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ON THE VELOCITY DEPENDENCE OF THE EROSION OF DUCTILE METALS BY SOLID PARTICLES AT LOW ANGLES OF INCIDENCE

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Summary

An earlier analytical study of the erosion of ductile metals by rigid abrasive grains was based on the volume removed when a grain cut into the surface. Among other things, this predicted a dependence of volume removal on particle velocity of the form, volume ~ $(\text{velocity})^n$, where $n = 2$. However, subsequent experimental work has shown that $n > 2$. Several alternative mechanisms of material removal have been proposed to explain this effect.

In this paper, the original analysis is reexamined. By making a more realistic assumption about the location of the forces during particle-surface interaction, values of the exponent n are predicted which are in the range observed experimentally.

Introduction

Some years ago, one of the present writers attempted to predict the volume removed in erosion by writing the equations of motion for a rigid grain cutting into a ductile surface[1]. The result which was discussed more completely in later work[2] is essentially

$$V \sim \frac{MU^2}{p} f(\alpha)$$

where V = volume removed from the surface

M = mass of eroding particles

U = particle velocity

p = horizontal component of flow pressure between particle and surface

$f(\alpha)$ = a function of α the angle of impact, measured from the surface to the particle velocity vector.

Many features of this simple analysis agree well with experiment. The volume removed is generally proportional to the mass of eroding particles, except for an incubation period for values of α approaching 90°. The particle size does not have an influence for rigid particles provided it is greater than about 100 μm. The flow pressure p for annealed face-centered cubic metals, during erosion at $\alpha = 20°$, has been shown to be proportional to their Vicker's Hardness[3]. Perhaps the most impressive feature of the analysis is that the variation of volume removal with angle is predicted quite closely for angles between 0 and about 45°. For larger angles of impingement, the mechanism of erosion ceases to be one of cutting and could be described, loosely, as one of "repeated indenting". Also, a modification of the cutting analysis to treat surfaces with curvature allows the formation of a ripple pattern at low values of α to be explained[4].

In view of the success of the analysis in treating these features of erosion at low angles of impingement it is curious that it generally underestimates the role of the particle velocity. The predicted relation $V \sim U^n$ with $n = 2$ is seen to be only a first approximation with actual values of n being noticeably larger. For example, Sheldon[5] reported values of $n = 2.36$ for electrolytic copper and 6061-0 aluminum eroded by 60 and 180 mesh SiC particles at $\alpha = 20°$. For a SAE 1215 steel also at $\alpha = 20°$ the values were $n = 2.48$ for 60 mesh and 2.69 for 180 mesh particles. In other work[3], also using 60 mesh SiC particles at $\alpha = 20°$, tests on a variety of metals showed values of the exponent n ranging from 2.05 to 2.44. From tests at $\alpha = 90°$ Tilly and colleagues[6] found values of n close to 2.3 for a variety of materials. They also stated that tests on an 11 per cent chromium steel at

$\alpha = 20°$ confirmed this velocity dependence.

Attempts to explain the observed values of $n = 2$ have been limited. Sheldon and Kanhere[7] have presented a model based on indentation which predicts $V \sim \frac{U^3}{H_v^{3/2}}$ while Tilly and colleagues[6] suggest that shattering of the particles and an increase in secondary erosion occur at higher velocities. While such mechanisms may be involved in special cases, they lack generality and do not appear appropriate for erosion of ductile metals at low angles by rigid grains.

Another possibility is that the thermal properties of the surface may be involved in the mechanism of erosion. This was disposed of by testing 1100-0 aluminum and titanium. Despite the difference in thermal diffusivities by a factor of about twenty five, the velocity exponents deduced from tests at $\alpha = 10°$ and velocities of 39 and 97 m/sec were very similar being 2.42 for Aluminum and 2.47 for Titanium. These tests were made in a "sand-blast" type apparatus originally developed by Sheldon[8] and particle velocities were measured using the rotating disc device of Ruff and Ives[9]. Another series of tests on the aluminum showed an exponent of 2.46 at 10°, 2.64 at 20°, 2.65 at 50° and 3.12 at 80°.

