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Michael A. Green

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As the energy of particle accelerators increases, the need for small high-field magnets increases. These magnets are most likely to be found in an accelerator's extraction system. The advent of slow extraction at CERN and Brookhaven has shown the need for high-current-density magnet design.

I define high-current-density water-cooled conductors as conductors in which the heat transfer from the copper to the water across the thermal boundary layer can no longer be ignored. I have arbitrarily set a lower limit of 5000 A/cm^2, in the copper, for high-current-density conductors. In general, the film heat transfer becomes more dominant as the length of the cooling circuit decreases.

High-current-density conductors are most likely to be found in extraction septum magnets of various types. The elimination of the magnetic field outside the magnet requires the total current flow to be restricted to the magnet gap itself. For the extraction of high-energy particles, the thickness of the conductor in the horizontal direction may be severely limited. The current density in this conductor is determined by its horizontal dimension. The vertical dimension of septum conductor has no effect on current density, because the total current in the conductor bundle is directly proportional to the vertical aperture of the magnet.

The use of high-current-density conductors should be avoided whenever possible for economic reasons. The optimum current density for minimum cost in a septum magnet occurs at current densities of less than 500 A/cm^2. The high-current-density magnet should be a minimum-field magnet with a small vertical aperture. In an extraction system with more than one magnet, the current density of each magnet conductor should be considered in cost-optimizing the extraction system.

Heat Transfer from a High-Current-Density Conductor

The current density of a conductor bundle may be defined as follows:

$$i/A = \frac{Ni}{\epsilon\, A_B} ,$$

$$Ni = \frac{10}{4\pi} B_0 h ,$$

where Ni = ampere-turn requirement,
   i/A = current density,
    ϵ = packing factor (including water passages),

B_0 = magnetic field (gauss),
  h = vertical aperture,
A_B = overall area of the bundle.

The heat generated per unit volume may be calculated once the current density is found (see Fig. 1),

$$P_c = (i/A)^2 \, Res$$

where P_c = heat generation per unit volume (W/cm^2),
   Res = resistivity.

The resistivity is a function of the temperature of the copper. The heat generation per unit volume will increase with temperature.

The generalized equation for conductive heat transfer, which includes transient effects and heat source, may be represented by

$$\nabla^2 T + \frac{P_c}{K} = \frac{c\rho}{K} \frac{\partial T}{\partial t} ;$$

when the system is in steady state the equation becomes Poisson's equation,

$$\nabla^2 T + \frac{P_c}{K} = 0 ,$$

where T = temperature,
   K = thermal conductivity,
   c = specific heat,
   ρ = density of the conductor,
   t = time.

The differential equation can be solved by several well-known techniques. In general, the heat conduction to the water can be neglected, particularly with standard conductors. There are cases in which the conductive heat transfer to the water tube surface should be considered. These cases usually can be solved from simplified one or two-dimensional equations.[1]

Current densities of 5000 A/cm^2 or more require that the convective heat transfer from the copper to the water be investigated. Fortunately, a large amount of experimental work has been done with convective heat transfer from copper tube walls to water. The empirical techniques for calculation of a heat transfer are well documented. The calculation can be made with an accuracy of 20%.

The basic convective heat transfer equation is

$$Q_c = h_c A (T_w - T_b) ,$$

where Q_c = heat transferred,
   h_c = heat-transfer coefficient,
   A = heat-transfer area,

T_w = temperature of the heated surface,
T_b = bulk temperature of the water.

The value of the heat-transfer coefficient is determined by a number of factors such as the velocity of the water, the hydraulic diameter of the tube, and the viscosity, density, and thermal conductivity of the water. These terms are generally lumped into two dimensionless numbers, the Prandtl and Reynolds numbers.

The Reynolds number of the water at its bulk temperature and velocity determines whether the flow is laminar or turbulent. In most cases, the flow will be turbulent. The heat transfer coefficient can be found as follows:

for laminar flow, $Re_b < 2300$,

$$h_c = 0.332 \frac{K_f}{D} Re_f^{0.5} Pr_f^{0.33} \;;$$

for turbulent flow, $Re_b > 5000$,

$$h_c = 0.023 \frac{K_f}{D} Re_f^{0.8} Pr_f^{0.33} \;;$$

where

$$Re_b = \frac{VD\rho_b}{\mu_b} \,,$$

$$Re_f = \frac{VD\rho_f}{\mu_f} \,,$$

$$Pr_f = \frac{C_f\mu_f}{K_f} \,,$$

$$D = \frac{4A_H}{P} \,,$$

and $\qquad T_f = 0.5 (T_b + T_w)$.

