Submitted to Physical Review

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September 1969

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
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ABSTRACT

It is shown how one can use the results of a nuclear structure calculation in the theory of nucleon-nucleus scattering. We restrict ourselves to the simple case of nucleon scattering on a hole nucleus. The correlations of the ground state have been included.

INTRODUCTION

In a previous work[1] we have developed the general theory of nucleon-nucleus scattering on a hole nucleus in the framework of linear response theory extending Migdal's approach[2] to the scattering problem. With a similar goal the unrenormalized RPA has been applied to the scattering problem using a schematic model.[3] The details and restrictions of these methods can be found in Refs. 1 and 3 as well as further references. It turns out that a calculation of the scattering process using an effective particle-hole interaction would be rather complicated, since one has to solve a complicated Fredholm problem.[1] Therefore, one has so far studied the problem only in the framework of a schematic model[1,3] where the corresponding Fredholm determinant degenerates. But it is well known, that the schematic model is only a poor approximation to the real situation.* For this reason we think one can obtain an improvement of the present status of the theory by including the results of the nuclear-structure calculation obtained with a normal effective particle-hole interaction. The deviations from the nuclear structure calculation--caused by the matrix elements of the interaction between continuum-bound and continuum-continuum single-particle states--will be treated in this work by a schematic approach. This implies that these special matrix elements can be approximately represented by a separable particle-hole force with the help of a fitting procedure to the real particle-hole force. One may get further improvement using perturbation theory for the difference of the particle-hole force and the separable force as a final step. In the first section we give a short summary of the nucleon-nucleus scattering theory on a hole nucleus in terms of Migdal's renormalized quantities. The explicit treatment of the model will then be given in the second section.

*See for instance Ref. 4.

I. GENERAL FORMALISM

It has been shown in Ref. 1, that the scattering state in the shell model representation is given by

$$\langle \phi^{(-)}_{k,i} | \Psi^{(+)}_{p,j} \rangle = (z_i z_k z_p z_j)^{-1/2} \langle 0 | \psi^+_i \psi_k \frac{i\eta}{E-H+i\eta} \psi^+_p \psi_j | 0 \rangle \, e^{i(\delta_p+\delta_k)} \tag{I.1}$$

and

$$\langle \phi_{r,i} | \Psi^{(+)}_{p,j} \rangle = (z_i z_r z_p z_j)^{-1/2} \langle 0 | \psi^+_i \psi_r \frac{i\eta}{E-H+i\eta} \psi^+_p \psi_j | 0 \rangle \, e^{i\delta_p} \tag{I.2}$$

Here, $|0\rangle$ denotes the normalized ground state of the compound system, $(H|0\rangle = E_0(A)|0\rangle$, $E_0(A) = 0)$, E is the energy of the scattering system $(E := \varepsilon_p - \varepsilon_j)$. ψ^+_μ and ψ_μ are the Schrödinger creation and annihilation operators, respectively, of a nucleon with the quantum number set μ fixed by an independent particle Hamiltonian H_S. Asymptotically they behave as standing waves. With z_μ we denote Migdal's renormalization constant. If it is necessary we will label the continuum states by p and k, the hole states by i and j, and the bound particle states by r and s $(\delta_r = \delta_s = 0)$. The independent-particle model states in (I.1) and (I.2) are defined by:

$$|\phi^{(+)}_{p,i}\rangle = \ell^{i\delta_p} (z_i z_p)^{-1/2} \psi^+_p \psi_i |0\rangle \tag{I.3}$$

and

$$|\phi_{r,i}\rangle = (z_i z_r)^{-1/2} \psi^+_r \psi_i |0\rangle \tag{I.4}$$

By comparison with the definition of the renormalized response function

$$\tilde{L}_{\alpha\mu\lambda\nu}(\omega) := -(z_\alpha z_\mu z_\lambda z_\nu)^{-1/2} \langle 0 | \{ \psi^+_\lambda \psi_\alpha (\omega - H + i\eta)^{-1} \psi^+_\nu \psi_\mu - \psi^+_\nu \psi_\mu (\omega + H - i\eta)^{-1} \psi^+_\lambda \psi_\alpha \} | 0 \rangle \tag{I.5}$$

