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Non-Adiabatic Berry's Phase For A Spin Particle In A Rotating Magnetic Field

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Abstract: The time-dependent Schrödinger equation for a spin particle in a rotating magnetic field is solved analytically by the cranking method and the exact solutions are employed to study non-adiabatic Berry's phase. A new expression for Berry's phase is given, which shows that Berry's phase is related to the expectation value of spin along the rotating axis and gives Berry's phase a physical explanation besides its gauge geometric interpretation. The new expression also presents a simple algorithm for calculating non-adiabatic Berry's phase for Hamiltonians which are non-linear functions of the SU(2) generators. It is shown that non-adiabaticity alters the time evolution ray and in turn changes its Berry's phase. For SU(2) dynamical group, non-adiabatic effect on Berry's phase manifests itself as spin-alignment (a phenomenon in nuclear physics) and spin-alignment quantization(observed recently in high spin nuclear physics) is related to Berry's phase quantization.

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Berry's phase [1] is a striking discovery in both theoretical and experimental physics in recent years. A great amount of interest has been generated by Berry's profound work. Soon after Berry's discovery, Simon [2] gave it a simple geometrical interpretation and related Berry's phase to the homological problem in the theory of fibre bundle. Berry's phase also finds its application in gauge theories [3]. In the meanwhile, a number of experiments have been reported, including observations on photons [4], neutrons [5], electrons [6], nuclear quadrupole resonances [7], laser interferometry [8] and molecular energy levels [9].

The naive definition of Berry's phase is based on the quantum adiabatic theorem [10]. Aharonov and Anandan [11] generalized Berry's results by dropping the adiabatic condition and identifying the time integral of expectation value of the Hamiltonian as the dynamical phase. Samuel and Bhandari [12] further extended Berry's phase to the cases of non-unitary and non-cyclic evolution by employing Pancharatnam's [13] idea of comparing phases of two arbitrary polarized light beams based on interference. The main conclusion of the above authors is that Berry's phase is a geometrical object in projective Hilbert space(so called ray space): it depends only on the path traced by the evolution ray in the projective Hilbert space and its value is determined by the curvature of the ray space and the traced path. The above development is important and includes non-adiabatic aspects of Berry's phase in principle. However, it is too general and formal to provide a deeper insight into and a tractable algorithm for calculating non-adiabatic Berry's phase.

On the other hand, up to now, most of experiments performed is designed to measure adiabatic Berry's phase, most of theoretical articles on Berry's phase addresses its geometrical aspects. It is true that Berry's phase in general conditions depends only on the path traced by the time evolution ray and the curvature of the ray space. To decribe a ray could be a geometrical problem, while to generate a ray and its time evolution is definitely a physical problem. As a time-evolution ray has been generated, its geometric properties in ray space is also specified and the Berry's phase can be calculated by pure geometric method, while the relevant dynamics which generated the ray could be forgotten.

However, as a whole physical problem, when the dynamics of a system generates a time-dependent physical state, it generates a special geometric object(a ray) at the same time. As the dynamical group is fixed, the geometry dictated by which is also specified. The dynamics, i.e., the Hamiltonian (which is a function of the dynamical group generators) can not change the geometry. However it can change a geometric object and its geometric properties (for example, one can't change Euclidean geometry. But one can change a circle into a triangle with different geometric properties). In such a sense, dynamics determines Berry's phase through determining the ray itself and its path. How dynamics effects a ray, its path and its Berry's phase, is a dynamical problem and non-adiabatic aspects of Berry's phase. The objective of this note is to address this subject.

In what follows we shall employ a soluble model to explore the subject. A particle with spin-j in a rotating magnetic field is an elegant model which is both of practical and theoretical interest. Its analytical solution can be obtained and the theoretical predictions can be tested by experiments.Berry has obtained the adiabatic solution of this model. In this note, we shall present a a systematic cranking method to solve the problem and the resulting exact solutions are used to study the non-adiabatic effect on Berry's phase.

