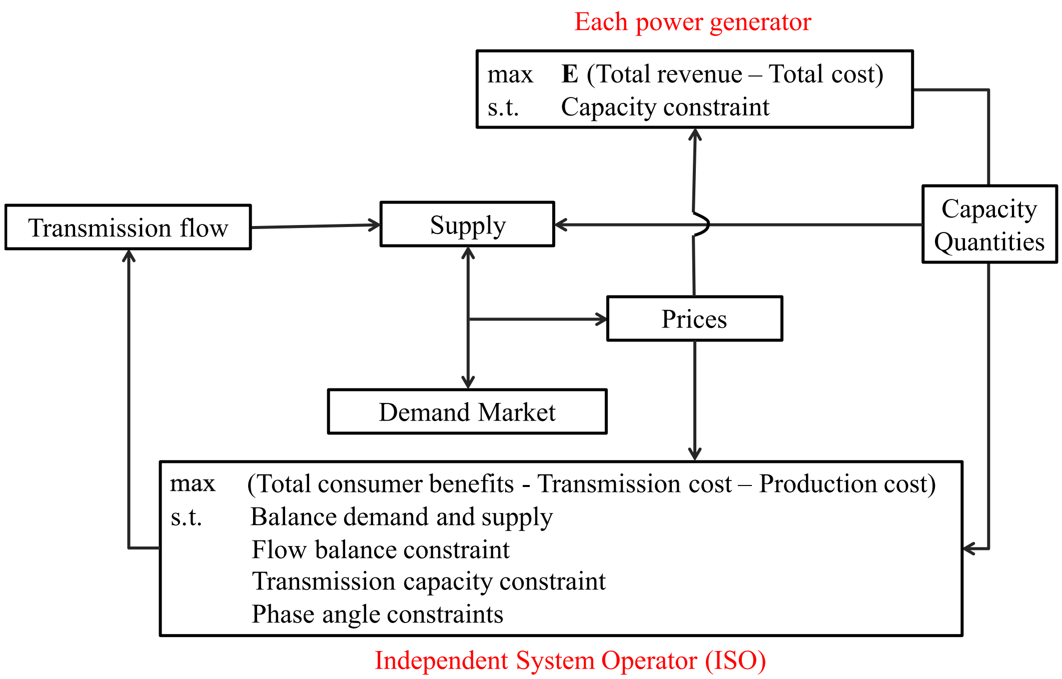

Figure 4.1: Conceptual Modeling Framework for Power System

strategy. This generator makes all these decisions to maximize the expected total profit while taking into account the decisions of other entities in the system. For each generation Firm $\forall i \in I$:

$$\underset{g_i^j(\boldsymbol{\xi}),c_i^j}{\text{maximize}} \quad -\sum_{k\in K}\sum_{j\in J_k}\phi_c(c_i^j) + \mathrm{E}_{\boldsymbol{\xi}}\left\{\sum_{k\in K}\sum_{j\in J_k}\left[\rho_k(\boldsymbol{\xi})g_i^j(\boldsymbol{\xi}) - \phi_g(g_i^j(\boldsymbol{\xi}),\boldsymbol{\xi})\right]\right\} \tag{4.2a}$$

$$\text{subject to} \quad g_i^j(\boldsymbol{\xi}) \le c_i^j,\ \forall j \in J_k, k \in K, \boldsymbol{\xi} \in \Xi; \tag{4.2b}$$

$$g_i^j(\boldsymbol{\xi}) \ge 0,\ \forall j \in J_k, k \in K, \boldsymbol{\xi} \in \Xi; \tag{4.2c}$$

$$c_i^j \ge 0,\ \forall j \in J_k, k \in K. \tag{4.2d}$$

where:

J_k : set of candidate investment locations connecting to access point[1] k, in-

[1]Any node in the power grid can be an access point as long as there is supporting transmission

dexed by j;

K : set of access points, indexed by k;

I : set of companies, indexed by i;

c_i^j : added capacity at location j of firm i;

g_i^j : production quantity by firm i at location j ;

ρ_k : ISO's electricity purchasing price at each accessing point k;

$\phi_c(\cdot)$: total capital cost function with respect to facility capacity;

$\phi_g(\cdot)$: total production cost function with respect to generation quantity and scenario;

$\boldsymbol{\xi}$: vector of uncertain parameters, whose support is denoted by Ξ.

Note that throughout the entire chapter, we denote vectors in lowercase bold font. The objective function (4.2a) maximizes the total profit of each firm, which is the total revenue minus the total capital and production costs. The decision variables include the capacity and generation amount at each potential production location. We assume a uniform nodal price (Locational Marginal Price), thus the total revenue is calculated by $\sum_{k\in K}\sum_{j\in J_k}\rho_k g_i^j$. Constraint (4.2b) ensures that the total electricity generated at a production facility does not exceed its production capacity. Note that for some renewable technologies such as solar or wind, the right-hand-side of constraint (4.2b) may be attached with a weather-related factor to reflect its effective capacity. The rest are non-negative restrictions. Note that to keep consistent time scale, we treat all agents' problems at an hourly basis.

infrastructure.

4.2.2.2 Modeling the decision of Independent System Operator (ISO)

The ISO decides the wholesale price and transmission flow of each transmission line to balance electricity supply and demand in the network instantaneously. Considering the non-profit nature of ISO, we set its goal as to maximize total social welfare. To capture congestion effect of transmission lines, we assume that transmission cost is a monotone increasing function of the transmitted flow quantity, which is a similar treatment as in [Hearn and Yildirim, 2002]. Denote the transmission network by $\mathcal{G} = (\mathcal{N}, \mathcal{V})$, where $\mathcal{N}$ is the set of nodes (indexed by n) and $\mathcal{V}$ is the set of links (indexed by a). Electricity from a supply (origin) node to a demand (destination) node is modeled as an O-D flow. Since ISO's decisions are operational, these can be adjusted based on the actual realization of future uncertainty. This means that all decision variables of ISO are scenario dependent, but for brevity, we do not carry ξ in the formulation below. ISO's problem, in a given scenario, is formulated as:

$$\underset{\boldsymbol{x},\boldsymbol{t}}{\text{minimize}} \quad \phi_t(\boldsymbol{v})^T\boldsymbol{v} + \sum_{i\in I}\sum_{k\in K}\sum_{j\in J_k}\phi_g(g_i^j) - \sum_{k\in K}\int_0^{d_k} w_k(s)\mathrm{d}s \tag{4.3a}$$

$$\text{subject to} \quad \boldsymbol{v} = \sum_{q\in Q}\boldsymbol{x}^q, \tag{4.3b}$$

$$A\boldsymbol{x}^q = t^q E^q, \ \forall q \in Q, \tag{4.3c}$$

$$(\boldsymbol{\rho}) \quad \sum_{q\in Q} t^q E^{q+} = \boldsymbol{g}, \tag{4.3d}$$

$$\sum_{q\in Q} t^q E^{q-} = \boldsymbol{d}, \tag{4.3e}$$

$$\boldsymbol{x}^q \geq 0, \ \forall q \in Q, \tag{4.3f}$$

$$t^q \geq 0, \ \forall q \in Q. \tag{4.3g}$$

where:

$\boldsymbol{v}$: aggregated link flow vector. Each element corresponds to a link;

$\boldsymbol{t}$: O-D flow vector. Each element corresponds to an O-D pair;

$\phi_t(\cdot)$: transmission cost function, which depends on link flow;

$\boldsymbol{\rho}$: wholesale price vector. Each element corresponds to a node;

$\boldsymbol{g}$: electricity supply vector. The j^{th} element corresponds to the total energy supplied by all companies at node j, that is $g_j = \sum_{i \in I} g_i^j$;

$\boldsymbol{x}^q$: link flow vector associated with OD pair q. Each element corresponds to a link ;

$\boldsymbol{d}$: electricity demand vector. Each element corresponds to a node;

d_k : total electricity demand at node k;

$w_k(\cdot)$: inverse demand function at node k;

A : node-link incidence matrix, whose rows correspond to nodes and columns correspond to links, with +1 indicates the starting node of a link and -1 the ending node.

Q : set of O-D pairs, indexed by q;

t^q : O-D flow associate with O-D pair q;

E^q : O-D incidence vector of O-D pair q with +1 at the origin and -1 at the destination;

E^{q+} : "O" incidence vector of O-D pair q with +1 at the origin;

E^{q-} : "D" incidence vector of O-D pair q with +1 at the destination.

The objective function (4.3a) maximizes (minimizes the negative value of) the total system surplus. The first term in function (4.3a) is the total transmission cost; the second term is the total production cost for all the electricity consumed in the system; the third term is the willingness to pay by all consumers. Constraint (4.3b) defines the aggregate link flow vector as the sum of all O-D flow vectors. Constraints (4.3c $\sim$ 4.3e) ensure the flow conservation at each node, including the supply and demand nodes[2]. The rest constraints set non-negative restrictions on flow and demand. With an emphasis on the long-term planning decision, we have chosen to omit the Kirchhoff's second law, phase angle constraint. We acknowledge that such simplification may lead to certain level of accuracy loss, e.g. loop flows may not be accounted in our model. We shall also point out that the market clearing conditions adopted in several studies, such as [Nagurney, 2006], are implied by the ISO formulation, which becomes clear in Section 4.2.4. Note that this model, different from the typical DC models used for short-term transmission network operation, incorporates elastic demand, which reflects long-term effect of market equilibrium.

Directly solving the above stochastic multi-agent optimization model can be numerically challenging. In Section 4.2.3, we show how the stochastic problem can be reduced to simpler problems through scenario decomposition. In Section 4.2.4, we convert each scenario problem, by using variational inequalities, to a traffic user equilibrium problem, for which efficient solution algorithms have been developed in the transportation literature.

4.2.3 Scenario Decomposition

There is a rich literature on scenario decomposition for solving large-scale stochastic programming problems via augmented Lagrangian method [Rockafellar, 1976]. Let us first recap an important concept, nonanticipavity [Rockafellar and Wets, 1991], which states that a reasonable policy should not require different actions relative to different scenarios if the scenarios are not distinguishable at the time

[2] For a small example illustrating these flow conservation constraints, please refer to Appendix C.

when the actions are taken. Let S be a discrete set of possible scenarios for ξ and $s(s \in S)$ denote an individual scenario with probability p^s. One may consider solving each scenario-dependent problem and denote its solution as $\boldsymbol{x}^s$ for each s. However, these solutions cannot be directly implemented, because at the time when an investment decision is made, one does not know yet which scenario is going to happen. In order to consolidate the s-dependent solutions to an *implementable* solution, we must impose the following nonanticipativity condition:

$$\boldsymbol{x}^s = \boldsymbol{x}^{s'}, \forall s \in S, s' \in S, s \neq s' \tag{4.4}$$

or equivalently

$$\boldsymbol{x}^s - \boldsymbol{z} = 0, \forall s \in S \tag{4.5}$$

where $\boldsymbol{z}$ is a vector of free variables.

