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Author

Del Prado Martin, Fermin Moscoso

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Scale-Invariance of Human Latencies

Fermin Moscoso del Prado Martín
(fermin.moscoso-del-prado@univ-provence.fr)

CNRS & Aix-Marseille Université I,
3 pl. Victor Hugo
13331 Marseilles, France

Abstract

Cognitive activity modulates the distribution of human response latencies in different ways, and this is a central tool in experimental psychology. An issue that has received little attention is the ‘baseline’ state of these latencies, against which different cognitive modulations could be compared. A recent theory predicts that the right tails of human response latency distributions should follow a power-law with a constant scaling parameter of two, independently of the task or participants under study. We demonstrate this through the analysis of six large-size databases of human latencies. This finding introduces a fundamental constant that describes the ‘resting state’ of the cognitive system, and opens the possibility of directly measuring the amount of information processing performed in a task or condition.

Keywords: Reaction Time; Power law; Bayesian inference; Complex system

Introduction

Response latencies [RLs] – the times taken to initiate or complete an action or task – are one of the most widely used measures to investigate the mechanisms subserving human cognitive processes. The right tails of RL distributions have received little attention in experimental psychology. This is because such very long RLs have traditionally been considered irrelevant for psychological tasks as they are likely to reflect ‘contingent’ neural events unrelated to the experiment. Therefore, current standard practice recommends discarding very long RLs as outliers (e.g., Luce, 1986; Ratcliff, 1993). In this study, we show that these right tails provide crucial information about the ‘baseline’ level of RLs against which cognitive processes can be compared. This finding introduces a fundamental constant of the cognitive system that links behavioral measures to their neurophysiological bases.

A pervading assumption in the literature is that RLs follow a distribution whose right tail decreases exponentially (Luce, 1986). RLs are ultimately by-products of the workings of the brain, and further, of the firing patterns of heavily *inter-connected* neurons. From this perspective, exponential tails would be a rather surprising outcome for RL distributions. They would imply that the RLs were generated by a Poisson process, that is, they would be *independent* events, despite the interconnections between the neurons that generated them.

More in line with the probably correlated origins of behavioral events, two recent theories have predicted that the right tails of RLs distributions should follow a power-law (Holden, Van Orden, & Turvey, 2009; Moscoso del Prado Martín, 2009). This is to say that for all times t greater than a certain t_{\min} , their probability density function should be that of a

Pareto distribution:

$$p(t) = \frac{\alpha - 1}{t_{\min}} \left(\frac{t}{t_{\min}} \right)^{-\alpha}, \quad \alpha > 1, t \geq t_{\min}, \quad (1)$$

where α is referred to as the *scaling parameter*, and it corresponds to the slope of the straight line that is formed by the density function when plotted on log-log scale. A more precise theoretical proposal (Moscoso del Prado Martín, 2009) is that RLs arise as the result of the ratio of two correlated normal variables: The excitability of the response effector, and the strength of the signal that excited it. Therefore the distribution of RLs should follow a normal ratio distribution (NRD; Fieller, 1932). This has the further implication that the power-law right tail should have a value of the scaling parameter of exactly two (Jan, Moseley, Ray, & Stauffer, 1999; Sornette, 2001).¹ Such a precise tail behavior would hold irrespective of the properties of the task. It would constitute a complete description of the RL distribution in this region, in the strong sense of having zero degrees of freedom. The scaling parameter value would therefore represent a fundamental constant of the cognitive system. Furthermore, it would group RLs with other well-known natural systems with identical properties, such as Ising models of ferromagnetic materials close to their critical temperature (Jan et al., 1999) or the intervening times between major earthquakes (Mega et al., 2004).

Materials

RLs in the far tails are by definition very rare. Obtaining estimates of the distributions in this region requires very large numbers of – ideally untruncated – responses. Fortunately, massive databases of human responses collected on the internet are currently becoming available, offering enough long RLs as to enable inferences on their tail distribution. We analyzed six such large-scale datasets of human responses across experimental tasks and modalities, and at different time ranges. The datasets included ocular fixation and blink durations during reading (*The Dundee Corpus*) (Kennedy, 2003), spontaneous ocular fixation durations while participants were inspecting photographs (*DOVES* database) (van der Linde, Rajashekar, Bovik, & Cormack, 2008), and a sample of different web-collected experiments extracted from the *PsychExperiments* web site (?, ?; McGraw, Tew, & Williams, 2000). This last set included two-choice decision reaction times to both auditory (tones) and visual (col-

¹See Moscoso del Prado Martín (2009) for details on this theory.

ors) stimuli, reaction times of participants performing a mental rotation task, and the times that participants took to exit a virtual maze.