The Hamiltonian of the model is

$$\hat{H}_0 = \vec{B} \cdot \vec{j} \,, \tag{1}$$

where the spin operator is

$$\vec{j} = (\hat{j}_x \,, \hat{j}_y \,, \hat{j}_z), \tag{2}$$

and $\hat{j}_i$ are $(2j+1) \times (2j+1)$ matrices. The magnetic field is

$$\vec{B} = \Omega (\sin\theta \,, 0 \,, \cos\theta), \quad \Omega = B\mu \,. \tag{3}$$

The precession of the magnetic field can be realized through cranking the Hamiltonian by a unitary transformation,

$$\hat{H}(t) = \exp\{-i\hat{j}_z \omega t\} \, \hat{H}_0 \, \exp\{i\hat{j}_z \omega t\} = \vec{B}(t) \cdot \vec{j} \,, \tag{4}$$

where the rotating magnetic field is

$$\vec{B}(t) = \Omega\,(\, \mathrm{Sin}\,\theta \;\; \mathrm{Cos}\,\omega t,\; \mathrm{Sin}\,\theta \;\; \mathrm{Sin}\,\omega t,\;\; \mathrm{Cos}\,\theta \,). \tag{5}$$

It is evident that

$$\hat{H}(0) = \hat{H}(T) = \hat{H}_0 \,, \qquad T = 2\pi/\omega \;. \tag{6}$$

The equation of motiom for a spin-j particle in a rotating magnetic field is

$$i\,\frac{\partial \psi(t)}{\partial t} = \hat{H}(t)\,\psi(t). \tag{7}$$

Let

$$\psi(t) = \exp\{-i\hat{j}_z\,\omega\, t\}\;\eta(t). \tag{8}$$

Then the equation of motion for $\eta(t)$ is

$$i\,\frac{\partial \eta(t)}{\partial t} = \hat{H}(\omega)\;\eta(t), \tag{9}$$

where the body-fixed Hamiltomian (Routhian) $\hat{H}(\omega)$ is

$$\hat{H}(\omega) = \hat{H}_0 - \omega\;\hat{j}_z \;\; = \vec{\bar{B}}\cdot\vec{j}\;, \tag{10}$$

and the renormalized magnetic field $\vec{\bar{B}}$ is

$$\vec{\bar{B}} = \bar{\Omega}\,(\,\mathrm{Sin}\,\bar{\theta}\;,\;\; 0 \;\;,\; \mathrm{Cos}\,\bar{\theta}\;), \tag{11a}$$

$$\bar{\Omega} = \Omega\,[\;1-2\frac{\omega}{\Omega}\,\mathrm{Cos}\,\theta + (\frac{\omega}{\Omega})^2\;]^{1/2}, \tag{11b}$$

with

$$\mathrm{Sin}\,\bar{\theta} = \mathrm{Sin}\,\theta \;\;/[\;1-2\frac{\omega}{\Omega}\,\mathrm{Cos}\,\theta + (\frac{\omega}{\Omega})^2\;]^{1/2}\;, \tag{11c}$$

$$\mathrm{Cos}\,\bar{\theta} = (\;\mathrm{Cos}\,\theta \; - \omega/\Omega\;)/[\;1-2\frac{\omega}{\Omega}\,\mathrm{Cos}\,\theta + (\frac{\omega}{\Omega})^2\;]^{1/2}\;. \tag{11d}$$

The solutions of (7) and (9) are

$$\eta(t) = \exp\{-i\hat{H}(\omega)t\}\;\eta(0), \tag{12a}$$

$$\psi(t) = \hat{U}(t)\,\psi(0), \tag{12b}$$

with

$$\hat{U}(t) = \exp\{-i\hat{j}_z\,\omega\, t\}\;\exp\{-i\hat{H}(\omega)t\}, \tag{12c}$$

Consider the evolution operator after one period T

$$\hat{U}(T) = \exp\{-i2\pi\,\hat{j}_z\}\,\exp\{-i\hat{H}(\omega)T\}. \tag{13}$$

Since $\hat{H}(\omega)$ possesses the symmetry

$$\exp\{-i2\pi\,\hat{j}_z\}\,\hat{H}(\omega)\,\exp\{i2\pi\,\hat{j}_z\} = \hat{H}(\omega), \tag{14}$$

