

UC Berkeley

IURD Working Paper Series

Title

Some Computer Programs for Quadrant Analysis

Permalink

<https://escholarship.org/uc/item/7wm2q61k>

Authors

Martin, Juan

Gomar, Norbert

Rogers, Andrei

Publication Date

1969-05-01

Peer reviewed

SOME COMPUTER PROGRAMS FOR QUADRAT ANALYSIS

by

Juan Martin
Norbert Gomar
Andrei Rogers

Working Paper No. 96

This research has been partially supported by a grant from the Economic Development Administration and the Committee on Research, University of California, Berkeley,

May, 1969

PREFACE

This paper is the fourth working paper of a group that have as their common concern the development of statistical methods for the analysis of the spatial structure of population and employment in urban areas and regions. It consists of a set of computer programs that analyze and simulate urban dispersion using quadrat methods.

The authors are indebted to Jorge Silvetti for assisting them in some of the data processing chores associated with this project.

This work is currently being conducted under the sponsorship of a grant from the Economic Development Administration of the U.S. Department of Commerce. A complete listing of the Working Papers under this grant appears on the back page of this paper. They deal with regional patterns, development, and policies.

Andrei Rogers
Center for Planning and
Development Research
University of California, Berkeley
May 1, 1969

Working Papers on Spatial Pattern Analysis

- #86 Andrei Rogers and Norbert G. Gomar, "Statistical Inference in Quadrat Analysis," October, 1968, to be published in Geographical Analysis.
- #91 Shlomo Angel, "Point Patterns in Urban Regions: Two Complementary Tests," January, 1969.
- #93 Andrei Rogers, "Quadrat Analysis of Urban Dispersion," March, 1969, to be published in Environment.
- #96 Juan Martin, Norbert Gomar and Andrei Rogers, "Some Computer Programs for Quadrat Analysis," May, 1969.

Andrei Rogers and Juan Martin, "Bivariate Quadrat Analysis of the Dispersion of Employment and Population in Urban Areas," forthcoming.

INTRODUCTION

This report contains the following seven computer programs of models and techniques that are frequently used in quadrat analysis:

1. PROGRAM SIMONE
2. PROGRAM SIMTWO
3. PROGRAM NNEIGH
4. PROGRAM CUNIV
5. PROGRAM FITUNIV
6. PROGRAM CBIV
7. PROGRAM FITBIV

The first two programs, SIMONE and SIMTWO, are simulation packages that will generate and map random, dispersed and clustered point patterns which asymptotically have the Poisson, binomial and negative binomial distributions, respectively.

The third program, NNEIGH, will compute nearest neighbor statistics and the reflexive characteristics of a given point pattern.

The fourth and fifth programs, CUNIV and FITUNIV, will superimpose a rectangular grid pattern of specified dimensions over a point pattern and then compute observed and expected frequency distributions. Expected frequency distributions are derived for the following univariate probability models:

- a) Poisson
- b) Binomial
- c) Negative-Binomial
- d) Neyman Type A
- e) Poisson-Binomial
- f) Poisson-Negative Binomial

Moment and maximum-likelihood estimation is used, and expected and observed values are compared using the chi square goodness-of-fit test.

Finally, the sixth and seventh programs, CBIV and FITBIV, will find the bivariate frequency distribution of a point pattern that distinguishes between two classes of objects and compute expected bivariate frequency distributions that may be compared to the observed one. Expected distributions are derived for the following bivariate probability models:

- a) Generalized Bivariate Correlated Poisson
- b) Compound Bivariate Correlated Poisson
- c) Bivariate Correlated Poisson
- d) Contagious Bivariate Negative Binomial
- c) Compound Bivariate Uncorrelated Poisson

Moment estimators are used for all distributions except the last one, and expected and observed values are compared using the chi square goodness-of-fit test.

In addition to presenting the listings of the seven computer programs, this report also provides listings of typical inputs and outputs for each of the programs.

PROGRAM SIMONE

PURPOSE

This program simulates two-dimensional point patterns that are:

- 1) random;
- 2) dispersed; and
- 3) clustered.

Given data describing the dimensions of the study area, the number of points of the pattern, and the stochastic process of which the pattern is a realization, the program:

- 1) generates the pattern by assigning the specified number of points to the cells of the study area as a realization of a random, dispersed or clustered stochastic mechanism;
- 2) counts the number of cells within each frequency class of the corresponding univariate distribution (i.e. the number of cells with 0,1,2,3,...N points);
- 3) randomly locates within each cell the points assigned to it; and
- 4) maps the simulated point pattern.

Reference

Rogers, A. (1969). Quadrat Analysis of Urban Dispersion. Environment, forthcoming.



SIMONE

10
RUN,S.

LGO.

```
PROGRAM SIMONE(INPUT,OUTPUT,PUNCH)
COMMON/CONST/NC,N/COORD/ABC(1000),ORD(1000)/LOCA/NGRAPH(6,6)/
1DIB/BLANK,PLUS,STAR,MALLA
COMMON P(3600),CELL(3600),KOUNTE(3600)
```

```
C
C *****
C SUBROUTINES
C
C LOCATI = LOCATES POINTS WITHIN THE CELLS
C DRAW   = GRAPHIC PRESENTATION OF THE PATTERN
C
C *****
C READ INPUT DATA
C *****
C     READ 1000,NPATT
C     READ 1200,BLANK,PLUS,STAR
C     DO 999 INDIC=1,NPATT
C     READ 1100,INDEX,IND,INDICE,M,N,NC,MALLA,NTIM,A,B,IVAR
C
C 998 CONTINUE
C
C     DO 999 KOUNT=1,NTIM
C     G=A/B
C
C *****
C ASSIGN INITIAL PROBABILITIES
C *****
C     DO 10 I=1,M
C     KOUNTE(I)=0
C     P(I)=I*A
C 10 CELL(I)=0.0
C
C     DO 11 I=1,NC
C     DO 11 J=1,NC
C 11 NGRAPH(I,J)=0
C
C *****
C SELECT N CELLS
C *****
C     DO 20 L=1,N
C     R=RANF(0.0)
C     J=NC
C 90 CONTINUE
C     IF(R-P(J))5,5,15
C 15 J=J+NC
C     GO TO 90
C 5 I=J-NC+1
C 70 CONTINUE
C     IF(R-P(I))25,25,35
C 35 I=I+1
C     GO TO 70
C 25 CELDA=I
```

3

```

C *****
C FIND THE COORDINATES OF THE CELL
C *****
    COLM=NC
    LK=CELDA/COLM
    ABSC=LK
    KL=CELDA-ABSC*COLM
    IF(LK.EQ.0)GO TO 95
    IF(KL.EQ.0)GO TO 60
    LL=LK+1
    KK=KL
    GO TO 80
60  LL=LK
    KK=NC
    GO TO 80
95  LL=1
    KK=KL
80  ABC(L)=LL
    ORD(L)=KK
    II=NC+1-KK
    NGRAPH(II,LL)=NGRAPH(II,LL)+1
    CELL(I)=CELL(I)+1.0

C *****
C *CONTROL*
C *****
    IF(INDEX.EQ.1)GO TO 20

C *****
C ASSIGN NEW PROBABILITIES CLUSTERED OR DISPERSED PATTERN
C *****
    LP=0
    IF(INDEX.EQ.3) GO TO 12

C
    DO 14 I=1,M
    IF(CELL(I).LT.G) GO TO 14
    LP=LP+1
14  CONTINUE
12  SU=0.0
    EL=L
    EM=M-LP

C
    DO 17 J=1,M
    IF(INDEX.EQ.2.AND.CELL(J).EQ.G) GO TO 16
    P(J)=SU+A+(-1.0)**INDEX*B*EL/EM-(-1.0)**INDEX*CELL(J)*B
    SU=P(J)
    GO TO 17
16  P(J)=SU
    LP=LP+1
17  CONTINUE
20  CONTINUE

C *****
C CONSTRUCT THE MATRIX TO BE PRINTED

```

C *****

```

    IA=0
    DO 18 I=1,NC
    DO 18 J=1,NC
    IB=NGRAPH(I,J)
    IF(IB.LE.IA) GO TO 13
    IA=IB
13  K=0
    L=1
    IF(NGRAPH(I,J).EQ.K) GO TO 18
19  K=K+1
    L=L+1
    IF(NGRAPH(I,J).LE.K.OR.L.EQ.N) GO TO 18
    GO TO 19
18  KOUNTE(L)=KOUNTE(L)+1
    GO TO(1,2,3),INDEX
    1 PRINT 2000
    GO TO 9
    2 PRINT 2100
    GO TO 9
    3 PRINT 2200
    9 PRINT IVAR,((NGRAPH(I,J),J=1,NC),I=1,NC)
    PRINT 3200
    IA=IA+1
    DO 21 I=1,IA
    J=I-1
    PUNCH 9100,KOUNTE(I)
21  PRINT 3300,J,KOUNTE(I)

```

C
C

```

    IF(IND.NE.1) GO TO 100
    CALL LOCATI
    CALL DRAW
100  CONTINUE
    IF(IND.NE.1)GO TO 997
    PRINT 2400
    PRINT 2500,(I,ABC(I),ORD(I),I=1,N)
997  IF(INDICE.NE.1)GO TO 999
    K=1
    PUNCH 9000,(ABC(I),ORD(I),K,I=1,N)
999  CONTINUE

```

C *****

C FORMAT DECLARATIONS

C *****

```

1000 FORMAT(1I2)
1100 FORMAT(3I1,5I5,2F10.0,1A10)
1200 FORMAT(3A1)
2000 FORMAT(31H1GENERATION OF A RANDOM PATTERN////)
2100 FORMAT(34H1GENERATION OF A DISPERSED PATTERN////)
2200 FORMAT(34H1GENERATION OF A CLUSTERED PATTERN////)
2400 FORMAT(18H1POINT COORDINATES///
    16H POINT,13X,8H ABSCISA,11X,9H ORDINATE/)
2500 FORMAT(1X,1I6,2F20.6)
3200 FORMAT(18H0FREQUENCY CLASSES//
    13H X,9X,5H F(X)/)

```

4

```
3300 FORMAT(1X,I2,I14)
9000 FORMAT(2F10.4,I110)
9100 FORMAT(1I10)
      STOP
      END
```

5

SUBROUTINE LOCATI

COMMON/CONST/NC,N/COORD/ABC(1000),ORD(1000)/LOCA/NGRAPH(6,6)

C *****

C LOCATE THE POINTS WITHIN EACH CELL

C *****

L=1

DO 30 I=1,NC

DO 30 J=1,NC

NPOINT=NGRAPH(I,J)

IF(NPOINT.EQ.0)GO TO 30

C

DO 31 K=1,NPOINT

R=RANF(0.0)

ABC(L)=J-1.0+R

R=RANF(0.0)

ORD(L)=I-1.0+R

31 L=L+1

30 CONTINUE

C

RETURN

END

```

SUBROUTINE DRAW
COMMON/CONST/NC,N/COORD/ABC(1000),ORD(1000)/DIB/BLANK,PLUS,STAR,
1MALLA
COMMON PLOTT(61,110)

```

```

C
C *****
C PRINT PATTERN
C *****

```

```

      MA=NC*MALLA
      FMB=MA
      MA=MA+1
      MB=FMB*1.66+1.5
      IF(MB.GT.110)GO TO 600
      M=MB+1
      DO 300 I=1,MA
      DO 350 J=1,MB
      PLOTT(I,J)=BLANK
      IF(J.EQ.1.OR.J.EQ.MB) PLOTT(I,J)=PLUS
      IF(I.EQ.1.OR.I.EQ.MA) PLOTT(I,J)=PLUS
350  CONTINUE

```

```

C
      IF(M.GT.110)GO TO 300
      DO 375 J=M,110
375  PLOTT(I,J)=BLANK
300  CONTINUE

```

```

C
      DO 400 K=1,N
      I=ORD(K)*MALLA+1.5
      J=ABC(K)*MALLA*1.66+1.5
      L=MA+1-I
      PLOTT(L,J)=STAR
400  CONTINUE

```

```

C
      PRINT 3000
      PRINT 3200,((PLOTT(I,J),J=1,110),I=1,MA)

```

```

C
      RETURN
600  PRINT 3100
      RETURN

```

```

C
C *****
C FORMAT DECLARATION
C *****

```

```

3000  FORMAT(18H1PATTERN GENERATED//)
3100  FORMAT(42H1PATTERN SIZE EXCEEDS 110 AVAILABLE SPACES)
3200  FORMAT(10X,110A1)
      END

```

INPUT DATA

Abbreviations:

- (A): alphanumeric field
- (I): integer value (must be right adjusted in its field)
- (F): floating point number (must be punched with a decimal)

Control card (1I2)

Columns 1-2 (I) NPATT: total number of patterns to be simulated

Drawing symbols card (3A1)

Columns 1 (A) BLANK: a blank space
 2 (A) PLUS : the addition symbol (+)
 3 (A) STAR : the asterisk symbol (*)

Pattern specifications card (3I1,5I5,2F10.0,1A10)

The total number of cards in this set must equal the number specified for NPATT in the control card above. Each card must contain the following information:

Columns 1 (I) INDEX: control variable to indicate the pattern to
 be generated
 = 1 (random)
 = 2 (dispersed)
 = 3 (clustered)

 2 (I) IND: control variable to indicate whether the
 location of points within each cell and a map
 of the pattern are required.
 = 1 (locate and draw map)
 = 0 (do not)

- 3 (I) INDICE: control variable to indicate whether the punching of the coordinates of the generated point pattern is required. If
- = 1 (punch X- coordinate and Y-coordinate in format (2F10.4))
 - = 0 (do not)
- 4-8 (I) M: total number of cells in the study area
- 9-13 (I) N: desired number of points for the pattern.
- 14-18 (I) NC: number of cells of the study area in the X-direction. The study area is assumed to be square.
- 19-23 (I) MALLA: dimension of the side of an individual cell. All cells are assumed to be square and of the same size.
- 24-28 (I) NTIM: number of realizations desired for the pattern.
- 29-38 (F) A: initial probability assigned to each cell.
($A=1/M$)
- 39-48 (F) B: variation of the probability assigned to each cell over time:
- Random: $A(t+1) = A(t)$
 - Dispersed: $A(t+1) = A(t) - B x(t)$
 - Clustered: $A(t+1) = A(t) + B x(t)$,
- where $x(t)$ is the number of points already in the cell at time t .
- 49-58 (A) IVAR: this is a variable format of the following type:
(1X,NCI2), i.e. suppose $NC = 20$, then starting in columns 49 the input must be: (1X,20I2)

NOTE: In order to be consistent with the present dimensions of the arrays in the program the following limits must be observed:

$$M \leq 3600$$

$$NC \leq 60$$

$$N \leq 1000$$

and the pattern can be drawn only if:

$$NC(MALLA)1.66 + 1.5 \leq 110$$

OUTPUT

Printed output:

- 1) Matrix containing the number of points which have been "located" within each cell. This is an (NC x NC) matrix.
- 2) Count of the number of cells within each frequency class of the corresponding univariate distribution (i.e., number of cells with 0,1,2,3,...,N points)

Optional printed output:

This output is produced only if the number 1 has been assigned to the control variable IND in the "pattern specifications" card (see Input Data).

- 3) X-coordinate and Y-coordinate of each of the points of the pattern, and the number of observations at each location.
- 4) Maps of the study area and the two-dimensional point patterns.

Punched output:

- a) Punched output of the information contained in 2 above (FORMAT(1I10)) for further use.

Optional punched output:

This output is produced only if the number 1 has been assigned to the control variables IND and INDICE in the "pattern specifications" card (see Input Data)

- b) Punched output of the information contained in 3 above (FORMAT (2F10.4, 1I10)) for further use.

C INPUT DATA EXAMPLE

C

2

*

111	100	200	10	4	1	0.01	(1X,1 13)
300	100	200	10		1	0.01	0.05 (1X,1 13)

C OUTPUT EXAMPLE

C OUTPUT EXAMPLE

GENERATION OF A RANDOM PATTERN

4	2	2	4	2	0	1	2	4	1
2	0	1	5	0	1	2	1	2	2
1	3	2	3	4	5	1	2	2	0
3	2	1	4	1	1	1	1	3	1
1	1	1	4	0	2	0	2	2	3
2	2	2	2	3	2	3	1	1	2
1	1	4	2	2	2	5	1	3	3
2	4	2	1	2	1	2	1	3	4
3	2	2	3	2	4	2	1	3	1
4	2	1	0	2	0	0	4	0	3

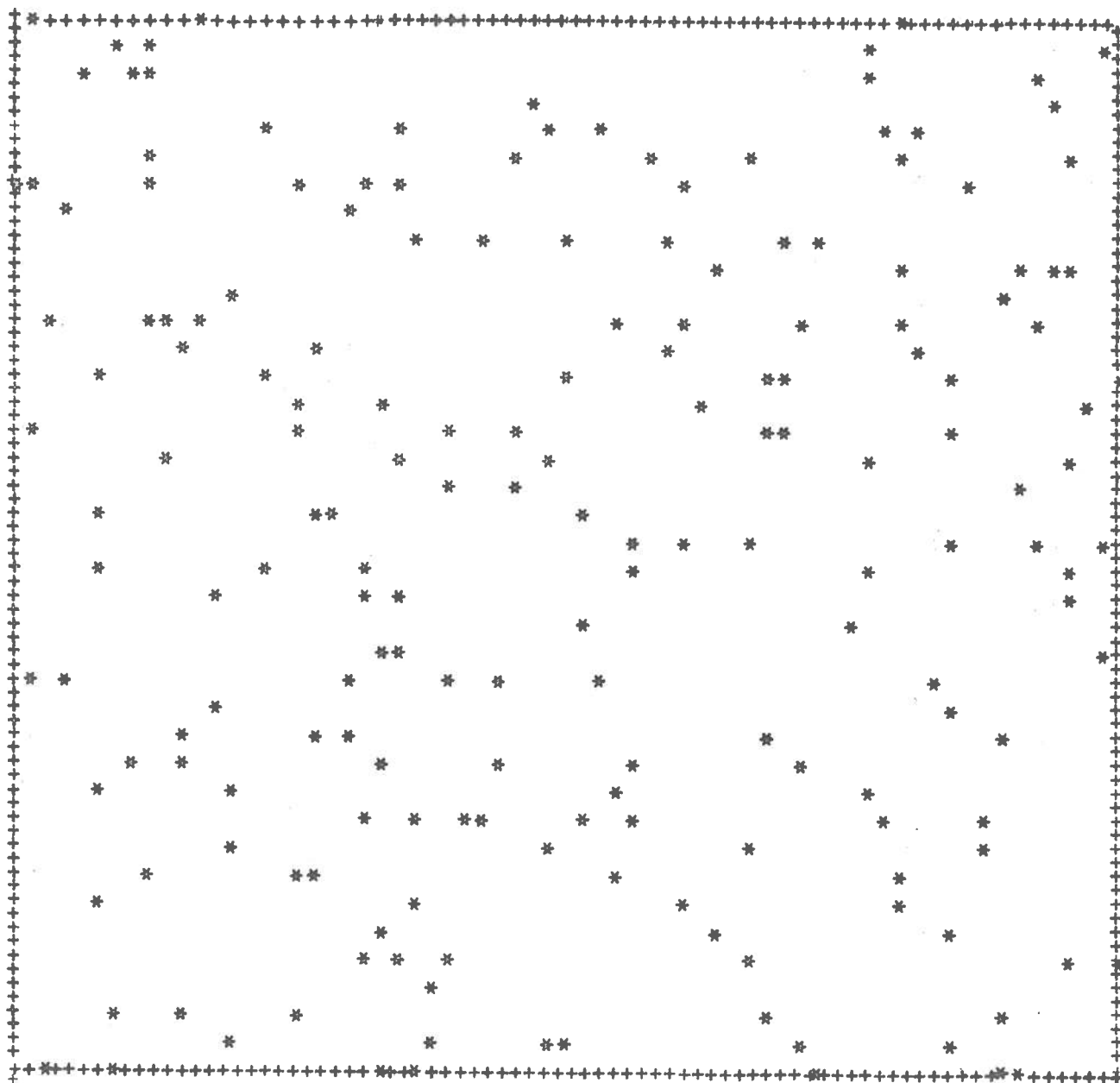
FREQUENCY CLASSES

X	F(X)
0	10
1	27
2	34
3	14
4	12
5	3

POINT COORDINATES

POINT	ABSCISA	ORDINATE
1	.932408	.188643
2	.849618	.286183
.	.	.
.	.	.
.	.	.
.	.	.
200	9.826263	9.755416

PATTERN GENERATED



GENERATION OF A CLUSTERED PATTERN

0	1	1	13	0	1	2	2	0	2
6	1	1	0	1	4	7	0	0	0
0	0	0	0	2	15	0	4	1	1
0	2	2	3	10	0	0	1	2	1
0	1	3	1	1	4	3	4	0	0
7	1	9	0	0	6	1	0	0	3
1	3	1	1	0	2	0	2	2	14
0	5	1	0	6	2	3	0	0	0
0	2	1	0	0	0	0	0	2	4
0	0	1	4	5	0	0	7	0	1

FREQUENCY CLASSES

X	F(X)
0	40
1	22
2	13
3	6
4	6
5	2
6	3
7	3
8	0
9	1
10	1
11	0
12	0
13	1
14	1
15	1

PROGRAM SIMTWO

PURPOSE

This program simulates two-dimensional point patterns that are:

- 1) negative binomial; and
- 2) Neyman's Type A.

