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Making Inferences and Classifications Using Categories That Are Not Linearly Separable

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Abstract

Previous research suggests that categories learned through classification focus on exemplar information, while categories learned by making predictive inferences focus on summary (i.e., prototype) information. To test this idea further, we demonstrated that it is more difficult to learn nonlinearly separable categories by making inferences than by classifying. This research also supports previous studies by indicating that different processes are likely to mediate inference and classification

In this paper, we examine the type of categorical information people assess in the process of obtaining inductive knowledge. Specifically, we investigate the extent to which abstract summary information about a category and specific information about individual exemplars of a category are used to make feature inferences.

Categories license inference in at least two ways. First, categories provide a summary representation of their members (e.g., a prototype). Given an unknown feature of a bird, for example, people may predict the value of that feature by referring to the bird prototype (Rips, 1975; Tversky & Kahneman, 1974; Yamauchi & Markman, 2000, in press). Another source of category-based induction comes from individual exemplars of a category. Many studies have shown that people classify items by retrieving information about specific exemplars from memory (Kruschke, 1992; Medin & Schaffer, 1978; Nosofsky, 1986). A similar process may be used to make feature inferences. In predicting an unknown feature of an item, people may predict characteristics of the new item based on exemplars stored in memory.

Studies investigating classification have shown that exemplar information plays a crucial role in making classification judgments. Research on inductive inference, however, reveals that category-level abstract feature information (e.g., prototypes) is crucial for inference. For example, Anderson and Fincham (1996) demonstrated that people are capable of predicting the value for one feature given the value of another, based on the overall correlation between the features in the study phase of the experiment, rather than on the basis of seeing those specific values during the study phase. Yamauchi and Markman (2000) further showed that varying the appearance of exemplars during learning disrupts classification, but not inference.

These findings suggest that, while classification and inference may be formally equivalent, they make use of Arthur B. Markman University of Texas, Austin Austin, TX 78712 markman@psy.utexas.edu

different kinds of information in practice (Yamauchi & Markman, 1998). In this paper, we extend this hypothesis and examine the idea that category-level summary information provides a basis for inference (e.g., prototypes), while exemplar information plays a major role in classification. In the following sections, we describe the inference and classification tasks that were employed in our experiments. Then, we examine the role of exemplar and prototype information in two experiments.

Classification and Inference

In our experiments, classification is defined as a practice in which an item is placed into one of two groups based on its attributes. Inference is defined as a practice in which an attribute of an item is predicted given the category label of the item as well as information about its other attributes. For example, classification as we define it is akin to the prediction of a category to which a person belongs (e.g., Democrat) having observed his attributes (e.g., supports affirmative action and favors reduced defense spending). Inference is akin to predicting an attribute of a person (e.g., supports affirmative action) given a category to which he belongs, and other known attributes (e.g., is a Democrat and favors reducing defense spending). We further define the term *category label* as a symbol that represents category membership by denoting a particular group of exemplars, and the term *category feature* as a symbol that denotes a characteristic of an exemplar. Classification requires the prediction of the category label based on the features of the item; inference requires the prediction of a category feature based on the information about other features and the category label.

In our experiments, subjects learn two categories (Table 1a) through a classification task or an inference task. On a classification trial, subjects are presented with a stimulus depicting the values of the form, size, color and position of the geometric figure, and they predict the category label of that stimulus (see Figure 1a). On an inference trial, subjects are presented with the values of the size, shape and position of the geometric figure along with the category label to which the stimulus belongs (e.g., Set A), and they predict the value of the missing feature (e.g., the color) (Figure 1b). On different trials, subjects predict different features. In this manner, classification and inference are formally equivalent if a category label is regarded as simply another feature (Anderson, 1990; but see Yamauchi & Markman, 1998, in press, for further discussion).



Is this figure in Set A or Set B?

Figure 1

If this figure is in Set A, then the item is either Green or Red. Is this item Green or Red?