With the preceding background we are led to reexamine the assumptions of the original cutting analysis. It will be shown that with a slight modification, velocity exponents are predicted which fall in the range observed experimentally and increase with increasing angle for a given range of velocities.

Original Analysis of Cutting

In the initial analysis[1], which we shall summarize for completeness, the configuration assumed was as shown in Fig. 1.

The assumptions made were:

1. Material is removed by rigid particles which do not fracture.
2. There is no initial rotation of the particles, which in a sense is an average condition. Since rotation of the particle may be shown to be small during cutting, this implies for polyhedral particles such as shown in Fig 1 that $X_T \simeq X + r\,\phi$, $Y_T \simeq Y$.
3. The ratio of the vertical force to the horizontal force on the particle during cutting is taken as a constant K. Based on grinding and scratching tests, a reasonable value for abrasive grains is $K = 2$.
4. A constant plastic flow pressure exists during cutting and its horizontal component is denoted by p.
5. Based on metal cutting observations, the depth over which metal contacts the particle is taken as twice the depth of cut, i.e. $\psi = L/Y_T = 2$ where L is shown in Fig. 1.
6. The volume removed is the product of the area swept out by the particle tip and the width b of the cutting face, i.e.

$$\text{Volume} = b \int Y_T \, dX_T = b \int^{t_c} Y_T \, \dot{X}_T \, dt$$

where t_c is the time at which cutting ceases and the dot denotes differentiation with respect to time, t.
7. A final assumption, implied but not stated in previous work, is that because the depth of cut is small compared to the particle size, the vertical and horizontal forces on the particle are taken to be located at its tip.

With these assumptions the equations of motion in the X, Y and ϕ directions may be written and the volume removal determined. This leads to:

$$V = \frac{c\,MU^2}{4p\left(1 + \frac{mr^2}{I}\right)}\left[\cos^2\alpha - \left(\frac{\dot{X}'_T}{U}\right)^2\right]$$

where

V = volume removed from surface,

M = mass of eroding particles,

m = mass of an individual particle,

I = moment of inertia of particle about its center of gravity,

r = average particle radius,

α = angle of impact,

U = particle velocity,

p = horizontal component of flow pressure,

c = fraction of particles cutting in idealized manner,

$\dot{X}'_T$ = horizontal velocity of tip of particle when cutting ceases.

An interesting feature which gives rise to the characteristic variation of V with α for ductile metals is that two possibilities exist for $\dot{X}'_T$. The first is that cutting ceases when the particle tip can no longer move forward i.e. $\dot{X}'_T = 0$. On the other hand, for low values of α the particle may leave the surface while it's tip is still moving horizontally. In this case it may be shown that

$$\dot{X}'_T = U\cos\alpha - \frac{2U}{P}\sin\alpha$$

where $\quad P = K \div (1 + mr^2/I)$

Thus we obtain

$$V = \frac{c\,MU^2}{4p\left(1 + \frac{mr^2}{I}\right)}\left[\cos^2\alpha\right]; \quad \dot{X}'_T = 0\ ,\ \alpha \geqslant \tan^{-1}\ \frac{P}{2}$$

$$V = \frac{c\,MU^2}{4p\left(1 + \frac{mr^2}{I}\right)} \frac{2}{P} \left[\sin 2\alpha - 2\,\frac{\sin^2\alpha}{P}\right] \; ; \quad \dot{X}_T' \text{ for } y_t = 0 \; , \; \alpha \leqslant \tan^{-1}\frac{P}{2}$$

The maximum volume removal occurs at $\tan 2\alpha = P$ while the two expressions are equal at the slightly higher angle given by $\tan \alpha = P/2$. Typically, $I \simeq mr^2/3$ and for $K \simeq 2$, $P \simeq 0.5$ and so the maximum erosion should occur at about $\alpha \simeq 13°$ and the transition in cutting modes at $\alpha \simeq 14°$.