we obtained for (I.1) the following expression for the scattering states:

$$\langle S_0 | S \rangle = -i\eta \, \tilde{L}_{kjip}(E) \quad , \tag{I.6}$$

$$\langle B_0 | S \rangle = -i\eta \, \tilde{L}_{rjip}(E) \tag{I.7}$$

Here, we have introduced the abbreviations

$$|S\rangle := e^{-i\delta_p} |\Psi^{(+)}_{p,j}\rangle \quad ,$$

$$|S_0\rangle := e^{-i\delta_k} |\phi^{(+)}_{k,i}\rangle \quad ,$$

and

$$|B_0\rangle = |\phi_{r,i}\rangle \quad . \tag{I.8}$$

Assuming that the zero-order response can be represented by the quasi-particle shell model response and the effective particle-hole interaction ("irreducible vertex part") to be weakly energy-dependent in the considered energy region, we have derived the following equations for $\tilde{L}$ [1]:

$$\tilde{L}_{\mu\alpha\beta\nu}(\omega) = \tilde{L}^{s}_{\mu\beta}(\omega) \, \{\delta_{\mu\nu}\delta_{\alpha\beta} - 2\pi \sum_{\rho\sigma} \tilde{I}_{\mu\rho\beta\sigma}(\omega) \, \tilde{L}_{\sigma\alpha\rho\nu}(\omega)\} \quad , \tag{I.9}$$

or

$$\tilde{L}_{\mu\alpha\beta\nu}(\omega) = \tilde{L}^{s}_{\nu\alpha}(\omega) \, \{\delta_{\mu\nu}\delta_{\alpha\beta} - 2\pi \sum_{\rho\sigma} \tilde{L}_{\mu\rho\beta\sigma} \, \tilde{I}_{\sigma\alpha\rho\nu}(\omega)\} \quad . \tag{I.10}$$

Here, $\tilde{I}$ is the effective particle-hole interaction (irreducible vertex in the particle-hole-channel). The shell model response is defined by

$$\tilde{L}^{s}_{\nu\mu}(\omega) := (n_\nu - n_\mu) \frac{n_\nu + n_\mu}{\omega + \varepsilon_\mu - \varepsilon_\nu - i\eta(n_\nu - n_\mu)} \tag{I.11}$$

where n_ν, n_μ are the quasiparticle occupation numbers. ε_ν denotes the single particle energy in the quantum state ν.

From the equations (I.6), (I.7), (I.9), and (I.11) we can now deduce the equations for the scattering states

$$\tilde{\rho}_{\nu\mu,S} = (n_\nu - n_\mu)\frac{1}{E_S + \varepsilon_\mu - \varepsilon_\nu - i\eta(n_\nu - n_\mu)} \{\delta_{p\nu}\delta_{j\mu}(-i\eta) - 2\pi \sum_{\sigma\lambda} \tilde{I}_{\nu\sigma\mu\lambda}(\omega)\, \tilde{\rho}_{\lambda\sigma,S}\} \quad (I.12)$$

where $\tilde{\rho}_{\nu\mu,M}$ the matrix element of the quasi-particle density matrix

$$\tilde{\rho}_{\nu\mu,M} = (z_\nu z_\mu)^{-1/2} \langle 0|\psi_\mu^+ \psi_\nu|M\rangle \quad (I.13)$$