$\hat{U}(T)$ and $\hat{H}(\omega)$ commute

$$[\,\hat{U}(T),\ \hat{H}(\omega)\,] = 0, \tag{15}$$

and both have common eigen-states,

$$\hat{U}(T)\,\eta_{jm} = \exp\{-i\,\Phi_m\}\,\eta_{jm}\ , \tag{16a}$$

$$\hat{H}(\omega)\,\eta_{jm} = E_m\,\eta_{jm}\ . \tag{16b}$$

For n periods, the solution is

$$\psi(nT) = [\,\hat{U}(T)\,]^n\,\psi(0) = \exp\{-i2n\pi\,\hat{j}_z\}\,\exp\{-in\hat{H}(\omega)T\,\}\psi(0). \tag{17}$$

As pointed out in a previous paper[14], there are two kinds of solutions:

i) Cyclic or recurrent solutions require that $\psi(0)$ be an eigen-state of $\hat{U}(T)$ (or $\hat{H}(\omega)$),

$$\psi(0) = \eta_{jm}\ . \tag{18}$$

This leads to

$$\psi(nT) = \exp\{-in\,\Phi_m\}\,\psi(0), \tag{19}$$

where the total phase Φ_m will be given later.

ii) Non-cyclic or non-recurrent solutions require

$$\psi(0) \neq \eta_{jm}\ , \tag{20}$$

and lead to

$$\psi(nT) \neq c\ \psi(0). \tag{21}$$

In this note we concentrate on cyclic solutions which are related to Berry's phase.

We proceed to solve the eigen equation (16). Suppose $|jm\rangle$, φ_{jm} and η_{jm} are eigen-states of $\hat{j}_z$, $\hat{H}_0$, and $\hat{H}(\omega)$ respectively,

$$\hat{j}_z \, | \, jm\rangle = m \, | \, jm\rangle \; , \tag{22}$$

$$\hat{H}_0 \; \varphi_{jm} = \epsilon_m \, \varphi_{jm} \; , \tag{23}$$

$$\hat{H}(\omega) \, \eta_{jm} = E_m \, \eta_{jm} \; . \tag{24}$$

Since

$$\hat{H}_0 = \Omega \exp\{-i\theta \, \hat{j}_y\} \; \hat{j}_z \exp\{i\theta \, \hat{j}_y\} \; , \tag{25}$$

$$\hat{H}(\omega) = \bar{\Omega} \exp\{-i\bar{\theta} \, \hat{j}_y\} \; \hat{j}_z \exp\{i\theta \, \hat{j}_y\} \; , \tag{26}$$

we have

$$\varphi_{jm} = \exp\{-i\,\theta\,\hat{j}_y\}\,|jm\rangle = \sum_{m'} D^j_{m'm} \; (0, \theta , 0)\,|jm\rangle \; , \tag{27}$$

$$\eta_{jm} = \exp\{-i\,\bar{\theta}\,\hat{j}_y\}\,|jm\rangle = \sum_{m'} D^j_{m'm} \; (0, \bar{\theta} , 0)\,|jm\rangle \; , \tag{28}$$

and

$$\epsilon_m = m\,\Omega \; , \tag{29}$$

$$E_m = m\,\bar{\Omega} \; . \tag{30}$$

We begin by calculating Berry's phase for the cyclic solution

$$\psi_m(t) = \exp\{-i\hat{j}_z\,\omega\, t\} \; \exp\{-i\hat{H}(\omega)t\}\,\eta_{jm} = \exp\{-iE_m\, t\} \; \exp\{-i\hat{j}_z\,\omega\, t\}\,\eta_{jm}$$

$$= \exp\{-iE_m\, t\} \; \sum_{m'} D^j_{m'm} \; (0, \bar{\theta} , 0) \; \exp\{-im'\omega\, t\}\,|jm'\rangle \; , \tag{31}$$

where $D^j_{m'm}$ is the Wigner function with rank j. After one period,

$$\psi_m(T) = \exp\{-iE_m\, T - i2m\pi\} \; \psi(0) \; , \tag{32}$$

and the total phase is

$$\Phi_m = E_m\, T + 2m\pi \; . \tag{33}$$

To calculate dynamical phase, we need the expectation value of $\hat{H}(t)$,

$$\mathcal{E}_m(t) = \langle \psi_m(t) | \hat{H}(t) | \psi_m(t) \rangle = \langle \eta_{jm} | \hat{H}(\omega) | \eta_{jm} \rangle + \omega \langle \eta_{jm} | \hat{j}_z | \eta_{jm} \rangle$$

