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DISCLAIMER

ORDERED HADRON S MATRIX*

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ABSTRACT

An ordered hadron S matrix is developed that accommodates an arbitrary number of "neighbors" for each particle. Automatic features are baryon-number conservation and zero triality. The ordered Hilbert space splits into a set of noncommunicating sectors each characterized by a "skeleton" graph whose external edges can be given a quark interpretation. Selection rules are found that generalize the OZI rules.

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INTRODUCTION

Sequential particle order, when added as a requirement to the standard S-matrix properties of Poincaré invariance, causality and unitarity, has been found to imply many of the successful quark-model predictions for mesons; quark-antiquark mesonic structure emerges as a manifestation of order without need to postulate the quark as a primitive constituent[1)]. At the same time, the pomeron and OZI-rule violations have become understood as unitarity-required deviations from perfect order; to understand such phenomena there is no need to introduce gluons. With this taste of the physical content that can flow from introduction of order into S-matrix theory, one is encouraged to seek a generalization of sequential order that will illuminate the complete hadron family, including baryons and exotics. This paper puts forward a candidate for a central ingredient of such a program: a general ordered unitary S matrix with consistent factorizable poles.

In our candidate ordered S matrix, unitarity will be shown to imply a 3-color hadronic spectrum whose leading families exhibit the attributes of $q\bar{q}$ mesons and qqq baryons. A well-defined set of higher hadron families with zero triality also emerges, the simplest of which corresponds to Rosner's exotics[2)]--sometimes called baryonium. In the ordered S matrix, baryonium states do not communicate with ordinary mesons, even when they share the same set of quantum numbers, because the two families correspond to different categories of order. The ordered Hilbert space splits into noncommunicating sectors, each characterized by a different type of order and sustaining a distinct family of hadrons.

Only a few special categories of ordered relationship turn out to be consistent with unitarity. Immediately ruled out will be hadrons whose relationships correspond to single-quark states; unitarity guarantees confinement. We shall see also that combining unitarity with order implies baryon-number conservation and is consistent with Fermi statistics for baryons. The 3-color character of the ordered hadron spectrum will stem from the representation of ordered relationships by graphs and the special role of the 3-vertex in graph theory.

This paper will describe the Hilbert-space contraction needed to make contact between the ordered S matrix and physical measurement, where particle order is not observable; the contraction involves the nonunitary "planar S matrix". We do not here present any analog of the topological expansion which for sequential ordering corrects the unitarity deficiency of the planar S matrix[3]. We do not here, therefore, propose a mechanism for baryonium decay into mesons; such a theory remains to be developed.

The theory proposed here marries the familiar S-matrix concept with the still mysterious notion of order. Although the extent to which these two concepts can be merged is a nontrivial question, we do not in this introductory paper dwell on well-known S-matrix properties that survive the merger, except for a brief summary in chapter VI where we also touch on internal quantum numbers. A treatment of both these topics as well as a more detailed and systematic account of the general theory will be presented in a separate paper by G. Weissmann. Another paper elaborating the mathematical structure of the theory is in preparation by J. P. Sursock. In the present paper we concentrate on the concept of order, proceeding in an intuitive style and presenting only the backbone of the argument.

II. GRAPHICAL REPRESENTATION OF ORDERED CHANNELS AND CONNECTED PARTS

The ordered S matrix acts in a Hilbert space specified not only by the momentum and spin of each particle but by assigning to the particles some definite order. The simplest type of order corresponds to placing particles in a linear sequence--attaching to each particle a predecessor and a successor; each particle, in other words, is "connected" to two other particles, the two connections being given an order that distinguishes them. It has been shown that for such a Hilbert space one may define a unitary cluster-decomposable S matrix whose connected parts satisfy the crossing property[4]. Poles of the ordered connected parts are consistently factorizable. The question addressed by this paper is whether it is possible to maintain such properties in a Hilbert space where ordering is more complex than simply sequential--some particles being "connected" to more than two other particles?

A general representation of particle order may be given in terms of connected graphs whose vertices are associated with particles. A graph formed from n vertices, connected through a certain number of edges, represents a particular order of n objects. (A "tadpole" edge connecting a vertex to itself is not admitted.) Any vertex may have several neighbors, but the collection of vertices is ordered in the sense that each has a unique set of neighbors and occupies a unique position in the graph. Associating particles with vertices, we shall find that unitarity will require the different edges converging at a particle vertex to somehow be distinguishable; this subset of edges, in other words, must itself be ordered.

We shall find it necessary to speak of three different but related types of graph: (1) "Amplitude graphs" to represent the order of connected parts. (2) "Channel graphs" to represent channel order. (3) "Particle graphs" to represent the order of the edges converging at a particle vertex. To the extent that one may speak of a single-particle channel, particle graphs constitute a special case of channel graphs, but the special case is so important as to warrant separate identification.

For sequential ordering, amplitude graphs have been shown to be oriented "rings" as illustrated in fig. 1(a) while channel graphs are simple open directed chains as illustrated in fig. 1(b) [4]. (Notice that our channel graphs include "dangling" edges at the ends of the chain.) The corresponding particle graph, illustrated in fig. 1(c) is almost trivial, but the two attached edges are mutually distinguished by direction. In general the existence of an S matrix means that specification of a channel pair--initial and final--determines uniquely the connected part for the intervening transition. A pair of channel graphs consequently should determine a unique amplitude graph. Conversely, ordered channels must be associated with graphs achieved by cutting amplitude graphs into two connected portions (henceforth to be designated a "bisection"). Such requirements place severe constraints on the acceptable categories of order. A glance at fig. 1 verifies the acceptability of oriented-ring amplitude graphs together with directed-chain channel graphs. Bisection here evidently permits unique "resewing".

Because each particle vertex corresponds to a single-particle channel, any particle vertex in an amplitude graph must be isolatable

by a bisection. This requirement immediately eliminates "pendant" vertices such as F in fig. 2(a) because the neighboring vertex B cannot be isolated by a bisection. A particle vertex must join at least two edges. When in sec. VIII below we identify edges with "quarks", it will be recognized that prohibition of pendant vertices is equivalent to quark confinement.

The graph of fig. 2(b) also is unacceptable as a representation of an ordered amplitude because the vertex B cannot be isolated. Henceforth we consider only graphs whose vertices are isolatable through bisection, but even with such a constraint many conceivable categories of order are excluded by the requirement that bisection of an amplitude graph lead to a pair of channel graphs with a unique resewing prescription. (For example, were directions omitted from the edges in fig. 1, uniqueness of channel-graph sewing would be lost.) To begin with, the only amplitude graphs we have found consistent with unique resewing are those that when bisected in all possible ways, always yield channel graphs with unique "spanning trees". By a spanning tree we mean a tree graph achieved by successively removing channel-graph edges so as to eliminate cycles (closed loops), at each stage erasing any vertex where only two edges meet (a 2-vertex). If tadpoles are created in the process, they are to be erased together with the connecting edge. Spanning trees are important because they exhibit the edges left dangling when an amplitude graph is bisected. Many different channel graphs will of course share the same spanning tree.

Figure 3(a) shows an example of an amplitude graph (with undirected edges) for which all of the associated channel graphs have unique spanning trees. Examples of these channel graphs and their spanning trees are displayed in fig. 3(b) and 3(c). Figure 4 shows an example of an amplitude graph for which at least one of the associated channel graphs has several different spanning trees.

To the extent that the spanning tree exhibits through its "twigs" the dangling channel-graph edges, the reader should find plausible that spanning-tree uniqueness is related to uniqueness of resewing. But a complete statement of the sewing prescription requires a way of distinguishing all the different "twigs" of a spanning tree, one from another. This problem we take up in the following section, together with the relation between channel-graph spanning trees and particle vertices.

It is not hard to establish that the requirement of unique spanning trees for channel graphs (all of whose vertices are isolatable through bisection) is equivalent to demanding that amplitude graphs be fully reducible. Reduction is defined by repeated application of the following two operations: (1) Remove 2-vertices. (2) If two vertices have several edges in common, replace the set of common edges by a single edge. Fully-reducible graphs may be simplified by these operations to a single loop. One sees that the amplitude graphs of figs. 1(a) and 3(a) are fully reducible, while that of fig. 4(a) is not.

III. ORIENTED PARTICLE VERTICES; COLORED EDGES

Because a single particle is a possible channel, a particle vertex is a special case of a spanning tree. Consequently the problem of distinguishing the twigs of a spanning tree and of distinguishing the edges attached to a particle vertex are essentially the same.