Through introducing an augmented Lagrangian function that adds a penalty of violating the nonanticipativity condition to the original objective function, Rockafellar and Wets [1991] developed a scenario-decomposition method, the progressive hedging (PH) method, for classic two-stage stochastic programming problems involving a single decision-maker. In this work, we extend the idea of scenario decomposition to multiple decision-maker cases.

Let $\boldsymbol{x}_i^s$ and $\boldsymbol{y}_i^s$ be the planning decision and the operational decision of player $i(\in I)$ in scenario $s(\in S)$, respectively. The stochastic multi-agent optimization problem can be reformulated as:

$$\begin{aligned}
&(\boldsymbol{x}_i^s, \boldsymbol{y}_i^s) = \operatorname*{arg\,min}_{\boldsymbol{x}_i^s, \boldsymbol{y}_i^s} \left\{ E\left[f_i(\boldsymbol{x}_i^s, \boldsymbol{x}_{-i}^s) + g_i(\boldsymbol{x}_i^s, \boldsymbol{x}_{-i}^s, \boldsymbol{y}_i^s, \boldsymbol{y}_{-i}^s, \xi) \right] \right\}, \\
&\text{s.t.} \qquad (\boldsymbol{x}_i^s, \boldsymbol{x}_{-i}^s) \in F, \qquad \boldsymbol{x}_i^s = \boldsymbol{z}_i, \qquad \text{and} \qquad (\boldsymbol{y}_i^s, \boldsymbol{y}_{-i}^s) \in G(\boldsymbol{x}_i^s, \boldsymbol{x}_{-i}^s, \xi), \\
&\forall i \in I, s \in S
\end{aligned}$$

For the i^{th} player, define

$$\mathscr{L}_i^s(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\omega}) = f_i^s(\boldsymbol{x}^s) + g_i^s(\boldsymbol{x}^s,\boldsymbol{y}^s) + \boldsymbol{\omega_i^{sT}}(\boldsymbol{x}_i^s - \boldsymbol{z}_i) + \tfrac{1}{2}\gamma\|\boldsymbol{x}_i^s - \boldsymbol{z}_i\|^2 \quad (4.6)$$

$$\mathscr{L}_i(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\omega}) = \textstyle\sum_{s\in S} p^s \mathscr{L}_i^s(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\omega}) \quad (4.7)$$

as the augmented Lagrangian, where $\boldsymbol{\omega_i^s}$ is the dual vector associated with the nonanticipativity constraints (4.5) and $\gamma > 0$ is a penalty parameter. Therefore, the augmented Lagrangian integrates the nonanticipativity constraints with the original objective function. The stochastic problem for player i becomes

$$\text{minimize} \quad \mathscr{L}_i(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\omega}) \quad \text{over all feasible } \boldsymbol{x}_i^s \text{ and } \boldsymbol{y}_i^s. \quad (4.8)$$

Due to the nonseparable penalty term $1/2\gamma\|\boldsymbol{x}_i^s - \boldsymbol{z}_i\|^2$ in (4.7), the problem cannot be decomposed directly. The PH method achieves decomposition by alternatingly fixing the scenario solutions $(\boldsymbol{x}_i^s, \boldsymbol{y}_i^s)$ and the implementable solution $\boldsymbol{z}_i$ in (4.8). The implementation procedure has been summarized in Algorithm 2. Recent successful applications of PH method in solving large-scale stochastic mix-integer problems can be found in [Chen and Fan, 2012, Fan and Liu, 2010, Waston and Woodruff, 2010]. As pointed out by Rockafellar and Wets [1991], parameter γ plays an important role in the convergence of the PH method in practice. In addition, several studies [Løkketangen and Woodruff, 1996, Mulvey and Vladimirou, 1991, 1992, Chen and Fan, 2012, Waston and Woodruff, 2010] reported some important factors that may influence the setting of penalty parameter γ. For example, it was suggested that an effective value for the penalty parameter should be close in magnitude to the coefficient of decision variable [Waston and Woodruff, 2010].

4.2.4 Analyzing Each Scenario-dependent Problem

Once the large-scale stochastic problem is decomposed, we need to iteratively solve many scenario-dependent deterministic problems. Each scenario-dependent problem itself is a multi-agent optimization problem, which is still computationally challenging. Next, we will show that, through creation of a virtual network and reformulation, we can convert the problem of interest to a traffic equilibrium problem, which allows us to exploit efficient algorithms developed by the transportation network science community. Of course, both multi-agent optimization and traffic equilibrium problems can be expressed using variational inequalities (VI). In some sense, it is not surprising that the two problems can be converted to each other, even though the equivalence is not apparent at first. For numerical implementation, one could directly rely on general purpose solvers designed for VI problems. On the other hand, there is an advantage in exploiting special problem structure, such as many efficient algorithms specifically developed for traffic equilibrium problems.

Let us first convert each player's optimization to a VI. Note that all the functions and variables are deterministic in each scenario dependent problem, therefore we do not carry the notation $\boldsymbol{\xi}$ in the following discussion. Assuming objective function (4.2) is concave and continuously differentiable, the model can be rewritten as the following VI[3]:

$$\sum_{k\in K}\sum_{j\in J_k}\Bigg\{-\left[\rho_k^* + \sum_{k'\in K}\left(\frac{\partial \rho_{k'}}{\partial g_i^j}\bigg|_{\boldsymbol{g}=\boldsymbol{g}^*}\sum_{j\in J_{k'}} g_i^{j*}\right) - \frac{\partial \phi_g}{\partial g_i^j}\bigg|_{g_i^j=g_i^{j*}} - \lambda_c^{ij*}\right]\left(g_i^j - g_i^{j*}\right)$$
$$-\left[-\frac{\partial \phi_c}{\partial c_i^j}\bigg|_{c_i^j=c_i^{j*}} + \lambda_c^{ij*}\right]\left(c_i^j - c_i^{j*}\right) + \left(-g_i^{j*} + c_i^{j*}\right)\left(\lambda_c^{ij} - \lambda_c^{ij*}\right)$$
$$\Bigg\} \geq 0, \forall\left(\boldsymbol{g}_i, \boldsymbol{c}_i, \boldsymbol{\lambda}_i\right) \in \mathscr{K}_i^1 \tag{4.9}$$

$$\mathscr{K}_i^1 \equiv \left\{(\boldsymbol{g}_i, \boldsymbol{c}_i, \boldsymbol{\lambda}_i) \in \mathbb{R}_+^{3l_i} | (4.2b) \text{ is satisfied}\right\}$$

[3]Note that since the wholesale prices depend on the production quantities, chain rule of differentiation should be used while taking derivatives to arrive at the VI.

where:

λ_c^{ij} : dual variable of capacity constraint of firm i on location j;

$\boldsymbol{g}_i$: vector that concatenates g_i^j variables;

$\boldsymbol{c}_i$: vector that concatenates c_i^j variables;

$\boldsymbol{\lambda}_i$: vector that concatenates λ_c^{ij} variables;

l_i : number of optional locations for each conpanies i;

$\mathscr{K}_i^1$: feasible set of firm i's decision.

Similarly, the ISO's problem can be expressed using VI as follows:

$$\begin{aligned}&\sum_{q\in Q}\left[\phi_t(\boldsymbol{v}^*)+\nabla\phi_t(\boldsymbol{v}^*)\boldsymbol{v}^*-A^T\lambda^{q*}\right]^T(\boldsymbol{x}^{\,q}-\boldsymbol{x}^{\,q*})\\&+\sum_{q\in Q}\left[E^{qT}\lambda^{q*}+\boldsymbol{\rho}^{\,*T}E^{q+}-w(\boldsymbol{d}^{*T})E^{q-}\right](t^q-t^{q*})\geq 0,\forall\boldsymbol{x}^{\,q},\boldsymbol{t}^{\,q}\in\mathscr{K}^2\end{aligned} \tag{4.10}$$

$$\mathscr{K}^2\equiv\{(\boldsymbol{x},\boldsymbol{t})\,|(4.3b)\sim(4.3g)\text{ is satisfied}\}$$

where:

$\nabla\phi_t(\cdot)$: Jacobian matrix of link cost function;

λ^q : dual vector associated with constraint (4.3c) of O-D pair q. Each row corresponds to a link;

$\mathscr{K}^2$: feasible set of ISO decision.

Note that the market clearing conditions [Nagurney, 2006] are implied by the ISO formulation: The second term in Equation (4.10) means that if the demand of OD pair q, t^q, is zero, then the wholesale price plus the transmission cost can be larger than the consumer willingness to pay; otherwise, the wholesale price plus the transmission cost must be equal to the consumer willingness to pay. In addition, note that in constraint (4.3c), ISO is required to balance demand and supply at all time, so the dual variable associated with this equality constraint is a free variable.

As stated before, the decisions of all participants in this system are interdependent and should be modeled simultaneously as a whole system. We state the system equilibrium more formally by the following definition.

Definition 3 *(Power System Equilibrium). The equilibrium state of a power system is that all generators achieve their own optimality (cf. (4.9)) and ISO achieves its optimality (cf. (4.10)).*

We claim the following Lemma, which provides the equivalent condition of the power system equilibrium conditions.

Lemma 3 *(Variational Inequality Condition for the Power System Equilibrium). The equilibrium conditions governing the power system equilibrium are equivalent to finding solutions satisfying the following variational inequality (4.11):*

$$
\begin{aligned}
&\sum_{k\in K}\sum_{j\in J_k}\left[-\sum_{k'\in K}\left(\left.\frac{\partial \rho_{k'}}{\partial g_i^j}\right|_{\boldsymbol{g}=\boldsymbol{g}^*}\sum_{j\in J_{k'}} g_i^{j*}\right)+\left.\frac{\partial \phi_g}{\partial g_i^j}\right|_{g_i^j=g_i^{j*}}+\left.\frac{\partial \phi_c}{\partial c_i^j}\right|_{c_i^j=c_i^{j*}}\right]\left(g_i^j-g_i^{j*}\right)\\
&+\left[\phi_t(\boldsymbol{v}^*)+\nabla\phi_t(\boldsymbol{v}^*)\boldsymbol{v}^*\right]^T(\boldsymbol{v}-\boldsymbol{v}^*)-w(\boldsymbol{d}^{*T})(\boldsymbol{d}-\boldsymbol{d}^*)\\
&+\sum_{k\in K}\sum_{j\in J_k}\left\{\left.\frac{\partial \phi_c}{\partial c_i^j}\right|_{c_i^j=c_i^{j*}}\left[\left(c_i^j-g_i^j\right)-\left(c_i^{j*}-g_i^{j*}\right)\right]\right.\\
&\left.-\lambda_c^{ij*}\left[\left(c_i^j-g_i^j\right)-\left(c_i^{j*}-g_i^{j*}\right)\right]+\left(c_i^{j*}-g_i^{j*}-0\right)\left(\lambda_c^{ij}-\lambda_c^{ij*}\right)\right\}\geq 0\\
&\forall\left(\boldsymbol{g}_i,\boldsymbol{c}_i,\boldsymbol{\lambda}_i\right)\in\mathscr{K}_i^1,\forall i,\forall\left(\boldsymbol{x},\boldsymbol{t}\right)\in\mathscr{K}^2
\end{aligned}
\tag{4.11}
$$

Proof. See Appendix B. □

Next we will show the VI problem in (4.11) is equivalent to a transportation network user equilibrium problem. Let us use a simple case illustrated in Figure 4.2 as an example to explain the construction of a virtual network corresponding to a traffic network equilibrium problem. In Figure 4.2, a virtual node C denotes an investment firm; F denotes a potential investment location or an existing generator. The link flow from a node C to a node F means the capacity that firm C invests at location F. For an existing generator, link flow of C-F is set to be the existing generation capacity. Each node P or U corresponds to a firm. The flows on link F-P and link F-U denote the electricity production quantity and the unused capacity of that firm at location F, respectively. Virtual node I is created to denote electricity that shares the same transmission infrastructure to access the existing power grid. Physical node A denotes an access point or a demand node in the power grid. In general, there are multiple access points and demand nodes in a power network. The flow on link P-I denotes the total electricity production of each firm, and the flow on link I-A or P-A denotes the transmission quantity between the corresponding locations.

