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Author
Bernardoni, Fabio

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Mapping the geometry of the E_6 group

Fabio Bernardoni[1*], **Sergio L. Cacciatori**[2†], **Bianca L. Cerchiai**[3‡] **and Antonio Scotti**[4§]

[1]Departament de Física Teòrica, IFIC, Universitat de València - CSIC
Apt. Correus 22085, E-46071 València, Spain.

[2] Dipartimento di Scienze Fisiche e Matematiche,
Università dell'Insubria,
Via Valleggio 11, I-22100 Como.

[3] Lawrence Berkeley National Laboratory
Theory Group, Bldg 50A5104
1 Cyclotron Rd, Berkeley CA 94720 USA

[4] Dipartimento di Matematica dell'Università di Milano,
Via Saldini 50, I-20133 Milano, Italy.

[5] INFN, Sezione di Milano, Via Celoria 16, I-20133 Milano.

Abstract

In this paper we present a construction for the compact form of the exceptional Lie group E_6 by exponentiating the corresponding Lie algebra $\mathfrak{e}_6$, which we realize as the the sum of $\mathfrak{f}_4$, the derivations of the exceptional Jordan algebra J_3 of dimension 3 with octonionic entries, and the right multiplication by the elements of J_3 with vanishing trace. Our parametrization is a generalization of the Euler angles for $SU(2)$ and it is based on the fibration of E_6 via a F_4 subgroup as the fiber. It makes use of a similar construction we have performed in a previous article for F_4. An interesting first application of these results lies in the fact that we are able to determine an explicit expression for the Haar invariant measure on the E_6 group manifold.

*Fabio.Bernardoni@ific.uv.es
†sergio.cacciatori@uninsubria.it
‡BLCerchiai@lbl.gov
§antonio.scotti@gmail.com

1 Introduction.

The Standard Model (SM) provides a very good description of elementary particle physics. However, despite its success there are reasons to go beyond it: for example the recent discovery of neutrino oscillations, the fine tuning of the mixing matrices, the hierarchy problem, the difficulty in including gravity and so on.

A starting point could be the fact that the renormalization flow of the coupling constants suggests the unification of gauge interactions at energies of the order of 10^{15} GeV, which can be improved (fine tuned) by supersymmetry. It is natural to expect the gauge group G of the GUT theory to be a simple group. Obviously, at low energies the GUT model must reproduce the SM physics, so that not only G should contain $SU(3)_c \times SU(2)_L \times U(1)_Y$ (the SM gauge group), but it should also predict the correct spectra after spontaneous symmetry breaking. The increasing accuracy in the analysis of particles spectra imposes even more restrictions in the possible choices for G. E.g., it is already known that the current estimate for the lower bound of the proton lifetime rules out some of the GUT models candidates. Recently, the particular structure of the neutrino mixing matrix seems to suggest that a good candidate for a GUT could be based on the semidirect product between the exceptional group E_6^4 and the discrete group S_4 [3, 6].
In general, while the local properties of the group G, which for a Lie group are all encoded in the corresponding Lie algebra, are enough to perform a perturbative analysis, in order to obtain non perturbative results, such as instantonic calculations or lattice simulations, a knowledge of the entire group structure is required, in particular of the invariant measure on the group in a suitable global parametrization. There are many ways to give such an explicit expression for the Haar measure on a Lie group. However for large dimensions it becomes quite hard to find a realization, which is able at the same time to provide both a reasonably simple form for the measure and an explicit determination for the range of the angles. In this paper we solve this problem for the exceptional Lie group E_6, by constructing a generalization of the Euler parametrization, with a technique we have introduced in [2] and fully developed in [1]. In section 2 we explain how the $\mathfrak{e}_6$ algebra can be represented using a theorem due to Chevalley and Schafer. The construction of the group and the determination of the corresponding Haar measure is made in section 3.

2 The construction of the $\mathfrak{e}_6$ algebra.

Our starting point for the construction of the exceptional algebra $\mathfrak{e}_6$ is a Theorem due to Chevalley and Schafer [5] which we rewrite here for convenience

Theorem 2.1 *The exceptional simple Lie algebra $\mathfrak{f}_4$ of dimension 52 and rank 4 over K is the derivation algebra $\mathfrak{D}$ of the exceptional Jordan algebra $\mathfrak{J}$ of dimension 27 over K. The exceptional simple Lie algebra $\mathfrak{e}_6$ of dimension 78 and rank 6 over K is the Lie algebra*

$$\mathfrak{D} + \{R_Y\} , \qquad TrY = 0 , \tag{2.1}$$

spanned by the derivations of $\mathfrak{J}$ and the right multiplications of elements Y of trace 0.

We will refer to [8] for the notations. Then the exceptional Jordan algebra is the algebra J_3. For our purposes K will be $\mathbb{R}$ or $\mathbb{C}$. The right multiplication is $R_Y(X) = Y \circ X$ and the trace is the sum of diagonal elements. The exceptional Jordan algebra is 27 dimensional, just like the principal fundamental representation of $\mathfrak{e}_6$.

In our previous paper [1] we determined the algebra of derivations $\mathfrak{D}$ and used it to obtain an irreducible representation of the exceptional algebra $\mathfrak{f}_4$. To obtain a representation of $\mathfrak{e}_6$ we only need to determine the matrix representation of R_Y. This is a very simple task and we solved it in $\mathbb{R}^{27}$ with the product inherited from J_3 by means of the linear isomorphism

$$\Phi : J_3 \longrightarrow \mathbb{R}^{27} \ , \quad A \mapsto \Phi(A) \ ,$$
$$\Phi(A) := \begin{pmatrix} a_1 \\ \rho(o_1) \\ \rho(o_2) \\ a_2 \\ \rho(o_3) \\ a_3 \end{pmatrix} \ , \tag{2.2}$$

where A is the Jordan matrix

$$A = \begin{pmatrix} a_1 & o_1 & o_2 \\ o_1^* & a_2 & o_3 \\ o_2^* & o_3^* & a_3 \end{pmatrix} \ , \tag{2.3}$$

and ρ is the linear isomorphism between the octonions $\mathbb{O}$ and $\mathbb{R}^8$ given by[1]

$$\rho : \mathbb{O} \longrightarrow \mathbb{R} \ , \quad o = o^0 + \sum_{i=1}^{7} o^i i_i \mapsto \rho(o) \ ,$$
$$\rho(o) := \begin{pmatrix} o^0 \\ o^1 \\ o^2 \\ o^3 \\ o^4 \\ o^5 \\ o^6 \\ o^7 \end{pmatrix} \ . \tag{2.4}$$

After choosing a 26 dimensional base of traceless Jordan matrices we used the Mathematica program in App.A to find the matrices which complete the base for $\mathfrak{f}_4$ to a base for the whole $\mathfrak{e}_6$ algebra. In particular if we work in the real case, we obtain the split form of $\mathfrak{e}_6$, with signature $(52, 26)$. However, multiplying the 26 added generators by i we find that the algebra remains real, but the Killing form becomes the compact one.

[1] See [1] for more details.

3 The construction of the group E_6

3.1 The generalized Euler parametrization for E_6

To give an Euler parametrization for E_6 we start by choosing its maximal subgroup. It is $H = F_4$, the group generated by the first 52 matrices. Let $\mathcal{P}$ be the linear complement of $\mathfrak{f}_4$ in $\mathfrak{e}_6$. We then search for a minimal linear subset V of $\mathcal{P}$, which, under action of $Ad(F_4)$, generates the whole $\mathcal{P}$. Looking at the structure constants we see that V can be chosen as the linear space generated by c_{53}, c_{70}. Note that they commute.
If we then write the general element g of E_6 in the form

$$g = \exp(\tilde{h}) \exp(v) \exp(h) \ , \quad h, \tilde{h} \in \mathfrak{f}_4 \ , \ v \in V \ , \tag{3.1}$$

we have a redundancy of dimension 28. As argued in [1] we expect to find a 28-dimensional subgroup of F_4 ,which, acting by adjunction, defines an automorphism of V. This can be done by noticing that V commutes with the first 28 matrices c_i, $i = 1, \dots, 28$, which generate an $SO(8)$ subgroup of F_4.
We found convenient to introduce the change of base

$$\tilde{c}_{53} := \frac{1}{2} c_{53} + \frac{\sqrt{3}}{2} c_{70} \ , \tag{3.2}$$

$$\tilde{c}_{70} := -\frac{\sqrt{3}}{2} c_{53} + \frac{1}{2} c_{70} \ . \tag{3.3}$$