Now the structure of most trees immediately provides some distinction between twigs. For the tree displayed in fig. 5, for example, twig α can be distinguished from the others. It is not possible, on the other hand, to distinguish twig β from twig γ unless we "color" the twigs. (Remember that our graphs merely describe connections and are not to be understood as projected on a plane.) We may kill two birds with one stone if we assign order to a set of three or more edges converging to a particle vertex by associating the particle "vertex" with an oriented colored tree graph built from 3-vertices. Assigning distinct colors to the three edges joined at each 3-vertex suffices for most tree graphs to assign unambiguous order to the twigs.* Figure 6 gives examples--each graph representing a distinct order for the twigs; changing color assignments changes the order. At the same time, certain tree graphs with an even number of 3-vertices possess a twofold symmetry that leaves, even when colored, a twofold ambiguity of order. Figure 7 gives examples. This twofold ambiguity may be removed by assigning a ($\pm$) "orientation" to the tree graph. We momentarily defer the notational matter of an orientation label on the graph, but note immediately that for particle vertices it is natural to expect the ($\pm$) orientation to be associated with charge conjugation (particle vs. antiparticle); such will indeed turn out to be the case. In any event we can completely distinguish any number

* Our use of the term "color" is that employed by mathematicians in graph theory (e.g., the 4-color theorem). The reader will quickly recognize a connection with "color" as employed in quantum chromodynamics, but we do not here attempt to make the connection precise.

of edges larger than 2, attached to a particle "vertex", by expanding the vertex to an oriented colored tree graph built from 3-vertices.

We do not yet fully understand the mathematical origin of the special role played by the 3-vertex (and 3 colors) in distinguishing an arbitrary number of converging edges. But given the consistency requirements of unitarity, the 3-color tree-graph characterization of order for the edges emanating from a particle "vertex" is the most general of which we are aware. A separate paper by one of us (J. P. Sursock) will connect the physical relationship between particles and antiparticles to the special role of the 3-vertex.

If all particle vertices with 3 or more attached edges are "expanded" through their tree-graph representations, we achieve "expanded amplitude graphs" and "expanded channel graphs" where only 2-vertices and 3-vertices appear. The amplitude graph of fig. 3(a), for example, might expand to that of fig. 8. Colors and orientations have here been omitted, for clarity, but each particle "vertex" carries a $(\pm)$ index and each of the edges that join at a 3-vertex carries a different color 1, 2, 3. It can be shown that every allowable expanded amplitude graph may be consistently colored according to this rule.

Corresponding to expanded channel graphs there will be expanded spanning trees and we now recognize the second bird slain by our stone: All expanded spanning trees with more than 2 twigs are built entirely from 3-vertices and colored edges. In this respect expanded spanning trees are similar to expanded particle "vertices", as demanded by unitarity. But what about orientation? Does an expanded channel graph built from oriented particle "vertices" have a well-oriented expanded spanning tree?

IV. DIRECTED EDGES; MATES

Let us require that an expanded amplitude graph be fully reducible, or equivalently, that any expanded channel graph obtained from a bisection have a unique expanded spanning tree. This requirement guarantees satisfaction of the corresponding requirement for unexpanded graphs. Now an easily-derived general property of a fully-reducible expanded amplitude graph is that the 3-vertices can be divided into two distinct sets so that (ignoring 2-vertices) every edge connects a 3-vertex in one set to a 3-vertex in the other. Since we have already been led to differentiate between two orientations $(\pm)$ of a particle vertex, let us see whether we might consistently attribute (+) orientation to all 3-vertices of one set and (-) orientation to all 3-vertices of the other set. That would mean for all types of expanded graph that the orientation of neighboring 3-vertices would alternate. We see that such a rule means two and only two orientations for each entire graph: Reversing the orientation of any 3-vertex requires reversal of all orientations.

We here recognize that expanded spanning trees carry a $(\pm)$ orientation of the same character as do particle "vertices". Satisfied is the unitarity requirement that the representation of order be consistent for single-particle and multi-particle channels. Also satisfied, if every twig of a spanning tree is distinguishable from every other twig, is the requirement of a unique sewing prescription for two channel graphs. This point will be elaborated in sec. V.

The fact that expanded amplitude graphs contain equal numbers of (+) and (-) oriented 3-vertices (no dangling edges) allows the assignment of a conserved quantum number +1 to a (+) vertex and -1 to a (-) vertex; it is natural to associate this conserved quantity with baryon number. The baryon number of a particle "vertex" is then obtained by expanding to reveal its order, as in figs. 9 and 10, and counting the excess (or deficiency) of (+) over (-) 3-vertices. Thus the particle vertices corresponding to figs. 9(a) and 9(b) have baryon number ± 1 while those of 9(c) have baryon number ± 2. The baryon number for both graphs in fig. 10 is zero. Particle vertices evidently are allowed for indefinitely-high baryon number.

A convenient property of expanded graphs is that they can be drawn on a plane with a clockwise (1, 2, 3) ordering of the colors at a (+) vertex and a counterclockwise color ordering at a (-) vertex. It must be remembered, however, that certain graphs which look different when drawn on a plane are equivalent.

The orientation of 3-vertices in an expanded graph may be expressed through a convention about the "direction" of edges joined at a 3-vertex. We may consistently assign to each edge a direction away from a (+) vertex and toward a (-) vertex. Figures 9 and 10, with the color index suppressed, illustrate this convention. Note that the use of both edge directions and ($\pm$) 3-vertex labels is redundant.

The association of a direction with each edge allows us to order a 2-vertex. Recall that the reduction concept requires a 2-vertex to be removable without altering the remaining order in a graph. If edges have "color" and "direction" we must then require continuity of both attributes for the edges that meet at a two vertex, as shown in fig. 11. One edge here may be regarded as connected to a "predecessor" vertex and one to a "successor" vertex. Our generalized ordering scheme in this fashion maintains for its 2-vertices the structure of the sequentially-ordered S matrix.

A fully-reducible expanded amplitude graph has the interesting and important property that each 3-vertex possesses a unique "mate". That is to say, in the reduction to a ring a stage inevitably is reached for each vertex where two of its three connecting edges are replaced by a single edge; these two edges both connect the vertex to a single other vertex. We refer to this vertex pair as "mates". It can be shown that the order of reduction does not affect the pairing. The two members of a mated pair evidently possess opposite orientations, one carrying baryon number +1 and the other -1. Recognition of the mating phenomenon will (in sec. IX) facilitate the handling of statistics when the Hilbert space is contracted to remove order as a degree of freedom.

V. SECTOR SKELETONS

Let us introduce the adjective "complementary" to describe the relationship between a pair of expanded graphs differing only in orientation and the term "skeleton" to denote an expanded spanning-tree graph from which particle labels have been removed--leaving only directed colored edges and 3-vertices. Now, when an expanded connected-part graph is cut into two expanded channel graphs, the two graphs necessarily possess complementary skeletons. Furthermore, since the external (dangling) edges of each skeleton are all distinguishable, there is a unique prescription for resewing. Figure 12 shows an example. Recognizing that the channel graph associated with a "bra" state-vector is the complement of the graph associated with the corresponding "ket" vector, we see that ordered connected parts are nonvanishing only when coupling two channels that share the same skeleton. The ordered Hilbert space, in other words, splits into noncommunicating sectors, each characterized by a different skeleton.

Among the channels belonging to any sector there will occur single-particles, corresponding to poles in the channel invariant, so the ordered "vertex" associated with a particle must coincide with the skeleton of the sector to which the particle belongs. Ordered particles are allowed to "decay" only into channels within its own sector.

In the sense of the foregoing, the sequentially-ordered S matrix possesses only one sector, with a trivial skeleton corresponding to fig. 11. Assuming that "ordinary" (nonexotic) mesons belong to the corresponding sector of our more general ordered S matrix, we see that ordered particles belonging to a sector with skeleton of the type of fig. 10(a) or 10(b), even though they have zero baryon number, may not decay into ordinary mesons by an ordered transition.

We now can see clearly why ordering of particle vertices was necessary in the first place. Particle vertices must correspond to sector skeletons and only if the external edges of sector skeletons are ordered will there be a unique way to "sew" together two ordered channel graphs in forming an ordered connected-part graph. S-matrix theory has an intrinsically circular quality with respect to poles (particles). We have chosen here to first introduce color and orientation in connection with particle vertices, but the necessity for a unique correspondence between connected parts and channel pairs demands a way to distinguish the different twigs of a spanning tree.