Table 1aThe category structure If periment 1

Leamin	g	Cate	goryA	1		CategoryB			
	F	S	С	Р		F	S	С	Р
A 1	1	1	1	1	B 1	0	0	0	0
A 2	1	0	1	0	B 2	1	0	1	1
A 3	0	1	0	1	B 3	0	1	0	0
Tranef									
A 4	0	1	1	1	B 4	1	0	0	0
A 5	1	1	0	1	B 5	0	0	1	0
A 6	1	1	1	0	B 6	0	0	0	1
					B 7	0	0	1	1
					B 8	1	1	0	0

TabelbLinearly Separable Categories

A 1	1	1	1	0	B 1	0	0	0	1
A 2	1	1	0	1	B 2	0	0	1	0
A 3	1	0	1	1	B 3	0	1	0	0
A 4	0	1	1	1	B 4	1	0	0	0
A 0	1	1	1	1	B 0	0	0	0	0
FrinS size c darPrositon									

In our previous studies (Yamauchi & Markman, 1998, 2000), we used linearly separable categories (see Table 1b) and found that these categories are easier to learn given an inference learning task than given a classification learning task. We reasoned that this result was obtained because inference relies on summary information about category members. The linearly separable categories have prototypes that summarize the feature values of the individual exemplars, although the prototype differs from all of the exemplars by a feature (e.g., A0&B0 in Table 1b). In this structure, additive combinations of feature values divide the two categories nicely; therefore, extracting prototypes from the two categories facilitates learning these categories.

Categories that are not linearly separable have a very different structure, and hence we expect a different pattern of performance on inference and classification tasks. A sample set of nonlinearly separable categories is shown in Table 1a. For these stimuli, subjects may find prototypes in the two categories (Category A=(1, 1, 1, 1) and Category B=(0, 0, 0, 0)). Nonetheless, this information is not useful for integrating category members in each category as no additive combination of feature values can predict category

coherence (Medin & Schaffer, 1978; Wattenmaker, Dewey, Murphy, & Medin, 1986). For example, the stimulus B2 differs minimally from the prototype in Category A but is included in Category B. In order to learn these categories, subjects need to remember the specific exemplars (see Medin & Schaffer, 1978). Because there are only 6 exemplars in the two categories, it is not difficult for subjects to store these exemplars in memory. It is difficult to learn to make feature inferences, however, because there is no abstract summary information that provides a good description of the categories. Thus, for nonlinearly separable categories, we expect a reversal in the ease of inference and classification relative to linearly separable categories, with the categories being difficult to learn and process through inference than through classification. We test this idea in Experiment 1.

Experiment 1

We used geometric figures as stimuli. All the stimuli varied along four binary feature dimensions: size (large, small), form (circle, triangle), position (left, right) and color (red, green). This structure is shown in Table la. These stimuli and the categories are equivalent to those employed by Medin and Schaffer (1978).

In Experiment 1, the subjects learn these two categories in one of two conditions: (1) Classification or (2) Inference.¹ In the Classification Learning condition, the subjects respond to classification questions. In the Inference Learning condition, the subjects respond to inference questions. Initially, no information about the categories is given to subjects in our studies, so that they have to learn the two categories incrementally by trial and error, based on the feedback that they receive after their response. The learning phase continues until subjects reach a criterion of 90% accuracy in three consecutive blocks (18 trials) or until they complete 30 blocks (180 trials).

Following the learning phase, we test the nature of this category representation using transfer trials, which consist of classifications and inferences of old stimuli that appeared during learning and new stimuli that did not appear during learning. In the transfer phase, all the subjects receive the same trials. Transfer stimuli were designed to explore the distinction between inference and classification. For example, the transfer stimuli, A4-A6 and B4-B6, deviate equally from the prototype of each category. Thus, subjects in Inference Learning should be able to classify these stimuli equally well after learning. These stimuli differ in the extent to which they share features with individual exemplars. The stimuli B4-B6 are highly similar to one exemplar in Category A and one exemplar in Category B. In contrast, the stimuli

¹ In our original experiment, we also included a Mixed condition, in which half trials consisted of classification questions and the remaining half were inference questions. Most scores obtained from the Mixed condition fell approximately midway between the Classification condition and the Inference condition. In order to focus on the distinction between inference and classification, we will not report the results from the Mixed condition in this paper.