Thus we have

$$g[x_1, \dots, x_{78}] = B[x_1, \dots, x_{24}] e^{x_{25}\tilde{c}_{53}} e^{x_{26}\tilde{c}_{70}} F_4[x_{27}, \dots, x_{78}] \ ,$$

where $B = F_4/SO(8)$. We chose for F_4 the Euler parametrization given in [1] so that we found for B

$$B[x_1, \dots, x_{24}] = B_{F_4}[x_1, \dots, x_{16}] B_9[x_{17}, \dots, x_{23}] e^{x_{24} c_{45}} \ , \tag{3.4}$$

where

$$B_{F_4}[x_1, \dots, x_{15}] = B_9[x_1, \dots, x_7] e^{x_8 c_{45}} B_8[x_9, \dots, x_{14}] e^{x_{15}\tilde{c}_{30}} e^{x_{16} c_{22}} \ , \tag{3.5}$$

$$B_9[x_1, \dots, x_7] = e^{x_1\tilde{c}_3} e^{x_2\tilde{c}_{16}} e^{x_3\tilde{c}_{15}} e^{x_4\tilde{c}_{35}} e^{x_5\tilde{c}_5} e^{x_6\tilde{c}_1} e^{x_7\tilde{c}_{30}} \ , \tag{3.6}$$

$$B_8[x_1, \dots, x_6] = e^{x_1\tilde{c}_3} e^{x_2\tilde{c}_{16}} e^{x_3\tilde{c}_{15}} e^{x_4\tilde{c}_{35}} e^{x_5\tilde{c}_5} e^{x_6\tilde{c}_1} \ , \tag{3.7}$$

and the tilded matrices are the ones introduced in [1].

3.2 Determination of the range for the parameters.

To determine the range of the parameters we will use the topological method we have developed in [2]. Let us first determine the volume of E_6 by means of the Macdonald formula.

3.2.1 The volume of E_6.

The rational homology of the exceptional Lie group E_6 is that of a product of odd dimensional spheres [10], $H_*(E_6) = H_*(\prod_{i=1}^{6} S^{d_i})$, with ([4])

$$d_1 = 3,\ d_2 = 9,\ d_3 = 11,\ d_4 = 15,\ d_5 = 17,\ d_6 = 23\ . \tag{3.8}$$

The simple roots of E_6 are

$$r_1 = L_1 + L_2\ , \tag{3.9}$$
$$r_2 = L_2 - L_1\ , \tag{3.10}$$
$$r_3 = L_3 - L_2\ , \tag{3.11}$$
$$r_4 = L_4 - L_3\ , \tag{3.12}$$
$$r_5 = L_5 - L_4\ , \tag{3.13}$$
$$r_6 = \frac{L_1 - L_2 - L_3 - L_4 - L_5 + \sqrt{3} L_6}{2}\ , \tag{3.14}$$

where L_i, $i = 1, \dots, 6$ is an orthonormal base for the Cartan algebra. The volume of the fundamental region is then

$$Vol(f_{E_6}) = \frac{2}{L}\ . \tag{3.15}$$

Indeed we computed the 36 positive roots, all having length $\sqrt{2}$. They have exactly the structure given in [8], with $L_i = e_i$, the canonical base of $\mathbb{R}^6$. The Macdonald formula [9], [11] gives for the volume of the compact form of E_6

$$Vol(E_6) = \frac{\sqrt{3} \cdot 2^{17} \cdot \pi^{42}}{3^{10} \cdot 5^5 \cdot 7^3 \cdot 11}\ . \tag{3.16}$$

3.2.2 The invariant measure on E_6.

With the chosen generalized Euler parametrization, the invariant measure on E_6 decomposes into the product of the measure of F_4 and the one on $M = E_6/F_4$. The invariant measure on F_4 was computed in [1] so that we need to compute here only the induced measure on M.
Using the notation of [1], let us define

$$J_M := \pi_{\mathcal{P}}(e^{-x_{26}\tilde{c}_{70}} e^{-x_{25}\tilde{c}_{53}} B[x_1, \dots, x_{24}]^{-1} d(B[x_1, \dots, x_{24}] e^{x_{25}\tilde{c}_{53}} e^{x_{26}\tilde{c}_{70}}))\ , \tag{3.17}$$

where $\pi_{\mathcal{P}}$ is the projection on the subspace generated by c_j, $j = 53, \dots, 78$. The metric induced on M by the bi-invariant metric on E_6 is then

$$ds^2_M = -\frac{1}{6} Trace(J_M \otimes J_M)\ , \tag{3.18}$$

and the invariant measure on E_6 is then

$$d\mu_{E_6} = |\det(J^j_{Mi})| d\mu_{F_4} \prod_{l=1}^{26} dx_l\ , \tag{3.19}$$

where J^j_{Mi} is the 26×26 matrix defined by

$$J_M = \sum_{i,j=1}^{26} J^j_{Mi} c_{i_j} dx^i \ , \tag{3.20}$$

with c_{i_j} a base $\{\tilde{c}_{53}, \tilde{c}_{70}, c_{54}, \ldots, c_{69}, c_{71}, \ldots, c_{78}\}$ of $\mathcal{P}$.
In order to compute $|det(J^j_{Mi})|$ it is convenient to introduce the notations

$$\omega[x,y,z] = e^{xc_{45}} e^{y\tilde{c}_{53}} e^{z\tilde{c}_{70}} \ , \tag{3.21}$$
$$J_\omega := \omega^{-1} d\omega \ , \tag{3.22}$$
$$J_9 := B_9^{-1} dB_9 \ , \tag{3.23}$$
$$J_{F_4} := B_{F_4}^{-1} dB_{F_4} \ , \tag{3.24}$$

so that

$$\begin{aligned} J_M[x_1, \ldots, x_{26}] & \\ = \omega[x_{24}, x_{25}, x_{26}]^{-1} & B_9[x_{17}, \ldots, x_{23}]^{-1} J_{F_4}[x_1, \ldots, x_{16}] B_9[x_{17}, \ldots, x_{23}] \omega[x_{24}, x_{25}, x_{26}] \\ + \omega[x_{24}, x_{25}, x_{26}]^{-1} & J_9[x_{17}, \ldots, x_{23}] \omega[x_{24}, x_{25}, x_{26}] + J_\omega[x_{24}, x_{25}, x_{26}] \ . \end{aligned} \tag{3.25}$$

Some remarks are in order now

1. the following relations are true

$$e^{-\alpha\tilde{c}_{53}} c_L e^{\alpha\tilde{c}_{53}} = \cos\alpha \ c_L + \sin\alpha \ c_{L+26} \ , \tag{3.26}$$

   if $L = 45, \ldots, 52$. Moreover $\tilde{c}_{70}$ commutes with c_I, $I = 45, \ldots, 52, 71, \ldots, 78$ and with $\tilde{c}_{53}$;

2. $\tilde{c}_{53}$ and $\tilde{c}_{70}$ commute with the $so(8)$ algebra generated by the matrices c_I, $I = 1, \ldots, 21, 30, \ldots, 36$;

3. the adjoint action of $e^{x_{24}c_{45}}$ on the above $so(8)$ algebra generates in addition the matrices c_J, $J = 46, \ldots, 52$;

4. from $J_{F_4} \in \mathfrak{f}_4$ and $B_9 \subset F_4$, it follows that

$$\tilde{J}_{F_4} := B_9^{-1} J_{F_4} B_9 \in \mathfrak{f}_4 \ ;$$

5. the adjoint action of ω on the $\mathfrak{f}_4$ matrices generates all the remaining matrices of $\mathfrak{e}_6$. In particular, the projection of $\omega^{-1} c_L \omega$, $L = 1, \ldots, 52$ on c_J, $J = 54, \ldots, 69$, is different from zero only if $L = 22, \ldots, 29, 37, \ldots, 44$;

6. the adjoint action of the $SO(8)$ group, corresponding to the above $so(8)$ algebra, gives a rotation both on the indexes $I = \{22, \ldots, 29\}$ and $J = \{37, \ldots, 44\}$. More precisely

$$SO(8)^{-1} c_I SO(8) = R_I{}^L c_L \ , \tag{3.27}$$
$$SO(8)^{-1} c_J SO(8) = \tilde{R}_J^K c_K \ , \tag{3.28}$$

where L runs from 22 to 29 and K runs from 37 to 44, and $R, \tilde{R}$ are both orthogonal matrices. To verify these it suffices to note that

$$e^{-xc_A} c_I e^{xc_A} = \cos\frac{x}{2}\ c_I \pm \sin\frac{x}{2}\ c_{I_A}\ , \tag{3.29}$$

$$e^{-xc_A} c_J e^{xc_A} = \cos\frac{x}{2}\ c_J \pm \sin\frac{x}{2}\ c_{J_A}\ , \tag{3.30}$$

where $A = 1, \ldots, 21, 30, \ldots, 36$, $I, I_A \in \{22, \ldots, 29\}$, $J, J_A \in \{37, \ldots, 44\}$. In particular $\det(R \otimes \tilde{R}) = 1$.