VI. PROPERTIES OF THE ORDERED S MATRIX

This section summarizes results that are developed and presented in detail in ref. (5). Having a specific form of order within the S-matrix framework, one can formulate an axiomatic system of postulates for ordered S-matrix theory analogous to that of usual S-matrix theory and deduce S-matrix properties. Although order requires adjusting certain concepts and their relationships, most S-matrix properties survive the transition to the ordered S matrix, albeit in modified form. This circumstance, plus the fact that the ordered S matrix passes stringent self-consistency requirements, provides an encouraging sign that we are on the right track.

We now summarize the axioms, and some properties derived therefrom. Lorentz invariance is introduced in the usual way, since a Lorentz transformation does not affect the order of the particles, but only their individual momenta and helicities.

Causality is postulated in a form that takes into account the order; one can deduce therefrom the analyticity of ordered amplitudes in and around the physical region except on certain lower-dimensional manifolds called Landau surfaces, whose location is determined. The main difference with usual S-matrix theory is that only a subset of the Landau singularities arising in usual S-matrix theory actually occur in the ordered case. For example, a given ordered amplitude has poles and normal threshold singularities only in channel-invariants corresponding to a bisection of the amplitude-graph. This simpler singularity structure leads, amongst other things, to simpler dispersion relations, simpler Regge structure, and to duality.

Ordered unitarity formally resembles usual unitarity: $S_{in} S^{\dagger}_{nf} = \delta_{if}$; but now i, n, and f refer to ordered states. In order even to formulate unitarity, we have had to introduce the symbol S_{in}, whereas the ordered amplitudes of the preceding sections corresponded to connected parts, i.e., T_{in}. Cluster decomposition allows S_{in} to be expressed as a sum of products of T_{jk}'s. How does cluster decomposition operate for the ordered S matrix? We postulate that ordered S-matrix elements, like connected parts, vanish except when connecting ordered channels that belong to the same sector, i.e. channels with the same colored sector skeleton. Any nonvanishing S-matrix element may then be represented by a pair of expanded channel graphs with complementary skeletons. An example is given in fig. 13 where one channel contains particles B, G, H, I, C and the other contains F, E, D. Note the expansion of the E vertex. Nonvanishing cluster products arise from all ways of cutting both skeletons through corresponding skeleton edges without disturbing individual particle vertices. Thus the S-matrix element of fig. 13 is the sum of the connected-part products shown in fig. 14.

On the basis of the postulates of Lorentz invariance, causality, unitarity and cluster decomposition one can, as in usual S-matrix theory, prove a variety of properties, such as:

(a) Landau singularities: All Landau surfaces of the ordered theory actually correspond to singularities; elsewhere, in and around the physical region, the ordered amplitudes are analytic. Discontinuity formulas can be derived for all Landau singularities.

(b) Pole factorization: Each pole residue factorizes into two factors in the usual way, each of the two factors being itself an

ordered amplitude. This circumstance permits identification of poles with external particles. Figure 15 gives an example of the residue structure of a pole J in the connected part of fig. 12, associated with the indicated cut. Note that the vertex associated with particle J has an order given by its sector skeleton.

(c) Crossing: If transitions between 2 different pairs of ordered channels correspond to the same (expanded) amplitude graph, then the 2 transition amplitudes are related by analytic continuation in the usual sense. Thus the connected part of the transition of fig. 13 is the continuation of the connected part of the transition of fig. 16, with the appropriate reinterpretation of certain ingoing particles as outgoing antiparticles, and vice versa.

(d) TCP, Hermitian analyticity, Froissart bounds, etc.

Duality properties, such as exchange degeneracy and the absence of Regge cuts, which are not general S-matrix properties and hold only approximately in nature, are exact results here due to the simpler singularity structure. In the same category are properties related to internal quantum numbers. We have seen that the skeleton classes (of ordered channels) form sectors: no channel from one skeleton class can communicate with a channel from another. With additive internal quantum numbers a further splitting of the Hilbert space occurs. One can show that irreducible sectors, and hence also particles, are characterized by a skeleton whose external edges are labeled by a discrete flavor index 1. A generalized set of OZI selection rules thus arises.

As discussed in sec. VIII, the resulting spectrum of hadrons is similar to the one predicted by quark-models. Symmetries like

SU(2) or SU(n), and the resulting multiplet structure, can be inserted by hand but have not yet been derived as natural consequences of the theory.

In summary, not only do the usual S-matrix properties persist in the ordered framework, but additional properties, in particular duality and quark-like structure, are found to hold.

VII. COLOR PERMUTATION SYMMETRY

Although channel graphs with different coloring correspond to different states of the ordered Hilbert space, channel graphs related by the systematic permutation of colors (e.g., $1 \leftrightarrow 2$, $3 \leftrightarrow 3$) are "equivalent". Figure 17 gives an example. A corresponding equivalence exists for amplitude graphs. Equivalence means that for any relationship (such as a discontinuity formula) obeyed by an amplitude belonging to a graph colored in one fashion, there will be a corresponding relationship for each of the $3!$ other amplitudes reached by color permutations. With a suitable phase convention, each of these six amplitudes has the same numerical value.

The number of different but equivalent colored skeletons may be less than 6, depending on the complexity of structure of the colorless skeleton. For instance, with no more than one flavor there is only one colored skeleton of the type of fig. 9(a) or 9(c). There are 3 each of the type of fig. 9(b), fig. 10(a) or fig. 11. The reader may verify that there are six permutation-equivalent skeletons of the type shown in fig. 10(b). (There are also two inequivalent ways of coloring such a skeleton.) Sector-skeleton equivalence means a

corresponding degeneracy for ordered particles. However, when the Hilbert space is contracted to remove order as a degree of freedom, one expects that some colored multiplets of degenerate ordered particles will turn out to correspond to only one physical particle.

VIII. ORDERED-PARTICLE CHARACTERISTICS; DIRECTED EDGES AS "QUARKS"

We have shown how baryon number arises in the ordered S matrix. We now point out that the opposite orientation of "initial" and "final" ("bra" and "ket") ordered channel graphs, together with crossing, requires identification of orientation-reversal with TCP. Applying this reasoning to a single-particle channel, one concludes that the tree-graph (ordered vertex) for an antiparticle is obtained from that for the corresponding particle by reversing the orientation. Such a rule ensures that if the particle has baryon number B, the antiparticle has baryon number -B.

Let us now classify the simpler ordered particles and the corresponding channel sectors according to two indices: baryon number and number of edges emanating from the particle vertex (or sector skeleton). Particles with baryon number 0 we optimistically designate with the symbol M, hoping that these will turn out to be related to mesons. Similarly those with baryon number 1 we designate with the symbol B (baryon) and those with baryon number 2 with the symbol D (deuteron). The number of edges will be shown as a subscript, as in fig. 18. It might be thought that the indicated indices uniquely determine the uncolored tree graph, but such is not in general the case. Figure 19 shows two distinct uncolored tree graphs that are both designated B_7. Physical interest for some time to come is nevertheless unlikely to go beyond the particle categories of fig. 18, for which our two indices suffice to identify the uncolored vertex.

Ordered particles in the M_2 sector will exhibit the same regularities of spectrum and slection rules as the mesons in the sequentially-ordered S matrix.* All the celebrated properties of the planar meson S matrix will be present[6)], properties often characterized as showing "quark-antiquark $(q\bar{q})$ structure". We may extend such terminology to the more general ordered S matrix by associating the term "quark" with a directed edge, attributing baryon-number $1/3$ to each such edge. To the extent that an ordered particle is characterized by the edges emanating from its vertex, it does no harm to speak of the particle as composed of a corresponding combination of quarks. No physical content is added thereby, but contact is afforded with a reservoir of familiar terminology. Notice that ordered particles necessarily have zero triality; confinement is assured.

