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Spectrum of QCD nucleation-site separations and primordial nucleosynthesis

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The spectrum of nucleation-site separations in the cosmic QCD phase transition is calculated within the framework of homogeneous nucleation theory. This spectrum has the property that the ratio of the standard deviation of the separations to the mean separation has the constant value of 0.466, independent of any QCD physics parameters. When the assumption is made that a duality exists between nucleation sites and baryon fluctuation sites, the same spectrum may be applied to baryon-number fluctuation separations. When nucleosynthesis results are averaged over this spectrum, the upper limit on $\Omega_b(2.735K/T_0)^3\times(H_0/50\ \mathrm{km\,s^{-1}Mpc^{-1}})^2$, as derived from the observed upper limit to the primordial ^{4}He abundance, drops from 0.8 to 0.56 in baryon inhomogeneous universes. In order for macroscopic baryon inhomogeneities to arise in the early Universe, persist to the era of nucleosynthesis, and give nucleosynthesis yields which allow Ω_b to be greater than the limit from standard big-bang nucleosynthesis, the QCD surface tension must be near its upper limit of around the cube of the coexistence temperature.

I. INTRODUCTION

Recent studies[1-3] have suggested that the Universe may not have been homogeneous in its distribution of baryons at the time of primordial nucleosynthesis. The cause of baryon inhomogeneity in the early Universe is most often taken to be the cosmic QCD phase transition.[4,5] Kajantie and Kukii-Suonio[6] and Fuller, Matthews, and Alcock[7] studied the nucleation of hadronic phase material within the context of homogeneous nucleation theory and concluded that macroscopic baryon fluctuations could have arisen during the phase transition. Detailed calculations of the subsequent evolution of the fluctuations showed that the resulting nucleosynthesis yields were often different from those in the homogeneous big bang.[7-16] For example, we found[16] that Ω_b, the present ratio of the baryon energy density to the critical energy density, as derived from observational constraints on ^{4}He and D, could be as large as 0.8 in baryon inhomogeneous universes for a present microwave background temperature of $T_0=2.735$ K and a Hubble constant of $H_0=50\ \mathrm{km\,s^{-1}\ Mpc^{-1}}$. (Hereafter any numerical value we give for Ω_b will be derived from these values for H_0 and T_0.) A drawback of these detailed calculations is that they assume that fluctuations are spaced uniformly. It is much more likely, however, that a distribution of fluctuation separations would occur. In this paper we use classical homogeneous nucleation theory and an independent-particle model to calculate the spectrum of nucleation-site separations. If we then assume a duality between nucleation sites and baryon-number fluctuations, this spectrum also applies to the spectrum of baryon fluctuation separations. We show how this spectrum might affect nucleosynthesis yields on a baryon inhomogeneous universe. Finally, we consider the possible constraints on QCD physics from primordial nucleosynthesis.

II. NUCLEATION OF HADRON PHASE AND SPECTRUM OF NUCLEATION-SITE SEPARATIONS

For small supercooling during the quark-hadron phase transition, the rate p at which bubbles of hadron phase are formed per unit volume is given by[7]

$$p(\eta)\approx CT_c^4\exp\left[\frac{-16\pi\sigma^3}{3T_cL^2\eta^2}\right], \tag{2.1}$$

where L is the latent heat per unit volume of the phase transition, σ is the free energy per unit area of the boundary associated with the bubble, T_c is the coexistence temperature for the quark and hadron phases, C is a coefficient of order unity, and η the "supercooling parameter" is given by

$$\eta=\frac{T_c-T}{T_c}. \tag{2.2}$$

Clearly $p=0$ when $T=T_c$ and then grows exponentially for $T<T_c$.

We now follow the picture of Kajantie and Kurki-Suonio[6] in the subsequent evolution of the phase transition. In this analysis the nucleated bubbles of hadronic material expand and drive a weak shock wave with velocity $V_S\approx 3^{-1/2}$ into the quark-gluon plasma. These shock waves reheat the quark-gluon plasma so that no further nucleation occurs in a region that has been traversed by a shock wave. (A potential modification to this picture is that passage of two or more shock waves may be required to reheat a region to the coexistence temperature. We discuss the effect of such a modification below.) The fraction of the Universe unaffected by nucleation during this time is thus

$$f(t)=\exp\left[-\int_{t_c}^{t} f(t')p(T')dt'\frac{4\pi}{3}V_S^3\left[\frac{T}{T'}\right](t-t')^3\right], \tag{2.3}$$

where t_c is the time the Universe first cools through $T=T_c$. Because the nucleation rate p rises rapidly with time, we may approximate Eq. (2.3) by

$$f(t)\approx\begin{cases}1-\int_{t_c}^{t}dt'p(T')\frac{4\pi}{3}V_S^3(t-t')^3 & \text{if } t<t_f,\\ 0 \quad \text{if } t\geq t_f,\end{cases} \tag{2.4}$$

where t_f is the time when the entire Universe has been reheated.