The following nomenclature is used:

K = thermal conductivity of the fluid,
C = specific heat of the fluid,
μ = viscosity,
V = bulk water velocity,
D = hydraulic diameter,
A_H = cross-section area of the tube,
P = wetted perimeter,
ρ = density of the fluid,
Re = Reynolds number,
Pr = Prandtl number.

The meaning of the subscripts:

f = water conditions at a temperature T_f
b = bulk water conditions at T_b
w = tube wall conditions.

There are a number of other methods for calculating the convective heat transfer. I have chosen these equations because of their connection with basic boundary-layer theory.[6] For other heat-transfer calculation methods, see references 2 and 5. The accuracy of all these methods decreases as the temperature difference between the tube wall and the bulk fluid increases.

For high-current-density conductors, the convective heat transfer plays an important role in determining the current capacity of a given conductor. The two factors which influence convective heat transfer most for a given fluid flow are the heat-transfer area and the allowable temperature difference between the wall and fluid (see Fig. 2).

The most important condition that affects the cooling of a magnet conductor is the length of the water circuit. For high-current-density conductors the length of the cooling circuit becomes even more important. In general, the conductor wall temperature should not be allowed to exceed 100°C. The heat-removal rate for a conductor is

$$Q_c = 4.19 \ VA_H C\rho (T_{out} - T_{in}) \,,$$

since the heat removal by the water equals the heat transferred by the water and the heat generated,

$$\frac{i^2 \ Re \ \ell}{A_c} = h_c A (T_w - T_b)$$

$$= 4.19 \ VA_H C\rho \ (T_{out} - T_{in})$$

and $\qquad T_w < 100°C$,

and the maximum

$T_b = T_{out}$
A_c = conductor cross-sectional area,
ℓ = length of conductor,
T_{out} = exit water temperature,
T_{in} = inlet water temperature.

As the current density increases, the water circuit length must be shortened, the hole size increased, or the water velocity increased. The temperature rise in the water per gram of conductor for various water-flow rates and current densities is shown in Fig. 3.

Special Problems Associated with High-Current-Density Magnets

There are several problems which become more important as the current density increases. These include short burnout time in the event of cooling failure, the pumping and handling of cooling water, and increased cavitation, erosion, corrosion, and electrolysis.

The burnout time is a function of current density and maximum allowable copper temperature. If there is no cooling water in the cooling channel, time to failure is given (see Fig. 4) by

$$t = \frac{4.19 \ \rho c \ (Tm - Twm)}{P_c} \,,$$

$$P_c = (i/A)^2 \ Res \,,$$

where
t = burnout time,
ρ = density of the conductor,
c = specific heat of the conductor,
Tm = maximum allowable temperature,
Twm = maximum copper temperature during normal operation,
P_c = heat generation rate (W/cm^3).

The burnout time calculated in the preceding paragraph is affected by water in the conductor and the stored energy of the magnet. The burnout times shown in Fig. 4 represent the maximum time allowed for the magnet to be shut off in case of cooling failure. If the conductor contains some cooling water, this time may be extended because of the heat capacity of the water. However, when the water in the conductor starts to boil, most of it will be forced out by the steam pressure generated from the boiling water.

The stored energy of the magnet may influence the failure time. If the amount of stored energy released in a section of uncooled conductor during magnet shutdown approaches the maximum allowable heat energy storage in the copper, the failure time will approach zero.

High-current-density magnets will require a fast-response temperature sensor on the conductor near the end of each cooling circuit. The temperature-sensing system should be fast enough to shut the magnet off before the conductor reaches burnout temperature.

The increase in current density in a water-cooled conductor results in changes in the handling of the cooling water. The heat generated in the conductor is directly proportional to the current density. Since the cooling water has only a limited heat capacity, the amount of cooling water needed must increase with current density. There are three ways that the flow rate can be increased. They include increasing the velocity of the cooling water, increasing the conductor hole area, and increasing the number of cooling circuits.

Increasing the velocity improves the heat transfer, but only at the expense of pumping power and increased erosion. Increasing the hole size increases the heat transfer but also results in increased pumping costs. Increasing the number of water circuits does not increase the cost of pumping, but results in increased manifolding costs.