Ordered particles communicating with the M_4 sector would be characterized in the above language as having $qq\bar{q}\bar{q}$ structure. These particles play the role of Rosner's exotics[2)], more recently called "baryonium" to emphasize the $B_3\bar{B}_3$ aspect of their structure. We have already stressed that an M_4 particle is forbidden to decay into any collection of M_2 particles. It can, however, decay into a combination of B_3 and $\bar{B}_3$ plus any number of M_2. Allowed decay

* Even though there will be $B_3\bar{B}_3$, $B_5\bar{B}_5$, etc. channels in the M_2 sector.

channels for particles in the M_6 category $(qqq\bar{q}\bar{q}\bar{q})$ must contain at least two B_3 and two $\bar{B}_3$ or, alternatively, two M_4. M_6 particles can thus maintain narrow widths up to even higher energies than M_4 particles. M_8, $M_{10}\cdots$ will continue the trend.

It should be noted that the ordered spectrum lacks many mesons with zero-quark, two-quark and four-quark structure that have been anticipated from QCD and (or) quark models[7,8,9)]. Our meson spectrum is relatively simple by comparison.

The story for sectors with baryon number 1 is similar. We expect to identify B_3 with "normal" qqq baryons, while B_5 is a $qq\bar{q}qq$ "exotic" which requires for an allowed decay channel at least two B_3 and one $\bar{B}_3$ or one B_3 and one M_4.

The particle-type D_6 is intriguing. Since its allowed decay requires at least $3B_3$ and $1\bar{B}_3$ or $2\bar{B}_3$ and $1M_4$, or $B_5 + B_3$, it seems unreasonable to expect any correspondence with the physical deuteron. We must then hope that the deuteron will arise from higher terms in some "topological expansion" that corrects the ordered S matrix.

Having associated edges with "quarks", the reader should have no difficulty in seeing the connection between our expanded amplitude graphs and the quark-line diagrams that have been extensively used in the recent literature[7,10,11)]. With appropriate constraints the two notations are compatible. Notice that we could draw "junction lines"[11)] between mates but that no new information is thereby conveyed.

IX. THE PLANAR AMPLITUDES

As was the case for sequential ordering[4] we pass to "planar" amplitudes by summing over ordered amplitudes. For a given set of ordered particles, we take the connected part of the planar amplitude to be the sum, with weights ϵ where $\epsilon = \pm 1$ as suggested by Fermi statistics, as explained below, of the connected parts of all of the ordered amplitudes for that set of ordered particles. In choosing the factors $\epsilon = \pm 1$, it is sufficient to look at the assignment of mates. In each expanded ordered amplitude, each 3 vertex of (+) orientation is mated to a specific 3-vertex of (-) orientation. Two ordered amplitudes with the same assignment of mates will contribute with the same sign to the planar amplitude; two ordered amplitudes with different assignments of mates will contribute with the same (the opposite) sign, if the mate assignments of one are related by an even (an odd) number of switches to the mate assignments of the other. For example, an ordered amplitude in which A is mated to B, and C is mated to D, contributes with a sign opposite to that of an ordered amplitude in which A is mated to D, and B is mated to C. It is easy to see that this rule guarantees that the planar amplitudes will obey the requirements of Fermi statistics.

These planar amplitudes, being sums of ordered amplitudes, will inherit the analyticity properties of the ordered amplitudes. They will not obey full unitarity; however, the pole terms will fully factorize. We anticipate that unitarity will be achievable through a "topological expansion" within which planar amplitudes will constitute the leading terms[3].

A final step in making contact with experiment requires an association between physical particles and ordered particles. In the case of sequential ordering the correspondence was one-to-one. With our generalized order the situation is more complex because of the color degree of freedom. Explicit consideration of flavor is necessary, and we do not further pursue the matter in the present paper.

X. CONCLUSION

At least two further steps must be taken before firm contact can be made with experiment: (1) The simultaneous roles of flavor and color in the ordered hadron spectrum must be understood. (2) A topological expansion of the physical S matrix must be developed to correct in principle the failure of the planar S matrix to satisfy unitarity. Hopefully there will be systematic mechanisms suppressing nonplanar expansion components for some portions of the physical hadron spectrum, so that these portions will bear a recognizable resemblance to the ordered spectrum. Work on both these problems is proceeding.

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FIGURE CAPTIONS

Fig. 1. Graphical representation of (a) a sequentially ordered (i.e. mesonic) amplitude, (b) sequentially ordered channels and (c) a single meson.

Fig. 2. Forbidden graphs. Factorizing particle B in either graph would result in cutting the graph into three connected parts. Graph (a) demonstrates the impossibility of isolating quarks as asympotic states.

Fig. 3. (a) An allowed amplitude graph with (b) some channel graphs obtained by bisection and (c) their corresponding spanning trees.

Fig. 4. Irreducible graph (a) characterized by the presence of at least one channel graph (b) with more than one spanning tree (c).

Fig. 5. A symmetric graph demonstrating the need for the edge-coloring and orientation that permit to distinguish between the twigs β and γ, β and ϵ, etc.

Fig. 6. Graphs with the property that the symmetry is broken by coloring the edges.

Fig. 7. Graphs requiring orientation of the vertices (or edges) to break a remaining symmetry.

Fig. 8. Vertex-expanded graph of fig. 3(a).

Fig. 9. Expanded particle vertices representing particles with baryon number (a) ± 1, (b) ± 1 and (c) ± 2, respectively.

Fig. 10. Expanded particle vertices representing particles with baryon number 0, but with a different number of 3-vertices.

Fig. 11. Edge orientation for 2-vertices representing mesons of the three different colora.

Fig. 12. In an ordered amplitude the resewing of the two states obtained by bisecting an amplitude is always unique.

Fig. 13. Graphs corresponding to 2 channels for which there is a nonvanishing S-matrix element.

Fig. 14. Cluster decomposition of the S-matrix element of fig. 13 into connected-part products.

Fig. 15. An example of the residue structure of the pole J in the connected part of fig. 12.

Fig. 16. Crossing properties of the ordered S matrix: the amplitude represented by the graph above is the analytic continuation of the amplitude represented in fig. 13.

Fig. 17. An example of a sector with several "color-equivalent" channels.

Fig. 18. Classification of particle types according to their topological representation.

Fig. 19. Two inequivalent tree graphs with the same number of external edges.