From the first three points, we see that the matrix J_M takes the form

$$M = \begin{pmatrix} A & 0 & 0 \\ * & C & 0 \\ * & * & D \end{pmatrix}\ , \tag{3.31}$$

where on the rows we indicate the coefficients of dx_I with $I = 1, \ldots, 26$ starting from the bottom, and on the columns the projections on $\tilde{c}_A$, c_L, following the order: $A = 53, 70$, $L = 71, \ldots, 78, 54, \ldots, 69$. The asterisks indicate the elements which do not contribute to the determinant:

$$\det J_M = detA \det C \det D\ .$$

In particular, A is a 3×3 block obtained by projecting J_ω on $\tilde{c}_{53}, \tilde{c}_{70}, c_{71}$. Point 1 implies

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sin x_{25} \end{pmatrix}\ , \tag{3.32}$$

so that

$$\det A = \sin x_{25}\ . \tag{3.33}$$

The C block is a 7×7 matrix obtained by projecting $\omega^{-1} J_9\ \omega$ on c_K, $K = 72, \ldots, 78$. From point 1 it follows that it can be obtained by multiplying the projection of $e^{-x_{24}c_{45}} J_9 e^{x_{24}c_{45}}$ on c_L, $L = 46, \ldots, 52$ by $\sin x_{25}$. If we call $\tilde{C}$ the matrix corresponding to such a projection, we find $\det C = \sin^7 x_{25} \det \tilde{C}$. The determinant of $\tilde{C}$ can be computed directly and gives

$$\det C = \sin x_{20} \cos x_{21} \cos x_{22} \sin^2 x_{22} \sin^2 x_{23} \cos^4 x_{23} \sin^7 x_{24} \sin^7 x_{25}\ . \tag{3.34}$$

The D block requires some further discussion. It is a 16×16 matrix obtained by projecting $\omega^{-1} \tilde{J}_{B_{F_4}} \omega$ on c_I, $I = 54, \ldots, 69$. First, from points 4, 5 of our remarks we see that only the c_L with $L = 22, \ldots, 29, 37, \ldots, 44$, in $\tilde{J}_{F_4}$ contribute to the determinant. Let us define the 16×16 matrix U with

$$U_A{}^B := -\frac{1}{6} \mathrm{Tr}\ (\omega^{-1} c_A \omega c_B)\ , \tag{3.35}$$

$$A = 22, \ldots, 29, 37, \ldots, 44\ , \tag{3.36}$$

$$B = 54, \ldots, 69\ . \tag{3.37}$$

Moreover let $\tilde{D}$ be the matrix obtained by projecting J_{F_4} on c_L with $L = 22, \ldots, 29, 37, \ldots, 44$, and

$$Q := \begin{pmatrix} R & 0 \\ 0 & \tilde{R} \end{pmatrix} , \tag{3.38}$$

where R and $\tilde{R}$ are the rotation matrices defined at point 6. Then we can deduce from points 4, 5 and 6 that

$$\det D = \det(U \cdot Q \cdot \tilde{D}) = \det U \det \tilde{D} . \tag{3.39}$$

The matrix U and its determinant are easily computed. Indeed, we have

$$\omega^{-1} c_{22} \omega = \cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{61} + \sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{69} + \ldots$$
$$\omega^{-1} c_{23} \omega = -\cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{57} - \sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{65} + \ldots$$
$$\omega^{-1} c_{24} \omega = -\cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{60} - \sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{68} + \ldots$$
$$\omega^{-1} c_{25} \omega = \cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{55} + \sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{63} + \ldots$$
$$\omega^{-1} c_{26} \omega = -\cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{59} - \sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{67} + \ldots$$
$$\omega^{-1} c_{27} \omega = \cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{58} + \sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{66} + \ldots$$
$$\omega^{-1} c_{28} \omega = \cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{56} + \sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{64} + \ldots$$
$$\omega^{-1} c_{29} \omega = -\cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{54} - \sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{62} + \ldots$$
$$\omega^{-1} c_{37} \omega = -\sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{54} + \cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{62} + \ldots$$
$$\omega^{-1} c_{38} \omega = -\sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{55} + \cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{63} + \ldots$$
$$\omega^{-1} c_{39} \omega = -\sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{56} + \cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{64} + \ldots$$
$$\omega^{-1} c_{40} \omega = -\sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{57} + \cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{65} + \ldots$$
$$\omega^{-1} c_{41} \omega = -\sin\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} + \frac{x_{25}}{2}\right) c_{58} + \cos\left(\frac{x_{24}}{2}\right) \sin\left(\frac{\sqrt{3}}{2} x_{26} - \frac{x_{25}}{2}\right) c_{66} + \ldots$$

$$\omega^{-1}c_{42}\omega = -\sin\left(\frac{x_{24}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}x_{26}+\frac{x_{25}}{2}\right)c_{59}+\cos\left(\frac{x_{24}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}x_{26}-\frac{x_{25}}{2}\right)c_{67}+\dots$$
$$\omega^{-1}c_{43}\omega = -\sin\left(\frac{x_{24}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}x_{26}+\frac{x_{25}}{2}\right)c_{60}+\cos\left(\frac{x_{24}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}x_{26}-\frac{x_{25}}{2}\right)c_{68}+\dots$$
$$\omega^{-1}c_{44}\omega = -\sin\left(\frac{x_{24}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}x_{26}+\frac{x_{25}}{2}\right)c_{61}+\cos\left(\frac{x_{24}}{2}\right)\sin\left(\frac{\sqrt{3}}{2}x_{26}-\frac{x_{25}}{2}\right)c_{69}+\dots$$

where ellipses stay for terms which vanish when projected on c_B, $B=54,\dots,61,70,\dots,78$. From these it follows

$$\det U = \sin^8\left(\frac{\sqrt{3}}{2}x_{26}+\frac{x_{25}}{2}\right)\sin^8\left(\frac{\sqrt{3}}{2}x_{26}-\frac{x_{25}}{2}\right)\ . \tag{3.40}$$

At this point we need to compute $\det\tilde{D}$. This can be done by noticing that J_{F_4} coincides exactly with the current J_M of F_4 in [1]. We then find

$$\begin{aligned}\det\tilde{D} = 2^7 \sin^{15}\frac{x_{16}}{2}\cos^7\frac{x_{16}}{2}\sin x_4\cos x_5\cos x_6\sin^2 x_6\cos^4 x_7\sin^2 x_7\sin^7 x_8\cdot\\ \cdot\sin x_{12}\cos x_{13}\cos x_{14}\sin^2 x_{14}\cos^2 x_{15}\sin^4 x_{15}\ .\end{aligned} \tag{3.41}$$

We can finally write down the invariant measure on the base space:

$$\begin{aligned}d\mu_M = 2^7 &\sin x_4\cos x_5\cos x_6\sin^2 x_6\cos^4 x_7\sin^2 x_7\sin^7 x_8\cdot\\ &\cdot\sin x_{12}\cos x_{13}\cos x_{14}\sin^2 x_{14}\cos^2 x_{15}\sin^4 x_{15}\ \ \sin^{15}\frac{x_{16}}{2}\cos^7\frac{x_{16}}{2}\cdot\\ &\cdot\sin x_{20}\cos x_{21}\cos x_{22}\sin^2 x_{22}\sin^2 x_{23}\cos^4 x_{23}\sin^7 x_{24}\cdot\\ &\cdot\sin^8 x_{25}\sin^8\left(\frac{\sqrt{3}}{2}x_{26}+\frac{x_{25}}{2}\right)\sin^8\left(\frac{\sqrt{3}}{2}x_{26}-\frac{x_{25}}{2}\right)\ .\end{aligned} \tag{3.42}$$

