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INCLUSION OF NUCLEAR-STRUCTURE CALCULATIONS IN NUCLEON-NUCLEUS SCATTERING\*

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ABSTRACT

It is shown how one can use the results of a nuclear structure calculation in the theory of nucleon-nucleus scattering. We restrict ourselves to the simple case of nucleon scattering on a hole nucleus. The correlations of the ground state have been included.

## INTRODUCTION

In a previous work<sup>1</sup> we have developed the general theory of nucleon-nucleus scattering on a hole nucleus in the framework of linear response theory extending Migdal's approach<sup>2</sup> to the scattering problem. With a similar goal the unrenormalized RPA has been applied to the scattering problem using a schematic model.<sup>3</sup> The details and restrictions of these methods can be found in Refs. 1 and 3 as well as further references. It turns out that a calculation of the scattering process using an effective particle-hole interaction would be rather complicated, since one has to solve a complicated Fredholm problem.<sup>1</sup> Therefore, one has so far studied the problem only in the framework of a schematic model<sup>1,3</sup> where the corresponding Fredholm determinant degenerates. But it is well known, that the schematic model is only a poor approximation to the real situation.\* For this reason we think one can obtain an improvement of the present status of the theory by including the results of the nuclear-structure calculation obtained with a normal effective particle-hole interaction. The deviations from the nuclear structure calculation--caused by the matrix elements of the interaction between continuum-bound and continuum-continuum single-particle states--will be treated in this work by a schematic approach. This implies that these special matrix elements can be approximately represented by a separable particle-hole force with the help of a fitting procedure to the real particle-hole force. One may get further improvement using perturbation theory for the difference of the particle-hole force and the separable force as a final step. In the first section we give a short summary of the nucleon-nucleus scattering theory on a hole nucleus in terms of Migdal's renormalized quantities. The explicit treatment of the model will then be given in the second section.

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\* See for instance Ref. 4.

## I. GENERAL FORMALISM

It has been shown in Ref. 1, that the scattering state in the shell model representation is given by

$$\langle \phi_{k,i}^{(-)} | \psi_{p,j}^{(+)} \rangle = (z_i z_k z_p z_j)^{-1/2} \langle 0 | \psi_i^+ \psi_k \frac{i\eta}{E-H+i\eta} \psi_p^+ \psi_j | 0 \rangle e^{i(\delta_p + \delta_k)} \quad (\text{I.1})$$

and

$$\langle \phi_{r,i} | \psi_{p,j}^{(+)} \rangle = (z_i z_r z_p z_j)^{-1/2} \langle 0 | \psi_i^+ \psi_r \frac{i\eta}{E-H+i\eta} \psi_p^+ \psi_j | 0 \rangle e^{i\delta_p} \quad (\text{I.2})$$

Here,  $|0\rangle$  denotes the normalized ground state of the compound system,  $(H|0\rangle = E_0(A)|0\rangle, E_0(A) = 0)$ ,  $E$  is the energy of the scattering system ( $E := \epsilon_p - \epsilon_j$ ).  $\psi_\mu^+$  and  $\psi_\mu$  are the Schrödinger creation and annihilation operators, respectively, of a nucleon with the quantum number set  $\mu$  fixed by an independent particle Hamiltonian  $H_S$ . Asymptotically they behave as standing waves. With  $z_\mu$  we denote Migdal's renormalization constant. If it is necessary we will label the continuum states by  $p$  and  $k$ , the hole states by  $i$  and  $j$ , and the bound particle states by  $r$  and  $s$  ( $\delta_r = \delta_s = 0$ ). The independent-particle model states in (I.1) and (I.2) are defined by:

$$|\phi_{p,i}^{(+)}\rangle = z^{i\delta_p} (z_i z_p)^{-1/2} \psi_p^+ \psi_i | 0 \rangle \quad (\text{I.3})$$

and

$$|\phi_{r,i}\rangle = (z_i z_r)^{-1/2} \psi_r^+ \psi_i | 0 \rangle \quad (\text{I.4})$$

By comparison with the definition of the renormalized response function

$$\tilde{L}_{\alpha\mu\lambda\nu}(\omega) := -(z_\alpha z_\mu z_\lambda z_\nu)^{-1/2} \langle 0 | \{ \psi_\lambda^+ \psi_\alpha (\omega-H+i\eta)^{-1} \psi_\nu^+ \psi_\mu - \psi_\nu^+ \psi_\mu (\omega+H-i\eta)^{-1} \psi_\lambda^+ \psi_\alpha \} | 0 \rangle \quad (\text{I.5})$$

we obtained for (I.1) the following expression for the scattering states:

$$\langle S_0 | S \rangle = -i\eta \tilde{L}_{kjip}(E) \quad , \quad (I.6)$$

$$\langle B_0 | S \rangle = -i\eta \tilde{L}_{rjip}(E) \quad (I.7)$$

Here, we have introduced the abbreviations

$$|S\rangle := e^{-i\delta_p} |\psi_{p,j}^{(+)}\rangle \quad ,$$

$$|S_0\rangle := e^{-i\delta_k} |\phi_{k,i}^{(+)}\rangle \quad ,$$

and

$$|B_0\rangle = |\phi_{r,i}\rangle \quad . \quad (I.8)$$

Assuming that the zero-order response can be represented by the quasi-particle shell model response and the effective particle-hole interaction ("irreducible vertex part") to be weakly energy-dependent in the considered energy region, we have derived the following equations for  $\tilde{L}^1$ :

$$\tilde{L}_{\mu\alpha\beta\nu}^S(\omega) = \tilde{L}_{\mu\beta}^S(\omega) \{ \delta_{\mu\nu} \delta_{\alpha\beta} - 2\pi \sum_{\rho\sigma} \tilde{I}_{\mu\rho\beta\sigma}(\omega) \tilde{L}_{\sigma\alpha\rho\nu}(\omega) \} \quad , \quad (I.9)$$

or

$$\tilde{L}_{\mu\alpha\beta\nu}^S(\omega) = \tilde{L}_{\nu\alpha}^S(\omega) \{ \delta_{\mu\nu} \delta_{\alpha\beta} - 2\pi \sum_{\rho\sigma} \tilde{L}_{\mu\rho\beta\sigma} \tilde{I}_{\sigma\alpha\rho\nu}(\omega) \} \quad . \quad (I.10)$$

Here,  $\tilde{I}$  is the effective particle-hole interaction (irreducible vertex in the particle-hole-channel). The shell model response is defined by

$$\tilde{L}_{\nu\mu}^S(\omega) := (n_\nu - n_\mu) \frac{n_\nu + n_\mu}{\omega + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)} \quad (I.11)$$

where  $n_\nu, n_\mu$  are the quasiparticle occupation numbers.  $\epsilon_\nu$  denotes the single particle energy in the quantum state  $\nu$ .

From the equations (I.6), (I.7), (I.9), and (I.11) we can now deduce the equations for the scattering states

$$\tilde{\rho}_{\nu\mu,S} = (n_\nu - n_\mu) \frac{1}{E_S + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)} \{ \delta_{\nu\mu} \delta_{j\nu}(-i\eta) - 2\pi \sum_{\sigma\lambda} \tilde{I}_{\nu\sigma\mu\lambda}(\omega) \tilde{\rho}_{\lambda\sigma,S} \} \quad (I.12)$$

where  $\tilde{\rho}_{\nu\mu,M}$  the matrix element of the quasi-particle density matrix

$$\tilde{\rho}_{\nu\mu,M} = (z_\nu z_\mu)^{-1/2} \langle 0 | \psi_\mu^+ \psi_\nu | M \rangle \quad (I.13)$$

The knowledge of these matrix elements is equivalent to the knowledge of the states of the compound system. By  $|M\rangle$  we denote a scattering state  $|S\rangle$  as well as a bound state  $|B\rangle$ . Since we also want to calculate the effect of the continuum single-particle states on the bound states of the compound system, we have to give the system of equations for the matrix elements  $\tilde{\rho}_{\nu\mu,B}^{1,2}$ . We derive these equations by inserting the spectral representation of the renormalized response function

$$\tilde{I}_{\nu\sigma\mu\beta}(\omega) = - \sum_M \left\{ \frac{\tilde{\rho}_{\nu\mu,M} \tilde{\rho}_{\beta\sigma,M}^*}{\omega - E_M + i\eta} - \frac{\tilde{\rho}_{\sigma\beta,M} \tilde{\rho}_{\mu\nu,M}^*}{\omega + E_M - i\eta} \right\} \quad (I.14)$$

into Eq. (I.9) or (I.10) and taking the limit  $\omega \rightarrow \pm E_B$ . Then the pole terms give the following set of equations:

$$\{E_B + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)\} \tilde{\rho}_{\nu\mu,B} = -(n_\nu - n_\mu) 2\pi \sum_{\alpha\beta} \tilde{I}_{\nu\alpha\mu\beta} \tilde{\rho}_{\beta\alpha,B} \quad , \quad (I.15)$$

$$\{-E_B + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)\} \tilde{\rho}_{\mu\nu,B}^* = -(n_\nu - n_\mu) 2\pi \sum_{\alpha\beta} \tilde{I}_{\nu\alpha\mu\beta} \tilde{\rho}_{\alpha\beta,B}^* \quad , \quad (I.16)$$



$$\{E_B + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)\} \tilde{\rho}_{\nu\mu,B}^* = -(n_\nu - n_\mu) 2\pi \sum_{\alpha\beta} \tilde{\rho}_{\beta\alpha,B}^* \tilde{I}_{\beta\mu\alpha\nu}, \quad (\text{I.17})$$

$$\{-E_B + \epsilon_\mu - \epsilon_\nu - i\eta(n_\nu - n_\mu)\} \tilde{\rho}_{\mu\nu,B} = -(n_\nu - n_\mu) 2\pi \sum_{\alpha\beta} \tilde{\rho}_{\alpha\beta,B} \tilde{I}_{\beta\mu\alpha\nu}, \quad (\text{I.18})$$

which are equivalent to (I.12).  $\sum_{\alpha\beta}$  means summation over the discrete states as well as integration over the continuum states. Insertion of the spectral representation into the equation of motion also gives the completeness relation:

$$\sum_M \{\tilde{\rho}_{\nu\mu,M} \tilde{\rho}_{\lambda\sigma,M}^* - \tilde{\rho}_{\sigma\lambda,M} \tilde{\rho}_{\mu\nu,M}^*\} = (n_\mu - n_\nu) \delta_{\nu\lambda} \delta_{\sigma\mu}, \quad (\text{I.18})$$

which can be proved by complex integration.