A4-A6 are highly similar to two exemplars in Category A, but are not similar to any of the exemplars in Category B (Medin & Schaffer, 1978, p. 218). Thus, subjects in Classification Learning (in contrast to those in Inference Learning) should classify the stimuli A4-A6 more accurately than the stimuli B4-B6. A similar prediction holds for the stimuli B7 (0, 0, 1, 1)and B8 (1, 1, 0, 0). These two stimuli are neutral with respect to the two prototypes. Both stimuli have two feature values consistent with Category A and two feature values consistent with Category B. However, they are highly similar to at least one of three exemplars of Category B (B7 is similar to B2, and B8 is similar to B3), but they are not similar to any of the exemplars of Category A. As a consequence, the stimuli B7 and B8 should be accurately classified into Category B as a function of exemplar storage during learning. Finally, because categories that are not linearly separable do not provide an accurate summary of category members, subjects in the two conditions should have difficulty making transfer inferences to new stimuli.

Participants and Materials. 49 subjects participated in this study. The data from 1 subject were lost due to an error in recording. In total, the data from 48 subjects (24 in each condition) were analyzed. Each category consisted of three exemplars that were shown during learning and transfer trials. In addition, there were eight new stimuli that were given only in the transfer phase. Two versions of the feature assignment were introduced in this experiment. In one version, the value of 0 was triangle and the value of 1 was circle. For color, the value of 0 was green and the value of 1 was red. For size, the value of 0 was small and the value of 1 was large. For position, the value of 0 was right and the value of 1 was left. In the other version, the values of form and size were reversed. Each stimulus was bounded by a 20.3 x 17.4 cm rectangular frame drawn with a solid black line on the computer screen.

Procedure. The experiment involved three phases — a learning phase, a filler phase and a transfer phase. In the learning phase, subjects were randomly assigned to one of two conditions — Classification and Inference. In the two conditions, subjects continued in the learning phase until they performed three consecutive blocks with a combined accuracy of 90% or until they completed 30 blocks (180 trials). A classification block consisted of presentations of six exemplars. One inference block consisted of one inference (along one of the four dimensions) for each of the six stimuli. In the two conditions, every exemplar appeared once in the feedback of each block. The order of stimulus presentation was determined randomly.

In Classification Learning, subjects saw one of the six stimuli and indicated the category to which it belonged by clicking a button with the mouse (Figure 1a). In Inference Learning, subjects inferred a value for one of the four feature dimensions while its category label and the remaining three feature values were depicted in the stimulus frame (Figure 1b). Different dimensions were predicted on different trials. Subjects responded by clicking one of two labeled buttons with the mouse. For each stimulus, the location of the correct choice was randomly determined. Following each response, feedback and the correct stimulus were presented on the screen for three seconds. The stimuli presented during feedback were identical in both the classification and inference tasks.²

After the learning trials, there was a brief filler task, and then all subjects carried out the same transfer tasks. In the transfer phase, subjects were first given classification transfer followed by inference transfer. The transfer stimuli consisted of 6 old stimuli and 8 new stimuli (Table 1a). All of which were shown both in the classification transfer task and in the inference transfer task. The order of stimulus presentation for each task was determined randomly. All the feature inferences were given in Inference learning. No feedback was given during transfer.

Results and Discussion

Overall, the basic results of Experiment 1 are consistent with our hypothesis (Table 2). With nonlinearly separable categories, inference was much more difficult than classification. This finding contrasts with previous research with linearly separable categories, where inference was easier than classification (Yamauchi & Markman, 1998).

In all, 17 subjects reached the learning criterion in the Inference Learning condition, and 22 subjects reached the criterion in the Classification Learning condition. Considering only those who reached the learning criterion, subjects in the Inference Learning condition (\underline{m} =15.8) required significantly more blocks during the learning phase than did subjects in the Classification Learning condition (\underline{m} =10.5), $\underline{t}(37)$ =3.32, \underline{p} <0.01, (Table 2).