The optimum solution involves increasing the velocity and hole size slightly as well as increasing the number of water circuits. The exact design would depend on the magnet under consideration. In general, increasing the number of water circuits results in the best solution. However, as an increased amount of convective heat transfer per unit volume of conductor is required, an increase in hole size and water velocity becomes increasingly advantageous.

The purity of the magnet cooling water for high-current-density magnets is far more important than it is for standard magnets. This is because the convective heat transfer is far more important than with standard magnets. Also, most high-current-density magnets have smaller cooling passages than the standard magnets. Cooling-water stoppages and partial stoppages will affect high-current-density magnet performance greatly.

Cavitation and erosion usually are not problems in conventional magnet designs. Cavitation can be avoided by not allowing the lowest pressure in the system to fall below the vapor pressure of the water. Erosion can usually be avoided by keeping the magnet water clean.

The use of dissimilar metals should be avoided as in conventional magnet cooling systems. The deplating problem is worse than with conventional-type magnets, because currents in the cooling water are higher and the conductor and its water channel generally are smaller in size. The copper removed from the conductor by deplating generally becomes a sludge which must be removed by the cooling water filtration system. Elevated temperatures of the copper result in the increased formation of deposits on the walls of the cooling tube. In general, most of the cooling system problems can be solved if the purity of the cooling water is maintained.

Optimization of Current-Carrying Capacity in Standard Rectangular Conductor

The current-carrying capacity of a standard conductor is a function of convective heat transfer and cooling circuit length. The maximum current-carrying capacity for a square conductor when the cooling-circuit length is zero (i.e., the heat capacity of the water is neglected) is

$$Q_{gen} = \frac{i^2\ Res\ \ell}{4.19\ A_c} = h_c\ A\Delta T_c = Q_{out} ,$$

where

$$A_c = b^2 - \frac{\pi D^2}{4} ,$$

$$A = \pi D \ell ,$$

$$h_c = \frac{0.023\ Re_f^{0.8}\ Pr_f^{0.33}\ K}{D}\ \text{for turbulent flow.}$$

If the film temperature drop is a maximum, namely $Tw \leq 100°C$, and T_b the inlet water temperature, then the Prandtl number will be a constant if we assume that

$$T_b = 30°C, \quad \text{then} \quad Pr_f^{0.33} = 1.4, \quad \text{hence}$$

$$Q_{out} = 0.032\ \pi Re_f^{0.8}\ K\ell\Delta T_c ,$$

$$Q_{gen} = Q_{out} ,$$

$$\frac{i_c^2\ Res\ \ell}{4.19\left(b^2 - \frac{\pi D^2}{4}\right)} = 0.032\ \pi\ Re_f^{0.8}\ K\ell AT_c ,$$

$$Re_f^{0.8} = \frac{V^{0.8} D^{0.8}}{v_f^{0.8}},$$

therefore

$$i_c^2 = X \left[b^2 D^{0.8} - \frac{\pi}{4} D^{2.8} \right] \Delta T_c$$

where the constant

$$X = \frac{0.135 \pi V^{0.8} K}{Re s \, v^{0.8}}$$

and $\quad \Delta T_c = \text{constant} \approx 70°C$.

The maximum current flow will occur when

$$\frac{di}{dD} = 0 ,$$

therefore maximum current-carrying capacity for a square conductor will occur when

$$D = 0.602 \, b .$$

When we no longer neglect the heat capacity of the water, the current-carrying capacity of the conductor goes down. The film-temperature drop must be decreased because of the rise in temperature of cooling water as it passes down the conductor. The sum of the film-temperature drop when the water circuit length is not taken into consideration is

$$\Delta T_c = \Delta T_B + \Delta T_P .$$

The required temperature rise in the cooling water to allow the film temperature drop to remain at ΔT_c is

$$\Delta T_R = \frac{4 i_c^2 \, Res \, \ell}{4.19 \, A_c V \pi D^2} ,$$

where ΔT_R is a linear function of ℓ. Since the sum of ΔT_c and ΔT_R is greater than ΔT_c, the current-carrying capacity of the conductor must be decreased. If we define

$$Y = \frac{\Delta T_R}{\Delta T_c} ,$$

we have the current-carrying capacity for a conductor with a nonzero water circuit length,

$$i = \frac{i_c}{\left[Y + 1 \right]^{\frac{1}{2}}} ;$$

then

$$\Delta T_P = \frac{\Delta T_R}{Y+1} = \frac{Y \Delta T_c}{Y+1} = T_{out} - T_{in} ,$$

$$\Delta T_B = \frac{\Delta T_c}{Y+1} = T_w - T_{out} ,$$

where $\quad$ b = side dimension of the square conductor,
$\quad A_c$ = copper area,