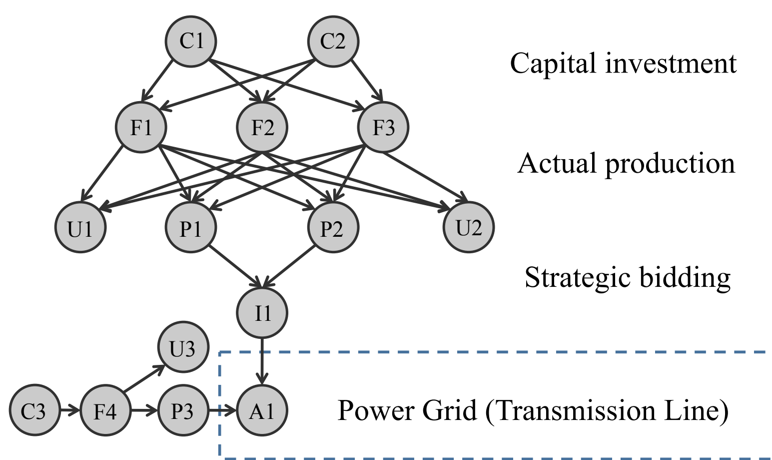

Figure 4.2: A Network Structure of the Problem

Theorem 3 *(Virtual Network Equivalence) The VI (4.11) is identical with the VI* *governing transportation user equilibrium of the virtual network shown in Figure 4.2 if link costs and demand market are defined in the following manner:*

- *For the links within "Capital investment" layer in Figure 4.2 (i.e. from Node C to Node F), the link cost is set to be the marginal capacity cost, i.e.* $\partial\phi_c(c_i^j)/\partial c_i^j$ *. In case of existing generators, the cost attached to this link is set to be zero.*

- *For links connecting Node F and Node P, the link cost is marginal production cost, i.e.* $\partial\phi_g(g_i^j)/\partial g_i^j$*.*

- *For links connecting Node F and Node U, the link cost represents the cost of shutting off unused generation capacity. In case such cost is negligible, it can be set to zero.*

- *For links connecting Node P and Node I, we interpret the link cost as strategic escalating of electricity price of each generator, which is set to* $-\sum_{k'\in K}\left(\frac{\partial\rho_{k'}}{\partial g_i^j}(\boldsymbol{g})\sum_{j\in J_{k'}} g_i^j\right)$ *(see Appendix D). Because we assume oligopoly competition, rather than perfect competition in the electricity supply industry, each firm will try to produce electricity at a level where the wholesale price equals to the marginal cost plus this term so that the profit is maximized.*

- *For links within the power grid, a marginal transmission cost is imposed by the ISO, i.e.* $\phi_t(v) + \nabla\phi_t(v)v$*.*

- *The demand functions of the nodes within the power grid are assumed to be given and depend on the retail price only, while the demand function of Node U is assigned zero despite of the value of capacity shadow price.*

Before we give the proof of Theorem 3, we introduce the following Theorem provided in [Nagurney, 2006]:

Theorem 4 *A travel link flow pattern and associated travel demand and disutility pattern is a traffic network equilibrium if and only if the variational inequality holds: determine* $(f^*, d^*, \lambda^*) \in \mathscr{K}^3$ *satisfying:*

$$\begin{aligned} &\sum_{a \in L} \phi_a (f^*) \times (f_a - f_a^*) - \sum_{n \in N} \lambda_n^* \times (d_n - d_n^*) \\ &+ \sum_{n \in N} [d_n^* - d_n(\lambda^*)] \times [\lambda_n - \lambda_n^*] \geq 0, \forall (\boldsymbol{f}, \boldsymbol{d}, \boldsymbol{\lambda}) \in \mathscr{K}^3 \end{aligned} \tag{4.12}$$

$$\mathscr{K}^3 \equiv \left\{ (\boldsymbol{f}, \boldsymbol{d}, \boldsymbol{\lambda}) \in I\!R_+^{|L|+2|N|} | \textit{there exist an } \boldsymbol{\chi} \textit{ satisfying (4.13) and (4.14)} \right\}$$

$$f_a = \sum_{p \in P} \chi_p \delta_{ap}, \forall a \in L \tag{4.13}$$

$$d_n = \sum_{p \in P_n} \chi_p, \forall n \in N \tag{4.14}$$

where:

N : *demand node set of virtue transportation network (indexed by n);*

L : *link set of virtue transportation network (indexed by a);*

P : *path set of virtue transportation network (indexed by p);*

δ_{ap} : *binary indicator,* $\delta_{ap} = 1$ *if link a is contained in path p, and* $\delta_{ap} = 0$ *otherwise;*

$\phi_a(\cdot)$: *link cost function of link a with respect to link flow;*

f_a : *link flow of link a;*

d_n : *demand at demand node n;*

$d_n(\cdot)$: *demand function at demand node n with respect to travel disutility;*

λ_n : *travel disutility at demand node n;*

χ_p : *path flow of path p;*

Notice that in Theorem 4, travel disutility is restricted to non-negative value, which is not applicable in power market, where price can become negative if necessary (e.g. the ISO may pay consumers to use electricity if supply exceeds demand and shutting down production facilities is too costly). So we propose the following Corollary to account for this situation.

Corollary 5 *(Unrestricted Locational Price). In a virtual transportation network where consumer could gain time (instead of spend time) to travel, a travel link flow, travel demand and disutility pattern (negative means utility) is a traffic network equilibrium if and only if it satisfies the following VI: determine* $(f^*, d^*, \lambda^*) \in \mathscr{K}^4$ *satisfying:*

$$\sum_{a \in L} \phi_a (f^*) \times (f_a - f_a^*) - \sum_{n \in N} \lambda_n^* \times (d_n - d_n^*) \geq 0, \forall (\boldsymbol{f}, \boldsymbol{d}, \boldsymbol{\lambda}) \in \mathscr{K}^4 \tag{4.15}$$

$$\mathscr{K}^4 \equiv \left\{ (\boldsymbol{f}, \boldsymbol{d}, \boldsymbol{\lambda}) \in I\!R_+^{|L|+|N|} \times I\!R^{|N|} | \textit{there exist an } \boldsymbol{\chi} \textit{ satisfying (4.13) and (4.14)} \right\}$$

Note that since the dual vector λ does not have sign restriction, its corresponding optimality condition is simply the original flow conservation constraints associated with it, which can be equivalently expressed by (4.13) and (4.14).

Now we propose the proof for Theorem 3.

Proof. See Appendix B. □

Note that in each iteration of the PH method, the objective function is updated by adding a Lagrange multiplier and a penalty term, i.e. $\boldsymbol{\omega_i^{sT}}(\boldsymbol{x}_i^s - \boldsymbol{z}_i) + \frac{1}{2}\gamma\|\boldsymbol{x}_i^s - \boldsymbol{z}_i\|^2$, which is a function of the planning decision variable. Therefore, the corresponding VI that needs to be solved during each iteration of the PH procedure should be modified as:

$$\sum_{k\in K}\sum_{j\in J_k}\left[-\sum_{k'\in K}\left(\frac{\partial\rho_{k'}}{\partial g_i^j}\bigg|_{\boldsymbol{g}=\boldsymbol{g}^*}\sum_{j\in J_{k'}} g_i^{j*}\right)+\frac{\partial\phi_g}{\partial g_i^j}\bigg|_{g_i^j=g_i^{j*}}+\frac{\partial\phi_c}{\partial c_i^j}\bigg|_{c_i^j=c_i^{j*}}+\omega_{ij}^{s*}\right.$$
$$\left.+\gamma\left(c_i^{js^*}-\bar{c}_i^{js*}\right)\right]\left(g_i^j-g_i^{j*}\right)+\left[\phi_t(\boldsymbol{v}^*)+\nabla\phi_t(\boldsymbol{v}^*)\boldsymbol{v}^*\right]^T(\boldsymbol{v}-\boldsymbol{v}^*)-w(\boldsymbol{d}^{*T})(\boldsymbol{d}-\boldsymbol{d}^*)$$
$$+\sum_{k\in K}\sum_{j\in J_k}\left\{\left[\frac{\partial\phi_c}{\partial c_i^j}\bigg|_{c_i^j=c_i^{j*}}+\omega_{ij}^{s*}+\gamma\left(c_i^{js^*}-\bar{c}_i^{js*}\right)\right]\left[\left(c_i^j-g_i^j\right)-\left(c_i^{j*}-g_i^{j*}\right)\right]\right.$$
$$\left.-\lambda_c^{ij*}\left[\left(c_i^j-g_i^j\right)-\left(c_i^{j*}-g_i^{j*}\right)\right]+\left(c_i^{j*}-g_i^{j*}-0\right)\left(\lambda_c^{ij}-\lambda_c^{ij*}\right)\right\}\geq 0$$
$$\forall\left(\boldsymbol{g}_i,\boldsymbol{c}_i,\boldsymbol{\lambda}_i\right)\in\mathscr{K}_i^1,\forall i,\forall\left(\boldsymbol{x},\boldsymbol{t}\right)\in\mathscr{K}^2 \tag{4.16}$$

The additional terms involving $\omega_{ij}^{s*}+\gamma\left(c_i^{js^*}-\bar{c}_i^{js*}\right)$ is attributed to the nonanticipativity condition. Therefore the link cost associated with C-F should be modified from $\partial\phi_c(c_i^j)/\partial c_i^j$ (see Theorem 3) to:

$$\text{modified C-F link cost}=\partial\phi_c(c_i^j)/\partial c_i^j+\omega_{ij}^s+\gamma\left(c_i^{js^*}-\bar{c}_i^{js}\right) \tag{4.17}$$

Based on the same network structure shown in Figure 4.2, we now have the PH-transportation network solution procedure for the stochastic problem as shown in Algorithm 6. Following this decomposition procedure, the original stochastic energy supply chain problem is converted to many scenario-dependent deterministic traffic network equilibrium problems, which can be solved efficiently by Frank-Wolf algorithm [LeBlanc et al., 1975], which is implemented in this study, or by other recent methods summarized in [Bar-Gera, 2010].