The data from *PsychExperiments* correspond in their greater part to latencies of psychology students performing the experiments during in-class sessions, but some residual component of latencies originating from persons ‘trying out’ the system is also present. In order to minimize in as much as possible the distortion of the distributions introduced by users testing the system, only RLs from participants whose number of responses in the database corresponds exactly the number of stimuli in the experiment, were kept. In addition, all RLs with values equal or smaller than zero, or for which some field of the downloaded file was contained an inconsistent value (*e.g.*, an incorrect task descriptor, *etc.*) were removed prior to the analyses.

The experimental RLs were measured to a precision of 1 msec. (Tew & McGraw, 2002), except for the ocular RLs from *DOVES*, which had a precision of 5 msec. (*i.e.*, the eye-tracking equipment used a sampling rate of 200 Hz.). A similar discretization was performed on the artificially generated datasets, simulating an experimental resolution of 1 msec. relative to a median of 9,278 msec. estimated by maximum likelihood.

Right Tails

The solid dots in the left panel of Figure 1 plot (in log-log scale) the histograms of the latencies in each of the datasets, aggregated across participants. Notice that all six distributions show a very similar pattern: The probabilities of the faster latencies rise to a peak, from which they decrease, gradually approaching a straight line. This straight line is the characteristic signature of a power-law distribution. Importantly, as predicted, the straight line components seem remarkably parallel across the six datasets, with a slope of approximately minus two (black dot-dashed lines). The right panel in Figure 1 further stresses this apparent invariance. It plots the corresponding distributions when the times have been divided by their medians. In this way, the principal scale-dependent component of the distributions – the location of their modes – is removed. After this simple normalization, we can distinguish three phases in the distributions. The early times rise to a peak, following very different patterns for each dataset. From the mode up to somewhere between five and forty times the median, there is a transition phase where the distributions gradually approach a power-law. The precise speed of convergence to the power-law varies depending on the properties of the participant and the task (Moscoso del Prado Martín, 2009; Holden et al., 2009). From this point onwards – as shown by the inset panel in the figure – the distribution of latencies is approximately the same, regardless of the particular experimental task. To confirm that this pattern holds when one considers only single-participant data, the figures also plot the histogram of the responses of an individual participant in

the Dundee dataset (open red circles).² Finally, in order to illustrate the theoretical prediction across the whole range of latencies, the figure also includes the theoretical density that would be predicted by an instance of the NRD with arbitrary parameters (black solid lines), and how the histogram from a sample of such would look like (grey open circles).

The histograms in Figure 1 seem consistent with the hypothesis that the right tails of latency distributions follow a power-law with a scaling parameter of two, and most certainly discard the traditional assumption of a light, exponential-type tail. However, other heavy-tailed distributions could also produce histograms with this appearance, and this has given rise to disagreements with respect to the precise nature of heavy tails in some datasets (Barabási, 2005; Barabási, Goh, & Vazquez, 2005; Stouffer, Malmgren, & Amaral, 2005; Oliveira & Barabási, 2005). Therefore, we need to compare our hypothesis with other possible distributions with similar right tails. Both log-normal and stretched-exponential (*i.e.*, Weibull) tailed distributions also give rise to very heavy tails (Limpert, Stahel, & Abbt, 2001; Mitzenmacher, 2004; Clauset, Shalizi, & Newman, 2007; Stouffer et al., 2005), and both have been proposed as plausible theoretical or empirical models for latency distributions (Logan, 1992; Luce, 1986). In addition, as we predict that the power-law should have a scaling parameter of exactly two, any other power-law with an arbitrary scaling parameter – not necessarily, but also including two – could be an alternative description (Holden et al., 2009).