$$= E_m + \omega \, \overline{j_z^m} \quad , \qquad (34)$$

where the spin alignment $\overline{j_z^m}$ is

$$\overline{j_z^m} = \langle \eta_{jm} | \hat{j}_z | \eta_{jm} \rangle = \langle \psi_m(t) | \hat{j}_z | \psi_m(t) \rangle$$

$$= \langle jm | \exp\{i\bar{\theta}\, \hat{j}_y\} \, \hat{j}_z \exp\{-i\bar{\theta}\, \hat{j}_y\} | jm \rangle$$

$$= \langle jm | [-\mathrm{Sin}\,\bar{\theta}\, \hat{j}_x + \mathrm{Cos}\,\bar{\theta}\, \hat{j}_z] | jm \rangle = m \,\mathrm{Cos}\,\bar{\theta} \quad . \qquad (35)$$

Thus the dynamical phase is

$$\Phi_m^d = \int_0^T \mathcal{E}_m(t)\, dt = E_m T + 2m\pi \,\mathrm{Cos}\,\bar{\theta} \quad , \qquad (36)$$

and Berry's phase is

$$\Phi_m^b = - (\Phi_m - \Phi_m^d) = -2m\pi (1 - \mathrm{Cos}\,\bar{\theta}) \qquad (37a)$$

$$= -2m\pi (1 - \overline{j_z^m} / m) \qquad (37b)$$

$$= -2m\pi \{ 1 - (\mathrm{Cos}\,\theta - \omega/\Omega) / [1-2\tfrac{\omega}{\Omega}\mathrm{Cos}\,\theta + (\tfrac{\omega}{\Omega})^2]^{1/2} \}. \qquad (37c)$$

Eqs.(37) indicate that Berry's phase is related to the spin expectation value along the rotating axis (i.e., spin alignment) and give Berry's phase a physical explanation besides its gauge geometric interpretation. The new expression for Berry's phase is useful in the calculation of non-adiabatic Berry's phase, since the calculation of spin alignment is a standard algorithm and therefore straightforward in a code of the cranking model[15], even though the Hamiltonian is a non-linear function of the dynamical group generators. In the adiabatic limit

$$\Phi_m^b \xrightarrow{\frac{\omega}{\Omega} \to 0} -2m\pi (1 - \mathrm{Cos}\,\theta), \qquad (38)$$

which recovers Berry's results.

It is interesting to exploit the relationship between spin alignment quantization and Berry's phase quantization. As spin alignment is quantized, i.e.,

$$\overline{j_z^m} = m' , \qquad m' = \text{integer or half-integer}, \qquad (39)$$

the corresponding Berry's phase assumes the value of $2N\pi$,

$$= -2\pi(m - m') = -2N\pi \ , \quad N = m - m', \qquad (40)$$

i.e., Berry's phase is also quantized. Eqs.(39) and (40) imply a fundamental fact that for the SU(2) dynamical group, spin alignment quantization is related to Berry's phase quantization. This result is important in nuclear high spin physics. As a result of a non-central collision between two nuclei, a fast rotation of the compound system about an axis perpendicular to the deformation axis may result. Due to the non-adiabatic effect, the spins of individual nucleons moving in the rotating deformed mean field will align along the rotation axis and a corresponding Berry's phase will be generated. If the Berry's phase is quantized (this is required by the stationary condition), the related spin alignment is also quantized. The consequence of Berry's phase quantization is as follows. Since $\exp\{-2N\pi i\} = 1$, the quantized Berry's phase has no effect on the time-dependent solutions. Yet such periodic solutions only possess dynamical phase. This property is essential for a solution to be stationary, since a stationary solution has only dynamical phase, no Berry's phase. We are thus led to the conclusion that as a quantum system responds to a rotating deformed mean field, the only way to keep its state stationary is to make its spin alignment quantized. This phenomenon was observed recently in high spin physics in superdeformed rotational bands and is surprising[16].