For the classification transfer of old stimuli, subjects given Classification Learning (<u>m</u>=0.92) were significantly more accurate than subjects given Inference Learning (<u>m</u>=0.69); <u>t</u>(37)=5.28, p<0.01. As predicted classification, but not inference, involves comparisons to exemplars. Subjects given Classification Learning classified the stimuli A4-A6 (<u>m</u>=0.76) more accurately than the stimuli B4-B6 (<u>m</u>=0.45), although the two sets of stimuli deviate equally from the prototype of each category; <u>t</u>(42)=3.73, <u>p</u><0.01. In contrast, there was no statistical difference in classification accuracy for the stimuli A4-A6 (<u>m</u>=0.63) and the stimuli B4-B6 (<u>m</u>=0.55) in subjects given Inference Learning; <u>t</u>(32)=0.77, <u>p</u>>0.1. Also as predicted, for the neutral stimuli B7 and B8, subjects in Classification Learning were more likely to classify these stimuli into Category B (<u>m</u>=0.61) than were subjects in

² The inference for the size of the stimuli B1 and B3 has two right answers. Given the inference question (0, ?, 0, 0), the response of the feature value 1 corresponds to the stimulus B3 and the response of the feature value 0 corresponds to the stimulus B1. We gave subjects a correct feedback irrespective of their responses for this question. This treatment should make inference learning faster, and thus functions against our hypothesis that inference learning requires more trials than classification learning for this category structure.

Inference Learning (\underline{m} =0.50), but this difference was not statistically significant; <u>t</u>(40)=1.04, <u>p</u>>0.10.

Classification Transfer								
	Old	New		Neutral				
		Average	<u>A4-A6</u>	<u>B4-B6</u>	<u>B7&B8</u>			
IL	0.69	0.59	0.63	0.55	0.44			
CL	0.92	0.61	0.76	0.45	0.61			
Inference Transfer								
	Old	New			Neutral			
		Average	$\Delta A_{-} \Delta 6$	B / B 6	D78.D8			
		Triverage	A4-A0	D4-D0	D/ADO			
IL	0.79	0.46	0.40	0.51	0.36			

Table2The main results of Exeriment1

IL: Inference Learning, CL: Classification Learning

For the neutral stimuli B7&B8, we measured the proportion that subjects classified the two stimuli into Category B.

For the inference transfer, subjects in the two conditions were about equally accurate in making feature inferences for old stimuli; Inference Learning, m=0.79, and Classification Learning, m=0.75. Their performance declined sharply given the inference transfer of new stimuli; Inference Learning, m=0.46, Classification Learning, m=0.50. The performance exhibited by subjects in Classification Learning was no better than a chance level; t(21)=0.11, p>0.1 (one-tail). The performance exhibited by subjects in Inference Learning was actually significantly below chance; t(16)=-2.36, p<0.05 (one-This poor performance contrasts with what we tail). observed in classification transfer, where performance on new items was significantly above chance in both learning conditions. These results are consistent with the view that categories that are not linearly separable provide little support for predictive inference.

The results of Experiment 1 support our view that it is difficult to make inferences for nonlinearly separable categories. Furthermore, the results indicate that inference and classification, two of the main functions of categories, differ significantly in the category information they utilize. In Experiment 2, we investigate this hypothesis further by examining a factor that distinguishes inference and classification.

Experiment 2

We have proposed that inference focuses on summary information about the category. In contrast, there is evidence that people who are trying to classify a set of items tend to focus on diagnostic information that reliably distinguishes between categories (Nosofsky, Palmeri, & Mckinley, 1994). For example, in sorting tasks people tend to divide the stimuli into groups on the basis of a single dimension, even when there is a clear family resemblance structure among the exemplars (Ahn & Medin, 1992; Medin, Wattenmaker, & Hampson, 1987). The hypothesis that classification tends to focus on diagnostic features and inference tends to focus on summary information received indirect support in our previous studies (Yamauchi & Markman, 1998, 2000, in press). In Experiment 2, we will test this idea more directly and scrutinize the distinction between inference and classification.