Note that the periods of the variables are 4π so that one should take the range $x_i=[0,4\pi]$ for $i=1,2,3,,9,10,11,17,18,19$. However, as in [1], it is easy to show directly from the parametrization that they can all be restricted to $[0,2\pi]$. The range of x_i is then

$$\begin{aligned}&x_1\in[0,2\pi]\ ,\quad x_2\in[0,2\pi]\ ,\quad x_3\in[0,2\pi]\ ,\quad x_4\in[0,\pi]\ ,\\ &x_5\in[-\frac{\pi}{2},\frac{\pi}{2}]\ ,\quad x_6\in[0,\frac{\pi}{2}]\ ,\quad x_7\in[0,\frac{\pi}{2}]\ ,\quad x_8\in[0,\pi]\ ,\\ &x_9\in[0,2\pi]\ ,\quad x_{10}\in[0,2\pi]\ ,\quad x_{11}\in[0,2\pi]\ ,\quad x_{12}\in[0,\pi]\ ,\\ &x_{13}\in[-\frac{\pi}{2},\frac{\pi}{2}]\ ,\quad x_{14}\in[0,\frac{\pi}{2}]\ ,\quad x_{15}\in[0,\frac{\pi}{2}]\ ,\quad x_{16}\in[0,\pi]\ ,\\ &x_{17}\in[0,2\pi]\ ,\quad x_{18}\in[0,2\pi]\ ,\quad x_{19}\in[0,2\pi]\ ,\quad x_{20}\in[0,\pi]\ ,\\ &x_{21}\in[-\frac{\pi}{2},\frac{\pi}{2}]\ ,\quad x_{22}\in[0,\frac{\pi}{2}]\ ,\quad x_{23}\in[0,\frac{\pi}{2}]\ ,\quad x_{24}\in[0,\pi]\ ,\\ &x_{25}\in[0,\frac{\pi}{2}]\ ,\quad -\frac{x_{25}}{\sqrt{3}}\le x_{26}\le\frac{x_{25}}{\sqrt{3}}\ .\end{aligned} \tag{3.43}$$

The remaining parameters x_j, $j=27,\dots,78$, will run over the range for F_4, as given in [1]. The volume of the whole closed cycle V so obtained is then

$$Vol(V) = Vol(F_4)\int_R d\mu_M = \frac{\sqrt{3}\cdot 2^{17}\cdot\pi^{42}}{3^{10}\cdot 5^5\cdot 7^3\cdot 11}\ , \tag{3.44}$$

where R is the range of parameters x_i, $i = 1, \ldots, 26$. This is the volume of E_6, so that we cover the group exactly once.[2]

4 Conclusions.

In this paper we have performed an explicit construction for the compact form of the simple Lie group E_6. This is particularly interesting, because recently it has been argued that this group could be the most promising for unification in GUT theories [3, 6]. To parameterize the group we have used the generalized Euler angles method, a technique we introduced in [2, 1] to give the most simple expression for the invariant measure on the group, while at the same time still being able to provide an explicit expression for the range of the parameters. Both these requirements are necessary in order to minimize the computation power needed for computer simulations, for example, of lattice models.

Our results can be easily extended to the GUT group $E_6^4 \rtimes S_4$. Also, a modified Euler parametrization could be applied to evidentiate the subgroups related to the correct symmetry breaking, see [3]. Finally, our parametrization could be used for a straightforward geometrical analysis of the exceptional Lie group E_6 and its quotients.

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[2]Obviously there is a subset of vanishing measure which is multiply covered.

A The $\mathfrak{e}_6$ matrices.

The matrices we found using Mathematica, and orthonormalized with respect to the scalar product $\langle a, b\rangle := -\frac{1}{6}Trace(ab)$, was computed by means of the followings programs.

The Mathematica program.

```
%%%% the octonionic products
QQ[1,1] = e[1];    QQ[1,2] = e[2];    QQ[1,3] = e[3];
QQ[1,4] = e[4];    QQ[1,5] = e[5];    QQ[1,6] = e[6];
QQ[1,7] = e[7];    QQ[1,8] = e[8];    QQ[2,1] = e[2];
QQ[2,2] = -e[1];    QQ[2,3] = e[5];    QQ[2,4] = e[8];
QQ[2,5] = -e[3];    QQ[2,6] = e[7];    QQ[2,7] = -e[6];
QQ[2,8] = -e[4];    QQ[3,1] = e[3];    QQ[3,2] = -e[5];
QQ[3,3] = -e[1];    QQ[3,4] = e[6];    QQ[3,5] = e[2];
QQ[3,6] = -e[4];    QQ[3,7] = e[8];    QQ[3,8] = -e[7];
QQ[4,1] = e[4];    QQ[4,2] = -e[8];    QQ[4,3] = -e[6];
QQ[4,4] = -e[1];    QQ[4,5] = e[7];    QQ[4,6] = e[3];
QQ[4,7] = -e[5];    QQ[4,8] = e[2];    QQ[5,1] = e[5];
QQ[5,2] = e[3];    QQ[5,3] = -e[2];    QQ[5,4] = -e[7];
QQ[5,5] = -e[1];    QQ[5,6] = e[8];    QQ[5,7] = e[4];
QQ[5,8] = -e[6];    QQ[6,1] = e[6];    QQ[6,2] = -e[7];
QQ[6,3] = e[4];    QQ[6,4] = -e[3];    QQ[6,5] = -e[8];
QQ[6,6] = -e[1];    QQ[6,7] = e[2];    QQ[6,8] = e[5];
QQ[7,1] = e[7];    QQ[7,2] = e[6];    QQ[7,3] = -e[8];
QQ[7,4] = e[5];    QQ[7,5] = -e[4];    QQ[7,6] = -e[2];
QQ[7,7] = -e[1];    QQ[7,8] = e[3];    QQ[8,1] = e[8];
QQ[8,2] = e[4];    QQ[8,3] = e[7];    QQ[8,4] = -e[2];
QQ[8,5] = e[6];    QQ[8,6] = -e[5];    QQ[8,7] = -e[3];
QQ[8,8] = -e[1];
%%%% the Jordan algebra product
Qm[x_, y_] := Sum[Sum[x[[i]]y[[j]]QQ[i, j], {i, 8}], {j, 8}]
QP[x_, y_] := {Coefficient[Qm[x, y], e[1]], Coefficient[Qm[x, y], e[2]],
    Coefficient[Qm[x, y], e[3]], Coefficient[Qm[x, y], e[4]], Coefficient[Qm[x, y], e[5]],
    Coefficient[Qm[x, y], e[6]], Coefficient[Qm[x, y], e[7]], Coefficient[Qm[x, y], e[8]]}
```

```
OctP[a_, b_] := {{Sum[QP[Part[Part[a, 1], i], Part[Part[b, i], 1]], {i, 3}] ,
      Sum[QP[Part[Part[a, 1], i], Part[Part[b, i], 2]], {i, 3}],
      Sum[QP[Part[Part[a, 1], i], Part[Part[b, i], 3]], {i, 3}]},
     {Sum[QP[Part[Part[a, 2], i], Part[Part[b, i], 1]], {i, 3}],
      Sum[QP[Part[Part[a, 2], i], Part[Part[b, i], 2]], {i, 3}],
      Sum[QP[Part[Part[a, 2], i], Part[Part[b, i], 3]], {i, 3}]},
     {Sum[QP[Part[Part[a, 3], i], Part[Part[b, i], 1]], {i, 3}],
      Sum[QP[Part[Part[a, 3], i], Part[Part[b, i], 2]], {i, 3}],
      Sum[QP[Part[Part[a, 3], i], Part[Part[b, i], 3]], {i, 3}]}}
OctPS[a_, b_] := 1/2(OctP[a, b] + OctP[b, a])
```