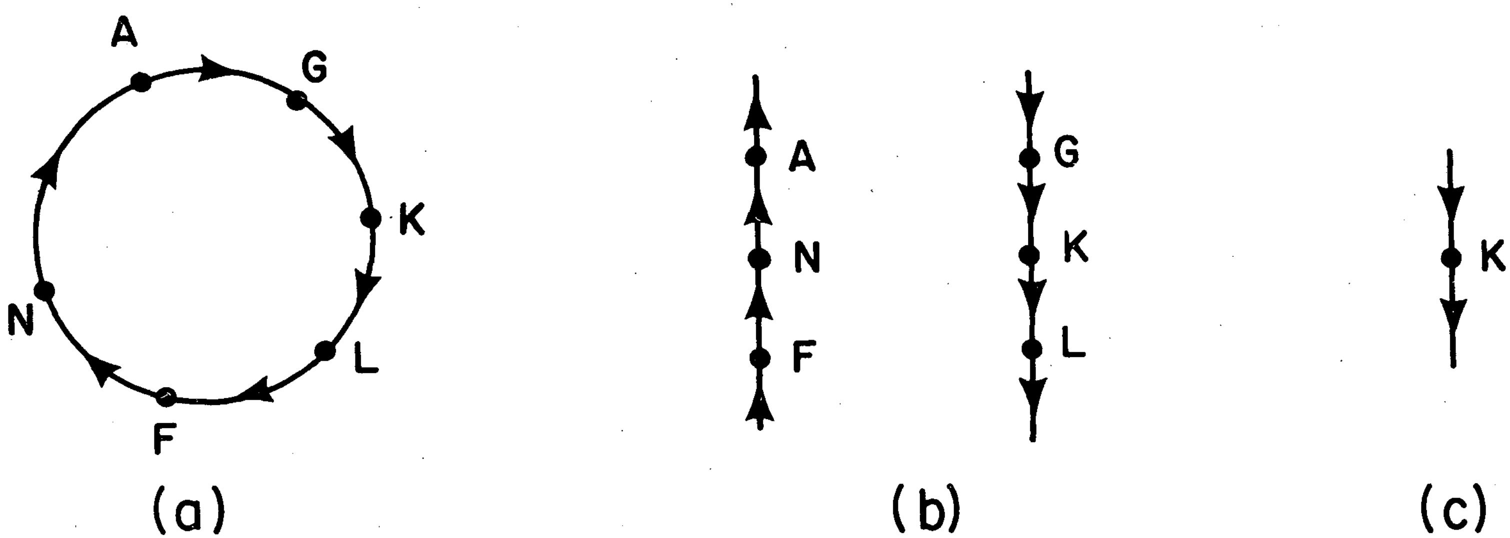


Fig. 1

XBL7712- 11113

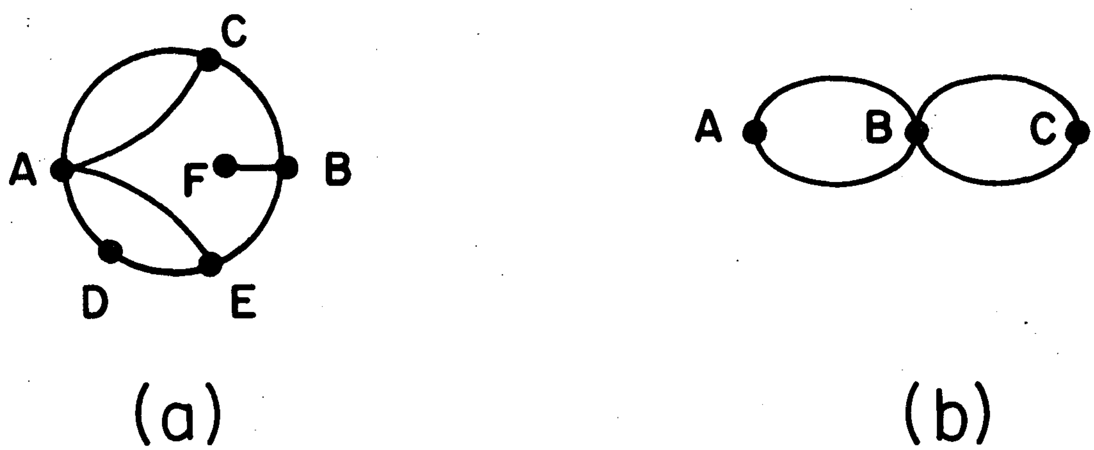


Fig. 2

XBL 7712- 11111

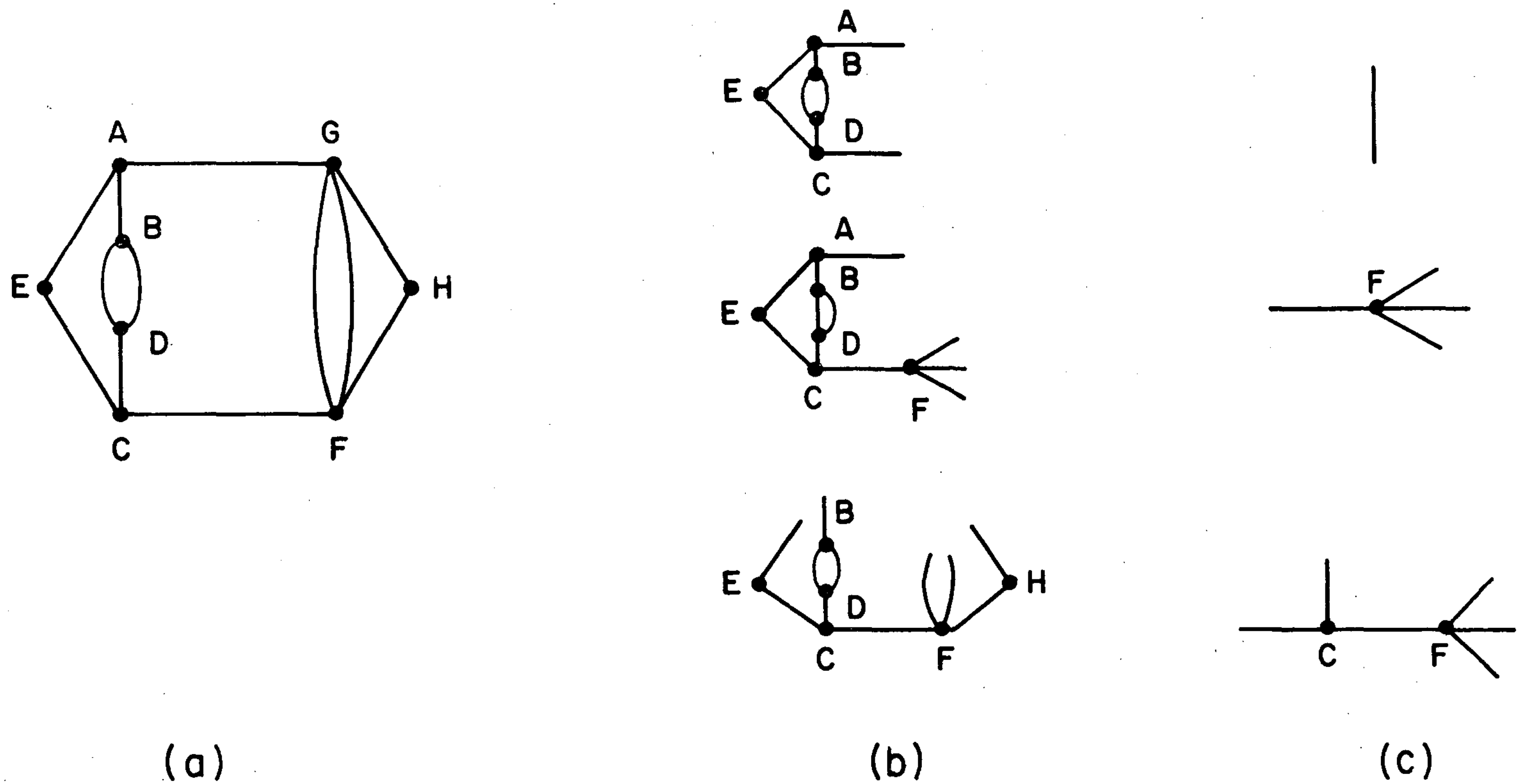


Fig. 3

XBL7712-11098

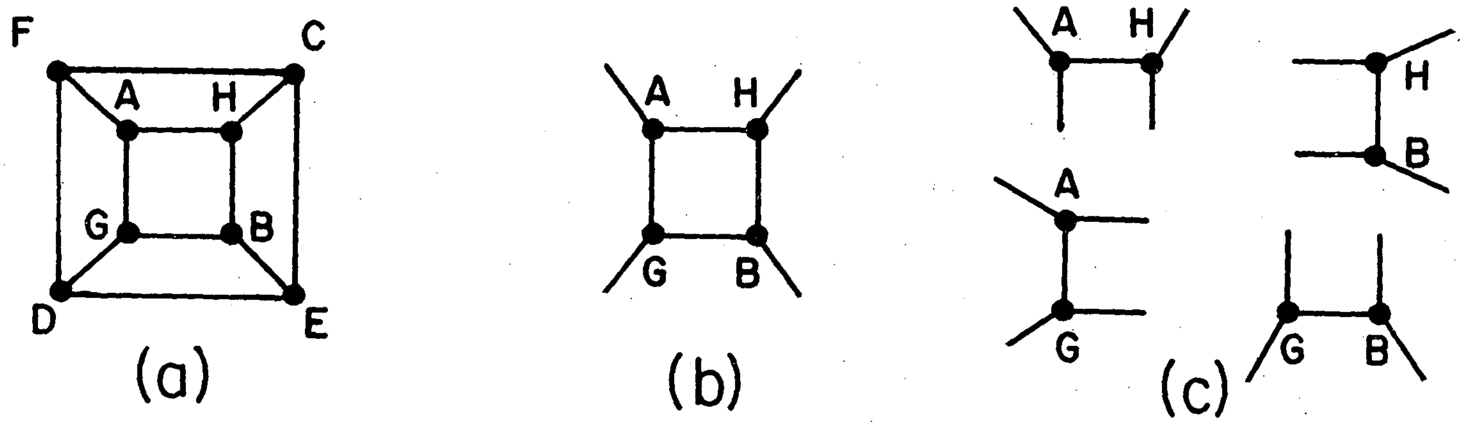


Fig. 4

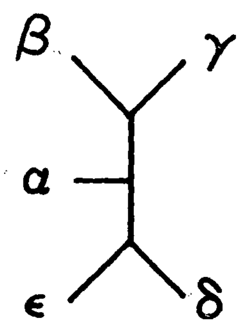