A = heat transfer area,
$v = \mu/\rho$ = kinematic viscosity,
D = tube diameter,
ΔT_c = the maximum allowable film temperature drop ($\approx 70°C$),
ΔT_R = temperature rise in the water required to remove the heat from a conductor carrying a current i_c,
ΔT_B = film temperature drop when the conductor carries i,
ΔT_P = water temperature rise when the current carries i,
i_c = maximum current that can be carried (very short sample),
i = current capacity when the circuit length is considered,
T_{in} = inlet water temperature,
T_{out} = outlet water temperature,
T_w = wall temperature ($\leq 100°C$).

The optimum water hole size is larger when the cooling circuit length is considered. The longer the water circuit, the larger the water hole. The solution presented here does not take into consideration the increase in Prandtl number and the decrease in Reynolds number as ΔT_B decreases. Fortunately, the effect of the change in ΔT_B is small.

Figures 5 and 6 show the current optimization for a 5×5-mm square conductor. The current-carrying capacity for four of these conductors is greater than for one 10×10-mm square conductor. A square conductor will perform better than a rectangular conductor of the same cross section. Heat transfer from the hot end of the conductor to the cold end of the conductor may be neglected except for very short conductors.[3]

Achievement of Very High Current Densities

In order to achieve current densities in the range of 20 000 A/cm^2 and above, nonstandard techniques must be used. There are three ways of achieving current densities in this range. They are: using high water velocities, using very short water circuit lengths, and increasing the ratio of heat transfer area to conductor volume (see Fig. 2).

In the design of very-high-current-density conductors, a combination of the three ways would be used. There are two basic conductor types which can be used in the very-high-current density range. These are multichannel conductors and cross-cooled conductors.

The multiple-channel conductor allows the heat transfer area to be increased, which increases the current-carrying capacity of the conductor. If the water velocity in the cooling channels is greater than 600 cm/sec and the cooling circuit length is 50 cm, the conductor shown in Fig. 7 should carry currents in excess of 12 000 A with current densities exceeding 20 000 A/cm^2. I feel that a conductor of this type can be built a technique that is similar to the one used for the CERN

multichannel conductor.

If a septum magnet could be designed to be a single-turn magnet, the cross-cooled conductor would be attractive. The conductor has a large surface-to-volume ratio as well as very short water circuit lengths. High water velocities should be possible if the manifolding is properly designed. The conductor shown in Fig. 8 should be able to carry currents in excess of 20 000 amperes. The greatest design problems, as I see it, associated with a cross-cooled magnet is the differential expansion between the heated conductor and the cooling manifolds.

Conclusions

Many of the problems associated with high-current-density septa are only briefly mentioned. Fabrication techniques developed in recent years make the construction of high-current-density magnets possible. The development of inorganic insulations is important for the construction of thin, slow-extraction septa.

In general, the following design criteria should be used in the design and fabrication of high current density magnets:

1. Avoid the use of high-current-density conductor whenever it is economically possible.

2. Minimize the vertical aperture of a high-current-density septum.

3. Do not allow the copper temperature at the tube wall to exceed the water boiling temperature, particularly for small cooling passages.

4. Optimize the conductor for its current-carrying capacity, not for maximum current density.

5. Minimize the number of turns in the magnet. Insulation takes up valuable space. Avoid multi-turn magnets in the very-high-current-density range (20 000 A/cm^2 or more).

6. Avoid the use of nonradiation-resistant insulations.

7. Use high-speed temperature sensors, which are essential at the hot end of each cooling circuit to prevent conductor burnout due to cooling failures.