%%%% correspondence between the Jordan algebra and $\mathbb{R}^{27}$

```
FF[AA_] := {Part[{Part[Part[Part[AA, 1], 1], 1]}, 1],
    Part[{Part[Part[Part[AA, 1], 2], 1]}, 1], Part[{Part[Part[Part[AA, 1], 2], 2]}, 1],
    Part[{Part[Part[Part[AA, 1], 2], 3]}, 1], Part[{Part[Part[Part[AA, 1], 2], 4]}, 1],
    Part[{Part[Part[Part[AA, 1], 2], 5]}, 1], Part[{Part[Part[Part[AA, 1], 2], 6]}, 1],
    Part[{Part[Part[Part[AA, 1], 2], 7]}, 1], Part[{Part[Part[Part[AA, 1], 2], 8]}, 1],
    Part[{Part[Part[Part[AA, 1], 3], 1]}, 1], Part[{Part[Part[Part[AA, 1], 3], 2]}, 1],
    Part[{Part[Part[Part[AA, 1], 3], 3]}, 1], Part[{Part[Part[Part[AA, 1], 3], 4]}, 1],
    Part[{Part[Part[Part[AA, 1], 3], 5]}, 1], Part[{Part[Part[Part[AA, 1], 3], 6]}, 1],
    Part[{Part[Part[Part[AA, 1], 3], 7]}, 1], Part[{Part[Part[Part[AA, 1], 3], 8]}, 1],
    Part[{Part[Part[Part[AA, 2], 2], 1]}, 1], Part[{Part[Part[Part[AA, 2], 3], 1]}, 1],
    Part[{Part[Part[Part[AA, 2], 3], 2]}, 1], Part[{Part[Part[Part[AA, 2], 3], 3]}, 1],
    Part[{Part[Part[Part[AA, 2], 3], 4]}, 1], Part[{Part[Part[Part[AA, 2], 3], 5]}, 1],
    Part[{Part[Part[Part[AA, 2], 3], 6]}, 1], Part[{Part[Part[Part[AA, 2], 3], 7]}, 1],
    Part[{Part[Part[Part[AA, 2], 3], 8]}, 1], Part[{Part[Part[Part[AA, 3], 3], 1]}, 1]}
```

```
FFi[vv_] :=
 {{{Part[vv, 1], 0, 0, 0, 0, 0, 0, 0}, {Part[vv, 2], Part[vv, 3], Part[vv, 4], Part[vv, 5],
    Part[vv, 6], Part[vv, 7], Part[vv, 8], Part[vv, 9]} , {Part[vv, 10], Part[vv, 11],
    Part[vv, 12], Part[vv, 13], Part[vv, 14], Part[vv, 15], Part[vv, 16], Part[vv, 17]}} ,
  {{Part[vv, 2], −Part[vv, 3], −Part[vv, 4], −Part[vv, 5], −Part[vv, 6],
    −Part[vv, 7], −Part[vv, 8], −Part[vv, 9]} , {Part[vv, 18], 0, 0, 0, 0, 0, 0, 0},
   {Part[vv, 19], Part[vv, 20], Part[vv, 21], Part[vv, 22],
    Part[vv, 23], Part[vv, 24], Part[vv, 25], Part[vv, 26]}} ,


  {{Part[vv, 10], −Part[vv, 11], −Part[vv, 12], −Part[vv, 13],
    −Part[vv, 14], −Part[vv, 15], −Part[vv, 16], −Part[vv, 17]} ,
   {Part[vv, 19], −Part[vv, 20], −Part[vv, 21], −Part[vv, 22], −Part[vv, 23],
    −Part[vv, 24], −Part[vv, 25], −Part[vv, 26]} , {Part[vv, 27], 0, 0, 0, 0, 0, 0, 0}}}


%%%% construction of the matrices
MT = { {{mt[1], 0, 0, 0, 0, 0, 0, 0}, {mt[2], mt[3], mt[4], mt[5], mt[6], mt[7], mt[8],
        mt[9]}, {mt[10], mt[11], mt[12], mt[13], mt[14], mt[15], mt[16], mt[17]}},
     {{mt[2], −mt[3], −mt[4], −mt[5], −mt[6], −mt[7], −mt[8], −mt[9]}, {mt[18], 0, 0,
        0, 0, 0, 0, 0}, {mt[19], mt[20], mt[21], mt[22], mt[23], mt[24], mt[25], mt[26]}},
     {{mt[10], −mt[11], −mt[12], −mt[13], −mt[14], −mt[15], −mt[16], −mt[17]},
        {mt[19], −mt[20], −mt[21], −mt[22], −mt[23], −mt[24], −mt[25], −mt[26]},
        {−mt[1] − mt[18], 0, 0, 0, 0, 0, 0, 0}}};

          h = Array[hh, 27];
          M = Array[mm, {27, 27}];
          imm = FF[OctPS[MT, FFi[h]]];
          Do[Do[mm[i, j]  = Coefficient[Part[imm, i], Part[h, j]], {i, 27}], {j, 27}]
          Do[econi[j] = D[M, mt[j]], {j, 26}];
          Mno[1] = econi[1];
          Do[Do[
             AA[j, i] = 0, {i, 26}], {j, 26}]
          Do[Do[AA[j, i] = −Tr[econi[j] .Mno[i]]/Tr[Mno[i] .Mno[i]], {i, j − 1}];
             Mno[j] = econi[j] + Sum[AA[j, i]Mno[i], {i, j − 1}], {j, 2, 26}]
          Do[Mnno[i] = −√6Mno[i]/Sqrt[Tr[Mno[i] .Mno[i]]], {i, 26}];
```

```
%%% rotation to give the irreducible 26 representation
XXX = {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0, 0, 0, 0, 0, 0, 0, 0}/Sqrt[2],
       {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
       {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, −2}/Sqrt[6],
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},
       {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1}/Sqrt[3]};
```

```
Do[cc[i + 52] = XXX.Mnno[i].Transpose[XXX], {i, 26}];
```

The matrices cc[i], i $= 53, \ldots, 78$ so obtained, must be added to the 52 matrices cc[i] determined in [1] to give a base of the $\mathfrak{e}_6$ algebra.

B Structure constants.

The structure constants ${s_{IJ}}^K$ are defined by $[C_I, C_J] = \sum_{K=1}^{78} {s_{IJ}}^K C_K$. We checked that the coefficients $s_{IJK} := {s_{IJ}}^K$ are completely antisymmetric in the indices. We write only the non-vanishing terms, up to symmetries, which are not yet written in [1]:

