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Author
Chai, Chen Chung

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Discretization Effects on First-Order OAM Beam Fidelity in Hexagonal Structured-Light Array

Chen Chai[1]
[1]Department of Electrical and Computer Engineering, University of California, Los Angeles, Los Angeles, CA 90024, USA
*cchai3@g.ucla.edu

ABSTRACT

In this paper we analyze how the finite number of channels in the hexagonal structured-light array of Lemons *et al.* affects the fidelity of a first-order orbital-angular-momentum donut beam, using simple interference modeling.

INTRODUCTION

We use structured light for optical trapping, advanced imaging, and communication, as it refers to beams whose amplitude, phase, and polarization are engineered to create specific spatial patterns [1]. One of the most important structured beams are orbital angular momentum (OAM) beams, which were one of the key example demonstrations in Lemons *et al.* The first-order OAM beam has a phase that winds 2π around the beam, which creates the ring-shaped, donut-like intensity profile in the far field, as illustrated in Fig. 1.

The motivation of this review paper is to study how the number of channels affects how clean and ideal the generated donut beams can be. When the approximation is poor, the donut beam can develop extra lobes or a partially filled center, which reduces the mode purity and can lead to poor performance in applications as mentioned above. The paper "Integrated structured light architectures" by Lemons *et al.* demonstrates a multi-beam laser system that can generate such structured beams using a hexagonal array of small beam channels. In their architecture, each channel can be controlled in phase, amplitude, polarization, and timing, and the channels are coherently combined to synthesize complex far-field patterns, including OAM beams and polarization textures. Because the number of channels is limited, some distortion and extra intensity features can appear in the synthesized beam.

In this review, I focus on how this discretization influences the quality of a first-order OAM beam generated by the hexagonal array described by Lemons *et al.* Using a simple scalar interference model, I consider arrays with different numbers of channels and compare the resulting far-field patterns to an ideal first-order OAM beam. By examining how the beam shape and a basic fidelity measure change with channel number, I aim to relate quantitative modeling results to the experimental demonstrations in the original paper and to highlight the trade-off between array complexity and beam quality in integrated structured-light systems.

METHODS

I use a simple scalar interference model that compares an ideal OAM reference field to discretized fields from a finite number of channels to analyze how discretization affects the quality of a first-order orbital angular momentum beam. In these cases I assume paraxial propagation and neglect polarization effects, so the problem reduces to the transverse complex field in a single plane. I use this model to compute the intensity pattern and compare it to the ideal OAM beam.

Ideal first-order OAM reference beam

I modeled the target beam as a first-order Laguerre-Gaussian mode in polar coordinates (r, θ) [eq. 1]. This is the ideal donut beam as described in Fig. 1.

$$E_{ideal}(r, \theta) \;=\; C\, r\, \exp\left(\frac{-r^2}{\omega^2}\right) \exp(i\theta) \qquad (2, \text{Eq.1})$$

Discrete array model of the structured-light architecture

To mimic the structured-light architecture of Lemons *et al.*, I represent the synthesized beam as the coherent sum of N channels located at positions on a ring that approximates the hexagonal array. The field is written as

$$E_{array}(x, y) \;=\; \sum_{k=1}^{N} A_k \exp\left(-\frac{(x - x_k)^2 + (y - y_k)^2}{w_0^2}\right) \exp(i\phi_k) \qquad (3, \text{Eq.2})$$

Fidelity metric to compare ideal vs. array

F represents how close the discretized beam is to the ideal OAM mode. I compute an overlap-based fidelity between the array field $E_{array}(x, y)$ and the ideal field $E_{ideal}(x, y)$. The fidelity is defined as

$$F = \frac{\left|\iint E_{array}(x, y)\, E^*_{ideal}(x, y)\, dx\, dy\right|^2}{\left(\iint |E_{array}(x, y)|^2\, dx\, dy\right) \left(\iint |E_{ideal}(x, y)|^2\, dx\, dy\right)} \qquad (4, \text{Eq.3})$$