Given data describing the dimensions of the study area, the parameter for the probability distribution of the location of primary points (parents), the parameter of the probability density function of the number of secondary points (off-spring) associated with each primary point, and the parameters for the dispersion function of secondary points, the program:

- 1) generates realizations of the underlying stochastic process; and
- 2) maps the resultant point patterns.

References:

- Karlin, S. (1966). A First Course in Stochastic Processes. Academic Press, New York and London, pp. 356-359.
- Thompson, H.R. (1954). A note on contagious distributions. Biometrika, 41, 268-271.

SIMTWO

RUN,S.
LGO.

```
PROGRAM SIMTWO (INPUT,OUTPUT,PUNCH)
COMMON X(500),Y(500)
COMMON/NORM/ABS,VARX,ORD,VARY,RO,B,R
COMMON/CONST/XDIM,YDIM,BLANK,PLUS,STAR
COMMON/DIB/SMAX
```

```
C
C *****
C SUBROUTINES
C
C POISSON = POISSON VARIATES GENERATOR
C GEOM =GEOMETRIC VARIATES GENERATOR
C CIRCULO =CIRCULAR DISPERSION
C NORMAL =NORMAL DISPERSION
C DRAW = GRAPHIC PRESENTATION OF THE PATTERN
C *****
C
C *****
C READ INPUT DATA
C *****
    READ 1000,NPATT
    READ 1005,BLANK,PLUS,STAR
    DO 100 KOUNT=1,NPATT
    SMAX=0.0
    READ 1001,INDEX,INDIKA,INDIKO,NTIM,XDIM,YDIM
    GO TO (1,2),INDEX
C
1  READ 1002,L,VARX,VARY,RO,Q
    C=1.0-RO**2
    B=SQRT(C)
    A=XDIM*YDIM
    D=6.2832*VARX**2
    AP=Q*A/(6.2832*VARY**2*(1.0-Q))
    GO TO 10
C
2  READ 1003,L,M,R
10 CONTINUE
C
C *****
C FIND NUMBER OF PARENTS AND PRINT PARAMETERS
C *****
    DO 101 IND=1,NTIM
    N=1
    CALL POISSON(L,IX)
    GO TO (3,4),INDEX
3  W=IX
    PAR=W/A
    AK=D*PAR
    AM=AP*AK
    PRINT 2001,L,VARX,VARY,RO,Q
    PRINT 2002,AK,AM
    GO TO 11
4  PRINT 2003,L,M,R
```

11 CONTINUE

C

C *****

C GENERATE PROGENY

C *****

DO 102 I=1,IX

R=RANF(0.0)

ABS=R*XDIM

R=RANF(0.0)

ORD=R*YDIM

X(N)=ABS

Y(N)=ORD

PRINT 2004,I,ABS,ORD

N=N+1

GO TO (5,6),INDEX

5 CALL GEOM(Q,K)

GO TO 12

6 CALL POISSON(M,K)

12 IF(K.GT.0)GO TO 13

PRINT 2005

GO TO 102

13 KA=N

KE=N+K-1

GO TO (7,8),INDEX

7 CALL NORMAL(KA,KE)

GO TO 14

8 CALL CIRCULO(KA,KE)

14 CONTINUE

PRINT 2006

DO 103 J=KA,KE

PRINT 2007,J,X(J),Y(J)

103 N=N+1

102 CONTINUE

N=N-1

IF(INDIKA.NE.1)GO TO 99

CALL DRAW(N)

99 IF(INDIKO.NE.1)GO TO 98

K=1

PUNCH 3000,(X(J),Y(J),K,J=1,N)

98 PRINT 2008

PRINT 2006

101 PRINT 2007,(J,X(J),Y(J),J=1,N)

100 CONTINUE

C

C *****

C FORMAT DECLARATION

C *****

1005 FORMAT(3A1)

1000 FORMAT(I5)

1001 FORMAT(3I1,2X,I5,2F10.0)

1002 FORMAT(1I5,4F10.0)

1003 FORMAT(2I5,1F10.0)

2001 FORMAT(37H1SIMULATION OF A NEG. BINOMIAL DISTR.//

119H POISSON PARAMETER=I11/

212H VARIANCE X=F18.4/

```
312H VARIANCE Y=F18.4/  
420H CORRELATION COEFF.=F10.4/  
521H GEOMETRIC PARAMETER=F9.4///)  
2002 FORMAT(13H PARAMETER K=F17.4/  
113H PARAMETER M=F17.4//)  
2003 FORMAT(43H1SIMULATION OF A NEYMAN TYPE A DISTRIBUTION//  
127H POISSON PARAMETER PARENTS=I11/  
227H POISSON PARAMETER PROGENY=I11/  
332H RADIUS FOR CIRCULAR DISPERSION=F11.4///)  
2004 FORMAT(16H0LOCATION PARENTI3/  
110H ABSCISA =F23.4/  
210H ORDENADA=F23.4//)  
2005 FORMAT(21H NO PROGENY GENERATED//)  
2006 FORMAT(7H0NUMBER,6X,7HABSCISA,5X,8HORDINATE//)  
2007 FORMAT(1X,I6,2F13.4)  
2008 FORMAT(18H1PATTERN GENERATED//)  
3000 FORMAT(2F10.4,I10)  
STOP  
END
```

SUBROUTINE POISSON(L,IX)

```

C *****
C GENERATION OF POISSON VARIATES
C *****
      IX=0
      ELE=L
      A=EXP(-ELE)
      P=1.0
1     R=RANF(0.0)
      P=P*R
      IF(P.LT.A) RETURN
      IX=IX+1
      GO TO 1
      END

```

SUBROUTINE GEOM(Q,K)

```

C *****
C GENERATION OF GEOMETRIC VARIATES
C *****
      QR=ALOG10(Q)
      R=RANF(0.0)
      TR=ALOG10(R)
      K=TR/QR
      RETURN
      END

```

SUBROUTINE CIRCULO(KA,KE)

```

COMMON X(500),Y(500)
COMMON/NORM/ABS,VARX,ORD,VARY,RO,B,R
COMMON/DIB/SMAX

```

```

C *****
C CIRCULAR DISPERSION
C *****
      DO 10 I=KA,KE
      Q=RANF(0.0)
      ALFA=6.2832*Q
      SALF=SIN(ALFA)
      CALF=COS(ALFA)
      Q=RANF(0.0)
      Q=Q*R
      X(I)=ABS+CALF*Q
      Y(I)=ORD+SALF*Q
10  CONTINUE
      SMAX=R
      RETURN
      END

```

```

SUBROUTINE NORMAL (KA,KE)
COMMON X(500),Y(500)
COMMON/NORM/UX,VARX,UY,VARY,RO,A,W
COMMON/DIB/SMAX
DIMENSION Z(2)

```

```

C *****
C BIVARIATE NORMAL DISPERSION
C *****
DO 20 L=KA,KE
DO 10 I=1,2
SUM=0.0

C
DO 5 J=1,12
R=RANF(0.0)
5 SUM=SUM+R
10 Z(I)=SUM-6.0
B=VARX*Z(1)
C=VARY*(RO*Z(1)+A*Z(2))
X(L)=B+UX
Y(L)=C+UY
IF(B.GT.C)C=B
IF(C.GT.SMAX)SMAX=C
20 CONTINUE
RETURN
END

```

```

SUBROUTINE DRAW (N)
COMMON X(500),Y(500)
COMMON/CONST/XDIM,YDIM,BLANK,PLUS,STAR
COMMON/DIB/SMAX
DIMENSION PLOTT (61,110)

```

```

C *****
C DRAW THE POINT PATTERN
C *****
MA=XDIM+2.0*SMAX+1.5
MB=(YDIM+2.0*SMAX)*1.66+1.5
IF(MB.GT.110)GO TO 600
M=MB+1

C
DO 300 I=1,MA
DO 350 J=1,MB
PLOTT(I,J)=BLANK
IF(J.EQ.1.OR.J.EQ.MB) PLOTT(I,J)=PLUS
IF(I.EQ.1.OR.I.EQ.MA) PLOTT(I,J)=PLUS
350 CONTINUE
IF(M.GT.110)GO TO 300

C
DO 400 J=M,110
400 PLOTT(I,J)=BLANK
300 CONTINUE

C
DO 500 K=1,N

```

```
I=SMAX+Y(K)+1.5  
J=(SMAX+X(K))*1.66+1.5  
L=MA+1-I
```

```
500 PLOTT(L,J)=STAR
```

C

```
PRINT 3000  
PRINT 3200,((PLOTT(I,J),J=1,110),I=1,MA)  
RETURN
```

```
600 PRINT 3100
```

```
3100 FORMAT(42H1PATTERN SIZE EXCEEDS 110 AVAILABLE SPACES)
```

```
3000 FORMAT(18H1PATTERN GENERATED//)
```

```
3200 FORMAT(10X,110A1)
```

```
RETURN
```

```
END
```

INPUT DATA

Abbreviations:

- (A): alphanumeric field
- (I): integer value (must be right adjusted in its field)
- (F): floating point number (must be punched with a decimal)

Control card (1I5)

Columns 1-5 (I) NPATT: total number of patterns to be generated.

Drawing Symbols (3A1)

Columns 1 (A) BLANK: a blank space

2 (A) PLUS : the addition symbol (+)

3 (A) STAR : the asterisk symbol (*)

Pattern control card (3I1,2X,1I5,2F10.0)

Columns 1 (I) INDEX: indicates the pattern to be simulated

- = 1 (negative Binomial)
- = 2 (Neyman's Type A)

2 (I) INDIKA: indicates whether the pattern is to be drawn

- = 1 (draw the pattern)
- = 0 (do not)

3 (I) INDIKO: indicates whether the punching of the coordinates of the generated pattern is desired.

- = 1 (punch X-coordinate and Y-coordinate in format (2F10.4))
- = 0 (do not)

6-10 (I) NTIM: indicates the number of realizations desired for the pattern.

22

11-20 (F) XDIM: dimension of the study area in the X-direction

21-30 (F) YDIM: dimension of the study area in the Y-direction

Pattern specifications card

Only one of the following cards is required, depending on the value specified for INDEX in the pattern control card:

a) Negative Binomial pattern (INDEX = 1) (1I5,4F10.0)

Columns 1-5 (I) L: parameter of the Poisson distribution that gives the number of primary points which are to be located at random.

6-15 (F) VARX: variance in the X-direction for the dispersion function

16-25 (F) VARY: variance in the Y-direction for the dispersion function

26-35 (F) RO: correlation(X,Y) for the dispersion function.

36-45 (F) Q: the parameter of the geometric distribution that gives the number of secondary points associated with each primary point.

b) Neyman's Type A pattern (INDEX=2) (2I5, 1F10.0)

Columns

1-5 (I) L: parameter of the Poisson distribution that gives the number of primary points which are to be located at random.

6-10 (I) M: parameter of the Poisson distribution that gives the number of secondary points to be associated with each primary point.

11-20 (F) R: radius of the circular dispersion function.

23

NOTE: The total number of Pattern control card-Pattern specification card pairs must equal the number assigned to the variable NPATT in the control card.

Example:

Control card (NPATT = 3)

Drawing symbols

{ Pattern control card (INDEX = 1)

{ Pattern specifications card (a)

{ Pattern control card (INDEX = 2)

{ Pattern specification card (b)

{ Pattern control card (INDEX = 2)

{ Pattern specification card (b)

OUTPUT

Printed output:

- 1) Parameters of the distribution
- 2) Coordinates of primary and secondary points (differentiating between primary and secondary points)
- 3) Coordinates of the two-dimensional point pattern and the number of observations associated with each location.

Optional printed output:

This output is produced only if the number 1 has been assigned to the variable INDIKA in the pattern control card (see Input Data)

- 4) actual mapping of the study area and the two-dimensional point pattern.

Punched output:

No punched output

Optional punched output:

This output is produced only if the number 1 has been assigned to the variable INDIKO in the pattern control card (see Input Data)

- a) Punches the information contained in 3 above in format (2F10.4, 1I10) for further use.

C INPUT DATA EXAMPLE

```

C
  2
*
100  2  50.  100.
   0  0.5  0.5  0.7  0.8
200  1  50.  100.
   20  5  1.0
C END OF INPUT DATA
  
```

C OUTPUT EXAMPLE

SIMULATION OF A NEG. BINOMIAL DISTR.

```

POISSON PARAMETER=      20
VARIANCE X=          .5000
VARIANCE Y=          .5000
CORRELATION COEFF=    .7000
GEOMETRIC PARAMETER=  .8000
  
```

```

PARAMETER K=          .0063
PARAMETER M=        80.0000
  
```

```

LOCATION PARENT 1
ABSCISA =          41.0685
ORDENADA=          51.7585
  
```

NUMBER	ABSCISA	ORDINATE
2	41.8828	51.3944
3	41.9067	52.4488
4	41.4042	52.0768
5	41.3144	52.0637
6	39.9332	50.2419
7	40.6897	51.2013

```

LOCATION PARENT 2
ABSCISA =          43.0031
ORDENADA=          63.2122
  
```

NUMBER	ABSCISA	ORDINATE
9	42.9988	63.1467

```

LOCATION PARENT 3
ABSCISA =          21.9547
ORDENADA=          95.5608
  
```

NUMBER	ABSCISA	ORDINATE
--------	---------	----------

11 22.2581 96.1857
LOCATION PARENT 4
ABSCISA = 15.3106
ORDENADA= 24.2391

NO PROGENY GENERATED

•
•
•
•

LOCATION PARENT 20
ABSCISA = 30.2011
ORDENADA= 33.6489

NUMBER	ABSCISA	ORDINATE
79	29.7595	33.1826
80	30.5261	33.6775
81	30.8755	34.2700
82	30.2907	33.7690
83	30.9291	34.5930
84	29.5931	32.8740
85	29.6699	33.2549

PATTERN GENERATED

NUMBER	ABSCISA	ORDINATE
1	41.0685	51.7585
2	41.8828	51.3944
3	41.9067	52.4488
4	41.4042	52.0768
•	•	•
•	•	•
•	•	•
•	•	•
85	29.6699	33.2549

SIMULATION OF A NEG. BINOMIAL DISTR.

POISSON PARAMETER= 20
 VARIANCE X= .5000
 VARIANCE Y= .5000
 CORRELATION COEFF= .7000
 GEOMETRIC PARAMETER= .8000

PARAMETER K= .0066
 PARAMETER M= 84.0000

LOCATION PARENT 1
 ABSCISA = 17.7364
 ORDENADA= 73.1860

NUMBER	ABSCISA	ORDINATE
2	18.5283	73.4378

LOCATION PARENT 2
 ABSCISA = 23.8354
 ORDENADA 65.2160

NUMBER	ABSCISA	ORDINATE
4	23.9502	64.9088
5	23.9233	64.6770
6	24.2316	65.6794
.	.	.
.	.	.
.	.	.
.	.	.

LOCATION PARENT 21
 ABSCISA = 28.1056
 ORDENADA= 71.4681

NUMBER	ABSCISA	ORDINATE
87	28.1440	71.1925
88	28.7027	72.3487

PATTERN GENERATED

NUMBER	ABSCISA	ORDINATE
--------	---------	----------

1	17.7364	73.1860
2	18.5283	73.4378
3	23.8354	65.2160
4	23.9502	64.9088
.	.	.
.	.	.
.	.	.
.	.	.
88	28.7027	72.3487

SIMULATION OF A NEWMAN TYPE A DISTRIBUTION

POISSON PARAMETER PARENTS= 20
 POISSON PARAMETER PROGENY= 5
 RADIUS FOR CIRCULAR DISPERSION= 1.0000

LOCATION PARENT 1

ABSCISA = 10.6191
 ORDENADA= 2.4807

NUMBER ABSCISA ORDINATE

2	10.5966	2.4840
3	10.6376	2.4755
4	10.6037	2.4832
5	10.6286	2.4747
6	10.6226	2.4855

LOCATION PARENT 2

ABSCISA = 38.1963
 ORDENADA= 72.8259

NUMBER ABSCISA ORDINATE

8	37.9243	72.8373
9	37.9494	72.6015
10	37.4877	72.6719

LOCATION PARENT 15

ABSCISA = 13.3103
 ORDENADA= 93.0396

NUMBER ABSCISA ORDINATE

88	13.3799	93.9378
89	13.8635	92.7612
90	14.0526	92.7882

PATTERN GENERATED

NUMBER ABSCISA ORDINATE

1	10.6191	2.4807
2	10.5966	2.4840
3	10.6376	2.4755
4	10.6037	2.4832

•
•
•
•
90

•
•
•
•
14.0526

•
•
•
•
92.7882

PROGRAM NNEIGH

PURPOSE

This program computes nearest neighbor statistics and finds other such properties of two-dimensional point patterns. Given the dimensions of the study area and the coordinates of the points in a pattern, the program:

- 1) computes the average distance to the 1st., 2nd., ..., Nth nearest neighbor; and
- 2) finds the reflexive properties of the point pattern.

References:

Clark, P.J. and Evans, F.C. (1954). Distance to nearest neighbor as a measure of spatial relationships in populations. Ecology, 35, 445-453.

Thompson, H.R. (1956). Distribution of distance to n-th neighbor in a population of randomly distributed individuals. Ecology, 37, 391-394.

N NEIGH

RUN,S.

LGO.

```

PROGRAM NNEIGH(INPUT,OUTPUT)
COMMON/COORD/ABC(500),ORD(500)/CONST/SCAL,N,NTH,IND,MAX,DENS/REFL/
INN(500,10),AVER(10)/DISTR/DNN(500,10)
C *****
C SUBROUTINES
C
C DISTNN = COMPUTES DISTANCES TO NEAREST NEIGHBORS
C REFLNN = FINDS REFLEXIVE PROPERTIES
C *****
C *****
C READ INPUT DATA
C *****
    DIMENSION D(500)
C
    99 READ 1000,IND,INDIKA,NTH,AX,OY
        GRA=AX**2+OY**2
        GRO=SQRT(GRA)
        MAX=GRO+0.5
C
    98 CONTINUE
        DO 15 I=1,500
            READ 1100,ABC(I),ORD(I)
            IF(ABC(I).LT.0) GO TO 16
    15 CONTINUE
C
    16 NCONTR=ABS(ABC(I))-1.0
        N=I-1
        XN=N
        DENS=XN/(AX*OY)
C
        GO TO(1,2,3,4),IND
    1 PRINT 2000
        GO TO 9
    2 PRINT 2001
        GO TO 9
    3 PRINT 2002
        GO TO 9
    4 PRINT 2003
    9 PRINT 2100,(I,ABC(I),ORD(I),I=1,N)
        CALL DISTNN
        IF (INDIKA.NE.1)GO TO 101
        CALL REFLNN
C
    101 IF(NCONTR.EQ.0) STOP
        GO TO (98,99),NCONTR
C
C *****
C FORMAT DECLARATION
C *****
    1000 FORMAT(2I1,I2,2F10.0)
    1100 FORMAT(2F10.4)

```

```
2000 FORMAT(47H1NEAREST NEIGHBOR ANALYSIS POISSON DISTRIBUTION////  
16H POINT,12X,8H ABSCISA,11X,9H ORDINATE/)  
2001 FORMAT(48H1NEAREST NEIGHBOR ANALYSIS BINOMIAL DISTRIBUTION////  
16H POINT,12X,8H ABSCISA,11X,9H ORDINATE/)  
2002 FORMAT(50H1NEAREST NEIGHBOR ANALYSIS N.BINOMIAL DISTRIBUTION////  
16H POINT,12X,8H ABSCISA,11X,9H ORDINATE/)  
2003 FORMAT(48H1NEAREST NEIGHBOR ANALYSIS OBSERVED DISTRIBUTION////  
16H POINT,12X,8H ABSCISA,11X,9H ORDINATE/)  
2100 FORMAT (1X,1I5,2F20.4)  
END
```

```

SUBROUTINE DISTNN
COMMON/COORD/ABC(500),ORD(500)/CONST/SCAL,N,NTH,IND,MAX,DENS/REFL/
1NN(500,10),AVER(10)/DISTR/DNN(500,10)
DIMENSION D(500)

C
RO=SQRT(DENS)
EXPE=0.5/RO
DO 12 L=1,NTH
12 AVER(L)=0.0

C
C *****
C CONSTRUCT THE DISTANCES MATRIX
C *****
DO 11 I=1,N
DO 10 J=1,N
IF(J.EQ.I) GO TO 20
A=(ABC(J)-ABC(I))**2+(ORD(J)-ORD(I))**2
D(J)=SQRT(A)
GO TO 10
20 D(J)=MAX
10 CONTINUE

C
C *****
C FIND THE NTH'S NEAREST NEIGHBOR FOR EACH POINT
C *****
DO 13 K=1,NTH
A=D(1)
L=1
DO 14 J=2,N
B=D(J)
IF(A.LE.B) GO TO 14
A=B
L=J
14 CONTINUE
NN(I,K)=L
DNN(I,K)=A
D(L)=MAX
13 AVER(K)=AVER(K)+A
11 CONTINUE

C
A=DNN(1,1)
DO 16 I=2,N
B=DNN(I,1)
IF(A.GE.B) GO TO 16
A=B
16 CONTINUE
SCAL=20./A**2

C
DO 15 K=1,NTH
15 AVER(K)=AVER(K)/N
QUOT=AVER(1)/EXPE

C
PRINT 3000,N,DENS,EXPE,AVER(1),QUOT
3000 FORMAT(28H1NEAREST NEIGHBOR STATISTICS//
118H NUMBER OF POINTS=I10/