8. Avoid many cooling system problems by maintaining the purity of the cooling water.

9. Investigate conductive heat transfer from the copper to the water when the current densities are very high or when the heat path passes through materials of low thermal conductivity.
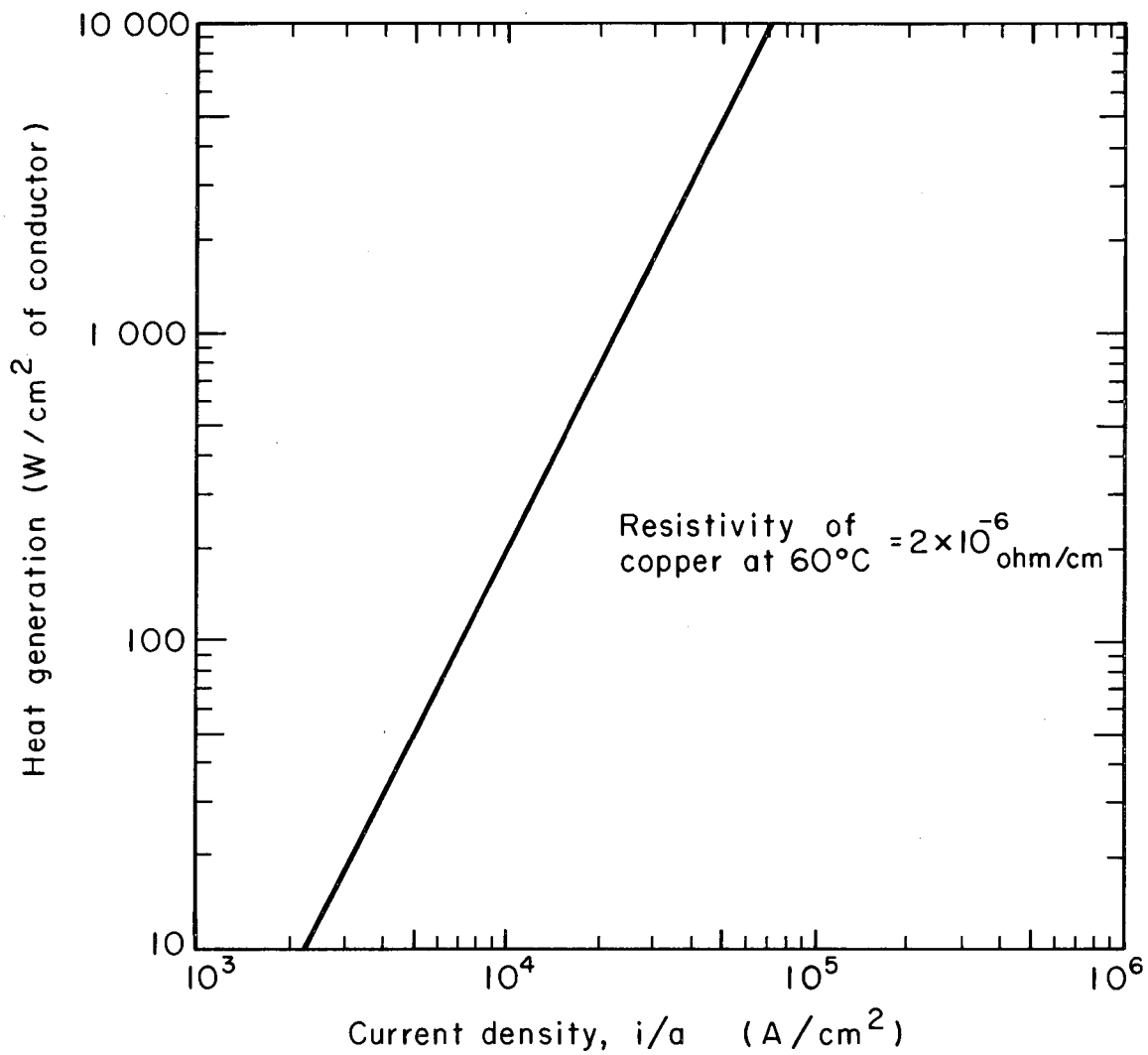
High-current-density magnets should be designed according to the basic laws of physics; experience is also very valuable when these magnets are built.

References

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[3] Green, Michael A., "Heat Transfer Properties of a 5-Millimeter Septum," Lawrence Radiation Laboratory Report UCID-2627, Sept. 1964.

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[5] Kreith, Frank, Principles of Heat Transfer (International Textbook Co., Scranton, Pennsylvania, 1960).