It is in fact not necessary to invoke a two-dimensional particle and the same equations can be deduced for a particle of arbitrary shape[10]. The problem then is what to chose for the value of ψ. In Reference 10, a factor of 2 rather than 4 appears in the denominator of the preceeding equations because the factor ψ was ignored and in essence taken as unity. However, for our subsequent development of a more realistic analysis, a two-dimensional model will be used for simplicity.

Modified Analysis of Cutting

In the original analysis, the forces were taken to act on the tip of the particle. Although this allows a simple solution to be obtained it is more realistic, as shown in Fig. 2, to locate the resultant force in the center of the material having contact with the particle. The symmetrical picture of two-dimensional cutting shown in the figure could be considered as an average condition for grains which are "tilted" in either direction as they strike the surface. In keeping with the assumption that the vertical force is twice the horizontal, i.e. $K = 2$, the projected contact area in the horizontal plane is taken as twice that in the vertical plane.

The equations of motion for the X and Y directions are unchanged by this modification and are:

$$m\ddot{X} + pY\,\psi\,b = 0 \tag{1}$$

$$m\ddot{Y} + Kp\,Y\psi\,b = 0 \tag{2}$$

The equation for angular rotation now becomes:

$$I\ddot{\phi} + p\psi bY(r-Y) - (Kp\psi Yb)\,2Y = 0 \tag{3}$$

where $p\psi bY$ = horizontal force, and $Kp\psi Yb$ = vertical force. The assumptions used in the above equations are identical to those used in the original analysis, with $Y_T = Y$ and $X_T = X + r\phi$.

Using the initial conditions that $Y(0) = 0$ and $\dot{Y}(0) = U\sin\alpha$ the solution of Eq. 2 is

$$Y(t) = \frac{U}{\beta}\sin\alpha\,\sin\beta t \tag{4}$$

where

$$\beta = \left(\frac{pK\psi b}{m}\right)^{1/2}$$

Substituting $Y(t)$ into Eq. (1), and using the boundary conditions $\dot{X}(0) = U\cos\alpha$ and $X(0) = 0$, $X(t)$ may be expressed as:

$$X(t) = \frac{U\sin\alpha}{\beta K}\sin\beta t + \left\{U\cos\alpha - \frac{U\sin\alpha}{K}\right\}t \tag{5}$$

The rotation of the particle is found to be

$$\phi(t) = \left\{\frac{p\psi b}{I}\right\}\left[\left\{\frac{dU^2\sin\alpha^2}{\beta^2}\right\}\left(\frac{1}{4}\,t^2 + \frac{1}{8\beta^2}\cos 2\beta t\right)\right.$$
$$\left. + \frac{rU\sin\alpha}{\beta^3}\sin\beta t - \frac{rU\sin\alpha}{\beta^2}t - \frac{d^2U^2\sin^2\alpha}{8\beta^4}\right]$$

where $d = 2K + 1$. It is assumed that for many particles the average initial values of ϕ and $\dot{\phi}$ are zero. Taking $I = \frac{1}{3}mr^2$ and $K = 2$ as typical values for angular particles, then

$$\phi(t) = \frac{15}{16}\frac{U^2\sin^2\alpha}{\beta^2 r^2}\left[2(\beta t)^2 + \cos 2\beta t - 1\right] + \frac{3}{2}\frac{U\sin\alpha}{r\beta}(\sin\beta t - \beta t) \qquad (6)$$

As before, the volume assumed to be removed by a single particle is $b\int_0^{t_c} Y_T\dot{X}_T dt$. Combining Eqs. 4, 5 and 6 the volume removal becomes

$$\frac{Vol}{b} = \frac{U\sin\alpha}{\beta}\int_0^{t_c}(\sin\beta t)\left[(U\cos\alpha - 2U\sin\alpha) + (2U\sin\alpha\cos\beta t)\right.$$
$$\left. - \frac{15}{8}\frac{U^2\sin^2\alpha}{r\beta}\sin^2\beta t + \left(\frac{15}{4}\frac{U^2}{r}\sin^2\alpha\right)t\right] dt$$