```

```
29H DENSITY=F19.5//  
337H AVERAGE DISTANCE TO NEAREST NEIGHBOR//  
420H EXPECTED IF RANDOM=F8.5/  
510H OBSERVED=F18.5/  
619H OBSERVED/EXPECTED=F9.5//)  
  RETURN  
  END
```

```

SUBROUTINE REFLNN
COMMON/CONST/SCAL,N,NTH,IND,MAX,DENS/REFL/NN(5 ,1 ),AVER(1 )
DIMENSION V(500,10),S(10),TH(10),DIFF(10)
C *****
C REFLEXIVE NEIGHBORHOOD PROPERTIES
C *****
DO 40 I=1,N
DO 40 J=1,NTH
II=NN(I,J)
KJ=NN(II,J)
IF(I.EQ.KJ) GO TO 41
V(I,J)=0.0
GO TO 40
41 V(I,J)=1.0
40 CONTINUE

C
C
B=0.6215
DO 60 J=1,NTH
S(J)=0.0
DO 61 I=1,N
61 S(J)=S(J)+V(I,J)
S(J)=S(J)/N
TH(J)=B**J
60 DIFF(J)=S(J)-TH(J)

C
PRINT 4000
PRINT 4100,(S(J),TH(J),DIFF(J),AVER(J),J,J=1,NTH)
4000 FORMAT(21HREFLEXIVE PROPERTIES///
114H SAMPLE VALUES,9X,19H TH.-RANDOM-VALUES,17X,11H DIFFERENCE,6X
2,22H AVERAGE N.N. DISTANCE,3X,15 HNEIGHBOR NUMBER/)
4100 FORMAT(1X,F13.6,3F28.6,I18)
RETURN
END

```

INPUT DATA

Abbreviations:

(I): integer value (must be right adjusted in its field)

(F): floating point number (must be punched with a decimal)

Control card (2I1,1I2,2F10.0)

Columns	1 (I)	IND: denotes the pattern to be analyzed
		= 1 (random)
		= 2 (dispersed)
		= 3 (clustered)
		= 4 (observed)
	2 (I)	INDIKA: indicates whether reflexive properties are desired
		= 1 (find reflexive properties)
		= 2 (do not)
	3-4 (I)	NTH: indicates the number of nearest neighbors to be considered for reflexive properties (if INDIKA equals zero, then NTH equals zero)
	5-14 (F)	AX: X-dimension of the study area (AX>maximum absolute difference between the X-coordinates of the point pattern)
	15-24 (F)	OY: Y-dimension of the study area (OY>maximum absolute difference between the Y-coordinates of the point pattern)

Coordinates of the point pattern: (2F10.4)

One card per point. Each card contains:

Columns	1-10 (F)	ABC: X-coordinate of a point
	11-20 (F)	ORD: Y-coordinate of a point

This set of cards must be ended with a card containing:

Columns	9-10 (I)	NCON: control variable
		= -1 (the preceeding point pattern was the last)
		= -2 (read in coordinates of the next point pattern. The values specified in the control card of the previous pattern are not modified)
		= -3 (read in new control card and coordinates for the next point pattern)

END OF INPUT DATA

Example

- 1) control card (pattern 1 and 2)
- 2) coordinates of the point pattern 1
NCON -2
- 3) coordinates of the point pattern 2
NCON -3
- 1) control card (pattern 3)
- 2) coordinates of the point pattern 3
NCON -1

NOTE: In order to be consistent with the present dimension of the arrays
in the program the following limits must be observed:

number of points ≤ 500

NTH ≤ 10

OUTPUT

Printed output:

- 1) Number of points and density of the pattern
- 2) Average distance to nearest neighbor of the observed pattern and of the one expected under the hypothesis of randomness. Also computes the observed over expected ratio of nearest neighbor distances.

Optional printed output:

This output is produced only if the number 1 has been assigned to the control variable INDIKA in the control card (see Input Data)

3) Reflexive properties

- of the observed pattern
- of the one expected under the hypothesis of randomness
- difference between observed and expected values
- average distance to 1st., 2nd.,...Nth nearest neighbor.

Punched Output

No punched output

C INPUT DATA EXAMPLE

```

11 5      10.0      10.0
  .9324    .1886
  .8496    .2862
  .3033    .3311
    .      .
    .      .
    .      .
    .      .
9.8263    9.7554
  
```

C END OF INPUT DATA

C OUTPUT EXAMPLE

NEAREST NEIGHBOR STATISTICS

NUMBER OF POINTS= 200
 DENSITY= 2.00000

AVERAGE DISTANCE TO NEAREST NEIGHBOR

EXPECTED IF RANDOM= .35355
 OBSERVED= .35466
 OBSERVED/EXPECTED= 1.00312

REFLEXIVE PROPERTIES

SAMPLE VALUES	TH.VALUES	DIFFERENCE	AVERAGE N.N.DISTANCE	NEIGHBOR NER.
.640000	.621500	.018500	.354656	1
.350000	.386262	.036262	.569992	2
.270000	.240062	.029938	.712424	3
.260000	.149199	.110801	.854581	4
.230000	.092727	.137273	.968388	5

PROGRAM CUNIVPURPOSE

Given a set of observations on a variable, say x , this program:

- 1) forms two dimensional arrays containing the number of observations of the variable X in each class of grid cells whose dimensions are specified in the control card; and
- 2) counts the number of cells in each frequency class for the corresponding univariate distribution.

The set of observed values may correspond to either a rectangular or square study region, the shape of the cells may be either rectangular or square, and the number of cells in the X -direction may differ from the number of cells in the Y -direction.

CUNIV

RUN,S.
LGO.

```

PROGRAM CUNIV (INPUT,OUTPUT,PUNCH)
COMMON INDEX,INT,MM,NN,M(60,60)
C
C *****
C READ DATA GIVEN BY COORDINATES AND FORM MATRIX
C *****
  DIMENSION XC(1000),YC(1000),N(1000),MO(60,60)
  DIMENSION IVAR(6),NSIZ(4),MSIZ(4)
  DO 555 KAPO=1,2
  READ 1000,INDEX
  GO TO (1,2),INDEX
1 READ 1001,NOBS,INT,XDIM,YDIM,(NSIZ(I),MSIZ(I),I=1,4)
  READ 1002,IVAR
  READ IVAR,(XC(I),YC(I),N(I),I=1,NOBS)
C
  DO 100 I=1,4
  IF(NSIZ(I).EQ.0.OR.MSIZ(I).EQ.0) GO TO 100
  A=NSIZ(I)
  B=MSIZ(I)
  NN=XDIM/A
  MM=YDIM/B
  A=NN
  B=MM
  KE=MSIZ(I)
  KI=NSIZ(I)
C
  DO 101 K=1,KE
  DO 101 J=1,KI
101 M(K,J)=0
  DO 102 L=1,NOBS
  II=YC(L)/B+1.0
  JJ=XC(L)/A+1.0
  KK=KE-II+1
102 M(KK,JJ)=M(KK,JJ)+N(L)
  CALL SUMM(KE,KI)
100 CONTINUE
  GO TO 555
C
C *****
C READ DATA GIVEN BY MATRIX AND FORM NEW MATRIX
C *****
  2 READ 1003,NR,NC,INT,(NSIZ(I),MSIZ(I),I=1,4)
  READ 1002,IVAR
  READ IVAR,((MO(I,J),J=1,NC),I=1,NR)
C
  DO 200 I=1,4
  IF(NSIZ(I).EQ.0.OR.MSIZ(I).EQ.0) GO TO 200
  IF(NSIZ(I).EQ.NC.AND.MSIZ(I).EQ.NR) GO TO 199
  MM=NR/MSIZ(I)
  NN=NC/NSIZ(I)
C
  DO 201 J=1,NC

```

```

      K=1
      NT=MM
      II=1
10    IA=0
      DO 202 L=K,NT
202   IA=IA+MO(L,J)
      M(II,J)=IA
      K=K+MM
      NT=K+MM-1
      II=II+1
      IF(NT.LE.NR) GO TO 10
201  CONTINUE
C
      ISIZ=MSIZ(I)
      DO 203 J=1,ISIZ
      K=1
      NT=NN
      JJ=1
11    IA=0
C
      DO 204 L=K,NT
204   IA=IA+M(J,L)
      M(J,JJ)=IA
      K=K+NN
      NT=K+NN-1
      JJ=JJ+1
      IF(NT.LE.NC) GO TO 11
203  CONTINUE
199  KE=MSIZ(I)
      KI=NSIZ(I)
      CALL SUMM (KE,KI)
200  CONTINUE
555  CONTINUE
C
C *****
C  FORMAT DECLARATION
C *****
1000 FORMAT(1I1)
1001 FORMAT(2I5,2F10.0,8I5)
1002 FORMAT(6A10)
1003 FORMAT(3I5,5X,8I5)
      STOP
      END

```

```

SUBROUTINE SUMM(NR,NC)
COMMON INDEX,INT,IM,IN,NX(60,60)
DIMENSION KOUNTE(30)

```

```

C *****
C COUNT UNIVARIATE
C *****
      DO 200 I=1,30
200 KOUNTE(I)=0
C
      DO 201 I=1,NR
      DO 201 J=1,NC
      K=0
      L=0
202 K=K+INT
      L=L+1
      IF(NX(I,J).LT.K.OR.L.EQ.30) GO TO 201
      GO TO 202
201 KOUNTE(L)=KOUNTE(L)+1
C *****
C FIND MAXIMUM CLASS INTERVAL
C *****
      J=0
      DO 150 I=1,30
      L=30-I+1
      J=J+KOUNTE(L)
      IF(J.GE.1) GO TO 175
150 CONTINUE
175 CONTINUE
C
      GO TO (1,2), INDEX
1 PRINT 500,IM,IN,NR,NC,INT
      GO TO 3
2 PRINT 550,NR,NC,INT
3 CONTINUE
C
      DO 250 I=1,L
      K=I-1
250 PRINT 501,K,KOUNTE(I)
      PUNCH 600,(KOUNTE(I),I=1,L)
C *****
C FORMAT DECLARATION
C *****
500 FORMAT(21H1UNIVARIATE FREQUENCY//
116H0SIZE OF SQUARES16,1H*13/
29H GRID OF 14,1H*13,2X,7HSQUARES/
416H CLASS INTERVAL=16//
37H0 X,6X,4HF(X)/)
550 FORMAT(21H1UNIVARIATE FREQUENCY//
19H GRID OF 14,1H*13,2X,7HSQUARES/
216H CLASS INTERVAL=16//
37H0 X,6X,4HF(X)/)
501 FORMAT(1X,16,1I10)

```

```
600 FORMAT(1I10)  
RETURN  
END
```

INPUT DATA

Abbreviations:

(A): alphanumeric field

(I): integer value (must be right adjusted in its field)

(F): floating point number (must be punched with a decimal)

Control card (111)

Columns	1	(I)	INDEX:	Specifies the form of the input data
			= 1	(each element of the data gives the coordinates of the location and the number of observations at that location.
			= 2	(the data is presented as a matrix of NR x NC cells and each element of the matrix represents the number of observations at the corresponding cell)

Observed distribution cards

Only one of the following (a or b) sets of cards is required, depending on the value specified for INDEX in the control card. This set of cards must follow the control card described above.

a) INDEX = 1

Distribution control card (2I5,2F10.0,8I5)

Columns	1-5	(I)	NOBS:	total number of observations
	6-10	(I)	INT:	grouping size for counting purposes
	11-20	(F)	XDIM:	dimension of the study area in the X-direction
	21-30	(F)	YDIM:	dimension of the study area in the Y-direction
	31-35	(I)	NSIZ(1)	Each pair NSIZ(1)-MSIZ(I) indicates
	35-40	(I)	MSIZ(1)	the desired number of columns and
	41-45	(I)	NSIZ(2)	rows, respectively, of the matrix on
	46-50	(I)	MSIZ(2)	which the count operation is to be
	51-55	(I)	NSIZ(3)	performed. A maximum of four
	56-60	(I)	MSIZ(3)	different pairs is allowed simul-
	61-65	(I)	NSIZ(4)	taneously.
	66-70	(I)	MSIZ(4)	

Format specification card (6A10)

Columns	1-60	(A)	IVAR:	this is the variable format in which the observed values are to be read, i.e., (2F10.4, 1I10) (must be punched with the parentheses)
---------	------	-----	-------	---

Observed values cards (IVAR)

Columns	(F)	XC:	X-coordinate of the location
	(F)	YC:	Y-coordinate of the location
	(I)	N:	number of observations at the location

b) INDEX = 2

Distribution control card (3I5,5X,8I5)

Columns	1-5	(I)	NR:	number of rows in the data matrix
	6-10	(I)	NC:	number of columns in the data matrix
	11-15	(I)	INT:	grouping size for counting purposes
	21-25	(I)	NSIZ(1)	Each pair NSIZ(I)-MSIZ(I) indicates
	26-30	(I)	MSIZ(1)	the desired number of columns and
	31-35	(I)	NSIZ(2)	rows, respectively, of the matrix on
	36-40	(I)	MSIZ(2)	which the count operation is to be
	41-45	(I)	NSIZ(3)	performed. A maximum of four
	46-50	(I)	MSIZ(3)	different pairs is allowed simul-
	51-55	(I)	NSIZ(4)	taneously.
	56-60	(I)	MSIZ(4)	

Format specification card (6A10)

Columns	1-60	(A)	IVAR:	this is the variable format in which the
				observed values are to be read, i.e., (10I4)
				(must be punched with the parentheses).

Observed values cards (IVAR)

Reads in the matrix ((MO(I,J), J=1, NC), I=1, NR) of observed values in format IVAR as specified above and follows the row order (1st row, 2nd row, ..., NRth row)

END OF INPUT DATA

NOTE: In order to be consistent with the present dimension of the arrays in the program the following limits must be observed:

XC(I) ≤ 1000	MO(I,J) ≤ 3600
YC(I) ≤ 1000	NR ≤ 60
N(I) ≤ 1000	NC ≤ 60

OUTPUT

Printed Output

- 1) Count of the number of cells within each frequency class of the corresponding univariate distribution (i.e., number of cells with INT, 2INT, 3INT, ..., iINT observations, where INT is the grouping size).

Punched Output

- a) Punches the information contained in 1) above for further use (Format (1110))

NOTE: These printed and the punched outputs are produced for every grid size as specified in the "distribution control card" (see Input Data).

C INPUT DATA EXAMPLE

C

1

```
      85      1      50.      100.      10      10      5      5
(2F10.4,1I10)
      41.0685      51.7585      1
      41.8828      51.3944      1
      41.9067      52.4488      1
      41.4042      52.0768      1
      .          .          .
      .          .          .
      .          .          .
      .          .          .
      29.6699      33.2549      1
```

2

```
      10      10      1      5      5      0      0      0      0
(10I4)
      4      2      2      4      2      0      1      2      4      1
      2      0      1      5      0      1      2      1      2      2
      1      3      2      3      4      5      1      2      2      0
      3      2      1      4      1      1      1      1      3      1
      1      1      1      4      0      2      0      2      2      3
      2      2      2      2      3      2      3      1      1      2
      1      1      4      2      2      2      5      1      3      3
      2      4      2      1      2      1      2      1      3      4
      3      2      2      3      2      4      2      1      3      1
      4      2      1      0      2      0      0      4      0      3
```

C END OF INPUT DATA

C

C OUTPUT EXAMPLE

UNIVARIATE FREQUENCY

SIZE OF SQUARES 10 * 5
GRID OF 10 * 10 SQUARES
CLASS INTERVAL= 1

X	F(X)
0	80
1	3
2	4
3	4
4	3
5	2
6	2
7	0
8	0
9	0
10	0
11	0
12	1
13	0

14	0
15	0
16	1

UNIVARIATE FREQUENCY

SIZE OF SQUARES 20*10
 GRID OF 5 * 5 SQUARES
 CLASS INTERVAL= 1

X	F(X)
0	11
1	3
2	0
3	0
4	2
5	1
6	3
7	2
8	0
9	1
10	0
11	0
12	1
13	0
14	0
15	0
16	1

UNIVARIATE FREQUENCY

GRID OF 5 * 5 SQUARES
 CLASS INTERVAL= 1

X	F(X)
0	0
1	0
2	0
3	1
4	0
5	1
6	5
7	4
8	4
9	5
10	1
11	2
12	1
13	1

PROGRAM FITUNIV

PURPOSE

This program fits the following univariate models to observed distributions:

- 1) Poisson
- 2) Binomial
- 3) Negative Binomial
- 4) Neyman's Type A
- 5) Poisson-Binomial
- 6) Poisson-Negative Binomial

Given an observed univariate distribution, the program:

- 1) computes statistics (mean, variance, variance/mean ratio) of the observed distribution;
- 2) estimates the parameters corresponding to the above models using moment and maximum likelihood methods; and
- 3) compares observed to expected values by means of the chi-square goodness-of-fit test.

References:

Rogers, A. (1965). A stochastic analysis of the spatial clustering of retail establishments. Journal of the American Statistical Association, 60, 1094-1103.

Rogers, A. and Gomar, N. (1969). Statistical inference in quadrat analysis. Geographical Analysis, forthcoming.

Rogers, A. (1969). Quadrat Analysis of Urban Dispersion. Environment, forthcoming.

FITUNIV

-RUN,S.

LGO.

,

PROGRAM FITUNIV (INPUT,OUTPUT)

REAL N

DIMENSION INDEX(6), NN(5)

COMMON F(100), PHI(100)/PARAM/U1, V, HK, NOBS, NSUP, N

C

C *****

C SUBROUTINES

C

C POISSON DISTRIBUTION

C POISSN = PARAMETERS AND FREQUENCY

C BINOMIAL DISTRIBUTION

C BINOM = PARAMETERS AND FREQUENCY

C NEGATIVE BINOMIAL DISTRIBUTION

C NEGBI = PARAMETERS AND FREQUENCY

C NEYMAN TYPE A DISTRIBUTION

C NTAPAR = PARAMETERS

C NTAFRQ = FREQUENCY

C POISSON BINOMIAL DISTRIBUTION

C PBPAR = PARAMETERS

C PBFRQ = FREQUENCY

C POISSON NEGATIVE BINOMIAL DISTRIBUTION

C PNBPAR = PARAMETERS

C PNBFRQ = FREQUENCY

C

C CHISQ = CHI SQUARE GOODNESS-OF-FIT TEST

C FACT = FACTORIAL FUNCTION

C *****

C

C

C

C *****

C READ DATA

C *****

603 READ 903, INDEX, NN, NSOP

903 FORMAT (6I1, 4X, 5I1, 15)

601 DO 10 I = 1,100

READ 901, F(I)

901 FORMAT (F10.0)

IF (F(I) .LT. 0) GO TO 11

10 CONTINUE

C

C *****

C COMPUTE SAMPLE STATISTICS

C *****

11 NCONTR = ABS(F(I)) - 1.

NSUP = NSOP + 1

NOBS = I - 1

IF (NSUP .GT. NOBS .OR. NSUP .EQ. 1) NSUP = NOBS

N = 0.

SUMRF = 0.

SUMR2F = 0.