We now express p as a function of time by expanding about $t=t_f$. This gives

$$\ln p(T)=\ln p(T_f)+\left[\frac{d\ln p}{dT}\right]_{T_f}\left[\frac{dT}{dt}\right]_{t_f}(t-t_f). \tag{2.5}$$

The relation between the age of the Universe t and the temperature is

$$t=\frac{K}{T^2}, \tag{2.6a}$$

where K is given in terms of the Planck mass $m_{\rm Pl}$ and the statistical weight of relativistic degrees of freedom g by

$$K=\left[\frac{45}{16\pi^3}\right]^{1/2} m_{\rm Pl}g^{-1/2}. \tag{2.6b}$$

For the range of σ and L considered here, g is well approximated by the relativistic degrees of freedom for bosons (g_b) and fermions (g_f) in a noninteracting u and d quark-gluon plasma, $g=g_b+\frac{7}{8}g_f=51.25$. Our nucleation results would not be qualitatively different if we used another expansion rate. The controlling factor here is the comparison of the relatively slow universal expansion with the rapid QCD time scale. From Eqs. (2.6a) and (2.6b) we find that

$$p(t)=p(t_f)\exp[-\alpha(t_f-t)], \tag{2.7}$$

where

$$\alpha=\frac{32\pi^2}{9}\left[\frac{4\pi}{5}g\right]^{1/2}\frac{1}{m_{\rm Pl}}\frac{\sigma^3T_c^4}{L^2(T_c-T_f)^3}. \tag{2.8}$$

From this expression for $p(t)$ we find for $t<t_f$ that

$$f(t)\approx 1-\frac{4\pi}{3}V_s^3p(t_f)\int_{t_c}^{t}dt'\exp[-\alpha(t_f-t')](t-t')^3. \tag{2.9}$$

We require that all of the Universe be reheated at $t=t_f$; therefore, $f(t_f)=0$ and, neglecting terms $\exp[-\alpha(t_f-t_c)]$ compared to terms $\exp[-\alpha(t_f-t)]$ again because of the rapid rise of p, we find

$$f(t)\approx 1-\exp[-\alpha(t_f-t)]. \tag{2.10}$$

At this point it is useful to define an effective nucleation rate $\psi(t)$ such that

$$\psi(t)=f(t)p(t). \tag{2.11}$$

$\psi(t)$ gives the rate of formation of hadron bubbles per unit volume corrected for the fact that only a fraction $f(t)$ of the Universe's volume is available for nucleation. From Eqs. (2.7) and (2.10) we find

$$\psi(t)=p(t_f)\{\exp[-\alpha(t_f-t)]-\exp[-2\alpha(t_f-t)]\}. \tag{2.12}$$

From this we may derive a normalized effective nucleation rate $\phi(t)$:

$$\phi(t)=\frac{\psi(t)}{\int_{t_c}^{t_f}\psi(t)dt}. \tag{2.13}$$

$\phi(t)$ is thus

$$\phi(t)\approx 2\alpha\{\exp[-\alpha(t_f-t)]-\exp[-2\alpha(t_f-t)]\}, \tag{2.14}$$

where we have again neglected the term $\exp[-\alpha(t_f-t_c)]$. Finally, the total number density of bubbles formed is given by

$$N=\int_{t_c}^{t_f}\psi(t)dt\approx\frac{\alpha^3}{16\pi V_S^3}. \tag{2.15}$$

Clearly as α grows, N grows rapidly. This of course reflects the fact that larger α gives more sudden nucleation.

With an effective nucleation rate now available, we may ask about the region of space associated with each nucleated bubble at $t=t_f$. A bubble formed at time t will drive out a shock wave into a spherical region that has a radius at time t_f given by

$$r(t)=V_S(t_f-t)\left[\frac{t_f}{t}\right]^{1/2}. \tag{2.16}$$

This is a combination of simple propagation of the shock wave outward and expansion of the Universe. Solution for t yields

$$t\approx t_f-\frac{r}{V_S}+O\left[\frac{r^2}{V_S^2t_f}\right]. \tag{2.17}$$

Because $t_f\gg t_f-t_c$, we may neglect the term of order $r^2/V_S^2t_f$. Now $\phi(t)$ gives the fraction of all bubbles that formed at time t. We therefore define $\theta(r)dr=\phi(t(r))dt$ which gives the fraction of all bubbles that have a shocked sphere of radius r surrounding them at time t_f. From Eqs. (2.14) and (2.17) $\theta(r)$ is given by

$$\theta(r)=\frac{2\alpha}{V_S}\left[\exp\left[\frac{-\alpha r}{V_S}\right]-\exp\left[\frac{-2\alpha r}{V_S}\right]\right]. \tag{2.18}$$

Since the entire Universe has been reheated at t_f to T_c, shocked spheres must touch or overlap. Let us consider two adjacent shocked spheres of radius r_1 and r_2 whose centers are separated by distance l. If $r_1\leq l$, then