Evaluating the integral from 0 to t_c, the predicted volume removal is

$$\frac{Vol}{b} = \frac{U^2\sin^2\alpha}{\beta^2}\left[\cos\beta t_c(2 - \cot\alpha) + \cot\alpha - \frac{3}{2} - \frac{1}{2}\cos 2\beta t_c\right] +$$
$$\frac{U\sin\alpha}{\beta}\left[\frac{15}{8}\frac{U^2\sin^2\alpha}{r\beta^2}\left(-\frac{1}{2}\sin\beta t_c + \frac{1}{6}\sin 3\beta t_c\right) + \frac{15}{4}\frac{U^2\sin^2\alpha}{r}\right. \qquad (7)$$
$$\left.\left(\frac{1}{\beta^2}\sin\beta t_c - \frac{1}{\beta}t_c\cos\beta t_c\right)\right].$$

This expression consists of the original solution, the first term, and a second term due to the change in location of the resultant force.

To determine the volume removal for the case when the tip of the particle is still moving horizontally as it leaves the surface, the condition on t_c is that $Y(t_c) = 0$ or $\beta t_c = \pi$. Using this value, Eq. 7 becomes:

$$\frac{Vol}{b} = \frac{U^2}{\beta^2}(\sin 2\alpha - 4\sin^2\alpha) + \frac{15}{4}\pi\frac{U^3\sin^3\alpha}{r\beta^3} \qquad (8)$$

The other possibility for the end of cutting is $\dot{X}_T(\beta t_c) = 0$. This condition can be expressed as

$$U\cos\alpha - 2U\sin\alpha + 2U\sin\alpha\,\cos\beta t_c - \frac{15}{8}\,\frac{U^2\sin^2\alpha}{\beta r}\,\sin(2\beta t_c)$$
$$+ \frac{15}{4}\,\frac{U^2\sin^2\alpha}{\beta r}\,(\beta t_c) = 0 \qquad (9)$$

The value of βt_c must be determined numerically for a given α and then used with Eq. 7 to predict volume removal.

We consider first the implications of Eq. 8. The ratio of the volumes removed for two velocities U_1 and U_2 is given by

$$\mathrm{Vol}_{U_2} \div \mathrm{Vol}_{U_1} = \left(\frac{U_2}{U_1}\right)^2 \frac{\left(\sin 2\alpha - 4\sin^2\alpha + \frac{15}{4}\,\pi\,\frac{U_2\sin^3\alpha}{\beta r}\right)}{\left(\sin 2\alpha - 4\sin^2\alpha + \frac{15}{4}\,\pi\,\frac{U_1\sin^3\alpha}{\beta r}\right)} \qquad (10)$$

For very small values of α, Eq. 10, reduces to $V \sim U^2$, the value given by the original analysis. Predictions for larger angles can be simplified by noting that the maximum depth of cut is $Y_{max} = \frac{U\sin\alpha}{\beta}$. Denoting the ratio of this depth to the particle radius as λ, the last terms in Eq. 10 become $\frac{15}{4}\,\pi\,\lambda\,\sin^2\alpha$. Although λ is a function of velocity and angle it is a convenient variable in that it is easily obtained from experiments.

If the ratio of the volume removal for two velocities is approximated by the power law $\mathrm{Vol} \sim U^n$, then

$$\left(\frac{U_2}{U_1}\right)^{n-2} = \frac{\sin 2\alpha - 4\sin^2\alpha + \frac{15}{4}\,\pi\,\frac{U_2}{U_1}\,\lambda_1\sin^2\alpha}{\sin^2\alpha - 4\sin^2\alpha + \frac{15}{4}\,\pi\,\lambda_1\,\sin^2\alpha} \qquad (11)$$

The value of λ_1 for a given velocity U_1 depends on the angle α. Typically for angles close to that for maximum erosion, for the velocities used in the tests we have been discussing, $\lambda_1 \simeq 0.1$ (i.e. a maximum depth of cut one-tenth of the particle radius). Taking this value as a guide the following table shows n values estimated for three values of λ_1, three velocity ratios and three angles.