$$
\begin{array}{cccc}
s_{1,54,55} = \frac{1}{2} \,, & s_{1,56,58} = -\frac{1}{2} \,, & s_{1,57,61} = -\frac{1}{2} \,, & s_{1,59,60} = -\frac{1}{2} \,, \\
s_{1,62,63} = -\frac{1}{2} \,, & s_{1,64,66} = \frac{1}{2} \,, & s_{1,65,69} = \frac{1}{2} \,, & s_{1,67,68} = \frac{1}{2} \,, \\
s_{1,71,72} = -1 \,, & s_{2,54,56} = \frac{1}{2} \,, & s_{2,55,58} = \frac{1}{2} \,, & s_{2,57,59} = -\frac{1}{2} \,, \\
s_{2,60,61} = -\frac{1}{2} \,, & s_{2,62,64} = -\frac{1}{2} \,, & s_{2,63,66} = -\frac{1}{2} \,, & s_{2,65,67} = \frac{1}{2} \,, \\
s_{2,68,69} = \frac{1}{2} \,, & s_{2,71,73} = -1 \,, & s_{3,54,58} = \frac{1}{2} \,, & s_{3,55,56} = -\frac{1}{2} \,, \\
s_{3,57,60} = -\frac{1}{2} \,, & s_{3,59,61} = \frac{1}{2} \,, & s_{3,62,66} = \frac{1}{2} \,, & s_{3,63,64} = -\frac{1}{2} \,, \\
s_{3,65,68} = -\frac{1}{2} \,, & s_{3,67,69} = \frac{1}{2} \,, & s_{3,72,73} = -1 \,, & s_{4,54,57} = \frac{1}{2} \,, \\
s_{4,55,61} = \frac{1}{2} \,, & s_{4,56,59} = \frac{1}{2} \,, & s_{4,58,60} = -\frac{1}{2} \,, & s_{4,62,65} = -\frac{1}{2} \,, \\
s_{4,63,69} = -\frac{1}{2} \,, & s_{4,64,67} = -\frac{1}{2} \,, & s_{4,66,68} = \frac{1}{2} \,, & s_{4,71,74} = -1 \,, \\
s_{5,54,61} = \frac{1}{2} \,, & s_{5,55,57} = -\frac{1}{2} \,, & s_{5,56,60} = \frac{1}{2} \,, & s_{5,58,59} = \frac{1}{2} \,, \\
s_{5,62,69} = \frac{1}{2} \,, & s_{5,63,65} = -\frac{1}{2} \,, & s_{5,64,68} = \frac{1}{2} \,, & s_{5,66,67} = \frac{1}{2} \,, \\
s_{5,72,74} = -1 \,, & s_{6,54,59} = \frac{1}{2} \,, & s_{6,55,60} = -\frac{1}{2} \,, & s_{6,56,57} = -\frac{1}{2} \,, \\
s_{6,58,61} = -\frac{1}{2} \,, & s_{6,62,67} = \frac{1}{2} \,, & s_{6,63,68} = -\frac{1}{2} \,, & s_{6,64,65} = -\frac{1}{2} \,, \\
s_{6,66,69} = -\frac{1}{2} \,, & s_{6,73,74} = -1 \,, & s_{7,54,58} = \frac{1}{2} \,, & s_{7,55,56} = -\frac{1}{2} \,, \\
s_{7,57,60} = \frac{1}{2} \,, & s_{7,59,61} = -\frac{1}{2} \,, & s_{7,62,66} = -\frac{1}{2} \,, & s_{7,63,64} = \frac{1}{2} \,, \\
s_{7,65,68} = -\frac{1}{2} \,, & s_{7,67,69} = \frac{1}{2} \,, & s_{7,71,75} = -1 \,, & s_{8,54,56} = -\frac{1}{2} \,, \\
s_{8,55,58} = -\frac{1}{2} \,, & s_{8,57,59} = -\frac{1}{2} \,, & s_{8,60,61} = -\frac{1}{2} \,, & s_{8,62,64} = -\frac{1}{2} \,, \\
s_{8,63,66} = -\frac{1}{2} \,, & s_{8,65,67} = -\frac{1}{2} \,, & s_{8,68,69} = -\frac{1}{2} \,, & s_{8,72,75} = -1 \,, \\
s_{9,54,55} = \frac{1}{2} \,, & s_{9,56,58} = -\frac{1}{2} \,, & s_{9,57,61} = \frac{1}{2} \,, & s_{9,59,60} = \frac{1}{2} \,, \\
s_{9,62,63} = \frac{1}{2} \,, & s_{9,64,66} = -\frac{1}{2} \,, & s_{9,65,69} = \frac{1}{2} \,, & s_{9,67,68} = \frac{1}{2} \,, \\
s_{9,73,75} = -1 \,, & s_{10,54,60} = \frac{1}{2} \,, & s_{10,55,59} = \frac{1}{2} \,, & s_{10,56,61} = -\frac{1}{2} \,, \\
s_{10,57,58} = -\frac{1}{2} \,, & s_{10,62,68} = \frac{1}{2} \,, & s_{10,63,67} = \frac{1}{2} \,, & s_{10,64,69} = -\frac{1}{2} \,, \\
s_{10,65,66} = -\frac{1}{2} \,, & s_{10,74,75} = -1 \,, & s_{11,54,59} = \frac{1}{2} \,, & s_{11,55,60} = \frac{1}{2} \,, \\
s_{11,56,57} = -\frac{1}{2} \,, & s_{11,58,61} = \frac{1}{2} \,, & s_{11,62,67} = -\frac{1}{2} \,, & s_{11,63,68} = -\frac{1}{2} \,, \\
s_{11,64,65} = \frac{1}{2} \,, & s_{11,66,69} = -\frac{1}{2} \,, & s_{11,71,76} = -1 \,, & s_{12,54,60} = \frac{1}{2} \,, \\
s_{12,55,59} = -\frac{1}{2} \,, & s_{12,56,61} = -\frac{1}{2} \,, & s_{12,57,58} = \frac{1}{2} \,, & s_{12,62,68} = \frac{1}{2} \,, \\
s_{12,63,67} = -\frac{1}{2} \,, & s_{12,64,69} = -\frac{1}{2} \,, & s_{12,65,66} = \frac{1}{2} \,, & s_{12,72,76} = -1 \,, \\
s_{13,54,57} = -\frac{1}{2} \,, & s_{13,55,61} = \frac{1}{2} \,, & s_{13,56,59} = -\frac{1}{2} \,, & s_{13,58,60} = -\frac{1}{2} \,, \\
s_{13,62,65} = -\frac{1}{2} \,, & s_{13,63,69} = \frac{1}{2} \,, & s_{13,64,67} = -\frac{1}{2} \,, & s_{13,66,68} = \frac{1}{2} \,, \\
s_{13,73,76} = -1 \,, & s_{14,54,56} = \frac{1}{2} \,, & s_{14,55,58} = -\frac{1}{2} \,, & s_{14,57,59} = -\frac{1}{2} \,, \\
s_{14,60,61} = \frac{1}{2} \,, & s_{14,62,64} = \frac{1}{2} \,, & s_{14,63,66} = -\frac{1}{2} \,, & s_{14,65,67} = -\frac{1}{2} \,, \\
s_{14,68,69} = \frac{1}{2} \,, & s_{14,74,76} = -1 \,, & s_{15,54,61} = \frac{1}{2} \,, & s_{15,55,57} = \frac{1}{2} \,, \\
s_{15,56,60} = \frac{1}{2} \,, & s_{15,58,59} = -\frac{1}{2} \,, & s_{15,62,69} = \frac{1}{2} \,, & s_{15,63,65} = \frac{1}{2} \,, \\
s_{15,64,68} = \frac{1}{2} \,, & s_{15,66,67} = -\frac{1}{2} \,, & s_{15,75,76} = -1 \,, & s_{16,54,60} = \frac{1}{2} \,, \\
s_{16,55,59} = -\frac{1}{2} \,, & s_{16,56,61} = \frac{1}{2} \,, & s_{16,57,58} = -\frac{1}{2} \,, & s_{16,62,68} = -\frac{1}{2} \,, \\
s_{16,63,67} = \frac{1}{2} \,, & s_{16,64,69} = -\frac{1}{2} \,, & s_{16,65,66} = \frac{1}{2} \,, & s_{16,71,77} = -1 \,, \\
s_{17,54,59} = -\frac{1}{2} \,, & s_{17,55,60} = -\frac{1}{2} \,, & s_{17,56,57} = -\frac{1}{2} \,, & s_{17,58,61} = \frac{1}{2} \,, \\
s_{17,62,67} = -\frac{1}{2} \,, & s_{17,63,68} = -\frac{1}{2} \,, & s_{17,64,65} = -\frac{1}{2} \,, & s_{17,66,69} = \frac{1}{2} \,, \\
s_{17,72,77} = -1 \,, & s_{18,54,61} = \frac{1}{2} \,, & s_{18,55,57} = \frac{1}{2} \,, & s_{18,56,60} = -\frac{1}{2} \,, \\
s_{18,58,59} = \frac{1}{2} \,, & s_{18,62,69} = \frac{1}{2} \,, & s_{18,63,65} = \frac{1}{2} \,, & s_{18,64,68} = -\frac{1}{2} \,, \\
s_{18,66,67} = \frac{1}{2} \,, & s_{18,73,77} = -1 \,, & s_{19,54,58} = -\frac{1}{2} \,, & s_{19,55,56} = -\frac{1}{2} \,, \\
s_{19,57,60} = -\frac{1}{2} \,, & s_{19,59,61} = -\frac{1}{2} \,, & s_{19,62,66} = -\frac{1}{2} \,, & s_{19,63,64} = -\frac{1}{2} \,, \\
s_{19,65,68} = -\frac{1}{2} \,, & s_{19,67,69} = -\frac{1}{2} \,, & s_{19,74,77} = -1 \,, & s_{20,54,57} = \frac{1}{2} \,,
\end{array}
$$