$r_2 \geq l - r_1$ since adjacent shocked spheres must at least touch. Also we must have $r_2 \leq l + r_1$, otherwise shock 2 would have passed nucleation site 1 before time $t_1 = t_f - r_1/V_S$, thereby preventing bubble 1 from forming. Therefore, the probability that within a volume of space we have a shocked sphere with a radius in the interval $r_1 \to r_1 + dr_1$ and with its center separated from that of an adjacent shocked sphere of radius r_2 by a distance in the range $l \to l + dl$, such that $l \geq r_1$, is

$$P(r_1, l \geq r_1) dr_1 dl \propto l^2 dl \, \theta(r_1) dr_1 \int_{l-r_1}^{l+r_1} \theta(r_2) dr_2 \ . \tag{2.19}$$

The $l^2 dl$ term is a geometrical factor giving the volume of the shell within which the center of the shocked sphere 2 may lie. If $r_1 \geq l$, then $r_2 \geq r_1 - l$, otherwise passage of shock 1 would have prevented nucleation at site 2 at $t_2 = t_f - r_2/V_S$. Also, we must again require $r_2 \leq l + r_1$. Thus, the probability for a shocked sphere of radius $r_1 \to r_1 + dr_1$ to have its center separated from that of an adjacent shocked sphere 2 by distance in the range $l \to l + dl$, such that $l \leq r_1$, is

$$P(r_1, l \leq r_1) dr_1 dl \propto l^2 dl \, \theta(r_1) dr_1 \int_{r_1 - l}^{r_1 + l} \theta(r_2) dr_2 \ . \tag{2.20}$$

The spectrum of separations of adjacent shocked spheres is thus given by

$$\Theta(l) dl = A \, dl \left[\int_0^l dr_1 P(r_1, l \geq r_1) + \int_l^\infty dr_1 P(r_1, l \leq r_1) \right] , \tag{2.21}$$

where A is the normalization constant. We identify $\Theta(l)$ with the spectrum of nucleation sites, which is thus

$$\Theta(l) = \frac{96}{185} \left[\frac{\alpha}{V_S} \right]^3 l^2 \left[e^{-\alpha l / V_S} \left[\frac{\alpha l}{2 V_S} - \frac{2}{3} \right] + e^{-2\alpha l / V_S} \left[\frac{\alpha l}{4 V_S} + \frac{2}{3} \right] \right] . \tag{2.22}$$

From Eq. (2.22) we may derive the mean separation l,

$$\langle l \rangle = \int_0^\infty l \Theta(l) dl = 4.38 \frac{V_S}{\alpha} , \tag{2.23}$$

and the variance

$$\sigma_l^2 = \int_0^\infty (l - \langle l \rangle)^2 \Theta(l) dl = 4.16 \left[\frac{V_S}{\alpha} \right]^2 . \tag{2.24}$$

We note that the ratio of the standard deviation to the mean is $\sigma_l / \langle l \rangle = 0.466$, a constant, and that the distribution $\Theta(l)$ is completely determined by the parameter V_S/α. Finally we note that the notation V_S/α for the QCD nucleation length scale is useful because it clearly identifies the role of the two effects giving rise to this length scale. α gives the purely QCD contribution to the length scale via the nucleation rate for hadron phase. V_S gives the rate at which the volume of regions near nucleated hadron bubbles are shut off from further nucleation. With this delineation of effects on the length scale readily apparent, we can estimate the effect of variations on the scenario we are considering. For example, if the speed of the shocks driven out by nucleated bubbles is some factor different from $3^{-1/2}$, the length scale will be that factor different from the length scales we discuss below. Similarly, if more than one shock passage is required to reheat regions of the Universe to the coexistence temperature, the shock speed will effectively be smaller, which will lead to smaller length scales.

We tested the spectrum in Eq. (2.22) by means of Monte Carlo calculations of the nucleation process. We allowed hadron bubbles to nucleate at random locations according to the rate in Eq. (2.12), making sure that a new bubble did not lie within the growing shocked sphere of a previously nucleated bubble. The calculation proceeded until the volume of space considered was completely reheated. The separations of the centers of adjacent shocked spheres were then calculated and binned. The histogram in Fig. 1 shows the results of a typical Monte Carlo calculation. The spectrum of Eq. (2.22) is the curve. Excellent agreement is seen, thereby confirming our analytic expression.