Table 4 shows the structure of the two categories used in Experiment 2. The categories consist of 3 exemplars each. The stimulus configuration AO(1, 1, 1, 1) summarizes Category A, and the stimulus configuration B0(1, 1, 0, 0)summarizes Category B because these feature values are dominant in each feature dimension of the two categories. In this category structure, the first two dimensions (form and size in Table 4) of the two prototypes are the same, so that they are not useful for distinguishing between the two categories. In contrast, the last two dimensions (color and position in Table 4) are more informative for distinguishing between the categories. Thus, if classification promotes attention to the features that differentiate the two categories, subjects in Classification Learning should attend more to feature information about color and position than to information about form and size. In contrast, because inference is assumed to focus on relations among features within a category, subjects given inference learning should be equally sensitive to the four feature dimensions.

This category structure is also useful for distinguishing the extent to which subjects assess a summary of the category as opposed to individual exemplars. In particular, subjects in Inference Learning should have difficulty acquiring these two categories because the stimulus A2 is the prototype of Category B, but is actually a member of Category A. Subjects in Inference Learning should also have trouble inferring features that do not correspond to the prototype stimuli of the two categories (which we call category-discordant features). For example, subjects in Inference Learning should exhibit less accurate performance for feature values that do not correspond to the prototype (the value 0 of Category A, and the value 0 of form and size in Category B and the value 1 of color and position of Category B). These factors, however, should not influence subjects in Classification Learning, because this task should focus people selectively on diagnostic features and individual exemplars.

Participants and Materials. Subjects were 48 members of the Columbia University community. The materials used for this experiment were the same kind of four-dimensional stimuli used for Experiment 1, but they were organized into a different category structure (Table 4). Each exemplar of a given category had two feature values in common and one feature value different from the rest of the members of that category. The prototype of Set A was (1, 1, 1, 1), which was also a member of the category (exemplar A1 in Table 4). The prototype of Set B was (1, 1, 0, 0), which was actually a member of category A (exemplar A2 in Table 4). The six exemplars from Table 4 were used for Classification Learning and classification transfer. Inference Learning and inference

transfer consisted of inferences of all the feature dimensions of the six exemplars (in total 24 different questions).

Table3The category structure used in Expriment2

	F	S	С	Р	_	F	S	С	Р
A 1	1	1	1	1	B 1	1	1	0	1
A 2	1	1	0	0	B 2	0	1	1	0
A 3	0	0	1	1	В3	1	0	0	0
A 0	1	1	1	1	B 0	1	1	0	0

Category-inaccordance fatures are show ninitalics.

A 0istheprototype@fategory A and B0 istheprototype

Gategory BhimSsieCc.ob;Pposion

Procedure. The basic procedure of this experiment was identical to that described in Experiment 1.

Results and Discussion

As predicted, learning these categories was particularly difficult for subjects given Inference Learning. All subjects (24) in Classification Learning, but only 8 subjects in Inference Learning reached the learning criterion. On average, subjects in Classification Learning spent 10.4 blocks, and subjects in Inference Learning spent 27.4 blocks in learning; t(46)>10.0, p<0.01. Because the number of subjects who reached the criterion differed considerably between Classification Learning and Inference Learning, we analyzed the transfer data from each learning condition separately.

In Classification Learning, subjects exhibited accurate performance for classification transfer (\underline{m} =0.94). Subjects' classification performance was generally high for all six stimuli. For the six transfer stimuli, the accuracy ranged from 88% to 96%. Subjects were also accurate in the classification of the stimulus A1 (\underline{m} =0.88), which is the prototype of category A (and a member of category A) as well as stimulus A2 (\underline{m} =0.92), which is the prototype of category B, but is actually a member of category A. During the transfer phase, subjects classified the stimulus A1 and the stimulus A2 equally well; \underline{Z} =-0.02, \underline{p} >0.1 (Table 4).

Subjects in Classification Learning were also accurate in inference transfer (\underline{m} =0.83). Consistent with our prediction, Classification Learning clearly led subjects to focus on the features that were useful for distinguishing between categories. Subjects in Classification Learning performed significantly better for the feature inferences of color and position (\underline{m} =0.86) than for form and size (\underline{m} =0.80); $\underline{t}(23)$ =1.83, p<0.05 (one-tailed).