Simple donut quality factor D

In addition to the overlap fidelity, I use a simple donut quality factor D to measure how dark the beam center is relative to its brightest point. Let $I(0)$ be the intensity at the exact beam center and I_{max} the maximum intensity in the pattern. Then

$$D = 1 - \frac{I(0)}{I_{max}}. \qquad (5, \text{Eq.4})$$

An ideal donut with a dark core has $I(0) \approx 0$ and therefore $D \approx 1$, while a partially filled center would give a smaller D. In the scalar model used here, both the ideal beam and the discretized beams have $I(0) = 0$ by symmetry, so $D = 1$ for all channel counts. As a result, D does not distinguish between different arrays in this work, and the main quantitative comparison relies on the fidelity F defined above.

RESULTS AND INTERPRETATION

Outcomes

The quantitative results are summarized in Table 1. The ideal Laguerre–Gaussian first-order OAM beam has a quality factor $D = 1$ and fidelity $F = 1$ by definition. Because $I(0) = 0$ for all the discretized beams in this model, the simple quality factor D defined in Eq. (5, Eq.4) is also equal to 1 for the 3-, 7-, 19-, and 37-channel arrays. This shows that D is not a useful discriminator here, so the discussion below focuses on the overlap fidelity F.

When the beam is synthesized with only 3 channels, the fidelity drops to $F = 0.239$. Increasing the channel count to 7, 19, and 37 raises the fidelity to $F = 0.536$, $F = 0.6271$, and $F = 0.6278$, respectively. Most of the improvement therefore happens between 3 and about 20 channels; beyond that, the gain in fidelity is very small.

Table 1. Quality metric D and overlap fidelity F for the ideal OAM beam and discretized beams synthesized with different numbers of channels.

Type of Field	$D = 1 - \frac{I(0)}{I_{max}}$	Fidelity F
Ideal Beam	1	1
3-channel array	1	0.239
7-channel array	1	0.536
19-channel array	1	0.6271
37-channel array	1	0.6278