Equation (2.22) is the spectrum of nucleation site separations. It is *not* necessarily the spectrum of baryon-number fluctuation separations since the quark-hadron phase transition has only begun. During the phase transition, hadron bubbles grow at the expense of the quark phase, percolate, and concentrate baryon number at sites between the original nucleation sites. It is the spectrum of the resulting baryon-number fluctuation separations that is relevant to discussions of big-bang nucleosynthesis. We will assume a duality between nucleation sites and baryon concentration sites; thus, the same spectrum is taken to describe both. This is probably a reasonable approximation because the rate of growth and percola-

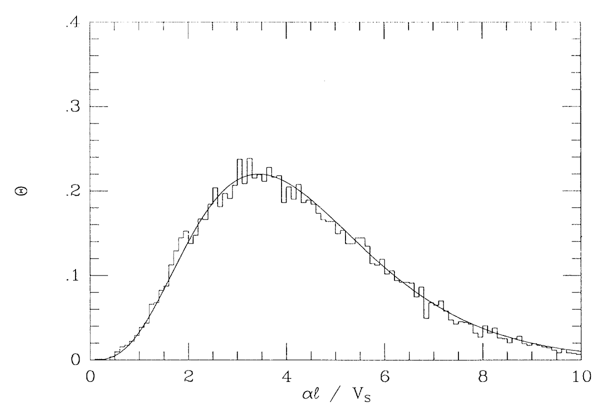

FIG. 1. The normalized spectrum of nucleation-site separation l arising in a QCD phase transition from a typical Monte Carlo calculation (histogram) and from Eq. (2.22) (curve). Note that V_S/α is the QCD nucleation length scale which is typically of order $\frac{1}{1000}$ of the horizon scale at the time of the phase transition.

tion of bubbles is regulated by the universal expansion which is slow compared to the time scale for the bubbles to assume an optimum spherical shape. Thus the regions of quark-gluon plasma should become bubbles located between nucleation sites after percolation. The effects of this spectrum on nucleosynthesis are discussed in the next section.

This picture will be modified if the relationship between nucleation sites and baryon-number fluctuations is not the simple duality assumed here. Under certain circumstances the nucleation process may become unstable to dendritic growth,[17] which probably would lead to baryon-number-fluctuation length scales smaller than those we have assumed from the duality between nucleation sites and baryon-number fluctuations. On the other hand, growing bubbles of hadronic material may coalesce, in which case the baryon-number-fluctuation length scales would probably be larger than those one would expect from duality. There may also be completely different scenarios for nucleation, for example, a perfect wetting scenario.[18] We will ultimately require detailed calculations with a thermodynamic model of both quark and hadron phases to determine the true nature of the nucleation process during the cosmic QCD phase transition.

III. EFFECTS ON NUCLEOSYNTHESIS

During the quark-hadron phase transition, the scale factor of the Universe expands by a factor β of order 40%. By our assumption of a duality between nucleation sites and baryon concentration sites, we have derived a spectrum of baryon fluctuation radii given by $\Theta'(l)$, where the prime indicates that V_S/α in Eq. (2.18) has been replaced by $\beta V_S/\alpha$ because of the expansion during the phase transition and l now refers to separations at $t=t_h$, the time at the end of the phase transition where the Universe is first completely hadronic. [We note that we can in fact consider β to give any modifications to the underlying QCD length scale during the phase transition. For example, if hadron bubbles coalesce during the phase transition, we may estimate the effect of this on the distribution $\Theta'(l)$ by taking β to be larger than it would be simply from universal expansion.] With the distribution of baryon-number-fluctuation separations at the end of the QCD phase transition available, we may now investigate the effects on big-bang nucleosynthesis.

The picture we currently have for the evolution after the phase transition is that the Universe cools until the differing mean free paths of neutrons and protons in the bath of hadrons, electrons, photons, and neutrinos lead to a segregation of the neutrons and protons such that high density, proton-rich and low density, neutron-rich regions develop. Nucleosynthesis then occurs in these different regions and the resulting abundances from the different regions ultimately mix to give the presently observed primordial abundances. Of course, nucleon diffusion occurs throughout nucleosynthesis; thus, for a complete calculation it is necessary to set up baryon concentrations on a grid with a distribution of separations as calculated above and follow the subsequent nucleosynthesis and diffusion. Such a calculation would be extremely large and time consuming. For the purpose of illustrating the effect of a spread in fluctuation radii, it is instead sufficient simply to average calculations made at fixed separation distance[16] over a distribution. Note that nucleon diffusion among fluctuations is now approximated by reflective outer boundary conditions for diffusion. In effect, we average over different Universes, each Universe with fluctuations of unique radius and nucleon diffusion among the fluctuations, weighted according to $\Theta'(l)$.

To effect this average, we assume that r, the radius of a baryon-number fluctuation, is given by $l/2$. We can then get dN'/dr, the number density of baryon fluctuations with radius in the range $r \to r+dr$ at time t_h. From dN'/dr it is then easy to show that the spectrum-averaged mass fraction of species is

$$\bar{X}_i = \int_0^\infty dr \frac{dN'}{dr} r^3 \bar{X}_i(r) \Bigg/ \int_0^\infty dr \frac{dN'}{dr} r^3 , \qquad (3.1)$$

where $\bar{X}_i(r)$ is the average mass fraction of species i resulting from a fluctuation of radius r at $t=t_h$. The $\bar{X}_i(r)$'s are available from our previous work.[16]

In Ref. 16 nucleosynthesis calculations were made as a function of radii of baryon-number fluctuations. These fluctuations were characterized by the ratio R of the densities of the high-baryon density region and a low-baryon density region. The ratio of the volume of the high-density region to the total volume was called f_v. The coupled baryon diffusion and nucleosynthesis equations were solved implicitly from a time just after the phase transition until after the epoch of nucleosynthesis.