Fig. 5

XBL 7712-11108

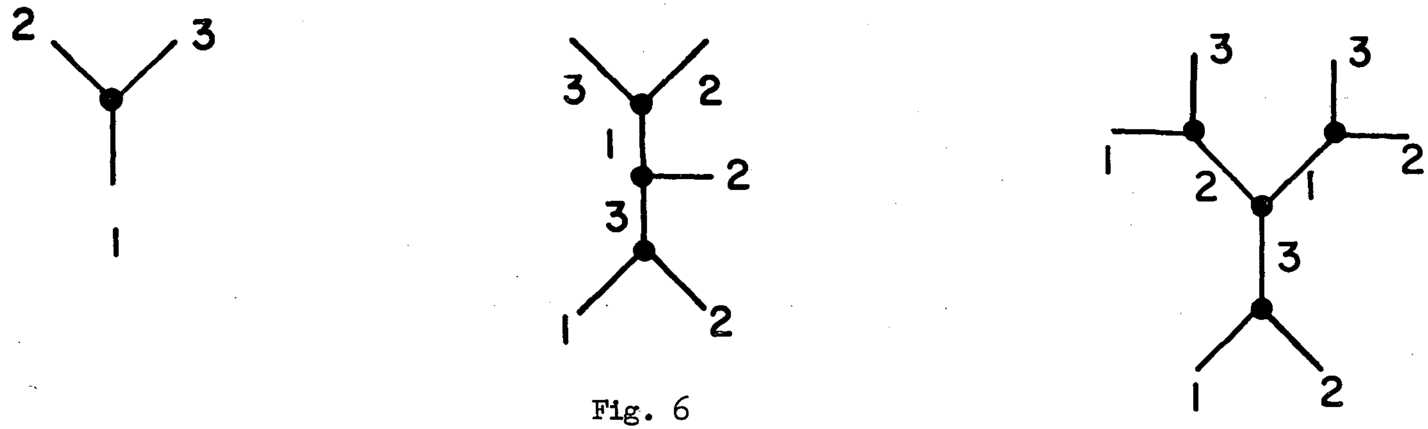


Fig. 6

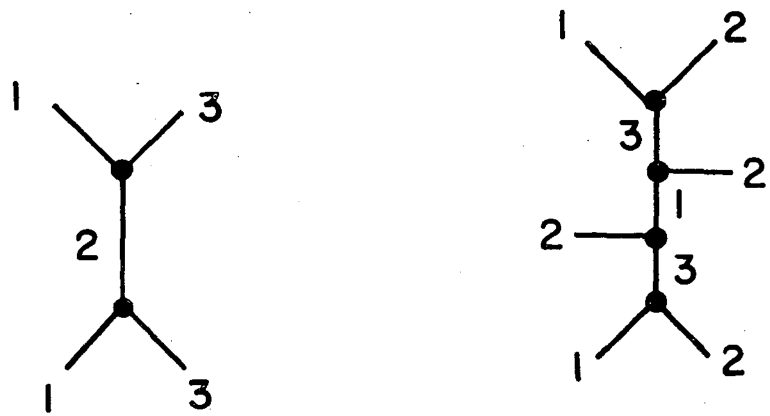


Fig. 7

XBL7712-11109

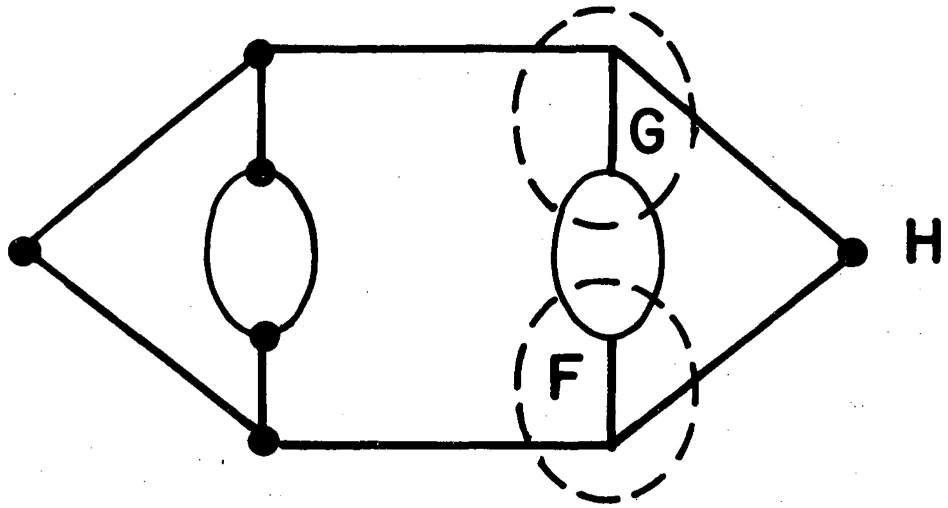
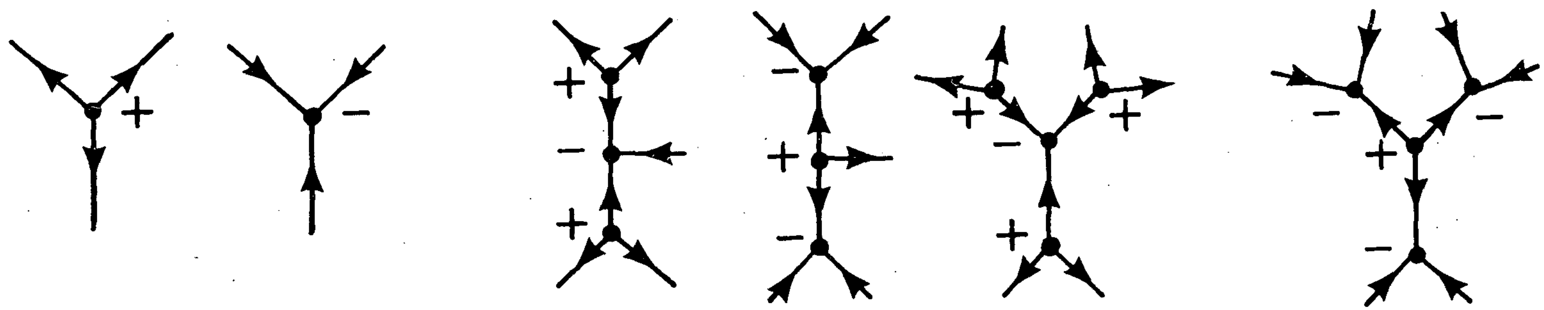


Fig. 8

XBL7712-11099



(a) (b) (c)