We shall note that in general the VI defined in (4.16) may have multiple solutions. For the numerical implementation reported herein, we consider only the single-solution case. Alternatively, one may consider a min-max formulation to seek the best investment decision in the equilibrium condition that returns the worst-case performance.

Algorithm 6 **PH-Transportation Network Solution Algorithm**

Step 1: Initialization

for each s in S **do**

update link cost according to Theorem 3

call Traffic Assignment Algorithm ▷ Such as Algorithm 5. For a path-base algorithm, see Appendix E.

$(\boldsymbol{c}^0, \boldsymbol{g}^0, \boldsymbol{\rho}^0, \boldsymbol{\lambda}_c^0) \leftarrow$ call Recover Decision Function ▷ See Appendix E

end for

$\boldsymbol{z}^0 \leftarrow \sum_{s \in S} p^s \boldsymbol{c}_s^0 \quad \boldsymbol{\omega}_s^0 \leftarrow \gamma(\boldsymbol{c}_s^0 - \boldsymbol{z}^0), \forall s \in S \quad \epsilon \leftarrow \sum_{s \in S} \|\boldsymbol{c}_s^0 - \boldsymbol{z}^0\| \quad \tau \leftarrow 0$

Step 2: PH-iteration

while $\epsilon \geq 10^{-4}$ **do**

for each s in S **do**

$\tau \leftarrow \tau + 1$

update link cost according to Theorem 3 and (4.17).

call Traffic Assignment Function ▷ Such as Algorithm 5. For a path-base algorithm, see Appendix E.

$(\boldsymbol{c}^\tau, \boldsymbol{g}^\tau, \boldsymbol{\rho}^\tau, \boldsymbol{\lambda}_c^\tau) \leftarrow$ call Recover Decision Function ▷ See Appendix E

end for

$\boldsymbol{z}^\tau \leftarrow \sum_{s \in S} p^s \boldsymbol{c}_s^\tau$

$\boldsymbol{\omega}_s^\tau \leftarrow \boldsymbol{\omega}_s^{(\tau-1)} + \gamma(\boldsymbol{c}_s^\tau - \boldsymbol{z}^\tau), \qquad \forall s \in S$

$\epsilon \leftarrow \sum_{s \in S} \|\boldsymbol{c}_s^\tau - \boldsymbol{z}^\tau\| + \sum_{s \in S} \|\boldsymbol{c}_s^\tau - \boldsymbol{c}_s^{\tau-1}\|$

end while

return $(\boldsymbol{c}, \boldsymbol{g}, \boldsymbol{\rho}, \boldsymbol{\lambda_c})$

4.3 Numerical Examples

4.3.1 A Simple Example for Illustration and Solution Validation

Example 1 is constructed to illustrate how the energy problem may be decomposed and converted to traffic network equilibrium problems. The example is intentionally set to be symmetric so that a benchmark solution can be easily obtained analytically, which then is used to validate the proposed solution procedure. This example includes two energy investment companies, one candidate investment location, and one electricity demand market. Two scenarios with equal probability are considered. Transmission cost is set to be zero and transmission capacity unlimited. The specifics of cost and demand functions are given in Table 4.1.

Table 4.1: Parameter Setting in Example 1

	Capital Cost Function ($)		Generation Cost Function($)		Demand Function ($/MWh)
Scenario	Firm 1	Firm 2	Firm 1	Firm 2	
1	$10 \times c_1$	$10 \times c_2$	$(g_1^{s_1})^2 + 30 \times g_1^{s_1}$	$(g_2^{s_1})^2 + 30 \times g_2^{s_1}$	$\rho = -D + 100$
2	$10 \times c_1$	$10 \times c_2$	$(g_1^{s_2})^2$	$(g_2^{s_2})^2$	$\rho = -D + 100$

Figure 4.3 shows the corresponding virtual network, with four paths: $p_1 = (C1, F1, P1, I1, A1)$, $p_2 = (C2, F1, P2, I1, A1)$, $p_3 = (C1, F1, U1)$, $p_4 = (C2, F1, U2)$. The solution yielded from our solution algorithm is given in Table 4.2. Each path in the virtual transportation network carries a physical meaning in the energy supply chain. For example, path flow on p_1 means the amount of power supplied by firm C1 from location F1 to demand market A1; path flow on p_3 represents the unused capacity of firm C1 at location F1. In addition to path flow, link flow also has a corresponding implication in the energy supply chain. For example, link flow from C1 to F1 represents the total capacity investment by firm C1 at location F1. The multiplier λ_{A1} tells us the marginal electricity price at node A1. The multipliers λ_{U1} and λ_{U2} tell us the marginal benefit of increasing one

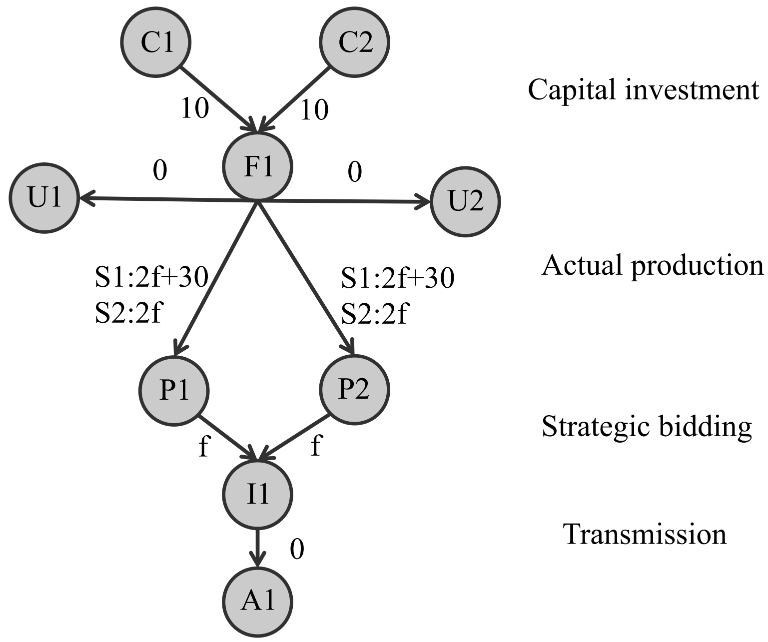

Figure 4.3: The Virtual Network for Example 1. (Note: the number attached to each arc is the assigned link cost defined in Theorem 3)

unit of production capacity at node F1 by C1 and C2, respectively. Using this correspondence, we extract the numerical solutions for the energy infrastructure investment problem, as shown in Table 4.3. Note that in this simple example, the consumer surplus is computed at the wholesale level. These results are consistent with Cournot-Nash equilibrium calculated analytically. The optimal solution to the stochastic multi-agent optimization problem suggests that each firm invest for a generation capacity of 16 units. As a comparison, if the company could wait until future uncertainty is revealed before making investment decision, the deterministic solutions would be 12 units for scenario 1 and 18 units for scenario 2.

Table 4.4 summarizes the numerical implementation details, including the parameter setting, computing environment, and computing time. The convergence pattern of the two scenario-dependent planning decisions is plotted in Figure 4.4, in which the termination criterion is reached within less than 30 iterations.

Table 4.2: Traffic Equilibrium Solutions for the Virtual Network

Items	Scenario 1	Scenario 2	Items	Scenario 1	Scenario 2
p_1 (MWh)	14	16	λ_{U1}(\$/MW)	0	20
p_2 (MWh)	14	16	λ_{U2}(\$/MW)	0	20
p_3 (MWh)	2	0	λ_{A1}(\$/MWh)	72	68
p_4 (MWh)	2	0			

Table 4.3: Power Market Equilibrium Results

Items	Firm 1		Firm 2	
Capacity (MW)	16		16	
Generation (MWh)	s_1 : 14	s_2 : 16	s_1 : 14	s_2 : 16
Capacity Shadow Price ($/MW)	s_1 : 0	s_2 : 20	s_1 : 0	s_2 : 20
Total Profit ($)	s_1 : 232	s_2 : 672	s_1 : 232	s_2 : 672
Expected Profit ($)	452		452	
Whole Sale Price ($/MWh)	s_1 : 72	s_2 : 68		
Consumer (at wholesale level) Surplus ($)	s_1 : 392	s_2 : 512		

Table 4.4: Numerical Implement Information

Item	Value
PH method parameter γ	1
Computing time	.218s
Computing tools	Matlab 2012b 64 bit (Mac Version)
Computing environment	Mac OSX, 2.3 GHz Intel Core i7, RAM 8GB

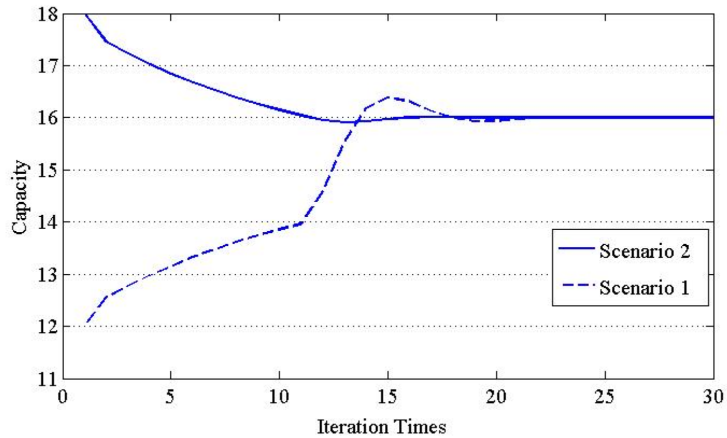

Figure 4.4: Convergence of the Planning Decision

4.3.2 A Realistic Case Study Based on SMUD Power Network

To draw meaningful practical implications from the theoretical results reported here, we implement our model and algorithm on a regional power network in Sacramento Municipal Utility District (SMUD). The transmission network consists of 25 nodes, 11 of which are demand nodes (Node 1∼11), and 65 links. The network structure is shown in Figure 4.5.