$s_{20,55,61} = -\frac{1}{2}$, $s_{20,56,59} = -\frac{1}{2}$, $s_{20,58,60} = -\frac{1}{2}$, $s_{20,62,65} = \frac{1}{2}$,
$s_{20,63,69} = -\frac{1}{2}$, $s_{20,64,67} = -\frac{1}{2}$, $s_{20,66,68} = -\frac{1}{2}$, $s_{20,75,77} = -1$,
$s_{21,54,55} = \frac{1}{2}$, $s_{21,56,58} = \frac{1}{2}$, $s_{21,57,61} = \frac{1}{2}$, $s_{21,59,60} = -\frac{1}{2}$,
$s_{21,62,63} = \frac{1}{2}$, $s_{21,64,66} = \frac{1}{2}$, $s_{21,65,69} = \frac{1}{2}$, $s_{21,67,68} = -\frac{1}{2}$,
$s_{21,76,77} = -1$, $s_{22,53,61} = \frac{1}{2}$, $s_{22,61,70} = -\frac{\sqrt{3}}{2}$, $s_{22,62,78} = -\frac{1}{2}$,
$s_{22,63,74} = -\frac{1}{2}$, $s_{22,64,77} = -\frac{1}{2}$, $s_{22,65,72} = \frac{1}{2}$, $s_{22,66,76} = -\frac{1}{2}$,
$s_{22,67,75} = \frac{1}{2}$, $s_{22,68,73} = \frac{1}{2}$, $s_{22,69,71} = \frac{1}{2}$, $s_{23,53,57} = -\frac{1}{2}$,
$s_{23,57,70} = -\frac{\sqrt{3}}{2}$, $s_{23,62,74} = \frac{1}{2}$, $s_{23,63,78} = -\frac{1}{2}$, $s_{23,64,76} = -\frac{1}{2}$,
$s_{23,65,71} = -\frac{1}{2}$, $s_{23,66,77} = \frac{1}{2}$, $s_{23,67,73} = \frac{1}{2}$, $s_{23,68,75} = -\frac{1}{2}$,
$s_{23,69,72} = \frac{1}{2}$, $s_{24,53,60} = -\frac{1}{2}$, $s_{24,60,70} = -\frac{\sqrt{3}}{2}$, $s_{24,62,77} = \frac{1}{2}$,
$s_{24,63,76} = \frac{1}{2}$, $s_{24,64,78} = -\frac{1}{2}$, $s_{24,65,75} = \frac{1}{2}$, $s_{24,66,74} = -\frac{1}{2}$,
$s_{24,67,72} = -\frac{1}{2}$, $s_{24,68,71} = -\frac{1}{2}$, $s_{24,69,73} = \frac{1}{2}$, $s_{25,53,55} = \frac{1}{2}$,
$s_{25,55,70} = \frac{\sqrt{3}}{2}$, $s_{25,62,72} = -\frac{1}{2}$, $s_{25,63,71} = \frac{1}{2}$, $s_{25,64,75} = -\frac{1}{2}$,
$s_{25,65,78} = -\frac{1}{2}$, $s_{25,66,73} = \frac{1}{2}$, $s_{25,67,77} = -\frac{1}{2}$, $s_{25,68,76} = \frac{1}{2}$,
$s_{25,68,74} = \frac{1}{2}$, $s_{26,53,59} = -\frac{1}{2}$, $s_{26,59,70} = -\frac{\sqrt{3}}{2}$, $s_{26,62,76} = \frac{1}{2}$,
$s_{26,63,77} = -\frac{1}{2}$, $s_{26,64,74} = \frac{1}{2}$, $s_{26,65,73} = -\frac{1}{2}$, $s_{26,66,78} = -\frac{1}{2}$,
$s_{26,67,71} = -\frac{1}{2}$, $s_{26,68,72} = \frac{1}{2}$, $s_{26,69,75} = \frac{1}{2}$, $s_{27,53,58} = \frac{1}{2}$,
$s_{27,58,70} = \frac{\sqrt{3}}{2}$, $s_{27,62,75} = -\frac{1}{2}$, $s_{27,63,73} = -\frac{1}{2}$, $s_{27,64,72} = \frac{1}{2}$,
$s_{27,64,77} = \frac{1}{2}$, $s_{27,66,71} = \frac{1}{2}$, $s_{27,67,78} = -\frac{1}{2}$, $s_{27,68,74} = -\frac{1}{2}$,
$s_{27,69,76} = \frac{1}{2}$, $s_{28,53,56} = \frac{1}{2}$, $s_{28,56,70} = \frac{\sqrt{3}}{2}$, $s_{28,62,73} = -\frac{1}{2}$,
$s_{28,63,75} = \frac{1}{2}$, $s_{28,64,71} = \frac{1}{2}$, $s_{28,65,76} = -\frac{1}{2}$, $s_{28,66,72} = -\frac{1}{2}$,
$s_{28,67,74} = \frac{1}{2}$, $s_{28,68,78} = -\frac{1}{2}$, $s_{28,69,77} = \frac{1}{2}$, $s_{29,53,54} = -\frac{1}{2}$,
$s_{29,54,70} = -\frac{\sqrt{3}}{2}$, $s_{29,62,71} = -\frac{1}{2}$, $s_{29,63,72} = -\frac{1}{2}$, $s_{29,64,73} = -\frac{1}{2}$,
$s_{29,65,74} = -\frac{1}{2}$, $s_{29,66,75} = -\frac{1}{2}$, $s_{29,67,76} = -\frac{1}{2}$, $s_{29,68,77} = -\frac{1}{2}$,
$s_{29,69,78} = -\frac{1}{2}$, $s_{30,54,61} = \frac{1}{2}$, $s_{30,55,57} = -\frac{1}{2}$, $s_{30,56,60} = -\frac{1}{2}$,
$s_{30,58,59} = -\frac{1}{2}$, $s_{30,62,69} = -\frac{1}{2}$, $s_{30,63,65} = \frac{1}{2}$, $s_{30,64,68} = \frac{1}{2}$,
$s_{30,66,67} = \frac{1}{2}$, $s_{30,71,78} = -1$, $s_{31,54,57} = -\frac{1}{2}$, $s_{31,55,61} = -\frac{1}{2}$,
$s_{31,56,59} = \frac{1}{2}$, $s_{31,58,60} = -\frac{1}{2}$, $s_{31,62,65} = -\frac{1}{2}$, $s_{31,63,69} = -\frac{1}{2}$,
$s_{31,64,67} = \frac{1}{2}$, $s_{31,66,68} = -\frac{1}{2}$, $s_{31,72,78} = -1$, $s_{32,54,60} = -\frac{1}{2}$,
$s_{32,55,59} = -\frac{1}{2}$, $s_{32,56,61} = -\frac{1}{2}$, $s_{52,57,58} = -\frac{1}{2}$, $s_{32,62,68} = -\frac{1}{2}$,
$s_{32,63,67} = -\frac{1}{2}$, $s_{32,64,69} = -\frac{1}{2}$, $s_{32,65,66} = -\frac{1}{2}$, $s_{32,73,78} = -1$,
$s_{33,54,55} = \frac{1}{2}$, $s_{33,56,58} = \frac{1}{2}$, $s_{33,57,61} = -\frac{1}{2}$, $s_{33,59,60} = \frac{1}{2}$,
$s_{33,62,63} = \frac{1}{2}$, $s_{33,64,66} = \frac{1}{2}$, $s_{33,65,69} = -\frac{1}{2}$, $s_{33,67,68} = \frac{1}{2}$,
$s_{33,74,78} = -1$, $s_{34,54,59} = -\frac{1}{2}$, $s_{34,55,60} = \frac{1}{2}$, $s_{34,56,57} = -\frac{1}{2}$,
$s_{34,58,61} = -\frac{1}{2}$, $s_{34,62,67} = -\frac{1}{2}$, $s_{34,63,68} = \frac{1}{2}$, $s_{34,64,65} = -\frac{1}{2}$,
$s_{34,66,69} = -\frac{1}{2}$, $s_{34,75,78} = -1$, $s_{35,54,58} = \frac{1}{2}$, $s_{35,55,56} = \frac{1}{2}$,
$s_{35,57,60} = -\frac{1}{2}$, $s_{35,59,61} = -\frac{1}{2}$, $s_{35,62,66} = \frac{1}{2}$, $s_{35,63,64} = \frac{1}{2}$,
$s_{35,65,68} = -\frac{1}{2}$, $s_{35,67,69} = -\frac{1}{2}$, $s_{35,76,78} = -1$, $s_{36,54,56} = \frac{1}{2}$,
$s_{36,55,58} = -\frac{1}{2}$, $s_{36,57,59} = \frac{1}{2}$, $s_{36,60,61} = -\frac{1}{2}$, $s_{36,62,64} = \frac{1}{2}$,
$s_{36,63,66} = -\frac{1}{2}$, $s_{36,65,67} = \frac{1}{2}$, $s_{36,68,69} = -\frac{1}{2}$, $s_{36,77,78} = -1$,
$s_{37,53,62} = -1$, $s_{37,54,71} = -\frac{1}{2}$, $s_{37,55,72} = \frac{1}{2}$, $s_{37,56,73} = \frac{1}{2}$,
$s_{37,57,74} = \frac{1}{2}$, $s_{37,58,75} = \frac{1}{2}$, $s_{37,59,76} = \frac{1}{2}$, $s_{37,60,77} = \frac{1}{2}$,
$s_{37,61,78} = \frac{1}{2}$, $s_{38,53,63} = -1$, $s_{38,54,72} = -\frac{1}{2}$, $s_{38,55,71} = -\frac{1}{2}$,
$s_{38,56,75} = -\frac{1}{2}$, $s_{38,57,78} = -\frac{1}{2}$, $s_{38,58,73} = \frac{1}{2}$, $s_{38,59,77} = -\frac{1}{2}$,
$s_{38,60,76} = \frac{1}{2}$, $s_{38,61,74} = \frac{1}{2}$, $s_{39,53,64} = -1$, $s_{39,54,73} = -\frac{1}{2}$,