In Inference Learning, we analyzed the data from all subjects, because only 8/24 subjects reached the learning criterion. First, the average performance for classification transfer by subjects in Inference Learning was $\underline{m}=0.70$. Unlike in Classification Learning, in Inference Learning there is a wide disparity between accuracy in classifying the stimulus A1 and the accuracy in classifying the stimulus A2. Subjects in Inference Learning accurately classified the prototype stimulus of Category A — A1(1, 1, 1, 1), $\underline{m}=0.83$ —

but not the prototype stimulus of Category B — A2(1, 1, 0, 0), <u>m</u>=0.46; <u>Z</u>=2.41, <u>p</u><0.01. This result suggests that subjects were focusing on information that summarized the categories rather than on information about specific exemplars.

Table4The main results ff xperiment2

Classification Transfer									
	A1	A2 All exemplars							
IL	0.83	0.46	0.70						
CL	0.88	0.92	0.94						
	Inference Transfer								
	F	S	С	Р					
IL	0.72	0.68	0.73	0.68					
CL	0.81	0.80	0.88	0.85					

F:form, S:size, C:color, P:position

Consistent with our prediction, subjects in Inference Learning did not differ in the feature inferences of form and size, as compared to the feature inferences of color and position (form & size, <u>m</u>=0.70, color & position, <u>m</u>=0.70). This result, combined with the results from Classification Learning, clearly indicates that inference and classification make use of different types of feature information.

Subjects in Inference Learning were not different in the inference transfer of Category-accordant features (m=0.71) (i.e., prediction of feature values that are the same as the value for the prototype of that category) and Categorydiscordant features (m=0.69) (i.e., prediction of features that have a different value than the prototype of the category); t(22)=0.62, p>0.10. A similar tendency appeared for subjects in Classification Learning; Category-accordance features (m=0.84) and Category-discordant features (m=0.82); t(22)=0.69, p>0.1. We applied the same analysis to the learning performance of subjects in Inference Learning. The results revealed that subjects' learning performance was significantly more accurate for Category-accordant features (m=0.63) than for Category-discordant features (m=0.56); t(22)=3.46, p<0.01. This analysis indicates that people find it difficult to make correct inferences for features that do not correspond to the category prototype during learning.

Taken together, The results of these studies support the hypothesis that nonlinearly separable categories are difficult to learn through inference. Our results also suggest that inference and classification promote a focus on different types of category information: The Classification Learning task guides subjects to focus on features that distinguish between categories; the Inference Learning task directs subjects to attend to the features that integrate the members within a category.

General Discussion

These studies demonstrate that it is easier to learn categories through classification than through inference when the categories are not linearly separable. This finding contrasts with earlier research with linearly separable categories, which found that inference learning was easier than classification learning. This finding reflects that summary category information is more important for inference than for classification. Our experiments, combined with the results from previous studies (Yamauchi & Markman, 1998, 2000, in press), suggest that the structure of a category is one of the major constraints on inductive inference. Unlike classification, inference requires feature information that relates the members of a category. Although some researchers argue that inference and classification are the same thing (e.g., Anderson, 1990), our results reveal that people exercise different strategies for the two tasks.

Why do people look for abstract summary information for inference, while they seek information about specific exemplars or diagnostic features for classification, even when they are given the same categories? This difference may follow from an intricate link between category representation and category functions. Classification is related to object identification and recognition (Nosofsky, 1986). Thus, it requires finding relationships between an individual exemplar and its category label. Once an object is identified, its overall feature information may become irrelevant except some features that are useful to distinguish between categories. In contrast, inference involves the prediction of missing feature values, and thus requires finding relationships between the category label and the features of the category (Gelman, 1986). In this case, the category identity of the object is known, and so information about the category features is needed to predict the value of missing features. Thus, differences in what is demanded in each task lead people to look for distinctions between groups given a classification task, and to seek commonalities within a group given an inference task.

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