SUMR3F = 0.

```

DO 30 I = 1,NOBS
IF (F(I) .EQ. 0) GO TO 30
R = I - 1
N = N + F(I)
SUMRF = SUMRF + R*F(I)
SUMR2F = SUMR2F + R**2*F(I)
SUMR3F = SUMR3F + R**3*F(I)
30 CONTINUE
U1 = SUMRF / N
U2 = (SUMR2F - N*U1**2) / (N - 1.)
U3 = (SUMR3F - 3.*U1*SUMR2F + 2.*N*U1**3) / (N - 3. - 2./N)
RATIO = U2 / U1
V = (U2 - U1) / U1
HK = U1 / V
TEST = (2.*U2**2 - U1*(U2 + U3)) / (U2**2 - U1*(U1 + U2 + U3))
PRINT 202, U1, RATIO, U2
202 FORMAT (1H1/13H SAMPLE MEAN , F15.5, 5X, 20HVARIANCE/MEAN RATIO ,
1F12.5 // 17H SAMPLE VARIANCE , F15.5)
PRINT 201, TEST
201 FORMAT (/70H MOMENT ESTIMATOR OF THE EXPONENT OF A POISSON BINOMIAL
DISTRIBUTION , F10.5//)

```

```

C
C *****
C CALL RELEVANT SUBROUTINE
C *****
DO 1000 I = 1,6
INDEXI = INDEX(I)
IF (INDEXI .EQ. 0) GO TO 111
GO TO (501, 502, 503, 504, 505, 506) INDEXI
501 CALL POISSN
GO TO 1000
502 CALL BINOM
GO TO 1000
503 CALL NEGBI
GO TO 1000
504 CALL NTAPAR
GO TO 1000
505 IF (NN(1) .EQ. 0) NN(1) = 2
DO 510 II = 1,5
IF (NN(II) .EQ. 0) GO TO 1000
CALL PBPAP (NN(II))
510 CONTINUE
GO TO 1000
506 CALL PNBPAR (TEST)
C
1000 CONTINUE
111 IF (NCONTR .EQ. 0) STOP
GO TO (601, 603) NCONTR
END

```

```

SUBROUTINE POISSN
  REAL N
  COMMON FR(100), PHI(100) / PARAM / U1, V, HK, NOBS, NSUP, N / FRQ
  1/ F(100)
C
C *****
C POISSON DISTRIBUTION - PARAMETERS AND FREQUENCY
C *****
  PRINT 101, U1
101 FORMAT (//10X, 33HFITTING OF A POISSON DISTRIBUTION //12X,
  117H MOMENT ESTIMATOR , 5X, 4HM = , F8.4)
  F(NSUP) = 0.
  DO 50 I = 2, NSUP
50 F(I - 1) = FR(I - 1)
  DO 60 I = NSUP, NOBS
60 F(NSUP) = F(NSUP) + FR(I)
C
  PHI(1) = EXP( -U1)*N
  SUM = PHI(1)
  DO 10 I = 3, NSUP
  XI = I - 2
  PHI(I - 1) = PHI(I - 2)*U1 / XI
  10 SUM = SUM + PHI(I - 1)
  PHI(NSUP) = N - SUM
  PRINT 100
100 FORMAT ( / 13X, 9H OBSERVED , 8X, 8HEXPECTED / 13X, 1 H FREQUENCY
  1, 7X, 9HFREQUENCY )
  CALL CHISQ(NSUP, 1)
  RETURN
  END

```

```

SUBROUTINE BINOM
  REAL N
  COMMON FR(100), PHI(100) / PARAM / U1, V, HK, NOBS, NSUP, N / FRQ
  1 / F(100)
C
C *****
C BINOMIAL DISTRIBUTION - PARAMETERS AND FREQUENCY
C *****
  XN = NOBS - 1
  P = U1 / XN
  NN = XN
  PRINT 101, NN, P
101 FORMAT (//10X, 44HFITTING OF A BINOMIAL DISTRIBUTION WITH N = , I3
  1// 13X, 17HMOMENT ESTIMATOR , 5X, 4HP = , F8.4)
  F(NSUP) = 0.
  DO 50 I = 2, NSUP
50 F(I - 1) = FR(I - 1)
  DO 60 I = NSUP, NOBS
60 F(NSUP) = F(NSUP) + FR(I)
C
  Q = 1. - P
  PHI(1) = N * Q**NN
  SUM = PHI(1)
  DO 10 I = 3, NSUP
  XI = I - 2
  PHI(I - 1) = (XN - XI + 1.) * P * PHI(I - 2) / (XI * Q)
10 SUM = SUM + PHI(I - 1)
  PHI(NSUP) = N - SUM
  PRINT 100
100 FORMAT ( / 13X, 9H OBSERVED , 8X, 8HEXPECTED / 13X, 1 H FREQUENCY
  1, 7X, 9HFREQUENCY )
C
  CALL CHISQ(NSUP, 1)
  RETURN
  END

```

```

SUBROUTINE NEGBI
  REAL N
  COMMON FR(100), PHI(100) / ANNEX / S(100) / PARAM / U1, V, HKK,
  1NOBS, NSUP, N / FRQ / F(100)
C
C *****
C NEGATIVE BINOMIAL DISTRIBUTION - PARAMETERS AND FREQUENCY
C *****
  HK = HKK
  IF (HK .LT. 0) GO TO 25
  PRINT 100, U1, HK
100 FORMAT (//10X, 43HFITTING OF A NEGATIVE BINOMIAL DISTRIBUTION //
  112X, 18H MOMENT ESTIMATORS, 5X, 4HM = , F8.4, 6X, 4HK = , F8.4)
  KIT=1
  GO TO 15
C
C *****
C MAXIMUM LIKELIHOOD ESTIMATORS
C *****
  26 INDEX = 0
  S(1) = N - FR(1)
  DO 10 J = 3, NOBS
10 S(J - 1) = S(J - 2) - FR(J - 1)
  5 R = HK / (HK + U1)
  Q = N * ALOG(R)
  QPRIME = N*(1./HK - 1./(HK + U1))
  DO 20 J = 2, NOBS
  XJ = J - 2
  Q = Q + S(J - 1) / (HK + XJ)
20 QPRIME = QPRIME - S(J - 1) / (HK + XJ)**2
  HK1 = HK - Q / QPRIME
  PRINT 200, HK, Q, QPRIME
200 FORMAT (90X, F8.4, 2F10.4)
  IF (HK1 .LT. 0) HK1 = HK / 2
  IF (ABS(HK1 - HK) .LT. 0.0001) GO TO 2
  HK = HK1
  INDEX = INDEX + 1
  IF (INDEX .EQ. 12) RETURN
  GO TO 5
  2 PRINT 300, U1, HK1
300 FORMAT (/30H MAXIMUM LIKELIHOOD ESTIMATORS, 5X, 4HM = , F8.4, 6X ,
  14HK = , F8.4)
C
C *****
C COMPUTE FREQUENCY DISTRIBUTION
C *****
  HK = HK1
  KIT=0
15 CONTINUE
  F(NSUP) = 0.
  DO 50 I = 2, NSUP
50 F(I - 1) = FR(I - 1)
  DO 60 I = NSUP, NOBS
60 F(NSUP) = F(NSUP) + FR(I)
C

```

60

```
R = HK / (HK + U1)
Q = 1. - R
PHI(1) = N*R**HK
SUM = PHI(1)
DO 9 I = 3, NSUP
  XI = I - 2
  PHI(I - 1) = PHI(I - 2)*Q*(HK + XI - 1.) / XI
9 SUM = SUM + PHI(I - 1)

C
  PHI(NSUP) = N - SUM
  PRINT 99
99 FORMAT ( / 13X, 9H OBSERVED , 8X, 8HEXPECTED / 13X, 1 H FREQUENCY
1, 7X, 9HFREQUENCY )

C
  CALL CHISQ(NSUP, 2)
  IF(KIT.EQ.1) GO TO 26
  RETURN
25 PRINT 101
101 FORMAT (//10X, 43HFITTING OF A NEGATIVE BINOMIAL DISTRIBUTION //
1* VARIANCE SMALLER THAN MEAN      MOMENT ESTIMATORS DO NOT EXIST*)
  HK = 100.
  GO TO 26
END
```

```

SUBROUTINE NTAPAR
  REAL N, M
  COMMON F(100), PHI(100) / ANNEX / G(100) / PARAM / U1, VV, XM,
  1NOBS, NSUP, N
C
C *****
C NEYMAN TYPE A DISTRIBUTION - PARAMETERS
C *****
  V = VV
  IF (V .LT. 0) GO TO 25
  M = XM
  PRINT 100, M, V
100 FORMAT (//10X, 41HFITTING OF A NEYMAN S TYPE A DISTRIBUTION // 12X
  1, 18H MOMENT ESTIMATORS , 5X, 4HM = , F8.4, 6X, 4HV = , F8.4)
  CALL NTAFRQ (V,M,2)
C
C *****
C MAXIMUM LIKELIHOOD ESTIMATORS
C *****
  26 INDEX = 0
  5 CALL NTAFRQ (V,M,1)
  Q = - N*U1
  QPRIME = 0.
  G(1) = PHI(2) / PHI(1)
  DO 10 I = 1,NOBS
  XI = I
  G(I + 1) = (XI + 1.) * PHI(I + 2) / PHI(I + 1)
  Q = Q + F(I)*G(I)
  10 QPRIME = QPRIME + F(I)*G(I)*(1./V - (V+1)*(G(I 1) G(I))/V**2)
  V1 = V - Q / QPRIME
  PRINT 200, V, Q, QPRIME
  200 FORMAT (90X, F8.4, 2F10.4)
C
  IF (V1 .LT. 0) V1 = V / 2
  M = U1 / V1
  IF (ABS(V1 - V) .LT. 0.0001) GO TO 2
  V = V1
  INDEX = INDEX + 1
  IF (INDEX .EQ.12) RETURN
  GO TO 5
  2 PRINT 300, M, V1
  300 FORMAT (//30H MAXIMUM LIKELIHOOD ESTIMATORS,5X,4HM = ,F8.4,6X,
  14HV = , F8.4)
  CALL NTAFRQ (V1,M, 2)
  RETURN
C
  25 PRINT 101
  101 FORMAT (//10X, 41HFITTING OF A NEYMAN S TYPE A DISTRIBUTION //
  1* VARIANCE SMALLER THAN MEAN      MOMENT ESTIMATORS DO NOT EXIST*)
  V = 10.
  M = U1 / V
  GO TO 26
  END

```

62

```

SUBROUTINE NTAFRQ (V, M, INDEX)
  REAL N, M
  COMMON FR(100), PHI(100) / PARAM / U1, VV, XM, NOBS, NSUP, N / FRQ
  1 / F(100)
C
C *****
C NEYMAN TYPE A DISTRIBUTION - FREQUENCY
C *****
  EX = EXP(-V)
  PHI(1) = EXP(M*(EX - 1.))*N
  LIMIT = NOBS + 2
  II = 2
999 TOTAL = 0.
  K = II - 1
C
  DO 11 J = 1,K
    L = II - J
  11 TOTAL = TOTAL + V**(J - 1)*PHI(L) / FACT(J - 1)
  XK = K
  PHI(II) = M*V*EX*TOTAL / XK
  IF(INDEX .EQ. 1 .OR. II .NE. (NSUP - 1)) GO TO 1
  F(NSUP) = 0.
C
  DO 50 I = 2,NSUP
  50 F(I - 1) = FR(I - 1)
  DO 60 I = NSUP,NOBS
  60 F(NSUP) = F(NSUP) + FR(I)
  SUM = PHI(1)
  DO 21 I = 3,NSUP
  21 SUM = SUM + PHI(I - 1)
C
  PHI(NSUP) = N - SUM
  PRINT 100
  100 FORMAT ( / 13X, 9H OBSERVED , 8X, 8HEXPECTED / 13X, 1 H FREQUENCY
  1, 7X, 9HFREQUENCY )
  CALL CHISQ(NSUP, 2)
  RETURN
C
  10 II = II + 1
  IF (II .LE. LIMIT) GO TO 999
  RETURN
  END

```

```

SUBROUTINE PBPAR (NN)
  REAL N, M
  COMMON F(100), PHI(100) / ANNEX / G(100) / PARAM / U1, PP, XM,
  1NOBS, NSUP, N
C
C *****
C POISSON BINOMIAL DISTRIBUTION - PARAMETERS
C *****
  XNN = NN
  PRINT 100, NN
100 FORMAT (/10X, 52HFITTING OF A POISSON BINOMIAL DISTRIBUTION WITH
  1N = , I2/)
C
C *****
C MOMENT ESTIMATORS
C *****
  INDEX = 0
  IF (PP .LT. 0) GO TO 118
  P = PP / (XNN - 1.)
  IF (P .GT. 1.) GO TO 128
  M = (XNN - 1.) * XM / XNN
  PRINT 102, M, P
102 FORMAT (/12X, 18H MOMENT ESTIMATORS, 5X, 4HM = , F8.4, 6X, 4HP = ,
  1F8.4)
  CALL PBFRQ (P, M, NN, 2)
C
C *****
C DOUBLE ZERO FREQUENCY ESTIMATORS
C *****
18 C = XNN * ALOG(F(1) / N) / U1
  IF (C .LT. -XNN) GO TO 138
  IF (C .GT. -1.) GO TO 148
  IF (NN .NE. 2) GO TO 51
  P = 2. + C
  M = U1 / (XNN * P)
  GO TO 99
51 P = 1.
  JINDEX = 0
  6 Q = C * P + 1. - (1. - P)**NN
  QPRIME = C + XNN * (1. - P)**(NN - 1)
  P1 = P - Q / QPRIME
  PRINT 200, P, Q, QPRIME
  IF (ABS(P1 - P) .LT. 0.0001) GO TO 98
  P = P1
  JINDEX = JINDEX + 1
  IF (JINDEX .EQ. 12 .OR. P .LT. 0.001) GO TO 321
  GO TO 6
98 P = P1
  M = U1 / (XNN * P)
99 PRINT 101, M, P
101 FORMAT (/33H SAMPLE ZERO FREQUENCY ESTIMATORS , 2X, 4HM = , F8.4,
  16X, 4HP = , F8.4)
C
C *****
C MAXIMUM LIKELIHOOD ESTIMATORS

```

C *****

```

5 CALL PBFRQ (P, M, NN, 1)
  Q = - N*U1
  QPRIME = 0.
  G(1) = PHI(2) / PHI(1)
  DO 10 I = 1, NOBS
    XI = I
    G(I + 1) = (XI + 1.) * PHI(I + 2) / PHI(I + 1)
    Q = Q + F(I)*G(I)
10  QPRIME = QPRIME + F(I)*G(I)*((XNN - 1.) / (XNN*P)   (XNN*P   P
    11.)*(G(I + 1) - G(I)) / (XNN*P**2))
    P1 = P - Q / QPRIME
    PRINT 200, P, Q, QPRIME
200 FORMAT (90X, F8.4, 2F10.4)
    IF ( P1 .LT. 0) P1 = P / 2
    IF (P1 .GT. 1) P1 = (1. + P) / 2
    M = U1 / (XNN*P1)
    IF (ABS(P1 - P) .LT. 0.0001) GO TO 2
    P = P1
    INDEX = INDEX + 1
    IF (INDEX .EQ. 12 .OR. P.LT. 0.001) RETURN
    GO TO 5
2 PRINT 300, M, P1
300 FORMAT (/30H MAXIMUM LIKELIHOOD ESTIMATORS, 5X, 4HM = , F8.4, 6X,
14HP = , F8.4)
    CALL PBFRQ (P1, M, NN, 2)
    RETURN
118 PRINT 119
119 FORMAT (/ * VARIANCE SMALLER THAN MEAN      MOMENT ESTIMATORS DO NOT
1 EXIST*)
    GO TO 18
128 PRINT 129
129 FORMAT ( /      * U2 IS TOO LARGE      POISSON BINOMIAL DISTRIBUTION
1IMPOSSIBLE BY MOMENT ESTIMATORS*)
    GO TO 18
138 PRINT 139
139 FORMAT (/      * FO IS TOO SMALL      POISSON BINOMIAL DISTRIBUTION IM
1POSSIBLE BY SAMPLE ZERO FREQUENCY ESTIMATORS*)
    GO TO 321
148 PRINT 149
149 FORMAT (/      * FO IS TOO LARGE      POISSON BINOMIAL DISTRIBUTION IM
1POSSIBLE BY SAMPLE ZERO FREQUENCY ESTIMATORS*)
321 P = .9
    M = U1 / (XNN * P)
    GO TO 5
END

```

```

SUBROUTINE PBFRQ (P, M, NN, INDEX)
REAL N, M
COMMON FR(100), PHI(100) / PARAM / U1, PP, XM, NOBS, NSUP, N / FRQ
1 / F(100)

```

C

C *****

C POISSON BINOMIAL DISTRIBUTION - FREQUENCY

```

C *****
  PHI(1) = N*EXP(M*((1. - P)**NN - 1.))
  LIMIT = NOBS + 2
  II = 2
999 TOTAL = 0.
  K = NN
  IF ((II - 1) .LT. NN) K = II - 1
  DO 11 J = 1,K
  L = II - J
11 TOTAL = TOTAL + FACT(NN)*P**(J - 1)*PHI(L) / (FACT(J - 1)*FACT(NN
1- J)*(1. - P)**(J - 1))
  XK = II - 1
  PHI(II) = M*P*(1. - P)**(NN - 1)*TOTAL / XK
  IF (INDEX .EQ.1 .OR. II .NE. (NSUP - 1)) GO TO 1
  F(NSUP) = 0.
C
  DO 50 I = 2,NSUP
50 F(I - 1) = FR(I - 1)
  DO 60 I = NSUP,NOBS
60 F(NSUP) = F(NSUP) + FR(I)
  SUM = PHI(1)
  DO 21 I = 3,NSUP
21 SUM = SUM + PHI(I - 1)
  PHI(NSUP) = N - SUM
  PRINT 100
100 FORMAT ( / 13X, 9H OBSERVED , 8X, 8HEXPECTED / 13X, 1 H FREQUENCY
1, 7X, 9HFREQUENCY )
  CALL CHISQ(NSUP, 2)
  RETURN
10 II = II + 1
  IF (II .LE. LIMIT) GO TO 999
  RETURN
END

```

```

SUBROUTINE PNBPAR (XK)
  REAL N, M, K, K1, KM, MM
  COMMON F(100), PHI(100) / ANNEX / G(100) / PARAM / U1, PP, XM,
  INOBS, NSUP, N
C
C *****
C POISSON NEGATIVE BINOMIAL DISTRIBUTION - PARAMETERS
C *****
  PRINT 100
  100 FORMAT (/10X, 51HFITTING OF A POISSON NEGATIVE BINOMIAL DISTRIBUTION ,/)
  INDEX = 0
  MOM = 0
C
C *****
C MOMENT ESTIMATORS
C *****
  IF (PP .LT. 0) GO TO 118
  IF (XK .GT. 0) GO TO 128
  MOM = 1
  KM = - XK
  PM = PP / (KM + 1.)
  MM = (KM + 1.) * XM / KM
  PRINT 102, KM, MM, PM
  102 FORMAT (/12X, 18HMOMENTS ESTIMATORS ,5X, 4HK = , F8.4, 6X, 4HM = ,
  1F8.4, 6X, 4HP = , F8.4)
  CALL PNBFRQ (PM, MM, KM, 2)
C
C *****
C RATIO FIRST TWO FREQUENCIES ESTIMATORS
C *****
  18 JINDEX = 0
  C = ALOG(U1 * F(1) / F(2)) / PP
  IF (C .GE. 1 .OR. C .LE. 0) GO TO 148
  P = 1. / C
  6 Q = C * P - ALOG(1. + P)
  QPRIME = C - 1. / (1. + P)
  P1 = P - Q / QPRIME
  PRINT 200, P, Q, QPRIME
  IF (ABS(P1 - P) .LT. 0.001) GO TO 98
  P = P1
  JINDEX = JINDEX + 1
  IF (JINDEX .EQ. 12 .OR. P .LT. 0.001) GO TO 789
  GO TO 6
  98 P = P1
  K = PP / P - 1.
  IF (K .LT. 0) GO TO 138
  M = U1 / (K * P)
  PRINT 101, K, M, P
  101 FORMAT (/ * MOMENTS AND RATIO FIRST TWO FREQUENCIES ESTIMATORS * 2X,
  14HK = , F8.4, 6X, 4HM = , F8.4, 6X, 4HP = , F8.4)
  CALL PNBFRQ (P, M, K, 0)
C
C *****
C MAXIMUM LIKELIHOOD ESTIMATORS

```

```

C *****
5 Q = - N*U1
  QPRIME = 0.
  G(1) = PHI(2) / PHI(1)
  DO 10 I = 1,NOBS
    XI = I
    G(I + 1) = (XI + 1.) * PHI(I + 2) / PHI(I + 1)
    Q = Q + F(I)*G(I)
10 QPRIME = QPRIME + F(I) * G(I) * ((K + 1.) / (K * P)   (K * P   P
   1+ 1.) * (G(I + 1) - G(I)) / (K * P**2))
  P1 = P - Q / QPRIME
  PRINT 200, P, Q, QPRIME
200 FORMAT (90X, F8.4, 2F10.4)
  IF (P1 .LT. 0.005) RETURN

C
  CALL PNBFRQ (P1,M, K ,1)
  TOTAL = 0.
  DO 20 I = 3,NOBS
    LIMIT = I - 2
    SUM = 0.
    DO 21 J = 1,LIMIT
      XI = I
      XJ = J
21 SUM = SUM + PHI(J + 1) * XJ *P1**(I - J- 1) / ((XI   XJ) * (XI
   1XJ - 1.) * (1. +P1)**(I - J - 1))
20 TOTAL = TOTAL + F(I) * SUM / PHI(I)
  K1 = TOTAL / (N * M * (P1- ALOG(1. +P1)))
  M = U1 / (K1 * P1)
  IF (ABS(P1 - P) .LT. 0.001 .AND. ABS(K1 - K) .LT. . 1 ) GO TO 2
  P = P1
  K = K1
  INDEX = INDEX + 1
  IF (INDEX .EQ.12) RETURN
  CALL PNBFRQ (P, M, K, 1)
  GO TO 5
2 PRINT 300, K1, M, P1
300 FORMAT (/30H MAXIMUM LIKELIHOOD ESTIMATORS, 5X, 4HK = , F8.4, 6X,
   14HM = , F8.4, 6X, 4HP = , F8.4)
  CALL PNBFRQ (P1,M, K1,2)
  RETURN

C
118 PRINT 119
119 FORMAT (/ * VARIANCE SMALLER THAN MEAN      MOMENT ESTIMATORS DO NOT
   1 EXIST*)
  GO TO 18
128 PRINT 129
129 FORMAT (/ * K IS NEGATIVE      POISSON NEGATIVE BINOMIAL DISTRIBUTIO
   1N IMPOSSIBLE BY MOMENTS ESTIMATORS*)
  GO TO 18
138 PRINT 139
139 FORMAT (/ * K IS NEGATIVE      POISSON NEGATIVE BINOMIAL DISTRIBUTIO
   1N IMPOSSIBLE BY RATIO FIRST TWO FREQUENCIES METHOD*)
  GO TO 789
148 PRINT 149

```

```

149 FORMAT (/ *      P = 0      POISSON NEGATIVE BINOMIAL DISTRIBUTIO
IN IMPOSSIBLE BY RATIO FIRST TWO FREQUENCIES METHOD*)

```

```

789 IF (MOM .EQ. 0) RETURN

```

```

      K = KM

```

```

      M = MM

```

```

      P = PM

```

```

      GO TO 5

```

```

      END

```

```

      SUBROUTINE PNBFRQ (P, M, K, INDEX)

```

```

      REAL N, M, K

```

```

      COMMON FR(100), PHI(100) / PARAM / V, PP, XM, NOBS, NSUP, N / FRQ
1 / F(100)

```

```

C *****
C *****

```

```

C POISSON NEGATIVE BINOMIAL - FREQUENCY

```

```

C *****

```

```

      PHI(1) = N * EXP(M * (1. / (1. + P)**K - 1.))