		Velocity Exponent n		
λ_1	$\frac{U_2}{U_1}$	$\alpha = 11°$	$\alpha = 15°$	$\alpha = 18°$
	2	2.250	2.377	2.492
.125	1.75	2.237	2.261	2.457
	1.5	2.224	2.343	2.436
	2	2.212	2.326	2.420
.100	1.75	2.198	2.311	2.402
	1.5	2.186	2.294	2.368
	2	2.168	2.267	2.358
.075	1.75	2.158	2.253	2.358
	1.50	2.148	2.239	2.333

Fortunately, the value predicted for the effective exponent n for a given angle and λ_1 is quite insensitive to the ratio U_2/U_1. This shows that a plot of vloume removal against velocity on a log-log scale should be essentially a straight line. The result is also relatively insensitive to the assumed value of λ_1. Turning to the other case, in which the particle tip stops moving in the horizontal direction at the end of cutting (i.e. $\dot{X}_T' = 0$), Eq. 9 may be solved numerically for the value of βt_c corresponding to a given α and λ. Rewritten in terms of λ and $\tau = \beta t_c$, Eq. 9 becomes

$$2\cos\tau - 2 + \cot\alpha + \frac{15}{4}\frac{(U\sin\alpha)\tau}{\beta r} - \frac{15}{8}\frac{U\sin\alpha}{\beta r}\sin 2\tau = 0$$

or

$$2\cos\tau - 2 + \cot\alpha + \frac{15}{4}\lambda\ \tau - \frac{15}{8}\lambda\sin 2\tau = 0 \qquad (12)$$

For $\lambda = 0$, Eq. 12 has no solution for $\alpha \leqslant \tan^{-1}(1/4)$, and in this range of angles the particle leaves the surface while still cutting. As λ increases, the angle at which the transition between the two types of cutting occurs also increases. For example, for $\lambda = 0.1$ this occurs at about 20° compared to about 14° for $\lambda = 0$. Evaluating Eq. 9 or Eq. 12 for example for $\alpha = 30°$, $\lambda = 0.1$ and 0.15, and substituting into Eq. 7 leads to the prediction that the exponent n for $\lambda_1 = 0.1$ and $U_2/U_1 = 1.5$ should be $n = 2.61$.

This value and those in the table agree well with the range of values reported in the literature. Our values for aluminum mentioned earlier $n = 2.46$ at $\alpha = 10°$ and 2.64 at 20° are somewhat higher than those in the table. However, they confirm one aspect of the predictions, namely, that the exponent n should increase with angle, at least in the region where this cutting mode of removal is occuring.

As a final point, the angle at which maximum erosion occurs can be estimated by differentiating Eq. 8 and equating the derivative to zero. That is

$$\frac{\partial}{\partial\alpha}\left(\sin 2\alpha - 4\sin^2\alpha + \frac{15\pi}{4}\ \frac{U\sin^3\alpha}{r\beta}\right) = 0 \qquad (13)$$

For the original analysis where the last term is absent the result is merely $\tan 2\alpha_{max} = \frac{1}{2}$ or $\alpha_{max} = 13°$ for the assumed values of $K = 2$ and $I = mr^2/3$. In the more general case we find

$$\tan 2\alpha_{max} = \frac{1}{2 - \frac{45}{16}\pi \frac{U\sin \alpha_{max}}{r\beta}} \quad (14)$$

and are faced with the problem of evaluating $U/r\beta$ to solve for α_{max}. As an approximation, if we first put $\lambda = \frac{U\sin\alpha}{r\beta}$ in Eq. 13 before differentiating then $\tan 2\alpha_{max} = \frac{1}{2 - \frac{30}{16}\pi\lambda}$ and for $\lambda = .1$ $\alpha_{max} = 17.7°$. This represents the angle for maximum erosion when the velocity is varied with angle to obtain constant λ. As another approximation, if we put $\frac{U\sin \alpha_{max}}{r\beta} = 0.1$ in Eq. 14 the value of the angle for maximum erosion becomes 20.9°. In either case we are led to expect a slight increase in the angle for maximum erosion as the velocity is increased. We recall that the angle for maximum erosion predicted by the original analysis is $\tan 2\alpha_{max} = \frac{K}{1 + \frac{mr^2}{I}}$. Since K and I can only be approximated, one cannot expect precise predictions for α_{max} even from the original analysis. Hence it is the shift in α_{max} with velocity which has to be studied to verify the preceding predictions. A large number of careful experimental observations would be required to examine this effect as the change in peak angle for say a fifty percent increase in velocity is only a few degrees. For this reason, we have not pursued this aspect experimentally.