Model Comparison Method

For an objective rationale to choose among the four possible explanations for the heavy tails, we used pairwise Bayes factors. These require the computation of the log-likelihoods for each candidate hypothesis. Our proposed distribution is fully specified, in the sense that it has no free parameters, thus the computation of the log-likelihood for a fixed value of t_{\min} is straightforward. However, the other three candidate hypotheses have either one (for the general power-law case) or two free parameters (the log-normal and Weibull tail cases). Therefore, estimation of their log-likelihoods requires integration over all possible values of the nuisance parameters, assuming some prior distribution of the later. This was achieved by numerical integration, assuming truncated uninformative (*i.e.*, Jeffreys’) priors for the free parameters. The truncation was designed to be as benevolent as possible to the three alternative hypotheses. With this goal we restricted the integration space to plausible values of the parameters: For the general power-law hypothesis, we assumed that the scaling parameter should take a value greater than one and smaller than six, as power-laws with scaling parameters greater than four are hardly ever encountered in natural phenomena (Clauset et al., 2007; Mitzenmacher, 2004;

²We chose participant for whom the database contained the largest number of events (participant “sd”), but the same pattern held across participants.

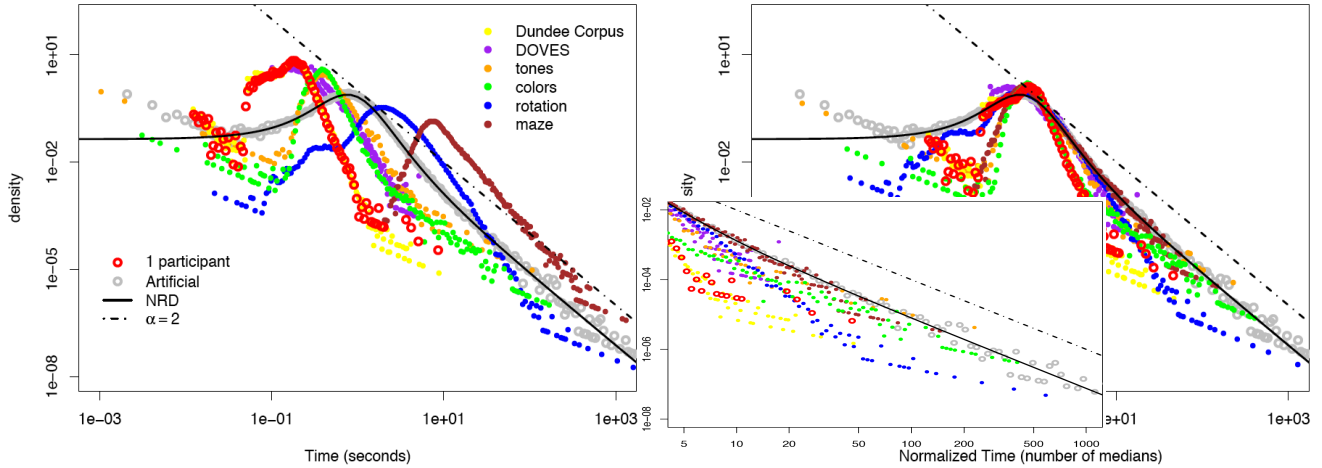


Figure 1: Histograms of the latencies in second in the six datasets plotted in log-log scale. The solid dots represent the six datasets aggregated across participants. The open red circles plot the histogram from a single participant from the *Dundee* dataset. The open grey circles plot a sample from an arbitrary instance of the NRD, whose density corresponds to the solid black lines. The dot-dashed black lines illustrate a slope of -2 . Both panels represent the same data either on the true time scale (left panel), or in the time scale normalized by the corresponding median (right panel). The inset on the right panel magnifies the power-law right tail of the distributions.

Newman, 2005). For the Weibull hypothesis, we assumed that the value of its shape parameter should never be above one, as this would imply an exponential tail or lighter, which cannot correspond to the pattern observed in the histograms. Finally, both the Weibull location and the log-normal location and scale parameters were restricted to values that would make the datapoints correspond to an actual right tail (*i.e.*, their mode should fall to the left of t_{\min}) and have a peak within the range of RLs. Note that these constraints actually increase the likelihood of the alternative hypotheses beyond the under-specification than is found in the literature.³ Therefore, our estimates of the evidence in support of our theory were conservative, slightly favoring the other possible hypotheses.