Four optional investment locations (Node 21∼24) are being considered, two of which are in remote areas (Node 21 and 22) with lower investment costs but also lower transmission resource; the other two locations (Node 23 and 24) are just the opposite. The two further locations are connected to Node 20 by a single transmission line; the two closer locations are connected to Node 2 and 3 via separate transmission lines. We consider two firms with different technologies as investors. Firm 2 has mature technologies whose production cost is certain, while Firm 1 represents emerging technology, whose future production cost is uncertain. We also assume that the investment cost of one firm is independent of the other

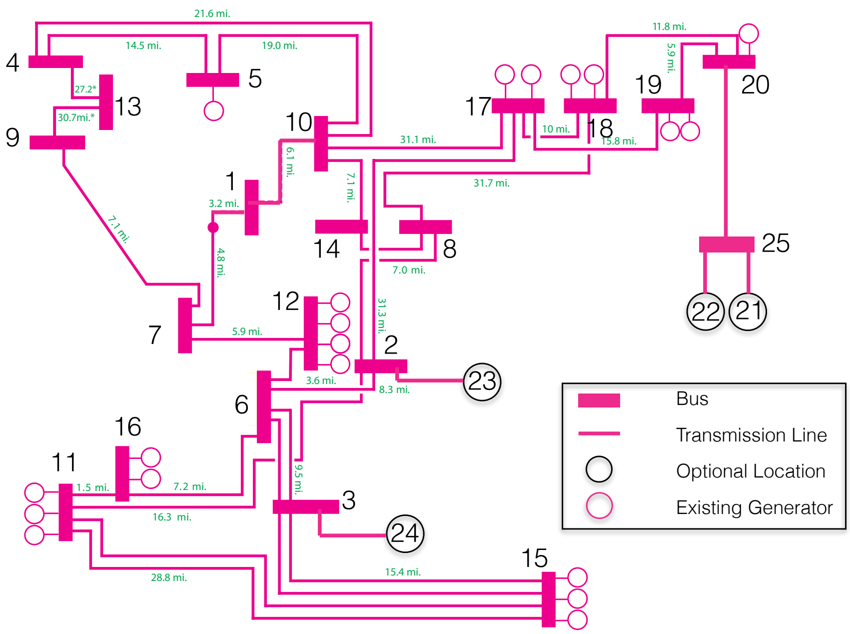

Figure 4.5: Sacramento Municipal Utility District (SMUD) Network

firm's decision[4]. The parameter values are given in Appendix 3. This setting is referred as base case in the following analysis.

An optimal solution is obtained using Algorithm 6 on the same computer as in Example 1, with a total computing time of 3312 seconds. The PH algorithm converges in 13 iterations with an absolute gap of 0.615 (see Figure 4.6). Each scenario-dependent problem within the PH algorithm is solved using Frank-Wolfe algorithm. See Figure 4.7 for its convergence pattern[5].

In Table 4.5, we examine the impacts of transmission network on investment decisions by comparing results from two cases: the base case, and the case where no transmission cost or constraint is considered (free-transmission Case). In the base case, both firms invest less in the further locations (location 21 and 22)

[4]Symmetric assumption and separable investment cost are not required in our model and algorithm.

[5] For the same scenario-dependent problems, PATH, a general-purpose optimization solver for complementarity problems, was unable to obtain solutions.

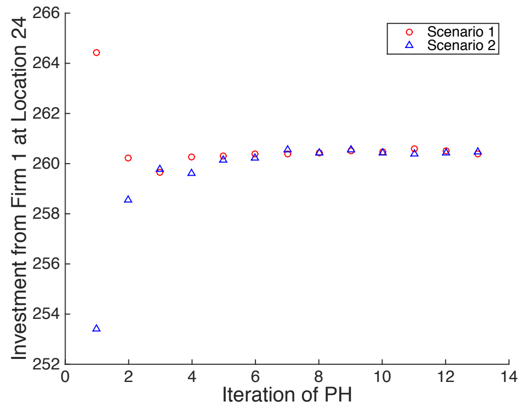

Figure 4.6: Convergence of PH Algorithm

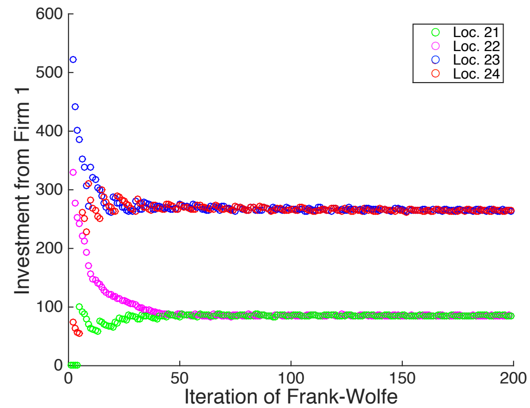

Figure 4.7: Convergence of Frank-Wolfe Algorithm

due to transmission restrictions and costs. However if the transmission network is ignored, the firms would increase their investment in the further locations to take advantage of cheaper capital cost. This comparison shows that ignoring transmission network may lead to poor investment recommendations. Therefore, a supply chain model that captures the essence of transmission network between supplies and demands is critical.

Table 4.5: Impacts of Transmission Network on Investment Decisions (MW)

Locations	**Base Case**		**Free-transmission Case**	
	Firm 1	**Firm 2**	**Firm 1**	**Firm 2**
21	84.0	84.2	217.1	215.1
22	84.0	83.3	216.9	214.7
23	260.2	258.8	215.9	213.8
24	260.5	258.4	215.9	214.0
Total	688.6	684.7	865.8	857.5

Next, we will use the proposed model to explore the impacts of oligopolistic competition on total investments, average electricity price (see Figure 4.8), and total system surplus (see Figure 4.9). The total system surplus is defined as the total consumer willingness-to-pay subtracts the total system cost. The consumer surplus is defined as the total consumer benefits subtracts the total electricity bill they pay. Thus we decompose total system surplus into three components: consumer surplus, generators profits (surplus) and transmission revenues[6]. We compare the results among three market types (cases): the base case, monopoly case (only Firm 2) and perfect-competition case. From Figure 4.8, with more competition involved in power supply side, lower electricity price and higher total investment can be expected. This is mainly due to the fact that electricity generally has low price elasticity of demand. Lacking competition will allow suppliers to exert market power by strategically withholding their investment (long term) and manipulate the market price (short term). From Figure 4.9, we can see that as market competition level increases, the total system surplus increases, the transmission revenues increases and the generator surplus decreases to zero. These results demonstrate that an energy planning model capturing oligopoly market is critical - simplifying an oligopolistic electricity market to either a central-planner case or a perfect market case would compromise the long-term investment decisions and thus the total system surplus.

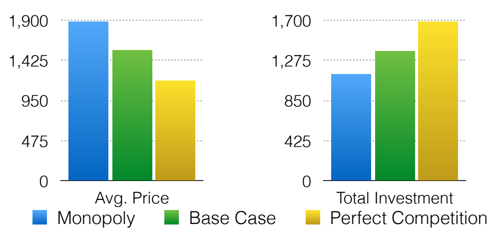

Figure 4.8: Impacts of Strategic Behavior on Price (\$/MWh) and Investment (MW)

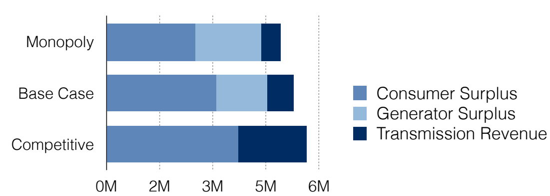

Figure 4.9: Impacts of Strategic Behavior on Total System Surplus (\$)

[6]In this example, ISO is allowed to make short term revenues from transmission services. But eventually, this revenue will be used for transmission investment so that ISO keeps long term profit neutral.

Finally, we explore the impacts of uncertainty. In Table 4.6, we compare results from the stochastic model (base case) and a deterministic approach. The deterministic approach takes the expected value of Firm 1's production cost as model input, in which case the two firms become symmetric. The results show that when there is no uncertainty about future technology, both firms reduce their investment. This is somehow counter intuitive because it is generally believed that uncertainty discourages industry from investing. In the investor's model, since the firms are allowed to adjust their production quantities in the operational stage (second stage of stochastic programming), they can always maintain a non-negative profit in each scenario. Therefore, firms are more "optimistic" when they make the first stage investment decisions - with uncertainty about future production cost, both firms will focus more on the good scenario for themselves. However, if the firms take a more risk-averse attitude instead of a risk-neutral one, we expect to have different results.

Table 4.6: Comparing Investment Decisions (MW) between Stochastic and Deterministic Approach

Locations	**Base Case**		**Case 4**		**Changes**	
	Firm 1	**Firm 2**	**Firm 1**	**Firm 2**	**Firm 1**	**Firm 2**
21	84.0	83.7	80.3	79.9	4.6%	4.7%
22	84.0	83.3	80.8	80.2	4.1%	3.8%
23	260.2	258.8	257.7	257.3	1.0%	0.6%
24	260.5	258.4	257.2	257.1	1.3%	0.5%
Total	688.6	684.2	675.9	674.6	1.9%	1.4%

4.4 Discussion

In this study, we use our general modeling framework N-SMOPEC to formulate energy infrastructure planning in a restructured electricity market. The main contribution is on the development of modeling and solution methods to address challenges brought by uncertainties and oligopolistic competition among energy producers over a complex network structure. Directly solving the stochastic multiple-agent model using general-purpose solvers may not be possible as we have demonstrated in the numerical example. To overcome the computational difficulty, we have combined two ideas. The first is using stochastic decomposition to convert a large-scale stochastic problem to many smaller scenario-dependent problems that are more easily solvable. The second is using variational inequalities to convert a multi-agent optimization problem to a single traffic equilibrium problem, allowing exploitation of efficient solution techniques that can typically over perform general-purpose solvers.

There are several directions for future research regarding this topic. One may explore the roles of risk attitudes and information quality on energy infrastructure investment strategies, which may be used to design efficient information sharing strategy across stakeholders in the system. In addition, with the connections established between the energy planning and traffic network equilibrium problems, one may extend the rich knowledge generated in the transportation literature to energy modeling. For example, knowledge about price of anarchy, congestion pricing, and dynamic equilibrium may be extended to energy system planning and policy related questions, such as how to influence individual energy investment decisions from user-optimal to system-optimal through economic incentives.

Chapter 5

Conclusions

5.1 Summary

The main contribution of this dissertation is on the establishment of a theoretical foundation for capturing the three main challenges, uncertainties, interdependencies and decentralization, during critical infrastructure systems analyses. Through two examples, we illustrate how our general modeling framework, N-SMOPEC, can be adapted to formulate the specific problems in transportation and energy. Each example is solved by decomposition based approach with convergence properties developed based on recent theoretical advances of variational convergence. We illustrate some knowledge from different domains, such as microeconomics, energy and transportation, can be shared to facilitate the formulation and solution process of seemingly unrelated problems, which could possibly foster communication between different fields. We have gained encouraging numerical results from medium-size testing networks.

5.2 Future Extensions

This research can be continued in several directions. In the remaining of this chapter, we outline some possible future extensions from three perspectives: modeling, computation and application. Some topics mentioned here are more direct extension of existing work presented in this dissertation, others are more fundamental and open.

5.2.1 Modeling

- System Dynamics

  In our applications, we only focus on the analysis of the equilibrium state of the systems in the long term. However, if the emphases are on learning effects and system dynamics during a transition state, one may want to adopt a multi-stage dynamic models. From modeling perspective, this extension is straightforward. But we should notice that different formulation may result in different computation burdens. Careful selection of modeling assumptions, e.g. number of stages, based on specific needs of studies are critical to balance the accuracy and efficiency.

- Information

  In this dissertation, information is assumed to be symmetric across all the decision makers. However, this is not necessarily valid in real applications. How to incorporate asymmetric information into the modeling framework? How asymmetric information is going to impact the overall system performance? Will information sharing be a good option for both the system and each individual player? Does there exist a information sharing mechanism that can lead to a Pareto improvement? These questions are all important in terms of both understanding and guiding system interactions. Answering these questions may need to include knowledge from game theory, information economics, mechanism design, etc.