Conclusions

For the range of angles in which erosion occurs by a cutting mechanism, say $\alpha < 45°$, the modified analysis predicts values for the velocity exponent which are in general agreement with those observed experimentally. The prediction that the exponent should increase with angle for a given range of velocities is also in accord with observations. The modified analysis also predicts a somewhat greater angle for maximum erosion than the original theory.

In subsequent work it will be shown that the crater shapes predicted by the modified analysis are not significantly different from those given by the original analysis. Hence the main contribution of this more complicated "modified analysis" is to explain more precisely the role of particle velocity. By doing this successfully it provides further support for the basic method of predicting erosion at low angles used in the original analysis.

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Figure Captions

1. Idealized two-dimensional model of a rigid grain cutting into a ductile metal.
2. Location of resultant forces on a rigid grain while cutting (A) Original Analysis (B) Modified Analysis.

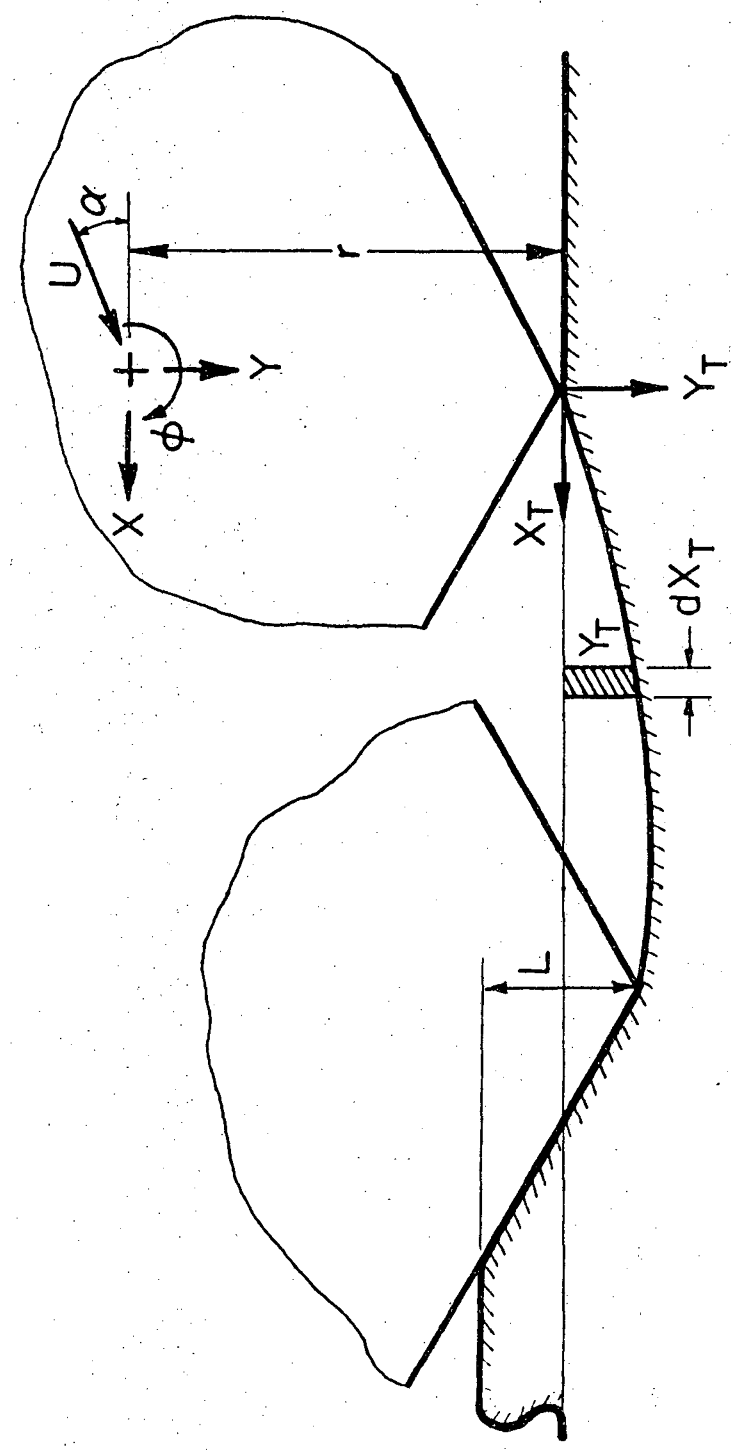

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Fig. 1

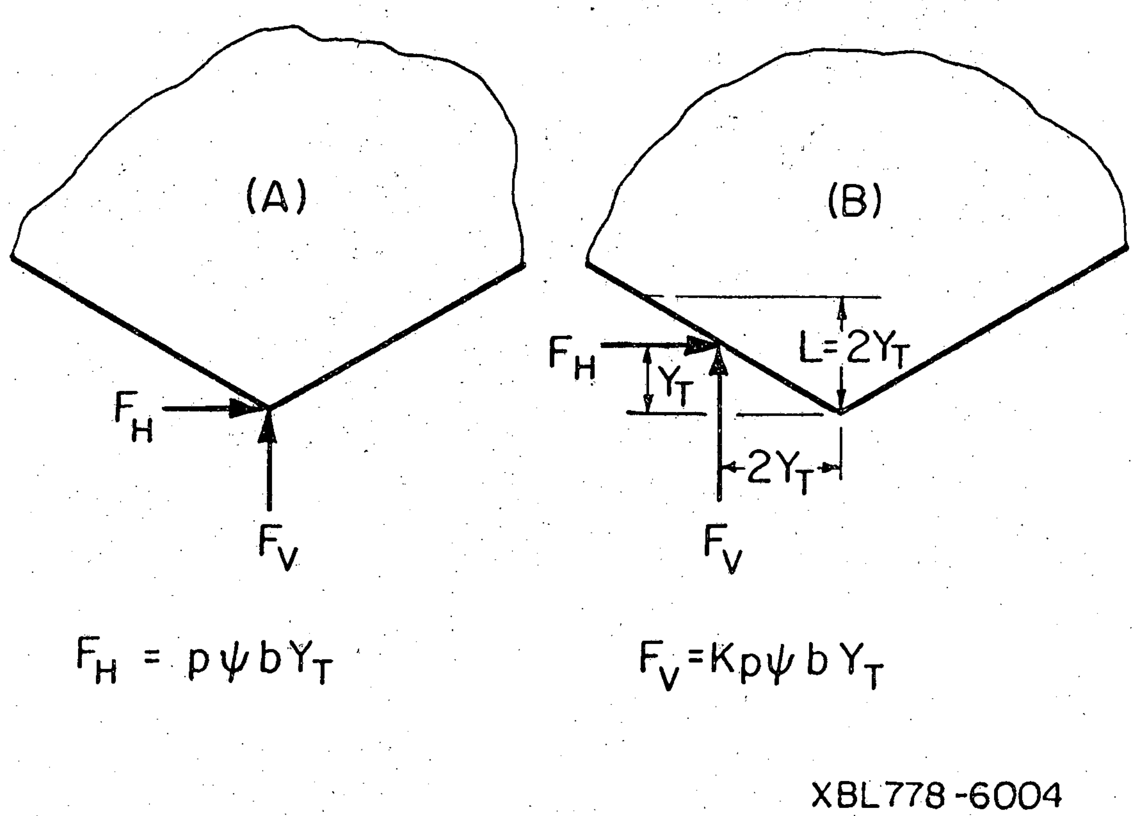

Fig. 2

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