The knowledge of these matrix elements is equivalent to the knowledge of the states of the compound system. By $|M\rangle$ we denote a scattering state $|S\rangle$ as well as a bound state $|B\rangle$. Since we also want to calculate the effect of the continuum single-particle states on the bound states of the compound system, we have to give the system of equations for the matrix elements $\tilde{\rho}_{\nu\mu,B}$.[1,2] We derive these equations by inserting the spectral representation of the renormalized response function

$$\tilde{L}_{\nu\sigma\mu\beta}(\omega) = -\sum_M \left\{\frac{\tilde{\rho}_{\nu\mu,M}\tilde{\rho}^*_{\beta\sigma,M}}{\omega - E_M + i\eta} - \frac{\tilde{\rho}_{\sigma\beta,M}\,\tilde{\rho}^*_{\mu\nu,M}}{\omega + E_M - i\eta}\right\} \quad (I.14)$$

into Eq. (I.9) or (I.10) and taking the limit $\omega \to \pm E_B$. Then the pole terms give the following set of equations:

$$\{E_B + \varepsilon_\mu - \varepsilon_\nu - i\eta(n_\nu - n_\mu)\}\, \tilde{\rho}_{\nu\mu,B} = -(n_\nu - n_\mu)\, 2\pi \sum_{\alpha\beta} \tilde{I}_{\nu\alpha\mu\beta}\, \tilde{\rho}_{\beta\alpha,B} \quad , \quad (I.15)$$

$$\{-E_B + \varepsilon_\mu - \varepsilon_\nu - i\eta(n_\nu - n_\mu)\}\, \tilde{\rho}^*_{\mu\nu,B} = -(n_\nu - n_\mu)\, 2\pi \sum_{\alpha\beta} \tilde{I}_{\nu\alpha\mu\beta}\, \tilde{\rho}^*_{\alpha\beta,B} \quad , \quad (I.16)$$

$$\{E_B+\varepsilon_\mu-\varepsilon_\nu-i\eta(n_\nu-n_\mu)\}\ \tilde{\rho}^*_{\nu\mu,B} = -(n_\nu-n_\mu)\ 2\pi \sum_{\alpha\beta} \tilde{\rho}^*_{\beta\alpha,B}\ \tilde{I}_{\beta\mu\alpha\nu} \quad , \qquad (I.17)$$

$$\{-E_B+\varepsilon_\mu-\varepsilon_\nu-i\eta\ (n_\nu-n_\mu)\}\ \tilde{\rho}_{\mu\nu,B} = -(n_\nu-n_\mu)\ 2\pi \sum_{\alpha\beta} \tilde{\rho}_{\alpha\beta,B}\ \tilde{I}_{\beta\mu\alpha\nu} \quad , \qquad (I.18)$$

which are equivalent to (I.12). $\sum_{\alpha\beta}$ means summation over the discrete states as well as integration over the continuum states. Insertion of the spectral representation into the equation of motion also gives the completeness relation:

$$\sum_M \{\tilde{\rho}_{\nu\mu,M}\ \tilde{\rho}^*_{\lambda\sigma,M} - \tilde{\rho}_{\sigma\lambda,M}\ \tilde{\rho}^*_{\mu\nu,M}\} = (n_\mu-n_\nu)\ \delta_{\nu\lambda}\delta_{\sigma\mu} \quad , \qquad (I.18)$$

which can be proved by complex integration.

II. TREATMENT OF THE MODEL

In our approach we want to take into account the solutions of a nuclear structure calculation. These solutions are known for many forms of the particle-hole interactions.* They are the result of the diagonalization of Eqs. (I.15)-(I.18) in a finite space, since all single-particle quantum numbers are restricted to bound states only. We denote the states resulting from this procedure by $|n\rangle$, the corresponding density matrix elements are $\tilde{\rho}_{\nu\mu,n}$, where ν and μ belong to the discrete part of the spectrum of H_S. We assume in our further procedure, that the $\tilde{\rho}_{\nu\mu,n}$ are known since we can either obtain them by a standard procedure or we can use the results of a previous calculation. With the help of (I.18) we can express the unknown $\tilde{\rho}_{\nu\mu,M}$ equivalently by the