- Big Data

  The value of increasing amount of data in infrastructure systems can also be incorporate into the system modeling. With more data, we are able to better estimate the system parameters, such as cost and demand. Or we can use the historical data to infer/validate some behavioral assumptions made in our model. For example, what is a reasonable market structure to assume? What's the risk attitude of each agents? Can we use the data to infer the incentive (objective) of each agent or to make educated guess about their information availability? In addition, how to integrate an optimization model and statistical model to take advantage of the strengths of both is another interesting topic.

5.2.2 Computation

- General Purpose Solver for N-SMOPEC

  Through two examples, we have shown that directly solving a multi-agent optimization problems with consideration of uncertainties and interdependencies are very challenging, because of both the large-scale and the non-convexity of the problem. We also observed that N-SMOPEC typically will lead to decomposition of the whole problem into small sub-task where existing efficient algorithm may be readily available. This provides hope to develop a general purpose solver, or to seek for a more general decomposition scheme that can guarantee certain level of efficiency and accuracy for N-SMOPEC problems.

- "Optimal" Decomposition Scheme

  On the other hand, we believe that the most efficient algorithm should be designed in a way taking the most advantage of the structure of individual problem. In addition, there is always a tradeoff between how quickly one subtask can be solved and how many times it needs to be solved. To identify an "optimal" decomposition scheme for different categories of N-SMOPEC problems is important to balance between these two tradeoffs. Providing some guidance or develop some prototypes for the decomposition process for certain problem structures will be helpful.

- Efficient Algorithms for Subproblems

  After decomposition, some of the sub-tasks are still computational challenging, such as traffic assignment problem. These sub-task typically need to be solve multiple times for convergence purpose. Therefore, we also need to incorporate the most cutting edge algorithm for the subtasks in order to improve the overall computational performance. On the other hand, developing parallel computing scheme for the subtasks will also be valuable in order to improve the overall computational performance.

5.2.3 Application

- Large-scale Case Study

  An immediate next step following this research is to implement our model in a real-world case study, where one may investigate relevant planning and policy questions, such as how to design economic/pricing mechanism to guide private investment towards a social-optimal outcome and how users' preference might influence business investment strategies.

- Risk Attitude

  Taking advantage of the capability of our general modeling framework to capture risk in the decision making process, one may want to explicitly consider the different risk attitudes of decision makers in the system. This is especially useful in CISs modeling because for some agents in critical infrastructure systems, such as government, security and reliability are typically a main consideration rather than the expected system outcomes.

- Other CISs

Although addressing issues in other CISs requires different domain expertise, the N-SMOPEC modeling framework captures the fundamental features shared by these problems: resource allocation over a network structure and non-cooperative decision entities who are interrelated and facing uncertainties. Therefore, it is hopeful that one can extend the general methodologies developed here to formulate other critical infrastructure systems, such as water resource, rail transportation, financial, and cyber systems.

In addition, we have demonstrate through two examples that fundamental knowledge on different CISs can be benefit the formulation or computation of each other. For example, with the connections established between the energy planning and traffic network equilibrium problems, one may extend the rich knowledge generated in the transportation literature to energy modeling. Knowledge about price of anarchy, congestion pricing, and dynamic equilibrium may be extended to energy system planning and policy related questions, such as how to influence individual energy investment decisions from user-optimal to system-optimal through economic incentives.

- Coupling Multiple CISs

  As EVs link the transportation and energy sectors more closely, it is inevitable that we will be facing more complex planning, engineering, and business problems. Integrating the study of these two systems could be potentially beneficial to both resilience and sustainable issues faced by the society. With our general modeling framework, both of these two systems can be described as the interactions of multiple decision makers over network structure facing uncertainties and interdependencies. The following are several research questions one can pursue in the short term: from a planning perspective, how to coordinate infrastructure planning process for the two systems to improve the resilience of both? How to couple the infrastructure development of alternative fuel vehicles and renewable energy to promote the adoption of each other? How can we integrate the design of both transportation and power infrastructure to support the concept of “smart city”; from an operational perspective, how can we improve the performance (carbon emission, congestion, safety, etc.) of transportation and energy systems by dynamic and locational pricing of charging? How to design an information sensing and sharing mechanism to guide the selfish behavior of decision makers toward a social optimal outcome. In the long term, it is valuable to incorporate other CISs, such as telecommunications, finance, etc, into this one modeling framework so that the intricate interactions between multiple CISs can be explicitly and elegantly captured.

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Appendix A

Data inputs

Table A.1: Base Case Link Capacity c_a (veh/h) and Free-flow Travel Time, FTT t_a^0 (min)

Link	FFT	Capacity	Link	FFT	Capacity	Link	FFT	Capacity	Link	FFT	Capacity	Link	FFT	Capacity
1	12	777	17	6	235	32	10	300	47	10	151	62	12	152
2	8	702	18	4	702	33	12	147	48	8	146	63	10	152
3	12	777	19	4	147	34	8	146	49	4	157	64	12	152
4	10	149	20	6	235	35	8	702	50	6	590	65	4	157
5	8	702	21	20	151	36	12	147	51	16	150	66	6	146
6	8	513	22	10	151	37	6	777	52	4	157	67	6	288
7	8	702	23	10	300	38	6	777	53	4	145	68	10	152
8	8	513	24	20	151	39	8	153	54	4	702	69	4	157
9	4	533	25	6	417	40	8	146	55	6	590	70	8	150
10	12	147	26	6	417	41	10	154	56	8	702	71	8	148
11	4	533	27	10	300	42	8	148	57	6	437	72	8	150
12	8	148	28	12	405	43	12	405	58	4	145	73	4	152
13	10	300	29	8	146	44	10	154	59	8	150	74	8	153
14	10	149	30	16	150	45	6	437	60	8	702	75	6	146
15	8	148	31	12	147	46	6	288	61	8	150	76	4	152
16	4	147												

Table A.2: Base Case Origin-Destination Travel Demand (veh)

O\D	1	7	14	20	24
2	100	100	100	100	100
11	100	100	100	100	100
13	100	100	100	100	100
19	100	100	100	100	100
21	100	100	100	100	100

Table A.3: Base Case Traveler's Utility Function

Coeff.	β_0	β_1	β_2	β_3	e	inc
Value	0	1	0	0.06	1	1
Unit	¢	¢/min	¢/kW	¢	kWh	¢

Table A.4: Link Capacity c_a (veh/h) and Free-Flow Travel Time, FTT t_a^0 (h)

Link	FFT	Capacity	Link	FFT	Capacity	Link	FFT	Capacity	Link	FFT	Capacity	Link	FFT	Capacity
1	1.8	602	17	0.9	1568	32	1.5	2000	47	1.5	1009	62	1.8	1012
2	1.2	901	18	0.6	4681	33	1.8	982	48	1.2	1027	63	1.5	1015
3	1.8	1202	19	0.6	980	34	1.2	975	49	0.6	1046	64	1.8	1012
4	1.5	1592	20	0.9	1568	35	1.2	4681	50	0.9	3936	65	0.6	1046
5	1.2	4681	21	3	1010	36	1.8	982	51	2.4	999	66	0.9	977
6	1.2	3422	22	1.5	1009	37	0.9	5180	52	0.6	1046	67	0.9	2063
7	1.2	4681	23	1.5	2000	38	0.9	5180	53	0.6	965	68	1.5	1015
8	1.2	2582	24	3	1010	39	1.2	1018	54	0.6	4681	69	0.6	1046
9	0.6	2825	25	0.9	2783	40	1.2	975	55	0.9	3936	70	1.2	1000
10	1.8	904	26	0.9	2783	41	1.5	1026	56	1.2	811	71	1.2	985
11	0.6	4685	27	1.5	2000	42	1.2	985	57	0.9	442	72	1.2	1000
12	1.2	1386	28	1.8	2702	43	1.8	2702	58	0.6	965	73	0.6	1016
13	1.5	1052	29	1.2	1027	44	1.5	1026	59	1.2	1001	74	1.2	1138
14	1.5	992	30	2.4	999	45	0.9	964	60	1.2	811	75	0.9	977
15	1.2	990	31	1.8	982	46	0.9	2063	61	1.2	605	76	0.6	1016
16	0.6	2162												

Table A.5: Travel Demand

Origin	1	2	4	5	10	11	13	14	15	19	20	21
Travel Demand	1292	1271	1218	1251	1351	1341	1146	1320	1411	1331	1170	1193
LMP	17.4	17.4	17.4	17.4	17.4	17.4	17.4	17.4	16.06	15.43	15.43	15.43

Table A.6: Traveler's Utility Function

Coeff.	β_0	β_1	β_2	β_3	e	inc
Value	0	1	0.000008	0.1	8.25	1
Unit	\$	\$/h	\$/kW	\$	kWh	\$

Table A.7: Capacity Cost Data

Node #	Firm 1	Firm 2
21	$24.3 \times c_1$	$24.3 \times c_2$
22	$24.3 \times c_1$	$24.3 \times c_2$
23	$46.1 \times c_1$	$46.1 \times c_2$
24	$46.1 \times c_1$	$46.1 \times c_2$

Table A.8: Generation Cost Data

Scenario #	Firm 1	Firm 2	Probability
1	$110 \times g_1$	$60 \times g_2$	0.5
2	$10 \times g_1$	$60 \times g_2$	0.5

Table A.9: Demand Function Parameters d_b and d_a(Demand Function is $d = -d_a * w + d_b$)

Node	1	2	3	4	5	6	7	8	9	10	11
Intercept (d_b)	202	78	318	167	180	277	293	183	148	363	333
Slope (d_a)	-0.075	-0.196	-0.048	-0.091	-0.085	-0.055	-0.052	-0.083	-0.103	-0.042	-0.046

Table A.10: Transmission Capacity c_t(Link Transmission Cost Function is $\phi_t = 10 * [1 + (v/c_t)^4)]$

Link #	From Node	End Node	Capacity	Link #	From Node	End Node	Capacity
1	1	7	307	34	8	2	309
2	1	10	319	35	11	2	478
3	2	6	319	36	6	3	478
4	2	8	309	37	6	3	319
5	2	11	478	38	5	4	289
6	3	6	478	39	10	4	467
7	3	6	319	40	13	4	319
8	4	5	289	41	10	5	319
9	4	10	467	42	12	6	319
10	4	13	319	43	9	7	319
11	5	10	319	44	12	7	319
12	6	12	319	45	14	8	744
13	7	9	319	46	13	9	319
14	7	12	319	47	14	10	638
15	8	14	744	48	15	11	638
16	9	13	319	49	15	11	638
17	10	14	638	50	15	3	638
18	11	15	638	51	15	3	478
19	11	15	638	52	6	16	478
20	3	15	638	53	11	16	319
21	3	15	478	54	17	2	319
22	16	6	478	55	17	10	303
23	16	11	319	56	17	19	331
24	2	17	319	57	17	18	319
25	10	17	303	58	8	18	303
26	19	17	331	59	20	18	309
27	18	17	319	60	20	19	307
28	18	8	303	61	21	25	1000
29	18	20	309	62	22	25	1000
30	19	20	307	63	23	2	400
31	7	1	307	64	24	3	400
32	10	1	319	65	25	20	300
33	6	2	319				

Appendix B

Proofs.