Fig. 9

XBL7712-11097

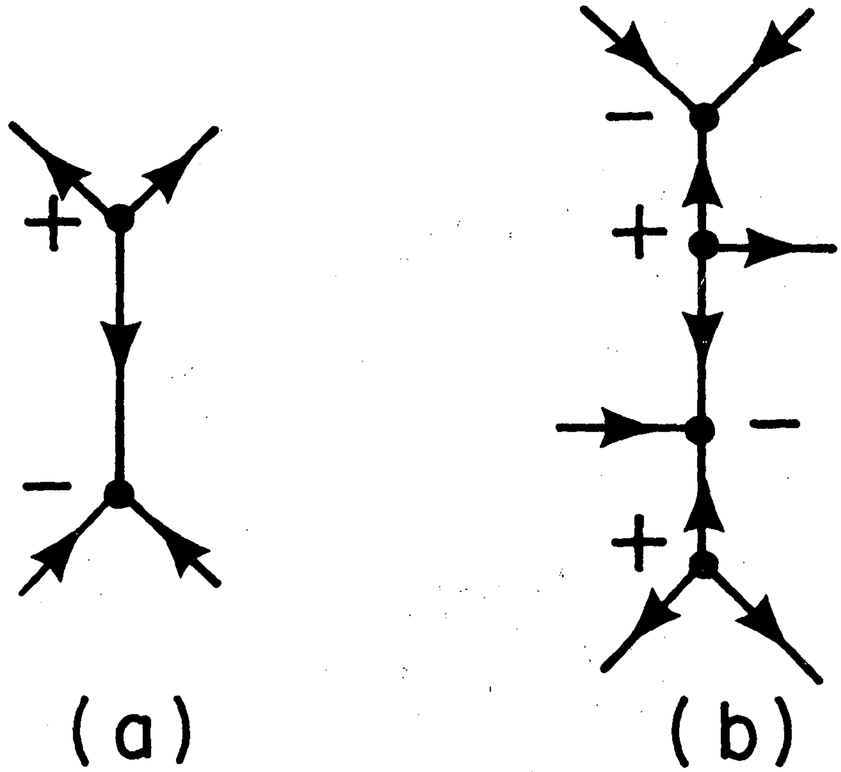


Fig. 10

XBL7712-11105

XBL7712-11101

Fig. 11

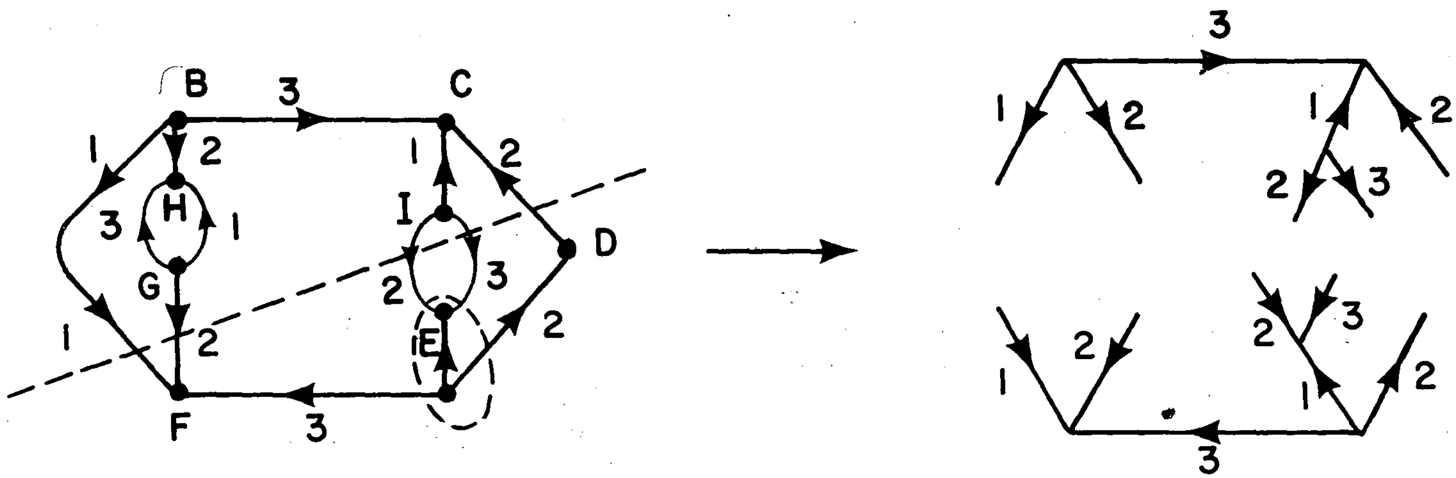


Fig. 12

XBL7712-11102

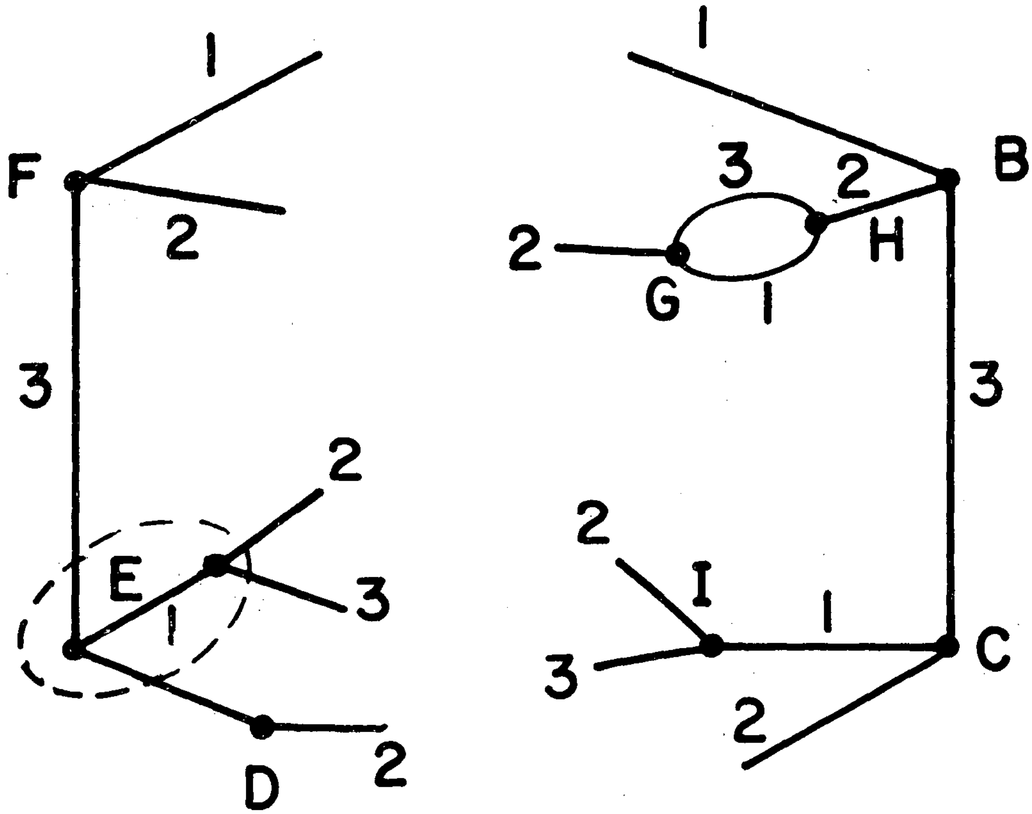


Fig. 13

XBL 7712-11106

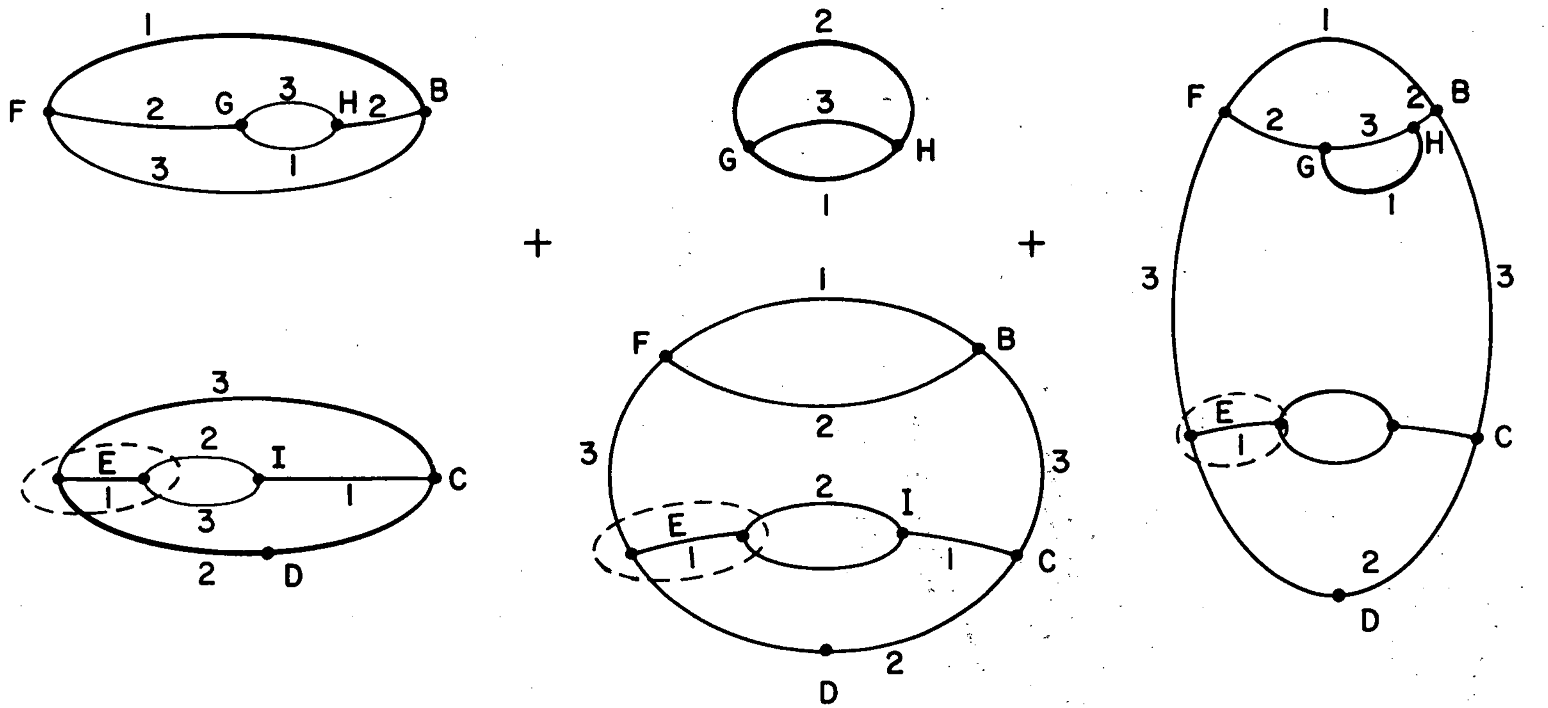


Fig. 14

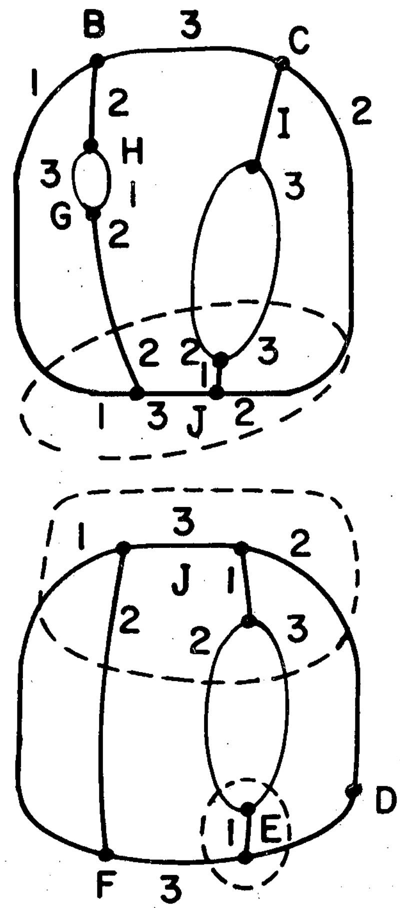


Fig. 15