```

```

      LIMIT = NOBS + 2

```

```

      II = 2

```

```

999 TOTAL = 0.

```

```

      KK = II - 1

```

```

      PROD = 1.

```

```

C

```

```

      DO 11 J = 1, KK

```

```

      L = II - J

```

```

      XJ = J

```

```

      PROD = PROD * (K + XJ - 1.)

```

```

11 TOTAL = TOTAL + PROD * P**(J - 1) * PHI(L) / (FACT(J - 1) * (1.
1 P)**(J - 1))

```

```

      XK = KK

```

```

      PHI(II) = M * P * TOTAL / (XK * (1. + P)**(K + 1.))

```

```

      IF (INDEX .EQ. 1 .OR. II .NE. (NSUP - 1)) GO TO 1

```

```

      F(NSUP) = 0.

```

```

C

```

```

      DO 50 I = 2, NSUP

```

```

50 F(I - 1) = FR(I - 1)

```

```

      DO 60 I = NSUP, NOBS

```

```

60 F(NSUP) = F(NSUP) + FR(I)

```

```

      SUM = PHI(1)

```

```

      DO 21 I = 3, NSUP

```

```

21 SUM = SUM + PHI(I - 1)

```

```

      PHI(NSUP) = N - SUM

```

```

      PRINT 100

```

```

100 FORMAT ( / 13X, 9H OBSERVED , 8X, 8HEXPECTED / 13X, 1 H FREQUENCY
1, 7X, 9HFREQUENCY )

```

```

      CALL CHISQ(NSUP, 3)

```

```

      IF (INDEX .EQ. 2) RETURN

```

```

10 II = II + 1

```

```

      IF (II .LE. LIMIT) GO TO 999

```

```

      RETURN

```

```

      END

```

```

SUBROUTINE CHISQ(NSUP, NPARAM)
COMMON FR(100), PHI(100) / ANNEX / NG(50), NG1(5) / FRQ / F(1)
C *****
C CHI-SQUARE GOODNESS-OF-FIT TEST
C *****
      K = 0
C *****
C GROUP DATA FOR MINIMUM EXPECTATION EQUAL 5
C *****
      DO 10 I = 1, NSUP
        IF (PHI(I) .GE. 5 .AND. K .EQ. 0) GO TO 10
        SUM = 0.
        DO 20 J = I, NSUP
          SUM = SUM + PHI(J)
          IF (SUM .GE. 5) GO TO 30
        20 CONTINUE
C
        IF (K .EQ. 0) GO TO 21
        NG(K) = NG(K) + NSUP - I + 1
        GO TO 110
      21 K = 1
        NG(1) = NSUP - I + 2
        GO TO 110
      30 K = K + 1
        NG(K) = J - I + 1
        I = J
      10 CONTINUE
C
      110 ISUM = 0
        IF (K .EQ. 0) GO TO 50
        DO 23 IJ = 1, K
          23 ISUM = ISUM + NG(IJ)
        50 NSAME = NSUP - ISUM
          NDF = NSAME + K - NPARAM - 1
C *****
C GROUP DATA FOR MINIMUM EXPECTATION EQUAL 1
C *****
          K1 = 0
          DO 510 I = 1, NSUP
            IF (PHI(I) .GE. 1 .AND. K1 .EQ. 0) GO TO 510
            SUM = 0.
            DO 520 J = I, NSUP
              SUM = SUM + PHI(J)
              IF (SUM .GE. 1) GO TO 530
            520 CONTINUE
C
            IF (K1 .EQ. 0) GO TO 521
            NG1(K1) = NG1(K1) + NSUP - I + 1
            GO TO 610
          521 K1 = 1
            NG1(1) = NSUP - I + 2
            GO TO 610

```

```

530 K1 = K1 + 1
    NG1(K1) = J - I + 1
    I = J
510 CONTINUE
610 ISUM = 0
    IF (K1 .EQ. 0) GO TO 550
    DO 523 IJ = 1,K1
523 ISUM = ISUM + NG1(IJ)
550 NSAME1 = NSUP - ISUM
    NDF1 = NSAME1 + K1 - NPARAM - 1
    L = K
    IF (L .GT. K1) L = K1
    LJ = L + 1
    IF (K .EQ. 0) GO TO 204
    IF (K1 .EQ. 0) GO TO 203

C
C *****
C PRINT GROUPED DATA
C *****
    DO 220 I = 1,L
        IJ = I - 1
220 PRINT 199, IJ, F(I), PHI(I), NG(I), NG1(I)

C
199 FORMAT (5X, I3, 6X, F6.0, 9X, F8.2, 80X, I3, 1 X, I3)
    IF (K .EQ. K1) GO TO 204
    IF (L .EQ. K1) GO TO 203
    DO 211 I = LJ,K1
        IJ = I - 1
211 PRINT 209, IJ, F(I), PHI(I), NG1(I)
209 FORMAT (5X, I3, 6X, F6.0, 9X, F8.2, 93X, I3)
    GO TO 204
203 DO 212 I = LJ,K
    IJ = I - 1
212 PRINT 219, IJ, F(I), PHI(I), NG(I)
219 FORMAT (5X, I3, 6X, F6.0, 9X, F8.2, 80X, I3)
204 LK = K1 + K - L + 2
    DO 221 I = LK,NSUP
        IJ = I - 2
221 PRINT 200, IJ, F(I - 1), PHI(I - 1)
200 FORMAT (5X, I3, 6X, F6.0, 9X, F8.2)
        NR = NSUP - 1
        PRINT 201, NR, F(NSUP), PHI(NSUP)
201 FORMAT (5X, I3, 1H+ , 5X, F6.0, 9X, F8.2)

C
C *****
C COMPUTE CHI-SQUARE FOR MINIMUM EXPECTATION EQUAL 5
C *****
    IF (NDF .LE. 0) GO TO 80
    CHI = 0.
    IF (NSAME .EQ. 0) GO TO 15
    DO 18 I = 1,NSAME
        SUM = (F(I) - PHI(I))**2 / PHI(I)
18 CHI = CHI + SUM
    IF (K .EQ. 0) GO TO 231
15 L = 0

```

```

C
DO 123 I = 1,K
NK = NG(I)
SUMF = 0.
SUMPFI = 0.
DO 124 J = 1,NK
NIJ = NSAME + L + J
SUMF = SUMF + F(NIJ)
124 SUMPFI = SUMPFI + PHI(NIJ)
L = L + NK
SUM = (SUMF - SUMPFI)**2 / SUMPFI
123 CHI = CHI + SUM
C
231 PRINT 300, NDF, CHI
300 FORMAT (20X, *NUMBER OF DEGREES OF FREEDOM*, 15 / 38X, *CHI SQUARE
1*, F8.3)
GO TO 1
80 PRINT 301, NDF
301 FORMAT (20X, *NUMBER OF DEGREES OF FREEDOM*, 15 / 3 X, *CHI SQUARE
1 TEST IMPOSSIBLE*)
1 IF (NDF1 .LE. 0) GO TO 800
C
C *****
C COMPUTE CHI-SQUARE FOR MINIMUM EXPECTATION EQUAL 1
C *****
CHI = 0.
IF (NSAME1 .EQ. 0) GO TO 815
DO 818 I = 1,NSAME1
SUM = (F(I) - PHI(I))**2 / PHI(I)
818 CHI = CHI + SUM
C
IF (K1 .EQ. 0) GO TO 931
815 L = 0
DO 923 I = 1,K1
NK = NG1(I)
SUMF = 0.
SUMPFI = 0.
DO 924 J = 1,NK
NIJ = NSAME1 + L + J
SUMF = SUMF + F(NIJ)
924 SUMPFI = SUMPFI + PHI(NIJ)
L = L + NK
SUM = (SUMF - SUMPFI)**2 / SUMPFI
923 CHI = CHI + SUM
931 PRINT 300, NDF1, CHI
RETURN
800 PRINT 301, NDF1
RETURN
END

```

FUNCTION FACT(N)

```

C
C *****

```

C FACTORIAL FUNCTION

C *****

FACT = 1.

IF (N .LE. 1) RETURN

DO 10 I = 2,N

X = I

10 FACT = FACT*X

RETURN

END

INPUT DATA

Abbreviations:

- (I): integer value (must be right adjusted in its field)
- (F): floating point number (must be punched with a decimal)

Control Card (6I1,4X,5I1)

Columns	1	(I) INDEX(1):	The vector INDEX(I) contains integers from
	2	(I) INDEX(2):	0 to 6 which indicate the models to which
	3	(I) INDEX(3):	the observed distribution is to be fitted.
	4	(I) INDEX(4):	If any of the elements of INDEX(I) are:
	5	(I) INDEX(5):	
	6	(I) INDEX(6):	
			=1 (the Poisson Distribution is fitted)
			=2 (the Binomial distribution is fitted)
			=3 (the Negative Binomial distribution is fitted)
			=4 (the Neyman's Type A distribution is fitted)
			=5 (the Poisson-Binomial distribution is fitted)
			=6 (the Poisson-Negative Binomial distribution is fitted)
			=0 (no model is fitted)
			(see example below)

Columns	11 (I)	NN(1): Specifies the value for the exponent
	12 (I)	NN(2): of the Poisson-Binomial distribution
	13 (I)	NN(3): (no more than five different values
	14 (I)	NN(4): are allowed)
	15 (I)	NN(5):

Examples:

INDEX(I) = [1,2,3,4,5,6] All six models are fitted and the
 NN(J) = [2,3,4,5,6] Poisson Binomial is fitted with
 exponents 2,3,4,5 and 6.

INDEX(I) = [1,4,0,0,0,0] Only the Poisson and the Neyman's
 NN(J) = [0,0,0,0,0] Type A models are fitted.

Note that when less than six models are desired, the rest of the vector
 INDEX must be filled in with zeros. The same applies for the vector NN
 whenever the Poisson-Binomial model is not requested.

Observed distribution cards

One card per value. Each card contains:

Columns	1-10 (F)	F(I): observed values for frequency classes (0 through maximum)
---------	----------	--

This set of cards must be ended with a card containing

Columns	9-10 (I)	NCON: control variable
		= -1 (the preceeding set of data was the last)
		= -2 (read in observed values for the next distribution and the previous control card is not modified)
		= -3 (read in new control card and observed values for the next distribution)

75

END OF INPUT DATA

Example

1) control card (distribution I and II)

2) observed distribution I

NCON = -2

3) observed distribution II

NCON = -3

1) control card (distribution III)

3) observed distribution III

NCON = -1

NOTE: In order to be consistent with the present dimension of the arrays
in the program, the following limit must be observed:

dimension of the vector $F(I) \leq 100$

OUTPUT

Printed output

1) Statistics of the observed distribution:

- mean
- variance
- variance/mean ratio

2) For each theoretical model:

- parameter estimation (using method of moments, maximum likelihood, sample zero frequency, ratio of first to second frequencies as relevant)
- observed and expected values
- chi-square goodness-of-fit test (this test is performed using a minimum expectation equal both to 5 and 1)

Punched output

No punched output.

C INPUT DATA EXAMPLE

C

123400 00000

10

27

34

14

12

3

-3

134000 00000

40

22

13

6

6

2

3

3

0

1

1

0

0

1

1

1

-1

C END OF INPUT DATA

C OUTPUT EXAMPLE

SAMPLE MEAN 2.00000 VARIANCE/MEAN RATIO .78788
 SAMPLE VARIANCE 1.57576
 MOMENT ESTIMATOR OF THE EXPONENT OF A POISSON BINOMIAL DISTRIBUTION .854

FITTING OF A POISSON DISTRIBUTION

MOMENT ESTIMATOR M= 2.0000

	OBSERVED FREQUENCY	EXPECTED FREQUENCY
0	10.	13.53
1	27.	27.07
2	34.	27.07
3	14.	23.04
4	12.	9.02
5+	3.	5.27

NUMBER OF DEGREES OF FREEDOM 4
 CHI SQUARE 5.562
 NUMBER OF DEGREES OF FREEDOM 4
 CHI SQUARE 5.562

FITTING OF A BINOMIAL DISTRIBUTION WITH N = 5

MOMENT ESTIMATOR P= .4000

	OBSERVED FREQUENCY	EXPECTED FREQUENCY
0	10.	7.78
1	27.	25.92
2	34.	34.56
3	14.	23.04
4	12.	7.68
5+	3.	1.02

NUMBER OF DEGREES OF FREEDOM 3
 CHI SQUARE 8.791
 NUMBER OF DEGREES OF FREEDOM 4
 CHI SQUARE 10.480

FITTING OF A NEGATIVE BINOMIAL DISTRIBUTION

VARIANCE SMALLER THAN MEAN MOMENT ESTIMATORS DO NOT EXIST

FITTING OF A NEYMAN'S TYPE A DISTRIBUTION

VARIANCE SMALLER THAN MEAN MOMENT ESTIMATORS DO NOT EXIST

SAMPLE MEAN 2.00000 VARIANCE/MEAN RATIO 4.54545
 SAMPLE VARIANCE 9.09091
 MOMENT ESTIMATOR OF THE EXPONENT OF A POISSON BINOMIAL DISTRIBUTION .3962

FITTING OF A POISSON DISTRIBUTION

MOMENT ESTIMATOR M= 2.0000

	OBSERVED FREQUENCY	EXPECTED FREQUENCY
0	40.	13.53
1	22.	27.07
2	13.	27.07
3	6.	18.04
4	6.	9.02
5	2.	3.61
6	3.	1.20
7	3.	.34
8	0.	.09
9	1.	.02
10	1.	.00
11	0.	.00
12	0.	.00
13	1.	.00
14	1.	.00
15+	1.	.00

NUMBER OF DEGREES OF FREEDOM 4
 CHI SQUARE 80.432
 NUMBER OF DEGREES OF FREEDOM 5
 CHI SQUARE 122.495

FITTING OF A NEGATIVE BINOMIAL DISTRIBUTION

MOMENT ESTIMATORS M= 2.0000 K= .5641

	OBSERVED FREQUENCY	EXPECTED FREQUENCY
0	40.	42.57
1	22.	18.73
2	13.	11.42
3	6.	7.62
4	6.	5.29
5	2.	3.77
6	3.	2.73
7	3.	1.99
8	0.	1.47
9	1.	1.09
10	1.	.81
11	0.	.61
12	0.	.46
13	1.	.35
14	1.	.26

80

15+ 1. .83

MAXIMUM LIKELIHOOD ESTIMATORS M= 2.0000 K= .6114

	OBSERVED FREQUENCY	EXPECTED FREQUENCY
0	40.	41.16
1	22.	19.27
2	13.	11.89
3	6.	7.93
4	6.	5.48
5	2.	3.87
6	3.	2.77
7	3.	2.01
8	0.	1.46
9	1.	1.07
10	1.	.79
11	0.	.58
12	0.	.43
13	1.	.32
14	1.	.24
15+	1.	.71

NUMBER OF DEGREES OF FREEDOM 4
 CHI SQUARE 1.467
 NUMBER OF DEGREES OF FREEDOM 9
 CHI SQUARE 5.009

FITTING OF A NEYMANS'S TYPE A DISTRIBUTION

MOMENT ESTIMATORS M= .5641 V= 3.5455

	OBSERVED FREQUENCY	EXPECTED FREQUENCY
0	40.	57.82
1	22.	3.34
2	13.	6.01
3	6.	7.33
4	6.	6.91
5	2.	5.50
6	3.	3.97
7	3.	2.77
8	0.	1.94
9	1.	1.37
10	1.	.97
11	0.	.68
12	0.	.46
13	1.	.31
14	1.	.21
15+	1.	.39

NUMBER OF DEGREES OF FREEDOM 4
 CHI SQUARE 78.832

81

NUMBER OF DEGREES OF FREEDOM 9
CHI SQUARE 125.03

MAXIMUM LIKELIHOOD ESTIMATORS M= 1.1439 V= 1.7483

	OBSERVED FREQUENCY	EXPECTED FREQUENCY
0	40.	38.87
1	22.	13.53
2	13.	14.19
3	6.	11.29
4	6.	7.95
5	2.	5.33
6	3.	3.46
7	3.	2.18
8	0.	1.34
9	1.	.80
10	1.	.46
11	0.	.27
12	0.	.15
13	1.	.08
14	1.	.04
15+	1.	.05

NUMBER OF DEGREES OF FREEDOM 4
CHI SQUARE 10.996
NUMBER OF DEGREES OF FREEDOM 7
CHI SQUARE 17.519

PROGRAM CBIVPURPOSE

Given sets of observations on two variables, say X and Y, the program:

- 1) forms two dimensional arrays containing the number of observations of each variable in each of the cells of a grid whose dimensions are specified in the control card; and
- 2) counts the number of cells in each frequency class for the corresponding bivariate distribution.

The sets of observed values may correspond to either a rectangular or a square study region, the shape of the cells may be either rectangular or square, and the number of cells in the X-direction may differ from the number of cells in the Y-direction.

47

CIBIV

83

RUN,S.
LGO.

```

PROGRAM CBIV (INPUT,OUTPUT,PUNCH)
COMMON INT,INK,M(60,60),NAUX(60,60)
DIMENSION XC(1000),YC(1000),N(1000),MO(60,60)
DIMENSION IVAR(6)
DO 555 KOUNT=1,2
READ 1000,INDEX
GO TO (1,2),INDEX

C
C *****
C READ DATA GIVEN BY COORDINATES AND FORM MATRIX
C *****
1 READ 1001,NOBS,INT,XDIM,YDIM,NSIZ,MSIZ
  READ 1002,IVAR
  READ IVAR,(XC(I),YC(I),N(I),I=1,NOBS)

C
  IF(NSIZ.EQ.0.OR.MSIZ.EQ.0) CALL EXIT
  A=NSIZ
  B=MSIZ
  NN=XDIM/A
  MM=YDIM/B
  A=NN
  B=MM
  KE=MSIZ
  KI=NSIZ

C
  DO 101 K=1,KE
    DO 101 J=1,KI
101  M(K,J)=0
    DO 102 L=1,NOBS
      II=YC(L)/B+1.0
      JJ=XC(L)/A+1.0
      KK=KE-II+1
102  M(KK,JJ)=M(KK,JJ)+N(L)
    GO TO (125,175), KOUNT

C
C *****
C READ DATA GIVEN BY MATRIX AND FORM NEW MATRIX
C *****
2 READ 1003,NR,NC,INT,NSIZ,MSIZ
  READ 1002,IVAR
  READ IVAR,((MO(I,J),J=1,NC),I=1,NR)

C
  IF (NSIZ.EQ.0.OR.MSIZ.EQ.0) CALL EXIT
  MM=NR/MSIZ
  NN=NC/NSIZ

C
  DO 201 J=1,NC
    K=1
    NT=MM
    II=1
10  IA=0
    DO 202 L=K,NT

```

```

202  IA=IA+MO(L,J)
      M(II,J)=IA
      K=K+MM
      NT=K+MM-1
      II=II+1
      IF(NT.LE.NR) GO TO 10
201  CONTINUE
C
      DO 203 J=1,MSIZ
      K=1
      NT=NN
      JJ=1
11   IA=0
      DO 204 L=K,NT
204  IA=IA+M(J,L)
      M(J,JJ)=IA
      K=K+NN
      NT=K+NN-1
      JJ=JJ+1
      IF(NT.LE.NC) GO TO 11
203  CONTINUE
      KE=MSIZ
      KI=NSIZ
C
C
      GO TO (125,175), KOUNT
125  INK=INT
      DO 150 I=1,KE
      DO 150 J=1,KI
150  NAUX(I,J)=M(I,J)
      GO TO 555
175  CALL SUMM (KE,KI)
555  CONTINUE
C
C *****
C FORMAT DECLARATION
C *****
1000 FORMAT(1I1)
1001 FORMAT(2I5,2F10.0,2I5)
1002 FORMAT(6A10)
1003 FORMAT(3I5,5X,2I5)
      STOP
      END