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*See for instance Refs. 5-7.

$$\tilde{\rho}_{ik,S} = -\frac{\lambda F_S w_{ik}}{E_S+\varepsilon_k-\varepsilon_i}\left\{1-\lambda\sum_n |F_n^0|^2 \frac{2E_n}{E_n^2-E_S^2}\right\} \tag{II.18}$$

where

$$F_S = \frac{w_{pj}}{1+\left\{\sum_{ki}\lambda|w_{ki}|^2\left(\frac{1}{E_S+\varepsilon_k-\varepsilon_i}-\frac{1}{E_S-\varepsilon_k+\varepsilon_i+i\eta}\right)\right\}\left[1-\lambda\sum_n|F_n^0|^2\frac{2E_n}{E_n^2-E_S^2}\right]} \tag{II.19}$$

Use of the normalization condition for the state $|B\rangle$ gives F_B as follows

$$F_B^2 = \lambda^{-2}\left\{\left[1-\lambda\sum_n|F_n^0|^2\frac{2E_n}{E_n^2-E_B^2}\right]^2\sum_{ki}|w_{ki}|^2\left[\left(\frac{1}{E_B-\varepsilon_k+\varepsilon_i}\right)^2-\left(\frac{1}{E_B+\varepsilon_k-\varepsilon_i}\right)^2\right]\right.$$
$$+\sum_{r,i}\left[\left|\sum_n\left(\frac{F_n^0}{E_B+E_n}\tilde{\rho}^*_{ir,n}+\frac{F_n^{0*}}{E_B-E_n}\tilde{\rho}_{ri,n}\right)\right|^2\right.$$
$$\left.\left.-\left|\sum_n\left(\frac{F_n^0}{E_B+E_n}\tilde{\rho}^*_{ri,n}+\frac{F_n^{0*}}{E_B-E_n}\tilde{\rho}_{ir,n}\right)\right|^2\right]\right\}^{-1} \tag{II.20}$$

Now, all unknown quantities have been expressed in terms of the solutions of the nuclear structure problem and the matrix elements between scattering-scattering states or scattering-bound states, respectively. The (complex) resonances are given by the solutions of Eq. (II.16). From (II.17) and (I.8) we can immediately now read off the wanted expressions for the S-matrix and the T-matrix obtaining:

$$S_{ki,pj}(E)_{pj} = e^{2i\delta_p}\delta_{kp}\delta_{ij} - 2\pi i\,\delta(E_{pj}-E_{ki})\,T_{ki,pj}(E_{pj}) \tag{II.21}$$

with

$$T_{ki,pj}(E) = e^{i(\delta_p+\delta_k)}\lambda\, w_{pj}\, w_{ki}$$
$$* \frac{1-\lambda\sum_n|F_n^0|^2\frac{2E_n}{E_n^2-E^2}}{1+\left\{\lambda\sum_{p'j'}|w_{p'j'}|^2\left[\frac{1}{\varepsilon_{p'}-\varepsilon_{j'}-E-i\eta}+\frac{1}{\varepsilon_{p'}-\varepsilon_{j'}+E}\right]\right\}\left(1-\lambda\sum_n|F_n^0|^2\frac{2E_n}{E_n^2-E^2}\right)} \tag{II.22}$$

Our formulas reduce to the corresponding ones of Ref. 3, where a different method has been used, if we neglect Migdal's renormalization and use a schematic model for the finite RPA, too. We are going to perform some calculations with the described method in the oxygen- and calcium-region using density-dependent forces for the bound-state calculation.[6]

The author would like to thank Dr. L. Garside, Professor N. Glendenning, and Professor C. Shakin for helpful discussions. The Deutsche Forschungsgemeinschaft, Germany supported this work by a grant.

FOOTNOTES AND REFERENCES

*Work performed under the auspices of the U. S. Atomic Energy Commission.

†On leave from the Sektion Physik der Universität Munich, Munich, Germany.

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