Figure 2 shows the effect of averaging over the spectrum of fluctuation radii. In this figure the dashed lines give the mass fraction of ^{4}He or the number density of a light species to that of H as a function of the fluctuation radius at a temperature $T=100$ MeV from Ref. 16. The particular fluctuation parameters are $R=10^6$ and $f_v^{1/3}=0.25$ with $\Omega_b=0.7$. The solid lines give the results averaged over fluctuation radii as prescribed in Eq. (3.1) plotted against $\beta V_S/\alpha$ at $T=100$ MeV. Note that features in the averaged curve occur at a length scale a factor of about three below that of the corresponding features in the unaveraged curves. This is due to the fact that the quantity $r^3 dN'/dr$ in the integral in Eq. (3.1) peaks around $r=3.13\beta V_S/\alpha$.

Clearly, averaging affects the maximum and minimum abundances. The minimum abundance of ^{4}He in Fig. 2 is raised by about 0.3% upon averaging. Similarly, the dip in ^{7}Li at $r=30$ m is raised with averaging by a factor of 1.3 at $\beta V_S/\alpha \approx 10$ m. The maximum abundance of D is not altered since the unaveraged D abundance is flat over a large range of radii.

These changes in maximum and minimum abundances can affect limits on the present value of Ω_b. If we take an upper limit on the primordial ^{4}He mass fraction of 0.254,[19] the unaveraged ^{4}He curve in Fig. 2 satisfies this constraint around $r=50$ m at $T=100$ MeV. The averaged ^{4}He curve, however, does not satisfy the observational constraint. This may also be seen in Fig. 3(a). Here the dashed line gives the lowest ^{4}He mass fraction

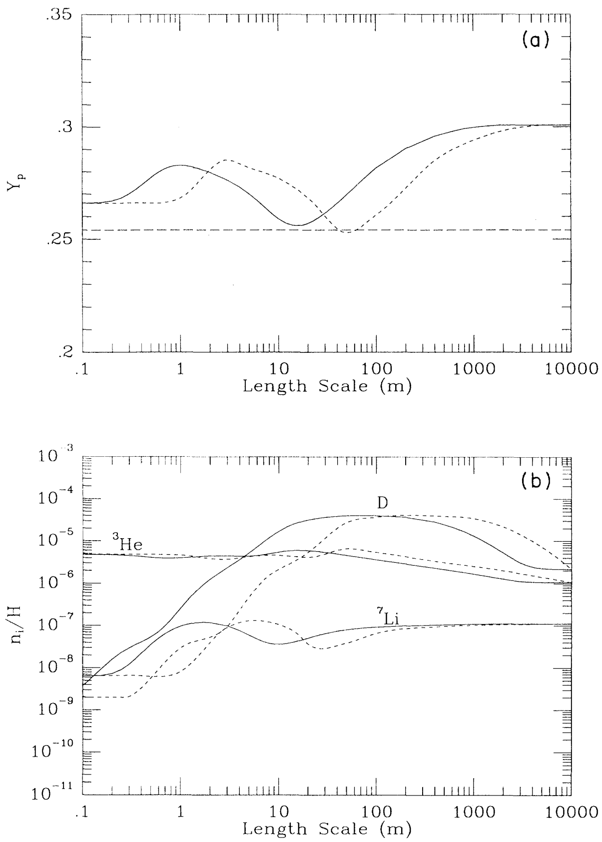

FIG. 2. Nucleosynthesis yields for an $R=10^6$, $f_v^{1/3}=0.25$ fluctuation with $\Omega_b=0.7$. The dashed curve shows the results plotted against the fluctuation radius in meters at a temperature of 100 MeV. These results averaged over the spectrum of fluctuation separations are shown as the solid curves, which are plotted against the length scale at the end of the QCD phase transition ($\beta V_S/\alpha$) in meters. (a) The ^{4}He mass fraction. The long-dashed line gives the observational upper limit on Y_p. (b) The number density of D, ^{3}He, and ^{7}Li relative to that of H.