Therefore, our estimates were conservative with respect to our target hypothesis. The threshold value t_{\min} was chosen by visual inspection of the histograms in Figure 1. In addition, these choices were validated by assessing whether the chosen value would be close to the one that would minimize the value of the Kolmogorov-Smirnoff statistic between the sample of RLs and a general power-law (Clauset et al., 2007), which in all cases suggested similar values. Note however, that this objective method can be problematic when one considers the additional upper truncation that is present in our data.

An additional problem that affects the comparison of the different theories is the right-truncation that is present in the

³For instance, power-laws with scaling parameter values as high as ten have been proposed in the RL literature (Holden et al., 2009). However, loosening our constrain on the values of the parameters led to significant increases of the evidence in favor of our theory.

data. This introduces a bias in favor of lighter tails: It favors either of the non-power-law distributions, or the general power-law with a high value of the scaling parameter. In both of the eye movement datasets, fixations are determined by computer algorithms that rely on aspects such as their duration, therefore implicitly introducing both left and right truncations. In the internet collected datasets, there is officially no right truncation in the data. However, most responses will be subject to an implicit upper bound dictated by the duration of the class. The residual of responses longer than one class session are very likely to originate in users testing the system, system failures in the client, *etc.*. To attenuate this problem, we assumed that all datasets had been right-truncated at the maximum RL observed. For the web collected datasets this is still an under-estimation of the real truncation point, leaving some advantage for the non-power-law distributions, and over-estimating the scaling parameter for power-laws.

Estimation of the evidence

For any two candidate models M_1 and M_2 , if $\mathbf{t} = (t_1, \dots, t_N)$ are the latencies in the dataset that are longer than t_{\min} , the evidence in favor of M_1 over M_2 is:

$$\begin{aligned} E(M_1, M_2 | \mathbf{t}) &= 10 \log_{10} \frac{p(M_1 | \mathbf{t})}{p(M_2 | \mathbf{t})} \\ &= 10 \log_{10} \frac{p(M_1)}{p(M_2)} + 10 \log_{10} \frac{p(\mathbf{t} | M_1)}{p(\mathbf{t} | M_2)}, \end{aligned} \quad (2)$$

where the second step is achieved by application of Bayes' Theorem, and $p(\mathbf{t} | M_i)$ is the likelihood of the datapoints for

model M_i . If *a-priori* we consider both models equally probable, the evidence reduces to the difference in log-likelihoods:

$$\begin{aligned} E(M_1, M_2 | \mathbf{t}) &= 10 [\log_{10} p(\mathbf{t} | M_1) - \log_{10} p(\mathbf{t} | M_2)] \\ &= 10 [\ell(\mathbf{t} | M_1) - \ell(\mathbf{t} | M_2)], \end{aligned} \quad (3)$$

We used decimal logarithms and the scaling factor of 10 so that the resulting evidence would be measured in decibels.

For the power-law with a pre-determined scaling parameter $\alpha = 2$, the computation of ℓ is straightforward:

$$\begin{aligned} \ell(\mathbf{t} | \alpha = 2) &= \log_{10} \prod_{i=1}^N p(t_i | t_{\min}, \alpha = 2) \\ &= \sum_{i=1}^N \log_{10} p(t_i | t_{\min}, \alpha = 2), \end{aligned} \quad (4)$$

where $p(t_i | t_{\min}, \alpha = 2)$ is the density function of a discrete power-law (normalized for an upper truncation at $\max\{\mathbf{t}\}$).

For an hypothesis M with free parameters $\theta = (\theta_1, \dots, \theta_k)$, the marginal log-likelihood of the hypothesis is given by:

$$\begin{aligned} \ell(\mathbf{t} | M) &= \log_{10} \int_{V(\theta)} p(\mathbf{t}, \theta | M) d\theta \\ &= \log_{10} \int_{V(\theta)} p(\mathbf{t} | \theta, M) p(\theta | M) d\theta, \end{aligned} \quad (5)$$

where $V(\theta)$ is the volume defined by the parameters, $p(\theta | M)$ is the prior on θ and:

$$p(\mathbf{t} | \theta, M) = \prod_{i=1}^N p(t_i | \theta, M), \quad (6)$$

where $p(t_i | \theta, M)$ is given by the density function of M .