```

45

```

SUBROUTINE SUMM (NR,NC)
COMMON INT2,INT1,NB(60,60),NA(60,60)
DIMENSION KOUNTE(60,60),MARGX(60),MARGY(60)
C
C *****
C FIND MAXIMUM CLASS INTERVAL FOR BOTH VARIABLES
C *****
      A=NA(1,1)
      B=NB(1,1)
      DO 46 I=1,NR
      DO 46 J=1,NC
      C=NA(I,J)
      IF(C.LE.A) GO TO 47
      A=C
47  D=NB(I,J)
      IF(D.LE.B) GO TO 46
      B=D
46  CONTINUE
C
      T1=INT1
      T2=INT2
      IX=A/T1+1.5
      IY=B/T2+1.5
      IF(IX.GT.60.OR.IY.GT.60) GO TO 98
      DO 200 I=1,60
      DO 200 J=1,60
200  KOUNTE(I,J)=0
C
C *****
C COUNT BIVARIATE
C *****
      DO 201 I=1,NR
      DO 201 J=1,NC
      K=0
      L=0
202  K=K+INT1
      L=L+1
      IF(NA(I,J).LT.K) GO TO 203
      GO TO 202
C
203  KK=0
      LL=0
204  KK=KK+INT2
      LL=LL+1
      IF(NB(I,J).LT.KK) GO TO 201
      GO TO 204
201  KOUNTE(L,LL)=KOUNTE(L,LL)+1
C
      PUNCH 501,IY,IX
      DO 120 I=1,IX
      DO 120 J=1,IY
120  PUNCH 500,KOUNTE(I,J)
C
C *****
C REDUCE TO 20*20 THE MATRIX KOUNTE TO BE PRINTED

```

86

```

C *****
  IF(IY.LE.20) GO TO 135
  DO 131 I=1,IX
  NVERT=0
  DO 130 J=20,IY
130 NVERT=NVERT+KOUNTE(I,J)
131 KOUNTE (I,20)=NVERT
C
135 IF(IX.LE.20) GO TO 136
  DO 132 J=1,20
  NHOR=0
  DO 133 I=20,IX
133 NHOR=NHOR+KOUNTE(I,J)
132 KOUNTE(20,J)=NHOR
C
136 CONTINUE
C
C *****
C COMPUTE MARGINALS
C *****
  DO 150 I=1,20
  MARX=0
  MARY=0
  DO 151 J=1,20
  MARX=MARX+KOUNTE(J,I)
  MARY=MARY+KOUNTE(I,J)
151 CONTINUE
  MARGX(I)=MARX
  MARGY(I)=MARY
150 CONTINUE
C
  PRINT 400,NR,NC,INT1,INT2
  PRINT 401,(((KOUNTE(I,J),J=1,20),MARGY(I)),I=1,2 )
  PRINT 402
  PRINT 403,(MARGX(I),I=1,20)
  50 CONTINUE
  GO TO 99
  98 PRINT 404,IX,IY
  99 CONTINUE
  RETURN
C
C *****
C FORMAT DECLARATION
C *****
400 FORMAT(19H1BIVARIATE COUNTING//
  19H MESH OF I3,1X,1H*I3,2X,7HSQUARES/
  218H CLASS INTERVAL 1=I6/
  318H CLASS INTERVAL 2=I6,85X,8H MARG. Y//)
401 FORMAT(1X,20I4,1I30/)
402 FORMAT(8H0MARG. X/)
403 FORMAT(1X,20I4)
404 FORMAT(25H0CHANGE DIMENSION KOUNTE(I2,1H,I2,1H)/)
500 FORMAT(1I10)
501 FORMAT(2I5)
  END

```

INPUT DATA

Abbreviations:

(A): alphanumeric field

(I): integer value (must be right adjusted in its field)

(F): floating point number (must be punched with a decimal)

Two similar sets of data are required as inputs to this program. The first corresponds to one of the variables, say Y, and the second corresponds to the other variable, say X.

Control card (111)

Columns	1 (I) INDEX: specifies the form of the input data.
---------	--

- = 1 (each element of the data gives the coordinates of one location and the number of observations at that location)
- = 2 (the data is presented as matrix of NR x NC cells and each element of the matrix represents the number of observations in the corresponding cell.)

Observed distribution cards:

Only one of the following (a or b) sets of cards is required, depending on the value specified for INDEX in the control card.

a) INDEX = 1

Distribution control card (2I5, 2F10.0, 2I5)

Columns

1-5	(I)	NOBS:	total number of observations
6-10	(I)	INT:	grouping size for counting purposes.
11-20	(F)	XDIM:	dimension of the study area in the X-direction.
21-30	(F)	YDIM:	dimension of the study area in the Y-direction.
31-35	(I)	NSIZ	} The pair NSIZ-MSIZ indicates the desired number of columns and rows, respectively, of the matrix on which the count opera- tion is to be performed.
36-40	(I)	MSIZ	

Format specification card (6A10)

Columns

1-60	(A)	IVAR:	this is a variable format in which the observed values are to be read (must be punched with the parentheses, i.e., (2F10.4, 1I10)).
------	-----	-------	--

Observed values cards (IVAR)

Columns

(F)	XC	:	X-coordinate of the location
(F)	YC	:	Y-coordinate of the location
(I)	N	:	number of observations at the location

89

b) INDEX = 2

Distribution control card (3I5, 5X, 2I5)

Columns

1-5	(I)	NR	:	number of rows in the data matrix
6-10	(I)	NC	:	number of columns in the data matrix
11-15	(I)	INT	:	grouping size for counting purposes
21-25	(I)	NSIZ	}	the pair NSIZ-MSIZ indicates the desired
26-30	(I)	MSIZ		
				of the matrix on which the count operation is to be performed.

Format specification card (6A10)

Columns

1-60	(A)	IVAR:	this is the variable format in which the observed values are to be read (must be punched with the parentheses, i.e., (10 I4)).
------	-----	-------	---

Observed values cards (IVAR)

Reads in the matrix $((MO(I,J), J=1, NC), I=1, NR)$ of observed values in format IVAR, as specified above, and in row order (1st row, 2nd row,, NRth row).

END OF INPUT DATA

NOTES:

a) Care should be taken to ensure that comparable matrices are used (matrices with the same number of rows and columns, respectively). In order to do that, NSIZ and MSIZ in distribution X must be equal to NSIZ and MSIZ in Y, respectively.

b) In order to be consistent with the present dimensions of the arrays in the program the following limits must be observed:

$$XC(I) \leq 1000$$

$$YC(I) \leq 1000$$

$$N(I) \leq 1000$$

$$MO(I,J) \leq 3600$$

$$NC \leq 60$$

$$NR \leq 60$$

OUTPUT

Printed output:

1) Count of the number of cells within each frequency class of the corresponding bivariate distribution.

(i.e., number of cells with

INT_x and INT_y, INT_x and 2INT_y,....., INT_x and jINT_y
 2INT_x and INT_y, 2INT_x and 2INT_y,....., 2INT_x and jINT_y

 iINT_x and INT_y, iINT_x and 2INT_y,....., iINT_x and jINT_y

observations, where:

INT_x: grouping size for X-distribution

INT_y: grouping size for Y-distribution

2) Marginal distributions of X and Y distributions

(i.e., number of cells with INT_x, 2INT_x,..... iINT_x observations,
 and number of cells with INT_y, 2INT_y,..... iINT_y observations)

Punched output

- a) Number of columns and rows of the matrix in 1 above (Format 2I5)
- b) Information contained in 1 above (Format 1I10)

C INPUT DATA EXAMPLE

2	11	11	1		11	11				
(1114)										
0	2	1	2	0	0	1	0	2	2	1
2	0	0	1	3	1	2	1	0	4	0
2	0	1	1	2	1	1	3	1	1	0
2	3	1	4	0	2	0	1	4	2	0
1	1	3	0	0	2	1	0	0	1	0
0	2	0	1	1	3	1	2	1	2	0
1	0	1	2	2	1	5	0	0	1	2
3	2	2	0	0	2	1	0	1	2	0
1	0	0	0	0	0	2	2	1	0	1
0	0	2	5	1	3	1	6	0	4	3
1	6	1	2	0	0	1	1	0	1	1

2	11	11	1		11	11				
(1114)										
0	1	0	1	1	2	0	0	1	0	0
3	1	3	1	3	0	2	2	0	1	2
1	0	0	2	0	2	1	0	1	0	1
0	3	2	5	0	0	0	1	1	2	0
1	0	0	0	0	1	0	1	0	1	1
0	1	1	1	1	1	3	1	0	1	1
1	2	0	0	2	0	0	0	0	1	3
2	1	1	0	1	3	1	3	0	0	0
0	0	2	0	1	0	3	0	2	1	0
1	0	0	1	0	3	0	1	0	2	2
1	2	0	2	0	1	3	1	1	0	1

C END OF INPUT DATA

C OUTPUT EXAMPLE

BIVARIATE COUNTING

MESH OF 11 * 11 SQUARES

CLASS INTERVAL 1= 1

CLASS INTERVAL 2= 1

MARG.

21	18	8	2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	50
13	14	10	1	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	42
4	5	4	2	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	17
2	2	4	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

MARG. X

40 39 26 8 4 2 2 0 0 0 0 0 0 0 0 0 0 0 0 0

PROGRAM FITBIV

PURPOSE

This program fits the following bivariate models to observed distributions:

- 1) Generalized Bivariate Correlated Poisson (GBCP)
- 2) Compound Bivariate Correlated Poisson (CBCP)
- 3) Bivariate Correlated Poisson (BCP)
- 4) Contagious Bivariate Negative Binomial (CBNB)
- 5) Compound Bivariate Uncorrelated Poisson (CBUP)

Given an observed bivariate distribution, the program:

- 1) computes statistics (mean and variance of each marginal distribution, covariance and correlation coefficient) of the observed distribution;
- 2) estimates the parameters of the above models using the method of moments (except for the last which uses the method of maximum likelihood); and
- 3) compares observed to expected values by calculating the chi-square goodness-of-fit test.

Reference :

Rogers, A. and Martin, J. (1969). Bivariate Quadrat Analysis of the Dispersion of Employment and Population in Urban Areas, Working Paper. Center for Planning and Development Research, University of California, Berkeley.

10

17th

95

```

RUN,S,,,,,20000.
LGO.
.
PROGRAM FITBIV(INPUT,OUTPUT,PUNCH)
COMMON F(30,30)
COMMON/DISTR/SAMPLE(60,60),XMARG(60),YMARG(60),NC,NR/PARAM/U1 ,U 1
1,U20,U02,U11/NB/T(120),ISUM,KE
DIMENSION INDEX(5)
C
C *****
C SUBROUTINES
C
C GENERALIZED BIVARIATE CORRELATED POISSON
C GBCP1 = PARAMETERS
C GBCP2 = FREQUENCY
C COMPOUND BIVARIATE CORRELATED POISSON
C CBCP1 = PARAMETERS
C CBCP2 = FREQUENCY
C BIVARIATE CORRELATED POISSON
C BICP1 = PARAMETERS
C BICP2 = FREQUENCY
C CONTAGIOUS BIVARIATE NEGATIVE BINOMIAL
C CBNB1 = PARAMETERS
C CBNB2 = FREQUENCY
C COMPOUND BIVARIATE UNCORRELATED POISSON
C CBUP1 = PARAMETERS
C CBUP2 = FREQUENCY
C
C CHITEST = CHI SQUARE GOODNESS-OF-FIT TEST
C PRINTT = PRINT OUTPUT
C OUTPUTT = PRINT OUTPUT IN FINAL TABLE FORM
C FACT = FACTORIAL FUNCTION
C *****
C
C
C      DO 8 I=1,60
C      DO 8 J=1,60
C      8 SAMPLE(I,J)=0.0
C
C      98 READ 1001,INDEX,IOPT1,IOPT2
C      1001 FORMAT(5I1,3X,2I1)
C      99 READ 1004,NC,NR
C      1004 FORMAT(2I5)
C      READ 1002,((SAMPLE(I,J),J=1,NC),I=1,NR)
C      1002 FORMAT(1F10.0)
C      READ 1003,CON
C      1003 FORMAT(1F2.0)
C      NCONTR=ABS(CON)-1.0
C
C *****
C COMPUTE MARGINAL DISTRIBUTIONS
C *****
C      DO 103 J=1,NC
C      SUM1=0.0
C      DO 102 I=1,NR

```

```

102 SUM1=SUM1+SAMPLE(I,J)
103 XMARG(J)=SUM1
C
DO 101 I=1,NR
SUM2=0.0
DO 100 J=1,NC
100 SUM2=SUM2+SAMPLE(I,J)
101 YMARG(I)=SUM2
C
C *****
C COMPUTE SAMPLE STATISTICS
C *****
OBS=0.0
SX1=0.0
SX2=0.0
SY1=0.0
SY2=0.0
SXY=0.0
C
DO 200 I=1,NC
IF(XMARG(I).EQ.0.0) GO TO 200
OBS=OBS+XMARG(I)
R=I-1
IF(R.EQ.0.0) GO TO 200
SX1=SX1+R*XMARG(I)
SX2=SX2+R**2*XMARG(I)
200 CONTINUE
C
DO 201 J=1,NR
Q=J-1
IF(YMARG(J).EQ.0.0.OR.Q.EQ.0.0) GO TO 201
SY1=SY1+Q*YMARG(J)
SY2=SY2+Q**2*YMARG(J)
C
DO 202 K=1,NC
R=K-1
IF(R.EQ.0.0.OR.SAMPLE(J,K).EQ.0.0) GO TO 202
SXY=SXY+R*Q*SAMPLE(J,K)
202 CONTINUE
201 CONTINUE
C
U10=SX1/OBS
U01=SY1/OBS
U20=(SX2-OBS*U10**2)/(OBS-1)
U02=(SY2-OBS*U01**2)/(OBS-1)
U11=SXY/OBS-U10*U01
SDX=SQRT(U20)
SDY=SQRT(U02)
RO=U11/(SDX*SDY)
C
ISUM=NC+NR
DO 1 I=1,ISUM
S=0.0
DO 2 K=1,I
L=I-K+1

```

```

      IF(K.GT.NR.OR.L.GT.NC) GO TO 2
      S=S+SAMPLE(K,L)
      2 CONTINUE
      1 T(I)=S/OBS
C *****
C CALL RELEVANT SUBROUTINE
C *****
      LLL=0
      DO 301 LL=1,5
      KE=3
      LLL=LLL+1
      INDEXI=INDEX(LL)
      IF(INDEXI.EQ.0) GO TO 111
C
      GO TO (501,502,503,504,505),INDEXI
501 CALL GBCP1
      GO TO 300
502 CALL CBCP1
      GO TO 300
503 CALL BICP1
      GO TO 300
504 CALL CBNB1(OBS)
      GO TO 300
505 CONTINUE
      DO 301 KE=1,2
      CALL CBUP1(OBS)
      300 CONTINUE
C
C *****
C AGGREGATE SAMPLE VALUES BELOW MINIMUM NUMBER OF OBSERVATIONS
C *****
      NOBS=5
C NOBS IS MINIMUM NUMBER OF OBSERVATIONS
C *****
      IC=0
      12 JC=NC-IC
      NVERT=0
      DO 10 I=1,NR
      DO 10 J=JC,NC
      10 NVERT=NVERT+SAMPLE(I,J)
      IF(NVERT.GE.NOBS) GO TO 11
      IC=IC+1
      GO TO 12
      11 IF(JC.EQ.NC) GO TO 13
      DO 14 I=1,NR
      AG=0.0
      AF=0.0
      DO 15 J=JC,NC
      AG=AG+SAMPLE(I,J)
      15 AF=AF+F(I,J)
      SAMPLE(I,JC)=AG
      14 F(I,JC)=AF
      NC=JC
      13 IR=0
      18 JR=NR-IR

```

```

NHOR=0
DO 16 J=1,JC
DO 16 I=JR,NR
16 NHOR=NHOR+SAMPLE(I,J)
IF(NHOR.GE.NOBS) GO TO 17
IR=IR+1
GO TO 18
17 IF(JR.EQ.NR) GO TO 21
DO 19 J=1,JC
EG=0.0
EF=0.0
DO 20 I=JR,NR
EG=EG+SAMPLE(I,J)
20 EF=EF+F(I,J)
SAMPLE(JR,J)=EG
19 F(JR,J)=EF
NR=JR
21 CONTINUE

C
IF(IOPT1.NE.1) GO TO 4
PUNCH 3000,((F(I,J),J=1,NC),I=1,NR)
3000 FORMAT(1F10.6)
4 CONTINUE

C
IF(IOPT2.EQ.1) GO TO 299
CALL PRINTT(OBS,INDEXI,RO,KE)
GO TO 301
299 CALL OUTPUTT(OBS,INDEXI,RO,KE,LLL)
301 CONTINUE

C
111 IF(NCONTR.EQ.0) STOP
GO TO (99,98),NCONTR
END

```

```

      SUBROUTINE GBCP1
      COMMON/PARAM/U10,U01,U20,U02,U11/NTA/EME,ENE,U,V,W
C *****
C PARAMETERS ESTIMATES BY THE METHOD OF MOMENTS
C *****
      EME=U20/U10-1.0
      ENE=U02/U01-1.0
      U=U10/EME
      V=U01/ENE
      W=U11/(EME*ENE)
      A=U
      IF(V.GE.U) GO TO 15
      A=V
15  IF(W.LE.A) GO TO 16
      W=A
16  CALL GBCP2
      RETURN
      END

      SUBROUTINE GBCP2
      COMMON F(30,30)
      COMMON/DISTR/SAMPLE(60,60),XMARG(60),YMARG(60),NC,NR/NTA/EME,ENE,U
1,V,W
C *****
C COMPUTE CONSTANTS
C *****
      A=EME+ENE
      A=EXP(-A)
      B=EXP(-EME)
      C=EXP(-ENE)
C
      C1=W*EME*A
      C2=(U-W)*EME*B
      C3=W*ENE*A
      C4=(V-W)*ENE*C
C
C *****
C COMPUTE INITIAL VALUES
C *****
      D=(U-W)*(B-1.0)+(V-W)*(C-1.0)+W*(A-1.0)
      F(1,1)=EXP(D)
C
C *****
C COMPUTE FIRST COLUMN
C *****
      DO 100 NX=2,NC
      IX=NX-1
      SUM1=0.0
      C11=C1/IX
      C12=C2/IX
C
      DO 50 I=1,IX

```

```
L=I-1
A=EME**(I-1)*F(1,NX-I)/FACT(L)
50 SUM1=SUM1+A
F(1,NX)=SUM1*(C11+C12)
100 CONTINUE

C *****
C COMPUTE EVERY ROW
C *****
DO 200 NX=1,NC
DO 200 NY=2,NR
IY=NY-1
SUM1=0.0
SUM2=0.0
C21=C3/IY
C22=C4/IY

C
DO 150 I=1,IY
L=I-1
DO 151 J=1,NX
K=J-1
A=ENE**(I-1)*EME**(J-1)*F((NY-I),(NX+1-J))/(FACT(L)*FACT(K))
151 SUM1=SUM1+A
B=ENE**(I-1)*F(NY-I,NX)/FACT(L)
150 SUM2=SUM2+B
200 F(NY,NX)=C21*SUM1+C22*SUM2
RETURN
END
```

SUBROUTINE CBCP1

COMMON/PARAM/U10,U01,U20,U02,U11/CCBP/CA,B1,B2,B12,A

C *****

C PARAMETER ESTIMATES BY THE METHOD OF MOMENTS

C *****

DELTA=U10/U01

ALFA=(U20-U01*DELTA)/(U01*DELTA**2)

CA=U01/ALFA

B12=DELTA*ALFA**2-U11/CA

IF(B12.GT.0.0) B12=-0.01

B2=-ALFA-B12

IF(B2.GT.0.0) B2=-0.01

B1=B2/DELTA+(1.0/DELTA-1.0)*B12

IF(B1.GT.0.0) B1=-0.01

A=1.0-(B1+B2+B12)

CALL CBCP2

RETURN

END

SUBROUTINE CBCP2

COMMON F(30,30)

COMMON/DISTR/SAMPLE(60,60),XMARG(60),YMARG(60),NC,NR/CCBP/CA,B1,B2
1,B12,A

C

C *****

C COMPUTE DENSITY

C *****

IF(NR.GE.NC) GO TO 10

NRP=NC

GO TO 20

10 NRP=NR

20 CONTINUE

DO 100 IR=1,NRP

DO 100 IS=1,IR

D=A**((1.-CA-IR)*B1**(IR-IS)*B12**(IS-1))

C=B1*B2/(A*B12)