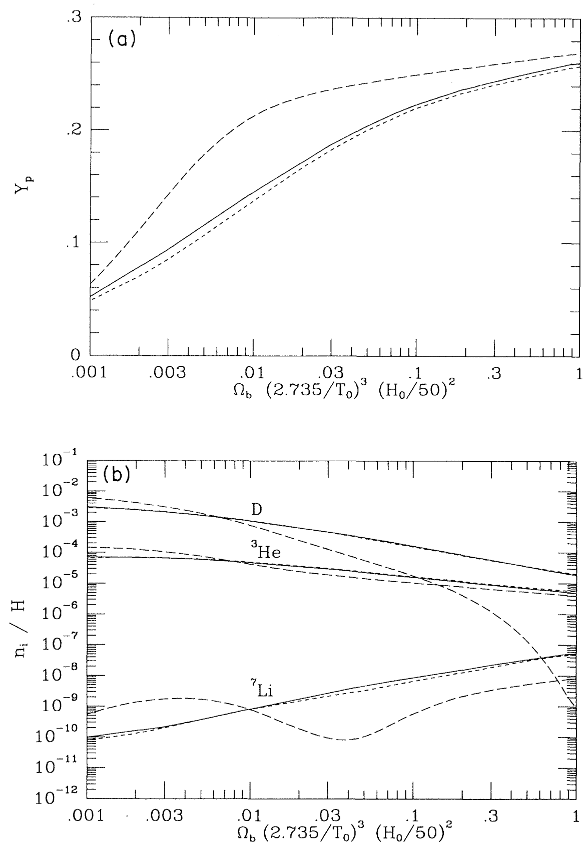

FIG. 3. (a) The minimum value of the ^{4}He mass fraction as a function of Ω_b for the unaveraged results (dashed curve) and the averaged results (solid curve). For comparison, the results of standard, homogeneous, big-bang nucleosynthesis are shown as the long-dashed curve. (b) The corresponding results for the other light elements at the same length scales as for the ^{4}He results.

for $R=10^6$, $f_v^{1/3}=0.25$, and the indicated values of Ω_b. The solid line gives the corresponding lowest ^{4}He mass fractions for these results averaged over fluctuation radii. The upper limit on Ω_b from the upper limit to the primordial ^{4}He mass fraction drops from 0.8 to 0.56 upon averaging over fluctuation-site distances. For comparison, the dashed line gives the results of standard, homogeneous big-bang nucleosynthesis.

The other light element abundances are shown in Fig. 3(b). Results are shown for the same length scales as for the ^{4}He results. The ^{7}Li curve is typically raised by a factor of 1.3 with averaging. The D and ^{3}He curves are only slightly affected since their abundance curves versus radius are usually flat in the regions of lowest ^{4}He mass fraction. Because galactic chemical evolution[20] and/or late-time hydrodynamics[21] may greatly alter the inferred primordial ^{7}Li abundance, we follow Ref. 16 and use ^{4}He and D observational abundances to constrain Ω_b. Since we found [16] $R=10^6$, $f_v^{1/3}=0.25$ fluctuations to be optimal for getting high Ω_b, we conclude that $\Omega_b \leq 0.56$ when the effects of averaging are included. When ^{7}Li formation and evolution is better understood, a considerably lower upper limit on Ω_b may be available.

IV. POSSIBLE CONSTRAINTS ON THE COSMIC QCD TRANSITION

It is possible to use our analysis to seek constraints on the fundamental QCD physics, using observed limits on the primordial helium mass fraction Y_p. This is, in effect, an inverse of the analysis presented in Sec. III above. As we shall see, the constraints depend directly on the magnitude of Ω_b. Independent information on the magnitude of Ω_b may in this be used to place constraints on the physics of the QCD phase transition.

The parameter that is constrained is the length scale $\beta V_S/\alpha$. Recall that V_S is probably just the speed of sound in the quark-gluon plasma (i.e., $3^{-1/2}$); β is the expansion factor during the phase transition (which depends upon the ratios of statistical weights); α is essentially the nucleation rate which depends on the latent heat and surface tension. We discuss α later in this section.

Figure 4 illustrates the allowed region in the $\Omega_b - \beta V_S/\alpha$ plane. Models to the left of the solid line have acceptable ^{4}He, namely, $Y_p \leq 0.254$. (The solid curve represents models with $R = 10^6$, $f_v^{1/3} = 0.25$.) The features of this curve can be understood by comparison with the upper panel of Fig. 3. The highest Ω_b obtainable is 0.56 at $\beta V_S/\alpha \approx 16 \times (100\ \mathrm{MeV}/T_c)$ m. This corresponds to the dip in the ^{4}He vs $\beta V_S/\alpha$ curve. The low constraint on Ω_b at $\beta V_S/\alpha \approx 1.0(100\ \mathrm{MeV}/T_c)$ m is due to the back diffusion bump in the ^{4}He curve. This is where fluctuation separations are sufficiently small that diffusion of neutrons back into the high-density regions of fluctuations leads to increased production of ^{4}He. For low $\beta V_S/\alpha$, the ^{4}He curve tends toward the homogeneous big-bang nucleosynthesis constraint (shown by the dashed line in Fig. 4) since fluctuation separations are so small that baryon diffusion homogenizes the Universe prior to nucleosynthesis. At very large $\beta V_S/\alpha$, fluctuation separations are so large that no diffusion occurs prior to or during nucleosynthesis. Consequently, the nucleosynthesis results are just the average of two separate Universes, one of high density and one of low density. For sufficiently low $\Omega_b (\approx 0.0019)$, even the separate Universes have $Y_p \leq 0.254$; therefore, the constraint curve is vertical at that Ω_b.