[6] Schlichting, Hermann, Boundary Layer Theory (McGraw-Hill Book Co., New York, 1960).
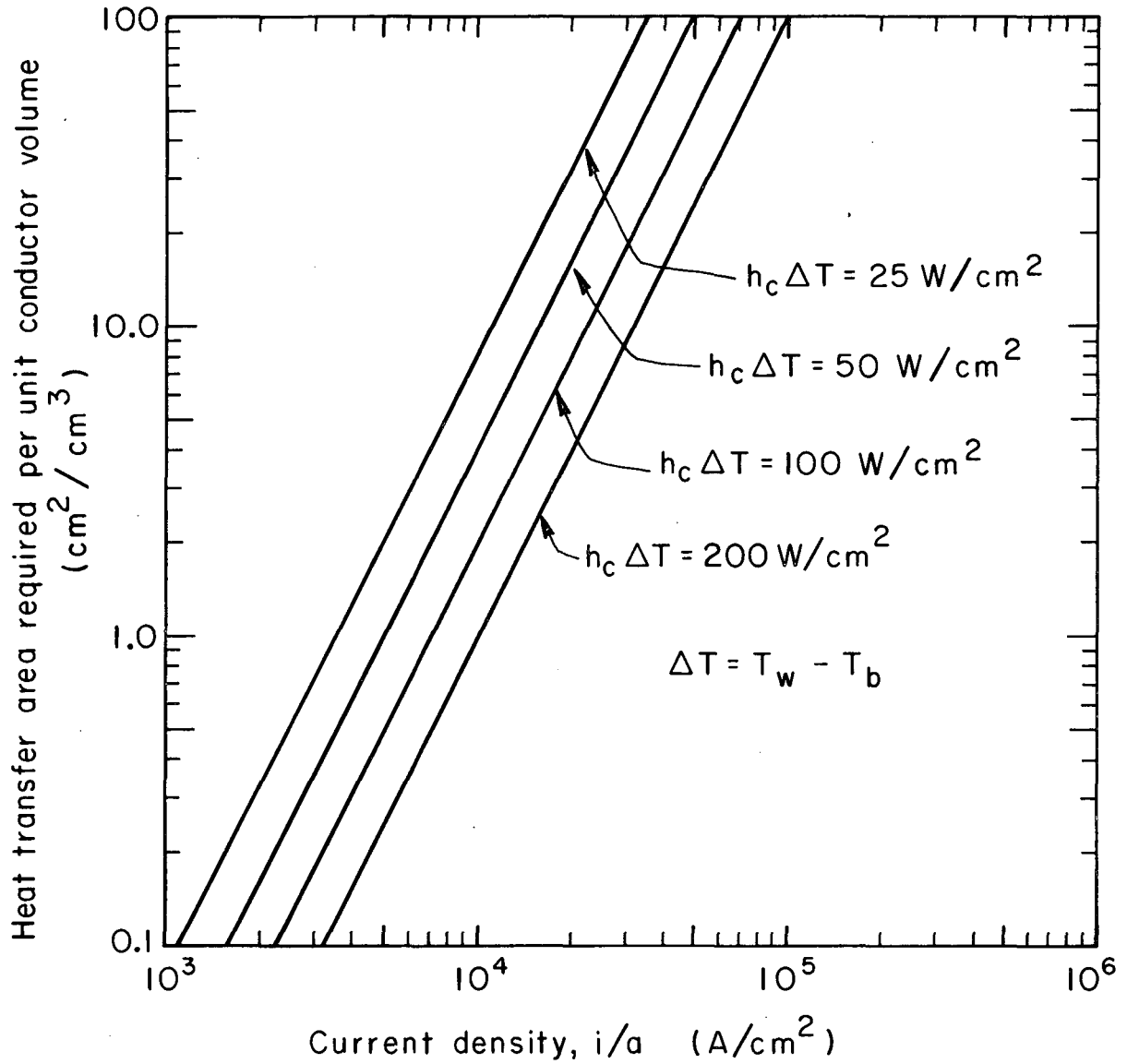
FIGURE LEGENDS

Fig. 1. Heat generated per cm^3 of the conductor as a function of current density.

Fig. 2. Connective heat-transfer area required per cm^3 of conductor as a function of current density for various $h_c \Delta T$.

Fig. 3. The water temperature rise in the cooling circuit per cm^3 of conductor as a function of current density for various water flow rates.

Fig. 4. The minimum time for conductor burnout as a function of current density for various failure temperatures.

Fig. 5. Maximum current flow verses hole diameter in a 5×5-mm water-cooled conductor for various water velocities. Maximum wall temperature = 100°C; maximum water temperature = 30°C; water circuit length not considered.