For each hypothesis, we estimated the integral in (5) numerically using a Montecarlo technique. We sampled 10^5 points from the prior distribution $p(\theta | M)$, computed the likelihood of \mathbf{t} using the sampled values of θ , and took the mean result as the marginal likelihood. For each of the three distributions, we used discrete versions of their densities (Clauset et al., 2007) truncated between t_{\min} and $\max\{\mathbf{t}\}$.

In order to minimize numerical errors, all the computations above were performed in logarithmic scale.

Results and Discussion

Table 1 summarizes the posterior evidence supporting the hypothesis that the right tails follow a power law distribution with a scaling parameter of exactly two over each of the other three candidate hypotheses. Notice, that for four out of the six aggregated datasets and for the individual participant analysis, the evidence supports our hypothesis over the three competing candidates (i.e., positive values in the table). In the remaining two cases (negative values, highlighted in bold), the best candidate distribution was still a power-law, albeit one with an arbitrary value of the scaling parameter. In both of these cases, it seems like the optimal value of the scaling parameter estimated under a general power-law hypothesis has a value above two (last row in the table).

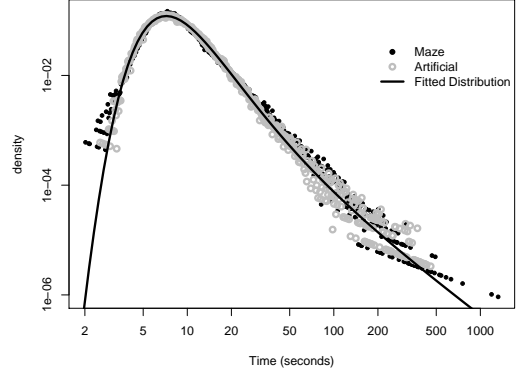


Figure 2: Histograms (in log-log scale) of the RLs in the Maze dataset (black solid dots) and of the simulated artificial dataset (grey open circles). The solid black line plots the fit to the data of the distribution from which the simulated points were sampled.

The model comparison method was particularly stringent on our target hypothesis. The implicit truncation present in the data could lead to the over-estimation of the scaling parameter that was found for two of the datasets. To investigate this possibility, we fitted an NRD to the RLs in the Maze dataset, as this was the one for which our theory appeared to show a worse performance. From the fitted distribution, we obtained a sample of the same size as the Maze dataset, sampling only points below 50 times the median (this is equivalent to an upper truncation at around eight minutes). The sample was discretized to simulate a measurement resolution of one msec. Fig. 2 compares the original data with the sample and the fitted distribution. Although these simulated data are undoubtedly originating from a power-law distribution with true scaling parameter of exactly two, applying the hypothesis testing procedure revealed a very similar pattern to what was observed in the maze data (see the last column of Table 1). All three alternative hypotheses seemed more probable than our target theory due to the advantage that truncation gives them. Given the quality of the fit in Figure 2, it seems likely that the same distortion took place in the Maze data. In sum, all datasets were consistent with our theoretical prediction. The theory was the better of the four candidate theories for the majority of the datasets studied, all of which showed power-law behavior with a scaling parameter value close to two.

Power-laws are often interpreted as evidence for Self-Organizing Criticality [SOC; cf., (Bak & Paczuski, 1995)], but several other mechanisms could also give rise to power-laws without the explicit need for self-organization (Newman, 2005; Sornette, 2001). In the domain of human RLs, some authors have argued for the presence of SOC using evidence for $1/f$ ‘pink’ noise in the frequency spectra of RLs (Gilden,

Table 1: Posterior evidence (in decibels) favoring the power-law with true scaling parameter value $\alpha = 2$ over the alternative hypotheses for each of the datasets analyzed, as well as for the artificially generated data simulating the Maze RLs. Positive values indicate support for the power-law with $\alpha = 2$, while negative values indicate evidence in favor of each of the alternative hypotheses. The first two rows indicate the value of t_{\min} in relation to the corresponding median, and the number of points above this threshold found in each dataset. The last row indicates the posterior estimate of α if ones assumed the general power-law hypothesis to be true.