Proof (Lemma 1). Firstly, the objective function (3.4a) is convex becasue it is a linear combination of three basic convex functions: (1) $f_1(x) = \int_0^x g(u)\mathrm{d}u$, with $g(u)$ being a positive and nondecreasing function, (2) $f_2(x) = x \ln x$ and (3) $f_3(x) = cx$. In addition, the constraints for problem (3.4) are all linear. Therefore, the optimization problem (3.4) is convex. Because of the differentiability of function (3.4a), the optimality conditions of problem (3.4) is equivalent to the following complementarity conditions in additions to constraints (3.4c $\sim$ 3.4f): $\forall a \in \mathcal{A}, r \in R, s \in S, k \in K^{rs}, p \in P^{rsk}$

$$0 \leq x_p \quad \perp \quad \sum_{a \in \mathcal{A}_p} t_a(\cdot) - \boldsymbol{\gamma}^T (B_{\hat{p}} + B_{\check{p}}) \geq 0 \tag{B.1a}$$

$$0 \leq \hat{x}_a^{rsk} \quad \perp \quad \gamma_a - A_a^T \hat{\boldsymbol{\lambda}} \geq 0 \tag{B.1b}$$

$$0 \leq \check{x}_a^{rsk} \quad \perp \quad \gamma_a - A_a^T \check{\boldsymbol{\lambda}} \geq 0 \tag{B.1c}$$

$$0 \leq q^{rsk} \quad \perp \quad \frac{1}{\beta_1}(ln(q^{rsk}) + \beta_3 \frac{\rho^k e^{rs}}{inc^{rs}} - \beta_2 \sum_{i \in I_k} c_i^s - \beta_0^k) + E^{rkT} \hat{\boldsymbol{\lambda}} + E^{ksT} \check{\boldsymbol{\lambda}} + \mu^{rs} \geq 0 \tag{B.1d}$$

We first show that the traffic flow solutions is Wardrop user equilibrium by proving the following two conditions.

1. **All the used paths connecting r, s, k have the same travel time.** $\forall r \in R, s \in S, k \in K^{rs}$, for those $\tilde{p} \in P^{rsk}$ with $x_{\tilde{p}} > 0$, $\sum_{a \in \mathcal{A}_{\tilde{p}}} t_a(\cdot) = \boldsymbol{\gamma}^T(B_{\hat{\tilde{p}}} + B_{\check{\tilde{p}}})$ (because of (B.1a)). Due to the following two conditions:

    - for $\tilde{a} \in \hat{\tilde{p}}$, i.e. $B_{\hat{\tilde{p}},\tilde{a}} = 1$, $\hat{x}_{\tilde{a}}^{rsk} > 0$ and therefore $\gamma_{\tilde{a}} = A_{\tilde{a}}^T \hat{\boldsymbol{\lambda}}$ (because of (B.1b)). So $\boldsymbol{\gamma}_{\tilde{a}}^T B_{\hat{\tilde{p}},\tilde{a}} = \hat{\boldsymbol{\lambda}}^T A_{\tilde{a}} B_{\hat{\tilde{p}},\tilde{a}}$
    - for $\tilde{a} \notin \hat{\tilde{p}}$, i.e. $B_{\hat{\tilde{p}},\tilde{a}} = 0$, $\boldsymbol{\gamma}_{\tilde{a}}^T B_{\hat{\tilde{p}},\tilde{a}} = \hat{\boldsymbol{\lambda}}^T A_{\tilde{a}} B_{\hat{\tilde{p}},\tilde{a}} = 0$

    we have $\boldsymbol{\gamma}^T B_{\hat{\tilde{p}}} = \hat{\boldsymbol{\lambda}}^T A B_{\hat{\tilde{p}}}$. Notice that $AB_{\hat{p}} = E^{rk}$, so $\boldsymbol{\gamma}^T B_{\hat{\tilde{p}}} = \hat{\boldsymbol{\lambda}}^T E^{rk}$.

    Same procedure, we have $\boldsymbol{\gamma}^T B_{\check{\tilde{p}}} = \check{\boldsymbol{\lambda}}^T E^{ks}$.

    So $\sum_{a \in \mathcal{A}_{\tilde{p}}} t_a(\cdot) = \boldsymbol{\gamma}^T(B_{\hat{\tilde{p}}} + B_{\check{\tilde{p}}}) = \hat{\boldsymbol{\lambda}}^T E^{rk} + \check{\boldsymbol{\lambda}}^T E^{ks} \doteq \tau^{rsk}$, which only dependents on r, s, k.

2. **All the unused paths connecting r, s, k have no smaller travel time than that of the used paths.** $\forall r \in R, s \in S, k \in K^{rs}$, for those $\tilde{p} \in P^{rsk}$ with $x_{\tilde{p}} = 0$, $\sum_{a \in \mathcal{A}_{\tilde{p}}} t_a(\cdot) \geq \boldsymbol{\gamma}^T(B_{\hat{\tilde{p}}} + B_{\check{\tilde{p}}})$ (because of (B.1a)). From (B.1b, B.1c), $\gamma_{\tilde{a}} \geq A_{\tilde{a}}^T \hat{\boldsymbol{\lambda}}$ and $\gamma_a \geq A_a^T \check{\boldsymbol{\lambda}}, \forall a$. So $\sum_{a \in \mathcal{A}_{\tilde{p}}} t_a(\cdot) \geq \boldsymbol{\gamma}^T(B_{\hat{\tilde{p}}} + B_{\check{\tilde{p}}}) \geq \hat{\boldsymbol{\lambda}}^T A B_{\hat{\tilde{p}}} + \check{\boldsymbol{\lambda}}^T A B_{\check{\tilde{p}}} = \hat{\boldsymbol{\lambda}}^T E^{rk} + \check{\boldsymbol{\lambda}}^T E^{ks} = \tau^{rsk}$.

Next, we show the OD-demand solutions are the service location choice with logit facility demand functions. This can be easily seen from (B.1d): for any k with $q^{rsk} > 0$, $\frac{1}{\beta_1}(ln(q^{rsk}) + \beta_3 \frac{\rho^k e^{rs}}{inc^{rs}} - \beta_2 \sum_{i \in I_k} c_i^s - \beta_0^k) + E^{rkT}\hat{\boldsymbol{\lambda}} + E^{ksT}\check{\boldsymbol{\lambda}} + \mu^{rs} = 0$. After reorganization,

$$
\begin{aligned}
q^{rsk} &= e^{\beta_0^k - \beta_1(E^{rkT}\hat{\boldsymbol{\lambda}} + E^{ksT}\check{\boldsymbol{\lambda}}) + \beta_2 \sum_{i \in I_k} c_i^s - \beta_3 \frac{\rho^k e^{rs}}{inc^{rs}} + \beta_1 \mu^{rs}} \\
&= e^{\beta_0^k - \beta_1 \tau^{rsk} + \beta_2 \sum_{i \in I_k} c_i^s - \beta_3 \frac{\rho^k e^{rs}}{inc^{rs}} + \beta_1 \mu^{rs}} \\
&= e^{U^{rsk} + \beta_1 \mu^{rs}}
\end{aligned}
$$

□

Proof (Lemma 2). If ρ^* is a maxinf-point of the Walrasian with $W(\rho^*, \cdot) \geq 0$, it follows that for all unit vectors $e^s = (0, \ldots, 1, \ldots)$, the s-th entry is 1, $W(\rho^*, e^s) \geq 0$ which implies $\mathrm{ES}_s(\rho^*) = 0$. □

Proof (Theorem 2).

The convergence of the maxinf sequence of points follows directly from the lopsided convergence ancillary tight of the augmented Walrasian sequence to the original Walrasian bifunction. Let $\varphi \in \Delta$ and $(\rho^\nu \in [0, M^\nu]) \to \rho \in I\!R^S_+$. Consider $\varphi^\nu \equiv \varphi$, $\nu \in I\!N$. The function $\rho \mapsto \mathrm{ES}(\rho)$ is lower semicontinuous (lsc) as $\{g_i(\cdot)\}$ and $\{q^{rs}(\cdot, c(\cdot))\}$ can be seen as argmin functions of strictly convex optimization problem (considering, for example, quadratic costs functions) [Rockafellar and Wets, 1998, Ex 1.19]. By definition of augmented Walrasian, one can get an upper bound by

$$W^\nu(\rho^\nu, \varphi^\nu) = \inf_{z \in \Delta} \left\{ W(\rho^\nu, z) + \frac{1}{2r^\nu} |\varphi - z|^2 \right\} \leq W(\rho^\nu, \varphi)$$

Then, using the lsc of $\mathrm{ES}(\cdot)$, $W(\cdot, \varphi)$ is an usc functions for every $\varphi \in \Delta$, and taking lim sup in the previous inequality

$$\limsup_\nu W^\nu(\rho^\nu, \varphi^\nu) \leq \limsup_\nu W(\rho^\nu, \varphi) \leq W(\rho, \varphi).$$

For any given $\rho \in I\!R^S_+$, eventually $\rho \in [0, M^\nu]$ for all ν sufficiently large, say $\nu \geq \nu_0$, since the sequence of parameters M^ν converges to ∞. Thus, considering the sequence ρ^ν being any point in $[0, M^\nu]$ for $\nu < \nu_0$, and $\rho^\nu = \rho$ when $\nu \geq \nu_0$, clearly $\rho^\nu \to \rho$. By the definition of W^ν, it is the inf-projection in the z-variable of the function $F^\nu(\varphi, z) = W(\rho, z) + \frac{1}{2r^\nu} |\varphi - z|^2$, a level bounded function in z locally uniform in φ. In virtue of [Rockafellar and Wets, 1998, Thm.1.17], $W^\nu(\rho, \cdot)$ is lsc. Thus, whatever sequence $(\varphi^\nu \in \Delta) \to \varphi \in \Delta$,

$$\liminf_\nu W^\nu(\rho^\nu, \varphi^\nu) \geq W(\rho, \varphi),$$

since for any $\varphi \in \Delta$, $W^\nu(\rho,\varphi) \to W(\rho,\varphi)$ as $\nu \to \infty$ and the conclusion follows from a standard diagonal argument.