SUM=0.0

C

DO 50 I=1,IS

L=IR-IS+I-1

J=IS-I

K=I-1

N=IR+I-2

Q=1

IF(N.EQ.0) GO TO 54

C

P=CA+N

DO 40 M=1,N

P=P-1

40 Q=Q*P

54 G=(-1.0)**(IR+I-2)*Q*C**((I-1)/(FACT(L)*FACT(J)*FACT(K)))

50 SUM=SUM+G

100 F(IR,IS)=D*SUM

102

°C
E=B2/B1
DO 200 IR=1,NR
IX=IR+1
IF(IX.GT.NC) RETURN
DO 200 IT=IX,NC
200 F(IR,IT)=E**((IT-IR)*F(IT,IR)
RETURN
END

```
SUBROUTINE BICP1
COMMON/PARAM/U10,U01,U20,U02,U11/DPOIS/C,B,D
```

```
C
C *****
C PARAMETER ESTIMATES BY THE METHOD OF MOMENTS
C *****
C
C   C=U10
C   B=U01
C   D=U11
C
C   DMAX=C
C   IF(B.GE.C) GO TO 1
C   DMAX=B
C   1 IF(D.LE.DMAX) GO TO 2
C     D=DMAX-0.01
C   2 CALL BICP2
C   RETURN
C   END
```

```
SUBROUTINE BICP2
COMMON F(30,30)
COMMON/DISTR/SAMPLE(60,60),XMARG(60),YMARG(60),NC,NR/DPOIS/C,B,D
```

```
C *****
C COMPUTE CONSTANTS
C *****
C   R=C+B-D
C   S=C-D
C   T=B-D
C   CONST=EXP(-R)
C
C *****
C COMPUTE FREQUENCY DISTRIBUTION
C *****
C   DO 1 I=1,NC
C     DO 1 J=1,NR
C       K=I
C       IF(J.GE.I) GO TO 10
C       K=J
C   10 CONTINUE
C
C   SUM=0.0
C   DO 2 L=1,K
C     II=I-L
C     JJ=J-L
C     LL=L-1
C     IF(S.EQ.0.0.AND.II.EQ.0) GO TO 3
C     U=S**II
C     GO TO 4
C   3 U=1.0
C   4 IF(T.EQ.0.0.AND.JJ.EQ.0) GO TO 5
C     V=T**JJ
C     GO TO 6
C   5 V=1.0
```

```
6 P=U*V*D**LL
  W=P/(FACT(II)*FACT(JJ)*FACT(LL))
2 SUM=SUM+W
1 F(J,I)=CONST*SUM
  RETURN
  END
```

105

77

```

SUBROUTINE CBNB1(OBS)
COMMON/PARAM/U10,U01,U20,U02,U11/NB/Q(120),J,KE/BCD/GAM,BET,AMAY
C *****
C PARAMETER ESTIMATES BY THE METHOD OF MOMENTS
C *****
      US1=0.0
      SM2=0.0
      DO 10 K=1,J
      G=K-1
      US1=US1+Q(K)*G
10  SM2=SM2+Q(K)*G**2
      US2=OBS*(SM2-US1**2)/(OBS-1.0)

      AMAY=U01/U10
      T=ALOG(US2)
      R=ALOG(US1)
      GAM=(T-R)/(1.0+AMAY)
      V=GAM*(1.0+AMAY)
      S=EXP(V)
      W=US1*GAM
      BET=W/(S-1.0)
      CALL CBNB2
      RETURN
      END

```

```

SUBROUTINE CBNB2
COMMON F(30,30)
COMMON/DISTR/SAMPLE(60,60),XMARG(60),YMARG(60),NC,NR/BCD/GAM,BET,A
1MAY
C *****
C COMPUTE CONSTANTS
C *****
      TT=BET*(1.0+AMAY)
      SS=GAM*(1.0+AMAY)
      RR=GAM*AMAY
      T=EXP(-TT)
      S=EXP(-SS)
      R=EXP(-RR)

C *****
C COMPUTE FREQUENCY DISTRIBUTION
C *****
      DO 1 I=1,NC
      DO 1 J=1,NR
      N=I+J-2
      Q=1.0
      IF(N.EQ.0) GO TO 2

      P=BET/GAM+N
      DO 3 M=1,N
      P=P-1.0

```

3 Q=Q*P

2 CONTINUE

F(J,I)=T*Q*(R-S)**(J-1)*(1.0-R)**(I-1)/(FACT(I-1)*FACT(J-1))

1 CONTINUE

RETURN

END

```

SUBROUTINE CBUP1(OBS)
COMMON/DISTR/SAMPLE(60,60),XMARG(60),YMARG(60),NC,NR/PARAM/XBAR,YB
1AR,U20,U02,U11/BNB/ALFA,BETA,ACAP/NB/Q(120),J,KE
IF(KE.EQ.2) GO TO 17
C
C *****
C PARAMETER ESTIMATES BY THE METHOD OF MOMENTS
C *****
    US1=0.0
    SM2=0.0
    DO 9 K=1,J
    G=K-1
    US1=US1+Q(K)*G
9  SM2=SM2+Q(K)*G**2
    US2=OBS*(SM2-US1**2)/(OBS-1.0)
C
    ACAP=YBAR/XBAR
    ALFA=US1**2/(US2-US1)
    BETA=(1.0+ACAP)*US1/(US2-US1)
    CALL CBUP2
    RETURN
C
17 CONTINUE
C *****
C MAXIMUM LIKELIHOOD PARAMETERS ESTIMATION
C *****
    W=Q(1)
    T=ALOG10(W)
    C=(-1.0)*T/(XBAR+YBAR)
    Z=0.
    A=0.1
    IF(C-1.)3,4,5
3  I=2
    GO TO 6
5  I=1
6  IF(I.EQ.1) A=(-A)
8  Z=Z+A
    T=C*Z
    Y1=ABS(T)
    T=1.0+Z
    P=ALOG10(T)
    Y2=ABS(P)
    B=Y1-Y2
    D=ABS(B)
    IF(D.LE.0.001) GO TO 4
    IF(I.EQ.1) B=(-B)
    IF(B.LT.0.0) GO TO 8
    A=A/10.
    I=I-(-1)**I
    GO TO 6
4  Z1=(XBAR+YBAR)/Z
7  T=1.0+(XBAR+YBAR)/Z1
    W=ALOG(T)
C
    S1=0.0

```

```

      S2=0.0
      DO 10 I=1,J
      X=0.0
      DO 11 K=1,I
11  X=X+Q(K)
      G=I
      V=(1.0-X)/(Z1+G-1.0)
      S=(1.0-X)/(Z1+G-1.0)**2
      S1=S1+V
      S2=S2+S
10  CONTINUE

```

```

C      F=W-S1
      T=Z1+XBAR+YBAR
      P=1./T-1./Z1+S2

```

```

C      Z=Z1
      Z1=Z-F/P
      U=Z1-Z
      U=ABS(U)
      IF(U.LE.0.0001) GO TO 15
      GO TO 7
15  ALFA=Z
      BETA=ALFA/XBAR
      ACAP=YBAR/XBAR
      CALL CBUP2
      RETURN
      END

```

```

      SUBROUTINE CBUP2
      COMMON F(30,30)
      COMMON/DISTR/SAMPLE(60,60),XMARG(60),YMARG(60),NC,NR/BNB/ALFA,BETA
1,ACAP

```

```

C      C=BETA+ACAP+1.0

```

```

C *****
C COMPUTE FREQUENCY DISTRIBUTION
C *****

```

```

      DO 1 I=1,NC
      DO 1 J=1,NR
      N=I+J-2
      Q=1.
      IF(N.EQ.0) GO TO 2

```

```

C      P=ALFA+N
      DO 13 M=1,N
      P=P-1.

```

```

13  Q=Q*P

```

```

C      2 S=2.0-(ALFA+J+I)
      F(J,I)=Q*BETA**ALFA*ACAP**(J-1)*C**S/(FACT(I-1)*FACT(J-1))
1  CONTINUE

```

109

RETURN
END

```

SUBROUTINE CHITEST(NP)
COMMON F(30,30)
COMMON/DISTR/SAMPLE(60,60),XMARG(60),YMARG(60),NC,NR/TEST/NDF(3),
1CHISQ(3),AMINEX
DIMENSION CHI(900),FRQ(900)
C *****
C AMINEX IS MINIMUM EXPECTATION
C *****
    AMINEX=2.
    M=NC*NR
    N=NC+NR
C
    DO 100 KOUNT=1,3
    L=1
    GO TO (40,50,60),KOUNT
C
C *****
C AGGREGATION ALONG ROWS
C *****
    40 CONTINUE
    DO 10 I=1,NR
    DO 10 J=1,NC
    CHI(L)=F(I,J)
    FRQ(L)=SAMPLE(I,J)
    10 L=L+1
    GO TO 70
C
C *****
C AGGREGATION ALONG COLUMNS
C *****
    50 CONTINUE
    DO 11 J=1,NC
    DO 11 I=1,NR
    CHI(L)=F(I,J)
    FRQ(L)=SAMPLE(I,J)
    11 L=L+1
    GO TO 70
C
C *****
C AGGREGATION ALONG DIAGONAL
C *****
    60 CONTINUE
    DO 12 I=1,N
    DO 12 K=1,I
    II=I-K+1
    IF(K.GT.NC.OR.II.GT.NR) GO TO 12
    CHI(L)=F(II,K)
    FRQ(L)=SAMPLE(II,K)
    L=L+1
    12 CONTINUE
C
C *****
C COMPUTE CHI SQUARE
C *****
    70 K=0

```

111

```

      J=1
14  SUM1=0.0
      SUM2=0.0
15  K=K+1
      SUM1=SUM1+CHI(K)
      SUM2=SUM2+FRQ(K)
      IF(K.EQ.M) GO TO 18
      IF(SUM1.LT.AMINEX) GO TO 15
      CHI(J)=SUM1
      FRQ(J)=SUM2
      J=J+1
      GO TO 14
C
18  CHI(J)=SUM1
      FRQ(J)=SUM2
      IF(CHI(J).GE.AMINEX) GO TO 19
      CHI(J-1)=CHI(J-1)+CHI(J)
      FRQ(J-1)=FRQ(J-1)+FRQ(J)
      J=J-1
19  NDF(KOUNT)=J-1-NP
      SUM=0.0
      DO 20 I=1,J
      B=(FRQ(I)-CHI(I))**2/CHI(I)
20  SUM=SUM+B
      CHISQ(KOUNT)=SUM
100 CONTINUE
      RETURN
      END

```

FUNCTION FACT(N)

```

C
C *****
C FACTORIAL FUNCTION
C *****
      FACT=1.
      IF(N.LE.1) RETURN
      DO 10 I=2,N
      X=I
10  FACT=FACT*X
      RETURN
      END

```

```

SUBROUTINE PRINTT(OBS,INDEXI,RO,KE)
COMMON F(30,30)
COMMON/PARAM/U10,U01,U20,U02,U11
COMMON/DISTR/SAMPLE(60,60),XMARG(60),YMARG(60),NC,NR/NTA/EME,ENE,U
1,V,W/CCBP/CA,B1,B2,B12,A/DPOIS/C,B,D/BNB/ALFA,BETA,ACAP
COMMON/TEST/NDF(3),CHISQ(3),AMINEX/BCD/GAM,BET,AMAY
DIMENSION XMTH(30),YMTH(30)

C
C *****
C PRINT TITTLE, STATISTICS AND PARAMETERS
C *****
PRINT 1500,RO,U10,U20,U11,U01,U02,OBS
GO TO (501,502,503,504,505),INDEXI
501 PRINT 2000,EME,ENE,U,V,W
NP=5
GO TO 5
502 PRINT 2001,CA,B1,B2,B12,A
NP=3
GO TO 5
503 PRINT 1999,C,B,D
NP=3
GO TO 5
504 PRINT 1997,GAM,BET,AMAY
NP=3
GO TO 5
505 NP=3
IF(KE.EQ.2) GO TO 507
506 PRINT 1998,ALFA,BETA,ACAP
GO TO 5
507 PRINT 1996,ALFA,BETA,ACAP
5 CONTINUE

C
C *****
C PRINT OBSERVED AND EXPECTED BIVARIATE DISTRIBUTION
C *****
Z=OBS
DO 1 I=1,NR
DO 1 J=1,NC
II=I-1
JJ=J-1
F(I,J)=F(I,J)*OBS
IF(I.EQ.NR.AND.J.EQ.NC) GO TO 1
PRINT 2002,II,JJ,SAMPLE(I,J),F(I,J)
Z=Z-F(I,J)
1 CONTINUE
F(NR,NC)=Z
PRINT 2003,II,JJ,SAMPLE(NR,NC),F(NR,NC)

C
CALL CHITEST(NP)

C
C *****
C PRINT CHI SQUARE TEST
C *****
PRINT 2050,AMINEX,NDF(1),CHISQ(1),NDF(2),CHISQ(2),NDF(3),CHISQ(3)
PRINT 2100

```

```

C
C *****
C PRINT OBSERVED AND EXPECTED MARGINALS
C *****
      DO 10 J=1,NC
      SUM1=0.0
      SUM3=0.0
      DO 11 I=1,NR
      SUM3=SUM3+SAMPLE(I,J)
11  SUM1=SUM1+F(I,J)
      XMARG(J)=SUM3
10  XMTH(J)=SUM1
C
      DO 12 I=1,NR
      SUM2=0.0
      SUM4=0.0
      DO 13 J=1,NC
      SUM4=SUM4+SAMPLE(I,J)
13  SUM2=SUM2+F(I,J)
      YMARG(I)=SUM4
12  YMTH(I)=SUM2
C
      IF(NR-NC)21,22,23
21  CONTINUE
      DO 14 I=1,NR
      J=I-1
14  PRINT 2101,J,XMARG(I),XMTH(I),J,YMARG(I),YMTH(I)
      K=NR+1
      DO 15 I=K,NC
      J=I-1
15  PRINT 2103,J,XMARG(I),XMTH(I)
      RETURN
C
22  CONTINUE
      DO 16 I=1,NR
      J=I-1
16  PRINT 2101,J,XMARG(I),XMTH(I),J,YMARG(I),YMTH(I)
      RETURN
C
23  CONTINUE
      DO 17 I=1,NC
      J=I-1
17  PRINT 2101,J,XMARG(I),XMTH(I),J,YMARG(I),YMTH(I)
      K=NC+1
      DO 18 I=K,NR
      J=I-1
18  PRINT 2102,J,YMARG(I),YMTH(I)
      RETURN
C
C *****
C FORMAT DECLARATIONS
C *****
1500 FORMAT(58H1BIVARIATE ANALYSIS OF OBSERVED DISTRIBUTION (CORR.COEFF
      3.=F6.4,1H)///
      16H E(X)=F10.4,5X,7HVAR(X)=F10.4,5X,9HCOV(X,Y)=F1 .4/

```

```

26H E(Y)=F10.4,5X,7HVAR(Y)=F10.4,5X,9HNER.OBS.=F6. //)
1996 FORMAT(40H0COMPOUND BIVARIATE UNCORRELATED POISSON//
942H PARAMETER ESTIMATES BY MAXIMUM LIKELIHOOD/
114X,5HALFA=F10.4/
214X,5HBETA=F10.4/
317X,2HA=F10.4///
434H OBSERVED AND EXPECTED FREQUENCIES//
55X,1HY,4X,1HX,10X,8HOBSERVED,10X,8HEXPECTED/)
1997 FORMAT(39H0CONTAGIOUS BIVARIATE NEGATIVE BINOMIAL//
945H PARAMETER ESTIMATES BY THE METHOD OF MOMENTS/
113X,6HGAMMA=F10.4/
214X,5HBETA=F10.4/
317X,2HA=F10.4///
434H OBSERVED AND EXPECTED FREQUENCIES//
55X,1HY,4X,1HX,10X,8HOBSERVED,10X,8HEXPECTED/)
1998 FORMAT(40H0COMPOUND BIVARIATE UNCORRELATED POISSON//
945H PARAMETER ESTIMATES BY THE METHOD OF MOMENTS/
114X,5HALFA=F10.4/
214X,5HBETA=F10.4/
317X,2HA=F10.4///
434H OBSERVED AND EXPECTED FREQUENCIES//
55X,1HY,4X,1HX,10X,8HOBSERVED,10X,8HEXPECTED/)
1999 FORMAT(29H0BIVARIATE CORRELATED POISSON//
945H PARAMETER ESTIMATES BY THE METHOD OF MOMENTS/
117X,2HA=F10.4/
217X,2HB=F10.4/
317X,2HD=F10.4///
434H OBSERVED AND EXPECTED FREQUENCIES//
55X,1HY,4X,1HX,10X,8HOBSERVED,10X,8HEXPECTED/)
2000 FORMAT(41H0GENERALIZED BIVARIATE CORRELATED POISSON//
945H PARAMETER ESTIMATES BY THE METHOD OF MOMENTS/
117X,2HM=F10.4/
217X,2HN=F10.4/
317X,2HU=F10.4/
417X,2HV=F10.4/
517X,2HW=F10.4///
634H OBSERVED AND EXPECTED FREQUENCIES//
75X,1HY,4X,1HX,10X,8HOBSERVED,10X,8HEXPECTED/)
2001 FORMAT(38H0COMPOUND BIVARIATE CORRELATED POISSON//
945H PARAMETER ESTIMATES BY THE METHOD OF MOMENTS/
117X,2HK=F10.4/
216X,3HB1=F10.4/
316X,3HB2=F10.4/
415X,4HB12=F10.4/
517X,2HA=F10.4///
634H OBSERVED AND EXPECTED FREQUENCIES//
75X,1HY,4X,1HX,10X,8HOBSERVED,10X,8HEXPECTED/)
2002 FORMAT(1X,2I5,2F18.4)
2003 FORMAT(1X,I5,1H+,I4,1H+,F17.4,F18.4///)
2050 FORMAT(39H-GOODNESS OF FIT (MINIMUM EXPECTATION =F2. ,1H)//
130X,18HDEGREES OF FREEDOM,5X,10HCHI SQUARE/
223H AGREGGATION ALONG ROWS,11X,I3,16X,F10.4/
326H AGREGGATION ALONG COLUMNS,8X,I3,16X,F10.4/
427H AGREGGATION ALONG DIAGONAL,7X,I3,16X,F10.4//)
2100 FORMAT(23H MARGINAL DISTRIBUTIONS//

```

115

15X,1HX,5X,8HOBSERVED,5X,8HEXPECTED,4X,1HY,5X,8HOBSERVED,5X,8HEXPEC
2TED/)

2101 FORMAT(1X,I5,2F13.4,I5,2F13.4)

2102 FORMAT(32X,I5,2F13.4)

2103 FORMAT(1X,I5,2F13.4)

END

```

SUBROUTINE OUTPUTT(OBS,INDEXI,RO,KE,LLL)
COMMON F(30,30)
COMMON/PARAM/U10,U01,U20,U02,U11
COMMON/DISTR/SAMPLE(60,60),XMARG(60),YMARG(60),NC,NR/NTA/EME,ENE,U
1,V,W/CCBP/CA,B1,B2,B12,A/DPOIS/C,B,D/BNB/ALFA,BETA,ACAP
COMMON/TEST/NDF(3),CHISQ(3),AMINEX/BCD/GAM,BET,AMAY
DIMENSION XMTH(30),YMTH(30),NUMB(30),PROB(30),P 5(3)

```

```

C *****
C SUBROUTINE TO PRINT OUTPUT IN FINAL TABLE FORM
C *****

```

```

DO 9 I=1,NC
9 NUMB(I)=I-1

```

```

C
Z=OBS
DO 1 I=1,NR
DO 1 J=1,NC
F(I,J)=F(I,J)*OBS
IF(I.EQ.NR.AND.J.EQ.NC) GO TO 1
Z=Z-F(I,J)
1 CONTINUE
F(NR,NC)=Z

```

```

C
DO 2 J=1,NC
SUM1=0.0
SUM3=0.0
DO 3 I=1,NR
SUM3=SUM3+SAMPLE(I,J)
3 SUM1=SUM1+F(I,J)
XMARG(J)=SUM3
2 XMTH(J)=SUM1

```

```

C
DO 4 I=1,NR
SUM2=0.0
SUM4=0.0
DO 6 J=1,NC
SUM4=SUM4+SAMPLE(I,J)
6 SUM2=SUM2+F(I,J)
YMARG(I)=SUM4
4 YMTH(I)=SUM2

```

```

C
IF(LLL.NE.1) GO TO 7

```

```

C
PROB(1)= 3.84
PROB(2)= 5.99
PROB(3)= 7.81
PROB(4)= 9.49
PROB(5)= 11.10
PROB(6)= 12.60
PROB(7)= 14.10
PROB(8)= 15.50
PROB(9)= 16.90
PROB(10)=18.30
PROB(11)=19.70
PROB(12)=21.00

```

```

PROB(13)=22.40
PROB(14)=23.70
PROB(15)=25.00
PROB(16)=26.30
PROB(17)=27.60
PROB(18)=28.90
PROB(19)=30.10
PROB(20)=31.40
PROB(21)=32.70
PROB(22)=33.90
PROB(23)=35.20
PROB(24)=36.40
PROB(25)=37.70
PROB(26)=38.90
PROB(27)=40.10
PROB(28)=41.30
PROB(29)=42.60
PROB(30)=43.80

```

```

C
PRINT 100
PRINT 101,(NUMB(I),I=1,NC)
PRINT 102

```

```

C
DO 10 I=1,NR
  II=I-1
10 PRINT 103,II,(SAMPLE(I,J),J=1,NC),YMARG(I)
  PRINT 104,(XMARG(J),J=1,NC),OBS
  PRINT 105,RO,U10,U01,U20,U02,U11
7 CONTINUE

```

```

C
GO TO (501,502,503,504,505),INDEXI
501 PRINT 200,EME,ENE,U,V,W
  NP=5
  GO TO 5
502 PRINT 201,CA,B1,B2,B12,A
  NP=3
  GO TO 5
503 PRINT 199,C,B,D
  NP=3
  GO TO 5
504 PRINT 197,GAM,BET,AMAY
  NP=3
  GO TO 5
505 NP=3
  GO TO (506,507),KE
506 PRINT 198,ALFA,BETA,ACAP
  GO TO 5
507 PRINT 196,ALFA,BETA,ACAP
5 CONTINUE