Fig. 6. Maximum current flow as a function of hole diameter in a 5×5-mm water-cooled conductor for various water circuit lengths. Water velocity = 450 cm/sec; maximum wall temperature = 100°C; maximum water temperature = 30°C.

Fig. 7. The multichannel conductor with the cooling water flowing lengthwise down the conductor.

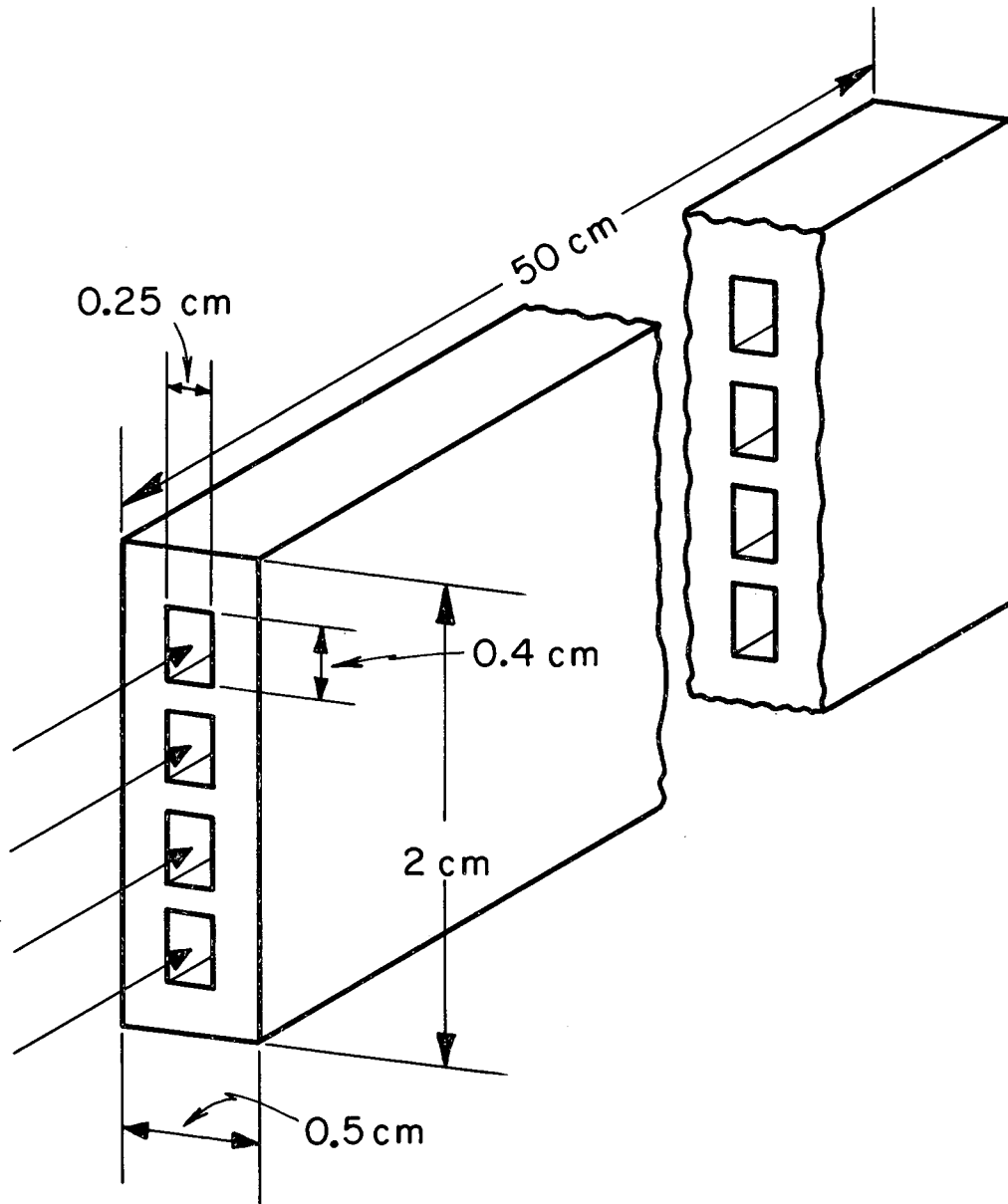Fig. 8. The multiple-channel cross-cooled conductor.
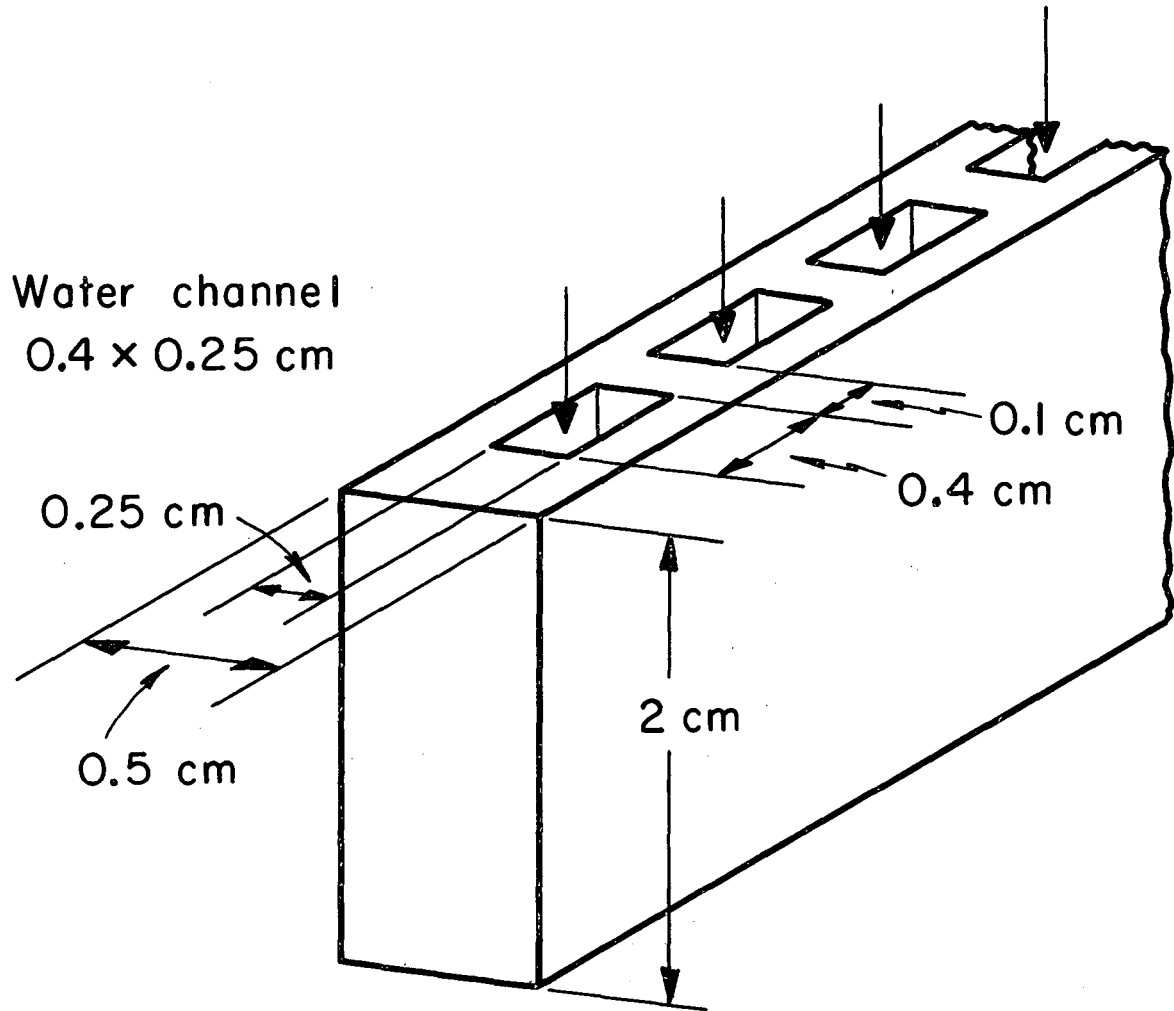
Fig. 1

MUB-7312

Fig. 2

Fig. 3

MUB-7314

Fig. 4

MUB-7315

Fig. 5

Fig. 6

MUB-7317

0.25 cm

50 cm

0.4 cm

2 cm

0.5 cm

MUB-7318

Fig. 7

Water channel
0.4 × 0.25 cm

0.25 cm

0.5 cm

0.1 cm

0.4 cm

2 cm

MUB-7319

Fig. 8