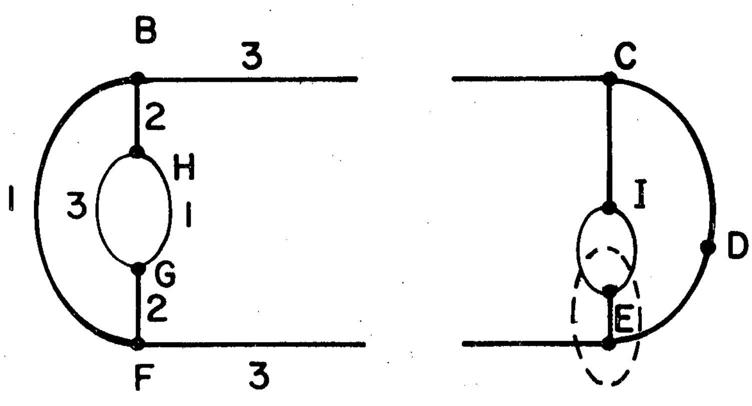


Fig. 16

XBL 7712-11103

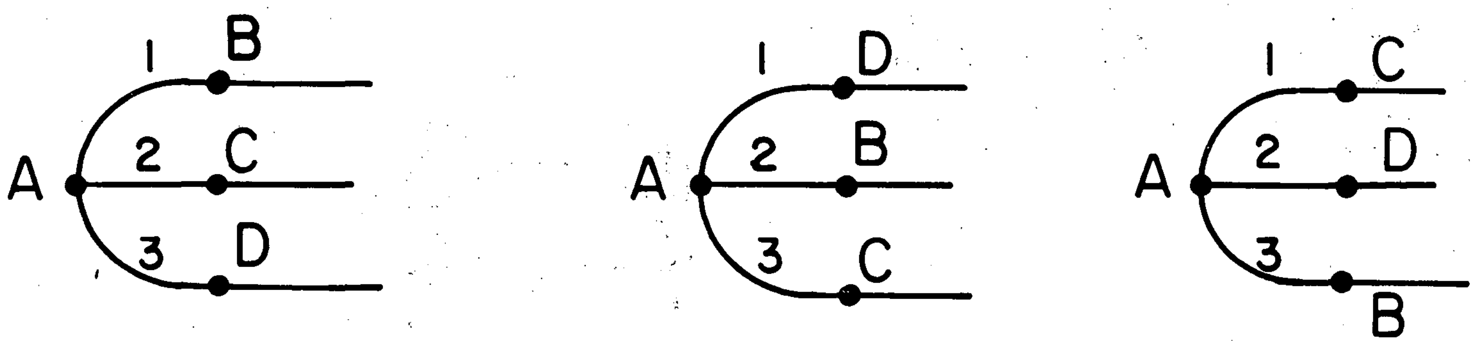


Fig. 17

XBL7712-11112

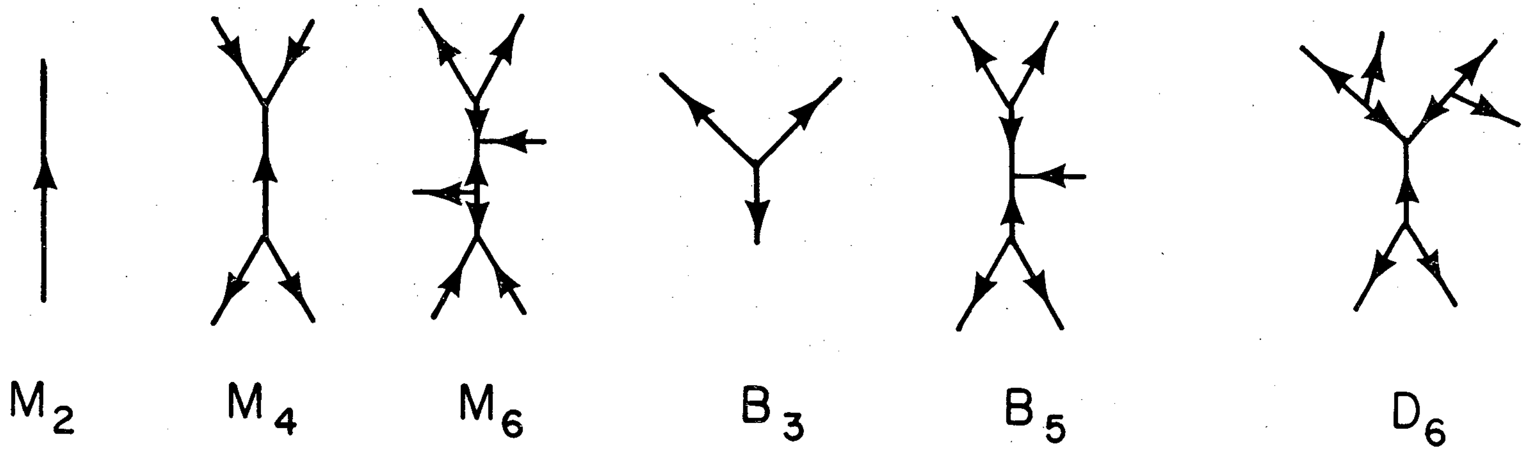
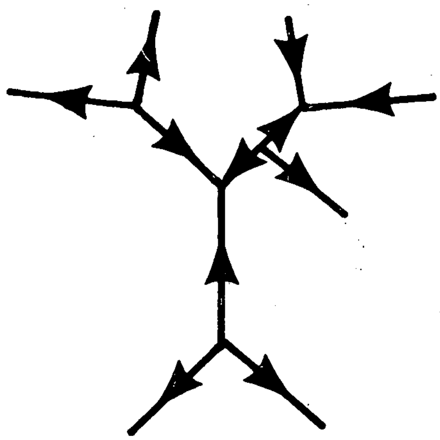
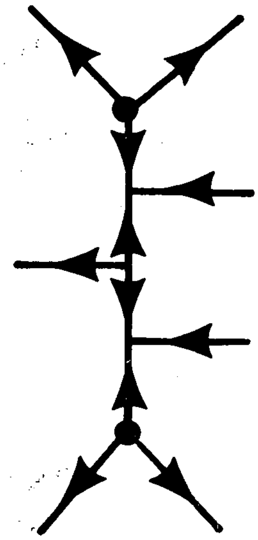


Fig. 18

XBL7712-11095



B_7



B_7

Fig. 19

XBL 7712-11096

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