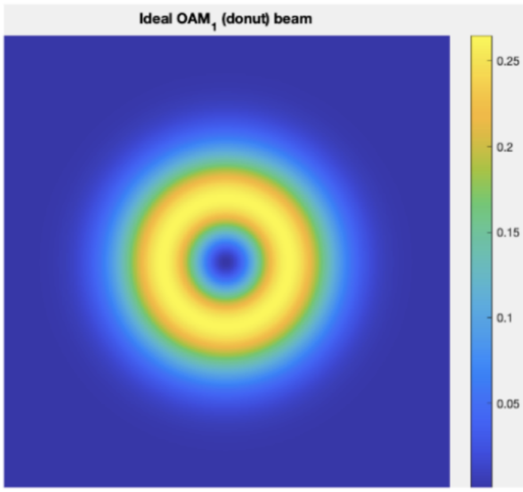

Fig. 2

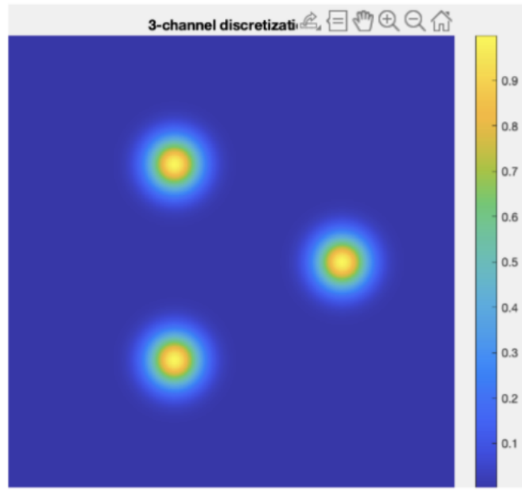

Fig. 3

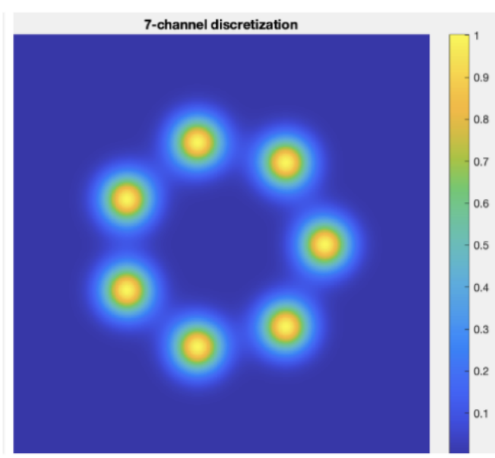

Fig. 4

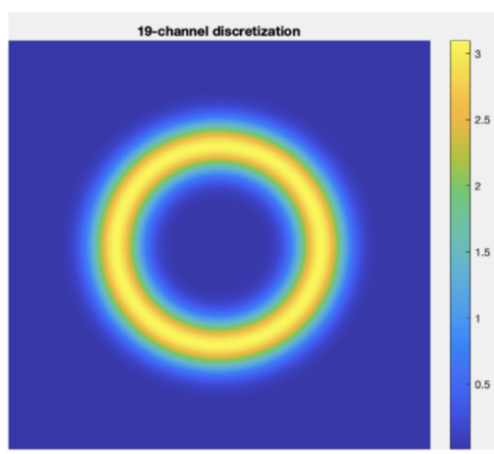

Fig. 5

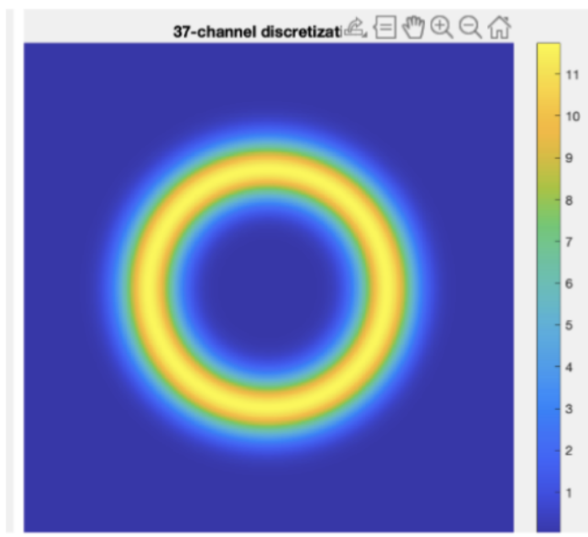

Fig. 6

Figure 2 shows the intensity profile of the ideal first-order OAM beam used as the reference. It has a dark core at the center and a smooth, almost perfectly circular ring. Figures 3–6 show the discretized beams generated by 3, 7, 19, and 37 channels. For the 3-channel case (Figure 3), the beam breaks into three isolated bright spots instead of forming a continuous ring, which explains the very low fidelity reported in Table 1. With 7 channels (Figure 4), the spots begin to overlap and a ring shape appears, but the ring is still strongly modulated and the center is not completely dark, so the fidelity remains far from 1.

In the 19-channel case (Figure 5), the beam looks much closer to the ideal donut: the ring is smoother and the center becomes darker, consistent with the higher fidelity of $F = 0.6271$. Finally, the 37-channel case (Figure 6) is only slightly smoother than the 19-channel case, and the fidelity increases by less than 10^{-3}. The combination of Table 1 and Figures 2–6 therefore shows that increasing the number of channels in the hexagonal array steadily improves the beam quality at first, but the improvement begins to saturate once the array has around twenty channels.

CONCLUSIONS

In this review I calculated how a finite, hexagonal array of beam channels can approximate an ideal first-order OAM donut beam. Using a simple scalar model and overlap fidelity as a metric, the simulations show that increasing the number of channels steadily improves the similarity to the ideal mode, but that the improvement begins to saturate once a higher channel count is reached. These results show that this hexagonal array design can only be improved to an extent. It would be valuable to explore other array geometries or extend this project to higher order OAM modes to fulfill more advanced use cases.

ACKNOWLEDGMENTS

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