The ancillary tightness condition follows directly from the invariance of the compact set Δ on the definition of the augmented Walrasian functions. Thus $W^\nu \to W$ when $\nu \to \infty$ ancilliary tight, and the conclusion is granted by [Jofré and Wets, 2014 (forthcoming, Thm.3.2]: every cluster point ρ^* of a the sequence of approximate ε^ν-maxinfpoints of the bifunctions W^ν is a maxinf-point of the lop-limit function W, and therefore an equilibrium price. □

Proof (Lemma 3). Combining VI (2) and (4), we have the following terms cancelled out:

1. $\sum_{q\in Q}\left[-A^T\gamma^{q*}\right]^T(\boldsymbol{x}^q-\boldsymbol{x}^{q*})$ and $\sum_{q\in Q}E^{qT}\gamma^{q*}(t^q-t^{q*})$
2. $\sum_{k\in K}\sum_{j\in J_k}\rho_k^*\left(g_i^j-g_i^{j*}\right)$ and $\sum_{q\in Q}\boldsymbol{\rho}^{\,*T}E^{q+}(t^q-t^{q*})$

The first cancellation is derived by Constraint (4.3c), and the second cancellation is derived by Constraint (4.3d). Then, add $-\left.\frac{\partial\phi_c}{\partial c_i^j}\right|_{c_i^j=c_i^{j*}}\left(g_i^j-g_i^{j*}\right)$ and subtract $-\left.\frac{\partial\phi_c}{\partial c_i^j}\right|_{c_i^j=c_i^{j*}}\left(g_i^j-g_i^{j*}\right)$, and reorganize the formulation in terms of variables $\boldsymbol{g}$ and $\boldsymbol{c}-\boldsymbol{g}$. Finally, after the use of Constraint (4.3b) and (4.3e) to substitute $(\boldsymbol{x},\boldsymbol{t})$ with $(\boldsymbol{v},\boldsymbol{d})$, VI (4.11) is derived. □

Proof (Theorem 3). There are two types of demand nodes in the virtual transportation network: the nodes within transmission network, denoted by "A"; and the virtual nodes representing unused capacity, denoted by "U". A-nodes do not have non-negativity constraint on price, so VI (4.15) is applied for these nodes (Corollary 5), while VI (4.12) is applied for U-nodes (Theorem 4). After algebraic simplification, the VI governing the virtual transportation network is identical with VI (4.11). □

Appendix C

Small Example Illustrating Flow Conservation Constraint (4.3b) ∼ (4.3e)

Network Structure

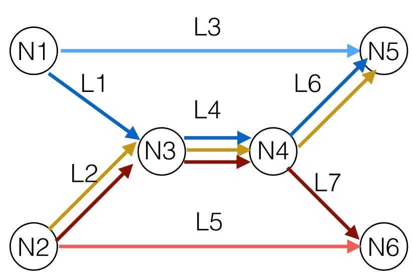

Figure C.1: Small Example Structure

OD and Path Information

Table C.1: Small Example Structure

OD #	OD	Path #	Path	Path Flow	Legend
Q1	N1-N5	P1	N1-N5	8	
		P2	N1-N3-N4-N5	2	
Q2	N2-N5	P3	N2-N3-N4-N5	5	
Q3	N2-N6	P4	N2-N3-N4-N6	3	
		P5	N2-N6	4	

Network Flow

$$t = \begin{bmatrix} 10 \\ 5 \\ 7 \end{bmatrix} \begin{matrix} \text{Q1} \\ \text{Q2} \\ \text{Q3} \end{matrix} \quad x = \begin{matrix} \text{Q1} & \text{Q2} & \text{Q3} \\ \end{matrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 3 \\ 8 & 0 & 0 \\ 2 & 5 & 3 \\ 0 & 0 & 4 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{matrix} \text{L1} \\ \text{L2} \\ \text{L3} \\ \text{L4} \\ \text{L5} \\ \text{L6} \\ \text{L7} \end{matrix} \quad v = \begin{bmatrix} 2 \\ 8 \\ 8 \\ 10 \\ 4 \\ 7 \\ 3 \end{bmatrix} \begin{matrix} \text{L1} \\ \text{L2} \\ \text{L3} \\ \text{L4} \\ \text{L5} \\ \text{L6} \\ \text{L7} \end{matrix} \quad g = \begin{bmatrix} 10 \\ 12 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \text{N1} \\ \text{N2} \\ \text{N3} \\ \text{N4} \\ \text{N5} \\ \text{N6} \end{matrix} \quad d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 15 \\ 7 \end{bmatrix} \begin{matrix} \text{N1} \\ \text{N2} \\ \text{N3} \\ \text{N4} \\ \text{N5} \\ \text{N6} \end{matrix}$$

Incidence Matrix

$$A = \begin{array}{c} \\ \\ \end{array}\begin{array}{ccccccc} \text{L1} & \text{L2} & \text{L3} & \text{L4} & \text{L5} & \text{L6} & \text{L7} \end{array}$$

$$A = \begin{bmatrix} -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{array}{l} \text{N1} \\ \text{N2} \\ \text{N3} \\ \text{N4} \\ \text{N5} \\ \text{N6} \end{array}$$

$$\begin{array}{ccc} \text{Q1} & \text{Q2} & \text{Q3} \end{array} \qquad \begin{array}{ccc} \text{Q1} & \text{Q2} & \text{Q3} \end{array} \qquad \begin{array}{ccc} \text{Q1} & \text{Q2} & \text{Q3} \end{array}$$

$$E = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \text{N1} \\ \text{N2} \\ \text{N3} \\ \text{N4} \\ \text{N5} \\ \text{N6} \end{array} \quad E^{+} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \text{N1} \\ \text{N2} \\ \text{N3} \\ \text{N4} \\ \text{N5} \\ \text{N6} \end{array} \quad E^{-} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \text{N1} \\ \text{N2} \\ \text{N3} \\ \text{N4} \\ \text{N5} \\ \text{N6} \end{array}$$

Appendix D

Calculation of $\partial\rho_{k'}/\partial g_i^j$

$\partial\rho_{k'}/\partial g_i^j$ can be computed from the ISO's optimization problem (4.3g), where $\boldsymbol{\rho}$ is the dual variables of constraint (4.3d) and $\boldsymbol{g}$ is parameters. $\partial\rho_{k'}/\partial g_i^j$ is essentially the derivative of dual variables with respect to right-hand side constants. We use the standard notations for convex optimization with linear constraints:

$$\min_{\boldsymbol{x}} \quad f(\boldsymbol{x}) \tag{D.1a}$$

$$\text{s.t.} \quad (\boldsymbol{\lambda}) \quad A\boldsymbol{x} = \boldsymbol{b} \tag{D.1b}$$

where $f(\boldsymbol{x})$ is a convex function and $\boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{\lambda}, \boldsymbol{b} \in \mathbb{R}^m$, to illustrate the calculating process and our goal is to calculate the Jacobian matrix $J_{\boldsymbol{\lambda}}(\boldsymbol{b})$.

Lagrangian of problem (D.1) is $\mathcal{L} = f(\boldsymbol{x}) - \boldsymbol{\lambda}^T(A\boldsymbol{x} - \boldsymbol{b})$. The optimality conditions of problem (D.1) is:

$$\nabla f(\boldsymbol{x}) - A^T\boldsymbol{\lambda} = \boldsymbol{0} \tag{D.2a}$$

$$A\boldsymbol{x} - \boldsymbol{b} = \boldsymbol{0} \tag{D.2b}$$

Take implicit derivatives of equations (D.2) with respect to $\boldsymbol{b}$:

$$\nabla_{\boldsymbol{x}}^2 f(\boldsymbol{x}^*(\boldsymbol{b}))J_{\boldsymbol{x}}(\boldsymbol{b}) - A^T J_{\boldsymbol{\lambda}}(\boldsymbol{b}) = \boldsymbol{0} \tag{D.3a}$$

$$AJ_{\boldsymbol{x}}(\boldsymbol{b}) - \boldsymbol{I} = \boldsymbol{0} \tag{D.3b}$$

The unknown variables in equations (D.3) are two Jacobian matrices, $J_{\boldsymbol{x}}(\boldsymbol{b})$ and $J_{\boldsymbol{\lambda}}(\boldsymbol{b})$. The total number of equations is equal to the total number of variables in equations (D.3), which is $nm + m^2$. Therefore, in most cases, $J_{\boldsymbol{\lambda}}(\boldsymbol{b})$ can be calculated given $\boldsymbol{b}$. Numerically, one can take an initial guess of $\boldsymbol{b}$ based on historical data and then solve equations (D.3) and Algorithm 6 iteratively until $J_{\boldsymbol{\lambda}}(\boldsymbol{b})$ converged.

Appendix E

Subroutine Pseudocode

Algorithm 7 **PH-Transportation Network Sub-Function**

function TRAFFIC ASSIGNMENT(path cost $\phi_p(\cdot)$, demand: $d_n(\cdot)$)

  $\tau = 0$ ▷ Initialize traffic assignment iteration index

  while $\epsilon \geq 10^{-4}$ **do**

    $\tau \leftarrow \tau + 1$

    $\chi_p^{\tau+1} \leftarrow \max\left\{0, \chi_p^\tau + \alpha_\tau\left(\lambda_n^\tau - \phi_p(x^\tau)\right)\right\}$ ▷ update path flow

    $\lambda_n^{\tau+1} \leftarrow \max\left\{0, \lambda_n^\tau + \alpha_\tau\left(d_n(\lambda^\tau) - \sum_{p\in P_n}\chi_p^\tau\right)\right\}$ ▷ update disutility

    $\epsilon \leftarrow \sum_{p\in P}\|\chi_p^{\tau+1} - \chi_p^\tau\| + \sum_{n\in N}\|\lambda_n^{\tau+1} - \lambda_n^\tau\|$

  end while

  return $(\boldsymbol{\chi},\boldsymbol{\lambda})$

end function

function RECOVER DECISION(path flow $\boldsymbol{\chi}$, travel disutility $\boldsymbol{\lambda}$)

  $f_a \leftarrow \sum_{p\in P} x_p\delta_{ap}, \forall a \in L$ ▷ get link flow

  $\boldsymbol{c} \leftarrow \boldsymbol{f}_{C-F}$ ▷ get investment decision

  $\boldsymbol{g} \leftarrow \boldsymbol{f}_{F-P}$ ▷ get production decision

  $\boldsymbol{\rho}$, $\boldsymbol{\lambda_c} \leftarrow \boldsymbol{\lambda}$ ▷ get electricity price and capacity shadow price

  return $(\boldsymbol{c},\boldsymbol{g},\boldsymbol{\rho},\boldsymbol{\lambda_c})$

end function

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Critical Infrastructure Systems:

Distributed Decision Processes over Network and Uncertainties

Abstract

Critical infrastructure systems (CISs) provide the essential services that are vital for a nation's economy, security, and health, but the analysis of CISs are challenged due to their inherent complexity. This dissertation focuses primarily on the system analysis of critical infrastructure systems, with a particular interest to address the modeling and computational challenges brought by uncertainties, interdependencies and distributed decision making of various components and stakeholders involved in CISs, so that a secure, reliable, efficient and resilient system can be further pursued. Through two examples, the first one is on electric vehicle charging infrastructure planning in a competitive market, and the second one is on power generators planning in a restructured electricity market, we illustrate how our general modeling framework, N-SMOPEC, can be adapted to formulate the specific problems in transportation and energy system. Each example is solved by decomposition based approach with convergence properties developed based on recent theoretical advances of variational convergence. Median size numerical experiments are implemented to study the performance of proposed method and draw practical insights. In addition, we have shown some knowledge from different domains, such as microeconomics, energy and transportation, can be shared to facilitate the formulation and solution process of seemingly unrelated problems of each other, which could possibly foster the communication between different fields and open up new research opportunities from both theoretical and practical perspectives.