## II. TREATMENT OF THE MODEL

In our approach we want to take into account the solutions of a nuclear structure calculation. These solutions are known for many forms of the particle-hole interactions.\* They are the result of the diagonalization of Eqs. (I.15)-(I.18) in a finite space, since all single-particle quantum numbers are restricted to bound states only. We denote the states resulting from this procedure by  $|n\rangle$ , the corresponding density matrix elements are  $\tilde{\rho}_{\nu\mu,n}$ , where  $\nu$  and  $\mu$  belong to the discrete part of the spectrum of  $H_S$ . We assume in our further procedure, that the  $\tilde{\rho}_{\nu\mu,n}$  are known since we can either obtain them by a standard procedure or we can use the results of a previous calculation. With the help of (I.18) we can express the unknown  $\tilde{\rho}_{\nu\mu,M}$  equivalently by the

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\* See for instance Refs. 5-7.

$$\tilde{\rho}_{ik,S} = - \frac{\lambda F_S w_{ik}}{E_S + \epsilon_k - \epsilon_i} \left\{ 1 - \lambda \sum_n |F_n^0|^2 \frac{2E_n}{E_n^2 - E_S^2} \right\} \quad (II.18)$$

where

$$F_S = \frac{w_{pj}}{1 + \left[ \sum_{ki} \lambda |w_{ki}|^2 \left( \frac{1}{E_S + \epsilon_k - \epsilon_i} - \frac{1}{E_S - \epsilon_k + \epsilon_i + i\eta} \right) \right] \left[ 1 - \lambda \sum_n |F_n^0|^2 \frac{2E_n}{E_n^2 - E_S^2} \right]} \quad (II.19)$$

Use of the normalization condition for the state  $|B\rangle$  gives  $F_B$  as follows

$$F_B^2 = \lambda^{-2} \left\{ \left[ 1 - \lambda \sum_n |F_n^0|^2 \frac{2E_n}{E_n^2 - E_B^2} \right]^2 \sum_{ki} |w_{ki}|^2 \left[ \left( \frac{1}{E_B - \epsilon_k + \epsilon_i} \right)^2 - \left( \frac{1}{E_B + \epsilon_k - \epsilon_i} \right)^2 \right] + \sum_{r,i} \left| \sum_n \left( \frac{F_n^0}{E_B + E_n} \tilde{\rho}_{ir,n}^* + \frac{F_n^{0*}}{E_B - E_n} \tilde{\rho}_{ri,n} \right) \right|^2 - \left| \sum_n \left( \frac{F_n^0}{E_B + E_n} \tilde{\rho}_{ri,n}^* + \frac{F_n^{0*}}{E_B - E_n} \tilde{\rho}_{ir,n} \right) \right|^2 \right\}^{-1} \quad (II.20)$$

Now, all unknown quantities have been expressed in terms of the solutions of the nuclear structure problem and the matrix elements between scattering-scattering states or scattering-bound states, respectively. The (complex) resonances are given by the solutions of Eq. (II.16). From (II.17) and (I.8) we can immediately now read off the wanted expressions for the S-matrix and the T-matrix obtaining:

$$S_{ki,pj}(E)_{pj} = e^{2i\delta_p} \delta_{kp} \delta_{ij} - 2\pi i \delta(E_{pj} - E_{ki}) T_{ki,pj}(E)_{pj} \quad (II.21)$$

with

$$T_{ki,pj}(E) = e^{i(\delta_p + \delta_k)} \lambda w_{pj} w_{ki} \frac{1 - \lambda \sum_n |F_n^0|^2 \frac{2E_n}{E_n^2 - E^2}}{1 + \left[ \lambda \sum_{p',j'} |w_{p'j'}|^2 \left[ \frac{1}{\epsilon_{p'} - \epsilon_{j'} - E - i\eta} + \frac{1}{\epsilon_{p'} - \epsilon_{j'} + E} \right] \right] \left( 1 - \lambda \sum_n |F_n^0|^2 \frac{2E_n}{E_n^2 - E^2} \right)} \quad (II.22)$$

Our formulas reduce to the corresponding ones of Ref. 3, where a different method has been used, if we neglect Migdal's renormalization and use a schematic model for the finite RPA, too. We are going to perform some calculations with the described method in the oxygen- and calcium-region using density-dependent forces for the bound-state calculation.<sup>6</sup>

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FOOTNOTES AND REFERENCES

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