Because the exact nature of the cosmic QCD phase transition is not known, we cannot constrain R or $f_v^{1/3}$ with great confidence. For now, we assume $1 \leq R \leq 10^6$. This means that we cannot rule out any length scale for $\Omega_b \leq 0.15$, the constraint from homogeneous nucleosynthesis. If Ω_b is larger than 0.15, however, we probably require some degree of baryon inhomogeneity in the early Universe. We also would require that these inhomogeneities have $\beta V_S/\alpha \approx 5-75 \times (100\ \mathrm{MeV}/T_c)$ m, with $\beta V_S/\alpha \approx 16 \times (100\ \mathrm{MeV}/T_c)$ m to get Ω_b as high as 0.56.

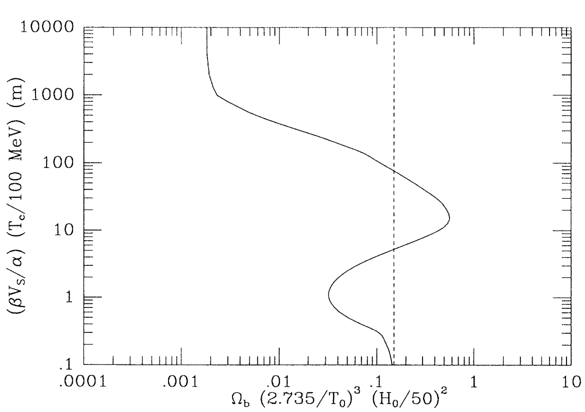

FIG. 4. The constraints on Ω_b and the length scale $\beta V_S/\alpha$ from observational limits on Y_p. Regions to the left of the solid curve give calculated $Y_p \leq 0.254$ for $R = 10^6$, $f_v^{1/3} = 0.25$ fluctuations. The dashed line gives the constraint from standard, homogeneous nucleosynthesis.

We may ask what sort of QCD physics is required to get these length scales, if the inhomogeneities arise from the QCD phase transition.

To get length scales in the QCD phase transition, we require some model for the latent heat L and surface energy σ. At present there are no definitive QCD calculations for these quantities. We can, however, identify two lines of attack in this problem: the independent-particle-plus-vacuum-energy difference model (a baglike model) and lattice gauge theory. The former model allows a quick, schematic survey of important cosmic phase transition parameter constraints. Inevitably these constraints will be crude at best and quite model dependent. Monte Carlo gauge lattice calculations show great promise but as yet do not have sufficient statistics. We will comment on results from both techniques.

In the baglike model, the latent heat L is given by

$$L = \frac{4\pi^2}{90} g_q T_c^4 \left[1 - \frac{1}{x} \right] , \tag{4.1}$$

where x is the ratio of spin degrees of freedom in the quark phase g_q to those in the hadron phase g_h, i.e., $x = g_q/g_h$. x is a temperature-dependent quantity, and for the present calculations we take the results of Ref. 7 for x, which include the complete spectrum of hadronic resonances. T_c we take as an unknown. From nucleosynthesis results we will attempt to get some constraints on σ for large Ω_b.

The inverse Hubble time at temperature T is

$$H = \left[\frac{4\pi^3}{45} \right]^{1/2} g^{1/2} \frac{T^2}{m_{\mathrm{Pl}}} , \tag{4.2}$$

where again g is the statistical weight of the relativistic degrees of freedom. In terms of the Hubble time, then α is given by

$$\alpha H^{-1} = \frac{32\pi}{3} \frac{\sigma^3}{T_c L^2 \eta_f^3} . \tag{4.3}$$

η_f is obtained from Eq. (2.2), where T_f is the temperature at which all of space has been reheated by shock waves driven by nucleation of hadronic phase. We obtain T_f by integrating Eq. (2.3) in conjunction with Eqs. (2.1) and (2.6). Typically we find

$$\eta_f \approx 0.4 \frac{\sigma^{3/2}}{L T_c^{1/2}} . \tag{4.4}$$

This is a factor of 3.5 smaller than the value quoted in Eq. (26) of Ref. 7 and leads to estimates of length scales that are a factor of $(3.5)^3 \approx 43$ less than in Ref. 7. Substitution of Eq. (4.4) into Eq. (4.3) yields

$$\alpha H^{-1} \approx \frac{32\pi}{3} (0.4)^{-3} \left[\frac{L}{T_c^4} \right] \left[\frac{T_c^3}{\sigma} \right]^{3/2} . \tag{4.5}$$