```

```

C
CALL CHITEST(NP)
PRINT 101,(NUMB(I),I=1,NC)
PRINT 102

```

```

C
DO 11 I=1,NR

```

```

      II=I-1
11 PRINT 103,II,(F(I,J),J=1,NC),YMTH(I)
C
      PRINT 104,(XMTH(J),J=1,NC),OBS
C
      DO 8 I=1,3
      K=NDF(I)
      IF(K.GT.30) K=30
8 P05(I)=PROB(K)
      PRINT 106,NDF(1),CHISQ(1),P05(1),NDF(2),CHISQ(2),P 5(2),NDF(3),CHI
      1SQ(3),P05(3),AMINEX
      RETURN
C
C *****
C FORMAT DECLARATIONS
C *****
100 FORMAT(1H1,9X,43HBIVARIATE ANALYSIS OF OBSERVED DISTRIBUTION//)
C
C *****
C THE NUMBER ABOVE THE ASTERISK INDICATES NUMBER OF COLUMNS
C *****
101 FORMAT(18X,1HX/
      119X, 9I8)
C
      **
102 FORMAT(1H+,24X, 9(8X),10HMARGINAL Y/
      **
      111X,1HY)
103 FORMAT(1H0,13X,I2,5X, 9F8.2,1F12.2)
C
      **
104 FORMAT(1H-,9X,11HMARGINAL X , 9F8.2,1F12.2)
C
      **
C *****
C
105 FORMAT(1H0/
      11H0,9X,17HSAMPLE STATISTICS//
      210X,12H CORR.COEFF.=F9.4/
      317X,5HE(X)=F9.4/
      417X,5HE(Y)=F9.4/
      515X,7HVAR(X)=F9.4/
      615X,7HVAR(Y)=F9.4/
      713X,9HCOV(X,Y)=F9.4)
106 FORMAT(1H-/
      11H-,9X,20HGOODNESS OF FIT (**)/
      239X,18HDEGREES OF FREEDOM,5X,10HCHI SQUARE,13X,5HP . 5/
      39X,23H AGGREGATION ALONG ROWS,11X,I6,13X,1F10.4,8X,1F1 .4/
      49X,26H AGGREGATION ALONG COLUMNS,8X,I6,13X,1F1 .4,8X,1F1 .4/
      59X,27H AGGREGATION ALONG DIAGONAL,7X,I6,13X,1F1 .4,8X,1F1 .4//
      69X,25H (**)MINIMUM EXPECTATION=F5.2)
196 FORMAT(1H1,9X,39HCOMPOUND BIVARIATE UNCORRELATED POISSON,6X,13HPAR
      9AMETERS(*)//
      155X,6HALFA =F10.4/
      255X,6HBETA =F10.4/
      355X,6HA =F10.4/
      41H0,54X,36H(*) ESTIMATION BY MAXIMUM LIKELIHOOD///)
197 FORMAT(1H1,9X,38HCONTAGIOUS BIVARIATE NEGATIVE BINOMIAL,7X,13HPAR

```

```

9METERS(*)//
155X,6HGAMMA=F10.4/
255X,6HBETA =F10.4/
355X,6HA     =F10.4/
41H0,54X,35H(*) ESTIMATION BY METHOD OF MOMENTS///)
198 FORMAT(1H1,9X,39HCOMPOUND BIVARIATE UNCORRELATED POISSON,6X,13HPAR
9AMETERS(*)//
155X,6HALFA =F10.4/
255X,6HBETA =F10.4/
355X,6HA     =F10.4/
41H0,54X,35H(*) ESTIMATION BY METHOD OF MOMENTS///)
199 FORMAT (1H1,9X,28HBIVARIATE CORRELATED POISSON,17X,13HPARAMETERS(*
9) //
155X,6HA     =F10.4/
255X,6HB     =F10.4/
355X,6HD     =F10.4/
41H0,54X,35H(*) ESTIMATION BY METHOD OF MOMENTS///)
200 FORMAT(1H1,9X,40HGENERALIZED BIVARIATE CORRELATED POISSON,5X,13HPA
9PARAMETERS(*)//
155X,6HEME   =F10.4/
255X,6HENE   =F10.4/
355X,6HU     =F10.4/
455X,6HV     =F10.4/
555X,6HW     =F10.4/
61H0,54X,35H(*) ESTIMATION BY METHOD OF MOMENTS///)
201 FORMAT(1H1,9X,37HCOMPOUND BIVARIATE CORRELATED POISSON,8X,13HPARAM
9ETERS(*)//
155X,6HK     =F10.4/
255X,6HB1    =F10.4/
355X,6HB2    =F10.4/
455X,6HB12   =F10.4/
555X,6HA     =F10.4/
61H0,54X,35H(*) ESTIMATION BY METHOD OF MOMENTS///)
END

```

INPUT DATA

Abbreviations:

- (I): integer value (must be right adjusted in its field)
 (F): floating point number (must be punched with a decimal)

Control card (5I1,3X,2I1)

Columns	1	(I)	INDEX(1)	} The vector INDEX(I) contains integers from 0 to 5, and they indicate the models which are to be fitted to the observed distribution. If any of the elements of INDEX(I)
	2	(I)	INDEX(2)	
	3	(I)	INDEX(3)	
	4	(I)	INDEX(4)	
	5	(I)	INDEX(5)	
				= 1 (the GBCP model is fitted)
				= 2 (the CBCP model is fitted)
				= 3 (the BCP model is fitted)
				= 4 (the CBNP model is fitted)
				= 5 (the CBUP model is fitted)
				= 0 (no model is fitted)
	9	(I)	IOPT1:	control variable to indicate whether the punching of the theoretical distribution is required for further use
				= 1 (punch distribution values in FORMAT (1F10.6))
				= 0 (do not)
	10	(I)	IOPT2:	control variable which specifies the format of the output
				= 1 (output in matrix form)
				= 0 (output in vector form)

Examples

INDEX(I) = [1,2,3,4,5] All five models are fitted. No punched
 IOPT1 = 0 output is produced and the printed
 IOPT2 = 0 output is in vector form.

INDEX(I) = [1,3,0,0,0] Only the GBCP and the BCP models are
 fitted. The values of these two
 theoretical distributions are punched,
 and the printed output is in matrix form.

NOTE: Whenever IOPT2 equals 1 the number of columns (NC) of the matrix
 after aggregation (see note at the end) needs to be specified (NC<10).
 This number must be punched in the space above the asterisks in
 formats 101,102,103,104, in subroutine OUTPUT (see program listing),
 i.g., suppose number of columns NC = 10, then 101 FORMAT (18X,1HX/19X,
 10I8).
 **

Dimension of the matrix of observed values (2I5)

Columns	1-5 (I)	NC: number of columns in observed distribution.
	6-10 (I)	NR: number of rows in observed distribution.

Matrix of observed values (1F10.0)

Insert one card for each observed value following the row order (first,
 all the numbers in the first row, second, all the numbers in the second row,
 etc.)

122

Each card contains:

Columns 1-10 (F) SAMPLE(I,J): observed value (I,J)

This set of cards must be ended with a card containing

Columns 1-2 (I) NCON: control variable

- = -1 (the preceeding set of data was the last)
- = -2 (read in dimension and observed values for the next distribution and the control card of the previous distribution is not modified)
- = -3 (read in new control card, and the dimension and observed values for the next distribution.)

END OF INPUT DATA

Example

- 1) control card (distribution I and II)
 - 2) dimension of the matrix of the observed distribution (distribution I)
 - 3) matrix of observed values (distribution I)
- NCON = -2
- 2) dimension of the matrix of observed distribution (distribution II)
 - 3) matrix of observed values (distribution II)
- NCON = -3
- 1) control card (distribution III)
 - 2) dimension of the matrix of observed distribution (distribution III)
 - 3) matrix of observed values (distribution III)
- NCON = -1

NOTES:

- a) The original data are grouped after computing the sample statistics in order to reduce computer time. The number of columns and rows is reduced until a given number of observations (NOBS) is reached. This value has been set equal to 5, but it can be modified by entering the desired value for this variable in the main program.
- b) The chi-square test is computed by grouping the expected values so as to obtain a minimum expectation (AMINEX) equal to 2. Again this value can be modified by entering the desired value for this variable in subroutine CHITEST.
- c) In order to be consistent with the present dimension of the arrays in the program, the following limits must be observed:

$$NC \leq 30$$

$$NR \leq 30$$

OUTPUT

A selection of the printed output can be made (see control card in Input Data). The information contained is the same in both cases, but its formal arrangement is different (see Output, Example).

Printed output

1) Statistics of the observed distribution

- mean (X)
- variance (X)
- mean (Y)
- variance (Y)
- covariance (X,Y)
- correlation coefficient

2) For each theoretical model:

- parameter estimates (using method of moments and/or maximum likelihood)
- observed and expected bivariate values
- observed and expected marginal distributions
- chi-square goodness-of-fit test.

Punched output

It is optional (see control card in Input Data)

a) For each theoretical model:

- expected values for each frequency class (Format (1F10.6))

125

C INPUT DATA EXAMPLE

C

13400 01

7 6

21

18

8

2

0

1

0

13

14

10

1

2

1

1

4

5

4

2

1

0

1

2

2

4

3

0

0

0

0

0

0

0

0

0

0

0

1

0

0

0

0

0

-1

C END OF INPUT DATA

PC OUTPUT EXAMPLE

BIVARIATE ANALYSIS OF OBSERVED DISTRIBUTION (CORR.COEFF.= .2867)

E(X)= 1.2645 VAR(X)= 1.7295 COV(X,Y)= .3872
 E(Y)= .9421 VAR(Y)= 1.0550 NER.OBS.= 121.

COMPOUND BIVARIATE CORRELATED POISSON

PARAMETER ESTIMATES BY THE METHOD OF MOMENTS

K= 3.4383
 B1= -.1923
 B2= -.2622
 B12= -.0118
 A= 1.4663

OBSERVED AND EXPECTED FREQUENCIES

Y	X	OBSERVED	EXPECTED
0	0	21.0000	32.4503
0	1	18.0000	19.9488
0	2	8.0000	7.9151
0	3	2.0000	2.5654
0	4	1.0000	.7383
1	0	13.0000	14.6341
1	1	14.0000	12.5139
1	2	10.0000	6.3608
1	3	1.0000	2.5140
1	4	4.0000	.8537
2	0	4.0000	4.2595
2	1	5.0000	2.9100
2	3	2.0000	1.3660
2	4	2.0000	.5378
3	0	2.0000	1.0127
3	1	2.0000	1.3529
3	2	4.0000	1.0021
3	3	3.0000	.5466
3+	4+	1.0000	2.8522

GOODNESS OF FIT (MINIMUM EXPECTATION = 2.)

	DEGREES OF FREEDOM	CHI SQUARE
AGREGGATION ALONG ROWS	10	18.7193
AGREGGATION ALONG COLUMNS	9	15.6235
AGREGGATION ALONG DIAGONAL	9	19.2867

MARGINAL DISTRIBUTIONS

X	OBSERVED	EXPECTED	Y	OBSERVED	EXPECTED
0	40.0000	52.3566	0	50.0000	63.6177
1	39.0000	38.4817	1	42.0000	36.8764
2	26.0000	18.1880	2	17.0000	13.7394
3	8.0000	6.9919	3	12.0000	6.7664
4	8.0000	4.9819			

BIVARIATE ANALYSIS OF OBSERVED DISTRIBUTION (CORR.COEFF.= .2867)

E(X)= 1.2645 VAR(X)= 1.7295 COV(X,Y)= .3872
 E(Y)= .9421 VAR(Y)= 1.0550 NER.OBS.= 121.

COMPOUND BIVARIATE UNCORRELATED POISSON

PARAMETER ESTIMATES BY THE METHOD OF MOMENTS

ALFA= 3.5837
 BETA= 2.8342
 A= .7451

OBSERVED AND EXPECTED FREQUENCIES

Y	X	OBSERVED	EXPECTED
0	0	21.0000	21.6794
0	1	18.0000	16.9662
0	2	8.0000	8.4913
0	3	28.0000	3.4513
0	4	1.0000	1.2405
1	0	13.0000	12.6415
1	1	14.0000	12.6537
1	2	10.0000	7.7146
1	3	1.0000	3.6972
1	4	4.0000	1.5307
2	0	4.0000	4.7141
2	1	5.0000	5.7482
2	2	4.0000	4.1321
2	3	2.0000	2.2811
2	4	2.0000	1.0689
3	0	2.0000	1.4276
3	1	2.0000	2.0526
3	2	4.0000	1.6996
3	3	3.0000	1.0620
3+	4+	1.0000	6.7474

GOODNESS OF FIT (MINIMUM EXPECTATION = 2.)

	DEGREES OF FREEDOM	CHI SQUARE
AGREGGATION ALONG ROWS	12	16.4488
AGREGGATION ALONG COLUMNS	13	13.9686
AGRAGGATION ALONG DIAGONAL	13	15.5768

MARGINAL DISTRIBUTIONS

X	OBSERVED	EXPECTED	Y	OBSERVED	EXPECTED
0	40.0000	40.4627	0	50.0000	51.8287
1	39.0000	37.4207	1	42.0000	38.2377
2	26.0000	22.0377	2	17.0000	17.944
3	8.0000	10.4915	3	12.0000	12.9891
4	8.0000	10.5875			

128

BIVARIATE ANALYSIS OF OBSERVED DISTRIBUTION (CORR.COEFF.= .2867)

E(X)= 1.2645 VAR(X)= 1.7295 COV(X,Y)= .3872
 E(Y)= .9421 VAR(Y)= 1.0550 NER.OBS.= 121.

COMPOUND BIVARIATE UNCORRELATED POISSON

PARAMETER ESTIMATES BY MAXIMUM LIKELIHOOD

ALFA= 3.6026
 BETA= 2.8491
 A= .7451

OBSERVED AND EXPECTED FREQUENCIES

Y	X	OBSERVED	EXPECTED
0	0	21.0000	21.6393
0	1	18.0000	16.9686
0	2	8.0000	8.4998
0	3	2.0000	3.4551
0	4	1.0000	1.2414
1	0	13.0000	12.6433
1	1	14.0000	12.6664
1	2	10.0000	7.7232
1	3	1.0000	3.6998
1	4	4.0000	1.5306
2	0	4.0000	4.7188
2	1	5.0000	5.7546
2	2	4.0000	4.1351
2	3	2.0000	2.2810
2	4	2.0000	1.0678
3	0	2.0000	1.4292
3	1	2.0000	2.0540
3	2	4.0000	1.6995
3	3	3.0000	1.0608
3+	4+	1.0000	6.7315

GOODNESS OF FIT (MINIMUM EXPECTATION = 2.)

	DEGREES OF FREEDOM	CHI SQUARE
AGREGGATION ALONG ROWS	12	16.4328
AGREGGATION ALONG COLUMNS	12	13.9523
AGREGGATION ALONG DIAGONAL	12	15.565

MARGINAL DISTRIBUTIONS

X	OBSERVED	EXPECTED	Y	OBSERVED	EXPECTED
0	40.0000	40.4306	0	50.0000	51.8 42
1	39.0000	37.4436	1	42.0000	38.2634
2	26.0000	22.0577	2	17.0000	17.9573
3	8.0000	10.4967	3	12.0000	12.9752
4	8.0000	10.5713			

C INPUT DATA EXAMPLE

C

25000 00

7 6

21

18

8

2

0

1

0

13

14

10

1

2

1

1

4

5

4

2

1

0

1

2

2

4

3

0

0

0

0

0

0

0

0

0

0

1

0

0

0

0

0

0

-1

C END OF INPUT DATA

C OUTPUT EXAMPLE

C

BIVARIATE ANALYSIS OF OBSERVED DISTRIBUTION

	X						
Y		0	1	2	3	4	MARGINAL Y
0		21.00	18.00	8.00	2.00	1.0	5 .
1		13.00	14.00	10.00	1.00	4.0	42.
2		4.00	5.00	4.00	2.00	2.0	17.
3		2.00	2.00	4.00	3.00	1.0	12.
MARGINAL X		40.00	39.00	26.00	8.00	8.0	121.

SAMPLE STATISTICS

CORR. COEFF. = .2867
 E(X) = 1.2645
 E(Y) = .9421
 VAR(X) = 1.7295
 VAR(Y) = 1.0550
 COV(X,Y) = .3872

GENERALIZED BIVARIATE CORRELATED POISSON

PARAMETERS(*)

EME = .3678
 ENE = .1197
 U = 3.4383
 V = 7.8685
 W = 3.4383

(*) ESTIMATION BY METHOD OF MOMENTS

	X						
Y		0	1	2	3	4	MARGINAL Y
0		19.48	15.13	8.65	4.02	2.42	49.7
1		14.09	12.75	8.00	4.01	2.65	41.5
2		5.94	6.03	4.09	2.12	1.56	19.81
3		2.47	2.82	2.08	1.20	1.42	9.99
MARGINAL X		41.98	36.73	22.82	11.42	8.05	121.

GOODNESS OF FIT (**)

	DEGREES OF FREEDOM	CHI SQUARE	P .
AGREGGATION ALONG ROWS	12	9.8	21.
AGREGGATION ALONG COLUMNS	12	8.3573	21.
AGREGGATION ALONG DIAGONAL	12	9.9518	21.
(**) MINIMUM EXPECTATION = 2.00			

131

BIVARIATE CORRELATED POISSON

PARAMETERS (*)

A = 1.2645
B = .9421
D = .3872

(*)ESTIMATION BY METHOD OF MOMEN

	X	0	1	2	3	4	MARGINAL Y
Y							
0		19.62	17.21	7.55	2.21	0.48	47. 7
1		10.89	17.15	10.86	4.15	1.12	44.16
2		3.02	6.87	6.33	3.25	1.11	2 .58
3		0.56	1.66	2.06	1.42	3.5	9.2
MARGINAL X		37.08	42.88	26.79	11.03	6.22	121.

GOODNESS OF FIT (**)

	DEGREES OF FREEDOM	CHI SQUARE	P .
AGREGGATION ALONG ROWS	11	13.3971	19.7
AGREGGATION ALONG COLUMNS	11	16.65 1	19.7
AGREGGATION ALONG DIAGONAL	11	15.7 71	

(**)MINIMUM EXPECTATION= 2.00

CONTAGIOUS BIVARIATE NEGATIVE BINOMIAL

PARAMETERS(*)

GAMMA= .2749
BETA = .9853
A = .7451

(*)ESTIMATION BY METHOD OF MOMEN

	X	0	1	2	3	4	MARGINAL Y
Y							
0		21.68	14.39	6.11	2.11	0.64	44.93
1		15.22	12.92	6.68	2.72	0.95	38.49
2		6.83	7.06	4.31	2.02	0.8	21. 2
3		2.49	3.04	2.13	1.13	7.77	16.56
MARGINAL X		46.22	37.41	19.23	7.97	10.17	121.

GOODNESS OF FIT (**)

	DEGREES OF FREEDOM	CHI SQUARE	P .
AGREGGATION ALONG ROWS	12	1 . 314	21.
AGREGGATION ALONG COLUMNS	13	22.3723	22.4
AGREGGATION ALONG DIAGONAL	13	23.9155	22.4

(**)MINIMUM EXPECTATION= 2.00

Center for Planning and Development Research
Working Papers Sponsored by the
Economic Development Administration

- #65 Ronald Choy, "A Review of the Role of Demographic Variables in Macro-Econometric Models," August, 1967.
- #66 Andrei Rogers, "The Aggregation Problem in Demography," October, 1967.
- #67 Michael Teitz, "Toward A Theory of Urban Public Facility Location," November, 1967.
- #70 Michael Teitz, "Locational Strategies for Competitive Systems," December, 1967.
- #72 William Alonso, "The Quality of Data and the Choice and Design of Predictive Models," December, 1967.
- #74 William Alonso, "Industrial Location and Regional Policy in Economic Development," February, 1968.
- #78 William Alonso, "Equity and Its Relation to Efficiency in Urbanization," July, 1968.
- #81 Andrei Rogers and Susan McDougall, "An Analysis of Population Growth and Change in Slovenia and the Rest of Yugoslavia," June, 1968.
- #84 Andrei Rogers, "Activity Allocation Models in Transportation Planning," September, 1968.
- #86 Andrei Rogers and Norbert G. Gomar, "Statistical Inference in Quadrat Analysis," October, 1968.
- #87 William Alonso, "Aspects of Regional Planning and Theory in the United States," October, 1968.
- #89 David Darwent, "Growth Pole and Growth Center Concepts: A Review, Evaluation and Bibliography," October, 1968.
- #90 William Alonso, "Beyond the Inter-Disciplinary Approach to Planning," November, 1968.
- #91 Shlomo Angel, "Point Patterns in Urban Regions: Two Complementary Tests," January, 1969.
- #92 Andrei Rogers and Susan McDougall Choy, "Multisectoral Models of Regional Demographic and Economic Growth," February, 1969.
- #93 Andrei Rogers, "Quadrat Analysis of Urban Dispersion," March, 1969.
- #96 Juan Martin, Norbert Gomar, and Andrei Rogers, "Some Computer Programs for Quadrat Analysis," May, 1969.
- #97 Andrei Rogers and Juan Martin, "Bivariate Quadrat Analysis of the Dispersion of Retail Establishments and Population in Urban Areas," May, 1969.