Now we consider spin alignment quantization of individual particles. For a particle to realize spin alignment quantization, a critical frequency is needed. To calculate it, let us consider the spin-1/2 particle and assume $\theta = \pi/2$. This model can simulate a nucleon in a deformed mean field. The axial symmetrically deformed potential plays the role of a magnetic field. The question is at what frequency the nucleon's spin alignment starts to be quantized. Suppose initially the nucleon spin is aligned along the deformed axis (x-axis), i.e.,

$$\psi(0) = \varphi_{+\frac{1}{2}} = \mathrm{Sin}\,\bar{\theta}/2\ \eta_{+\frac{1}{2}} + \mathrm{Cos}\,\bar{\theta}/2\ \eta_{-\frac{1}{2}}\ , \qquad (41)$$

then it is not difficult to show that

$$\psi(t) = \exp\{-i\hat{j}_z\omega t\}\ (\mathrm{Sin}\,\bar{\theta}/2\ \exp\{-iE_+t\}\eta_{+\frac{1}{2}} + \mathrm{Cos}\,\bar{\theta}/2\ \exp\{-iE_-t\}\eta_{-\frac{1}{2}}), \qquad (42)$$

and

$$\langle\psi(t)|j_z|\psi(t)\rangle = 1/2\ \{(\omega/\Omega)/[1+(\omega/\Omega)^2]\}\ [1-\cos\bar{\Omega}t] \geqslant 0. \qquad (43)$$

Since

$$\max\ (1-\cos\bar{\Omega}t) = 2, \qquad (44)$$

the spin alignment quantization, i.e.,

$$\max\ \langle\psi(t)|j_z|\psi(t)\rangle = 1/2\ , \qquad (45)$$

leads to

$$(\omega/\Omega)/[1+(\omega/\Omega)^2] = 1/2\ , \qquad (46)$$

which has the solutions

$$\omega = \Omega\ . \qquad (47)$$

Thus the critical frequency i.e., the minimum solution is

$$\omega_c = \min\ \omega = \Omega\ . \qquad (48)$$

Eq.(43) indicates that a nucleon initially with spin z-component zero, i.e., $\langle\psi(0)|j_z|\psi(0)\rangle = 0$, after having been put in a rotating deformed mean field, will acquire a non-zero z-component of spin. This phenomenon, called spin alignment, indicates that non-adiabatic effect on Berry's phase manifests itself as spin alignment. Yet, since two nuclei's collision is a short time non-stationary process, the collision will cause quantum transitions. According to quantum mechanics, quantum transition caused by short time perturtion always happens from one stationary state to another stationary state. As indicated above, to generate a stationay state, the corresponding Berry's has to be quantized. And this in turn leads to spin alignment quantization. In a mean field picture, for the whole deformed nucleus, each nucleon feeling the rotating deformed field will contribute an amount of spin alignment, which may and may not be quantized. But the total contribution must be quantized to populate a stationary rotational state. If a particular nucleon couples to the deformed potential rather weakly and the rotation frequency reaches a critical value to break the coupling, this nucleon's spin alignment will be quantized and the spin alignment quantized nucleon does'nt contribute to the collective rotation and the moment of inertia. The above statement is proved to be true within the framework of the cranking shell model[17].

We conclude with a summary of the information contained in this note. The time-dependent Schrödinger equation for a spin particle in a rotating magnetic field is solved analytically by the cranking method developed in nuclear physics and the exact solutions are employed to study non-adiabatic Berry's phase. A new expression for non-adiabatic Berry's phase is given, which shows that Berry's phase is related to the expectation value of spin along the rotating axis and gives Berry's phase a physical explanation besides its gauge geometric interpretation. The new expression also presents a simple algorithm for calculating non-adiabatic Berry's phase for Hamiltonians which are non-linear functions of the SU(2) generators. For the SU(2) dynamical group, the non-adiabatic effect on Berry's phase manifests itself as spin-alignment (a phenomenon in nuclear physics) and spin-alignment quantization (observed recently in high spin nuclear physics) is related to Berry's phase quantization. For a spin-1/2 particle the critical frequency of spin alignment quantization has been calculated which can be used in the description of a nucleonic spin coupled to a rotating deformed mean field.

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