	Dundee (whole set)	DOVES	Tones	Colors	Rotation	Maze	Dundee (participant "sd")	Artificial
$t_{\min}/\text{median}(t)$	10	10	5	5	40	10	5	10
Number of $t \geq t_{\min}$	33	57	133	544	31	458	27	366
Log-Normal (general)	5.5 dB	2 dB	-6.5 dB	13.5 dB	13 dB	-257.5 dB	9 dB	-15 dB
Weibull (general)	2 dB	2 dB	31 dB	43 dB	26 dB	-241.5 dB	11.5 dB	-7 dB
Power-Law (general)	2 dB	3 dB	-22.5 dB	9.5 dB	5.5 dB	-261 dB	5.5 dB	-21 dB
'optimal' value of α	2.56	1.82	2.50	1.92	2.27	2.99	2.17	2.48

Thornton, & Mallon, 1995; Van Orden, Holden, & Turvey, 2003; Thornton & Gilden, 2005), but this evidence is currently subject to discussion (*e.g.*, Wagenmakers, Farrell, & Ratcliff, 2005; Farrell, Wagenmakers, & Ratcliff, 2006).

The fixed scaling parameter of two is common to a prototypical model of a system that is known to be in a critical state: The Ising model of a magnet (Jan et al., 1999). As ours, this model is also described by an NRD, originating from the fractional change in magnetization ($\Delta m/m$). At a small environment around the critical temperature, the Ising model exhibits power-law behavior, but very small deviations from the critical temperature restrict the power-law to the very far tails (Jan et al., 1999). This is very much what we observe in human RLs. The power-law behavior settles at the extreme right tails, between five and forty times the median RL in a particular task. Rather than evidence for SOC, our results in fact argue for a system that has been pushed slightly away from its critical point. This suggests that, at rest, the system is likely to be in a state which could be characterized as SOC, but the presentation of stimuli disturbs this criticality. This picture is consistent with recent work on electro-physiology. Human (and animal) neural oscillations are generally at a critical state, characterized by both power-laws and $1/f$ noise patterns, but transient synchronization of neural assemblies during cognitive processing can temporarily disturb this criticality (Buzsáki & Draguhn, 2004).

Figure 3 illustrates this last point further. The plot breaks down the histograms of the responses of the *Dundee* dataset into three groups. The first one, plotted in red, corresponds to the durations of actual ‘cognitively motivated’ fixations at the text during reading. Notice that although the NRD fit to this data is very good, at the far-right the distribution has not yet settled at the power-law. At the other extreme, the purple dots plot the histogram of the duration of blinks during

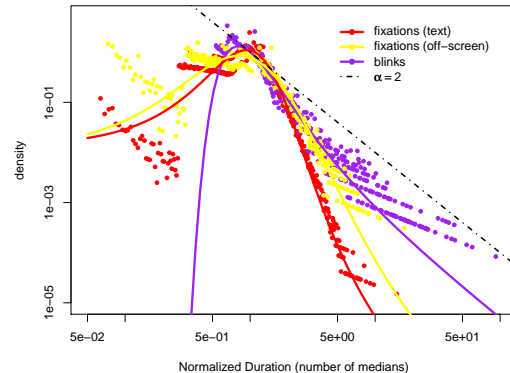


Figure 3: Histograms (in log-log scale) of the text fixation durations (red dots), out of screen fixation durations (yellow dots), and blink durations (purple dots) from the Dundee corpus. The solid lines illustrate the prediction that would be made by an NRD distribution fit to each subset. The dash-dotted black line illustrates a power law with scaling parameter 2.

reading. Discounting some minor cognitive influences, these blinks are in a large proportion truly spontaneous events, such as the ones that the system could produce in a resting state. Accordingly, blinks reach the stable power-law much earlier than text fixations. At an intermediate point, the yellow dots plot the histogram of the duration of fixations that focused at some point out of the screen. Many of these are likely to be motivated by external events, such as a movement in the room, a noise, *etc.*, but they will also have an important spontaneous component. These durations converge faster than the

text ones as they are more likely to be spontaneous events. At the same time, they show a much slower convergence than the blinks due to their stronger cognitive component. In all three cases, an NRD provides a very good account of the data, from the early to the very late times.

Conclusion

It comes as no surprise that human behavior, given its neural origins, should be best described by a complex system. The precise characterization of the critical or resting state of the cognitive system defines the baseline against which cognitive processes can be compared, analogous to the baselines defined for electro-physiological (Buzsáki & Draguhn, 2004) and haemodynamic activity in the brain. Measuring the magnitudes of deviations from this resting state elicited by different conditions can provide a direct measure of the amounts of information processing they involve, considered here as a relaxation in return to the critical state.

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