From this equation we may see that V_S/α is roughly $\frac{1}{1000}$ of the horizon size. Finally we note that β, the expansion parameter in the phase transition is

$$\beta \approx x^{1/3} ,$$

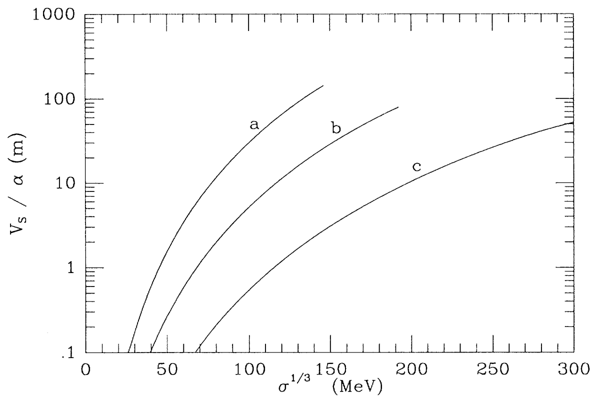

FIG. 5. The QCD nucleation length scale V_S/α as calculated for (a) $T_c = 75$ MeV, (b) $T_c = 100$ MeV, and (c) $T_c = 150$ MeV.

where again x is the ratio of spin degrees of freedom in the quark and hadron phases.

Figure 5 shows V_S/α as a function of $\sigma^{1/3}$(MeV) for the indicated T_c's, as calculated from Eq. (4.3) by numerical integration of Eq. (2.3). We then use a β of 1.4 to compare this length scale to the length scales from nucleosynthesis. In order to get $\beta V_S/\alpha \approx 16 \times (100\ \text{MeV}/T_c)$ m for high Ω_b, we require $\sigma^{1/3} \approx 85$ MeV for $T_c = 75$ MeV, $\sigma^{1/3} \approx 121$ MeV for $T_c = 100$ MeV, and $\sigma^{1/3} \approx 185$ MeV for $T_c = 150$ MeV. To get $\beta V_S/\alpha \approx 5-75 \times (100\ \text{MeV}/T_c)$ m, the range for which $\Omega_b \geq 0.15$ for $R = 10^6$, $f_v^{1/3} = 0.25$ fluctuations, we require $\sigma^{1/3} \approx 65-123$ MeV for $T_c = 75$ MeV, $\sigma^{1/3} \approx 91-274$ MeV for $T_c = 100$ MeV, and $\sigma^{1/3} \approx 141-270$ MeV for $T_c = 150$ MeV. In other words, $\sigma^{1/3}$ must be greater than or roughly equal to $0.9 T_c$ in order that $R = 10^6$, $f_v^{1/3} = 0.25$ fluctuations arise which can then give $\Omega_b > 0.15$ as determined from primordial ^{4}He yields. Other fluctuations (different R and $f_v^{1/3}$) give somewhat different ranges on $\beta V_S/\alpha$ for acceptable nucleosynthesis.[16] These variations in length scales are typically no more than a factor 3, however, so we still must have $\sigma^{1/3}$ near T_c for $\Omega_b > 0.15$ from big-bang nucleosynthesis.

We note in passing that there have been a few suggestions on how to estimate σ in finite-temperature lattice QCD calculations.[22-24] At present the results point to a limit of $\sigma^{1/3} \leq T_c$. Some workers find $\sigma^{1/3} \leq 0.62 T_c$.[25] Such a low surface tension would make it difficult for baryon inhomogeneous Universes to have $\Omega_b > 0.15$ unless hadron bubbles coalesce during the QCD phase transition. Lattice QCD techniques are full of promise and represent the only hope for a reliable theoretical estimate for σ.

V. CONCLUSIONS

We have calculated the spectrum of nucleation-site separations within the context of homogeneous nucleation theory and of the small supercooling scenario. This spectrum was found to have a ratio of the spread in separation to the mean separation equal to 0.466, independent of the QCD physics parameters. When a duality between nucleation sites and fluctuations is assumed, the same spectrum applies to fluctuation radii. We showed how to average nucleosynthesis yields over this spectrum. One important result is that the upper limit on Ω_b based upon the ^{4}He abundance constraint drops from 0.8 to 0.56 when averaging over the fluctuation separation spectrum is taken into account. Finally, we found that if we use the baglike model to compute length scales for baryon-number fluctuations, we typically need $\sigma^{1/3}$ near T_c in order to get fluctuations which would allow Ω_b to be larger than the limit from standard, homogeneous nucleosynthesis.

In closing, we should enumerate the assumptions involved in deriving these results in an effort to clarify areas in which work needs to be done. First, we have assumed the small supercooling limit for the nucleation process. Other scenarios for the nucleation process exist,[18] and it is imperative that the nature of this nucleation be clarified for accurate conclusions about the plausibility of the occurrence of macroscopic baryon-number fluctuations in the QCD phase transition. We have assumed a duality between nucleation sites and fluctuations. Detailed calculations of the dynamics of the quark-hadron phase transition are necessary to determine whether such a duality is in fact realized. In averaging nucleosynthesis results, we did not account for different degrees of baryon diffusion among fluctuations of different radii. A correct treatment of this would require extremely large-scale calculations which are probably beyond present computing capabilities. We have also assumed that all fluctuations have the same amplitude, geometry, and width in averaging nucleosynthesis yields. At this point we ignore the variation of R and $f_v^{1/3}$ with fluctation radii until detailed calculations of the phase transition are performed.

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