$$
\begin{array}{llll}
s_{39,55,75} = \frac{1}{2} \ , & s_{39,56,71} = -\frac{1}{2} \ , & s_{39,57,76} = -\frac{1}{2} \ , & s_{39,58,72} = -\frac{1}{2} \ , \\
s_{39,59,74} = \frac{1}{2} \ , & s_{39,60,78} = -\frac{1}{2} \ , & s_{39,61,77} = \frac{1}{2} \ , & s_{40,53,65} = -1 \ , \\
s_{40,54,74} = -\frac{1}{2} \ , & s_{40,55,78} = \frac{1}{2} \ , & s_{40,56,76} = \frac{1}{2} \ , & s_{40,57,71} = -\frac{1}{2} \ , \\
s_{40,58,77} = -\frac{1}{2} \ , & s_{40,59,73} = -\frac{1}{2} \ , & s_{40,60,75} = \frac{1}{2} \ , & s_{40,61,72} = -\frac{1}{2} \ , \\
s_{41,53,66} = -1 \ , & s_{41,54,75} = -\frac{1}{2} \ , & s_{41,55,73} = -\frac{1}{2} \ , & s_{41,56,72} = \frac{1}{2} \ , \\
s_{41,57,77} = \frac{1}{2} \ , & s_{41,58,71} = -\frac{1}{2} \ , & s_{41,59,78} = -\frac{1}{2} \ , & s_{41,60,74} = -\frac{1}{2} \ , \\
s_{41,61,76} = \frac{1}{2} \ , & s_{42,53,67} = -1 \ , & s_{42,54,76} = -\frac{1}{2} \ , & s_{42,55,77} = \frac{1}{2} \ , \\
s_{42,56,74} = -\frac{1}{2} \ , & s_{42,57,73} = \frac{1}{2} \ , & s_{42,58,78} = \frac{1}{2} \ , & s_{42,59,71} = -\frac{1}{2} \ , \\
s_{42,60,72} = -\frac{1}{2} \ , & s_{42,61,75} = -\frac{1}{2} \ , & s_{43,53,68} = -1 \ , & s_{43,54,77} = -\frac{1}{2} \ , \\
s_{43,55,76} = -\frac{1}{2} \ , & s_{43,56,78} = \frac{1}{2} \ , & s_{43,57,75} = -\frac{1}{2} \ , & s_{43,58,74} = \frac{1}{2} \ , \\
s_{43,59,72} = \frac{1}{2} \ , & s_{43,60,71} = -\frac{1}{2} \ , & s_{43,61,73} = -\frac{1}{2} \ , & s_{44,53,69} = -1 \ , \\
s_{44,54,78} = -\frac{1}{2} \ , & s_{44,55,74} = -\frac{1}{2} \ , & s_{44,56,77} = -\frac{1}{2} \ , & s_{44,57,72} = \frac{1}{2} \ , \\
s_{44,58,76} = -\frac{1}{2} \ , & s_{44,59,75} = \frac{1}{2} \ , & s_{44,60,73} = \frac{1}{2} \ , & s_{44,61,71} = -\frac{1}{2} \ , \\
s_{45,53,71} = -\frac{1}{2} \ , & s_{45,54,62} = -\frac{1}{2} \ , & s_{45,55,63} = -\frac{1}{2} \ , & s_{45,56,64} = -\frac{1}{2} \ , \\
s_{45,57,65} = -\frac{1}{2} \ , & s_{45,58,66} = -\frac{1}{2} \ , & s_{45,59,67} = -\frac{1}{2} \ , & s_{45,60,68} = -\frac{1}{2} \ , \\
s_{45,61,69} = -\frac{1}{2} \ , & s_{45,70,71} = -\frac{\sqrt{3}}{2} \ , & s_{46,53,72} = -\frac{1}{2} \ , & s_{46,54,63} = -\frac{1}{2} \ , \\
s_{46,55,62} = \frac{1}{2} \ , & s_{46,56,66} = \frac{1}{2} \ , & s_{46,57,69} = \frac{1}{2} \ , & s_{46,58,64} = -\frac{1}{2} \ , \\
s_{46,59,68} = \frac{1}{2} \ , & s_{46,60,67} = -\frac{1}{2} \ , & s_{46,61,65} = -\frac{1}{2} \ , & s_{46,70,72} = -\frac{\sqrt{3}}{2} \ , \\
s_{47,53,73} = -\frac{1}{2} \ , & s_{47,54,64} = -\frac{1}{2} \ , & s_{47,55,66} = -\frac{1}{2} \ , & s_{47,56,62} = \frac{1}{2} \ , \\
s_{47,57,67} = \frac{1}{2} \ , & s_{47,58,63} = \frac{1}{2} \ , & s_{47,59,65} = -\frac{1}{2} \ , & s_{47,60,69} = \frac{1}{2} \ , \\
s_{47,61,68} = -\frac{1}{2} \ , & s_{47,70,73} = -\frac{\sqrt{3}}{2} \ , & s_{48,53,74} = -\frac{1}{2} \ , & s_{48,54,65} = -\frac{1}{2} \ , \\
s_{48,55,69} = -\frac{1}{2} \ , & s_{48,56,67} = -\frac{1}{2} \ , & s_{48,57,62} = -\frac{1}{2} \ , & s_{48,58,68} = \frac{1}{2} \ , \\
s_{48,59,64} = \frac{1}{2} \ , & s_{48,60,66} = -\frac{1}{2} \ , & s_{48,61,63} = \frac{1}{2} \ , & s_{48,70,74} = -\frac{\sqrt{3}}{2} \ , \\
s_{49,53,75} = -\frac{1}{2} \ , & s_{49,54,66} = -\frac{1}{2} \ , & s_{49,55,64} = \frac{1}{2} \ , & s_{49,56,63} = -\frac{1}{2} \ , \\
s_{49,57,68} = -\frac{1}{2} \ , & s_{49,58,62} = \frac{1}{2} \ , & s_{49,59,69} = \frac{1}{2} \ , & s_{49,60,65} = \frac{1}{2} \ , \\
s_{49,61,67} = -\frac{1}{2} \ , & s_{49,70,75} = -\frac{\sqrt{3}}{2} \ , & s_{50,53,76} = -\frac{1}{2} \ , & s_{50,54,67} = -\frac{1}{2} \ , \\
s_{50,55,68} = -\frac{1}{2} \ , & s_{50,56,65} = \frac{1}{2} \ , & s_{50,57,64} = -\frac{1}{2} \ , & s_{50,58,69} = -\frac{1}{2} \ , \\
s_{50,59,62} = \frac{1}{2} \ , & s_{50,60,63} = \frac{1}{2} \ , & s_{50,61,66} = \frac{1}{2} \ , & s_{50,70,76} = -\frac{\sqrt{3}}{2} \ , \\
s_{51,53,77} = -\frac{1}{2} \ , & s_{51,54,68} = -\frac{1}{2} \ , & s_{51,55,67} = \frac{1}{2} \ , & s_{51,56,69} = -\frac{1}{2} \ , \\
s_{51,57,66} = \frac{1}{2} \ , & s_{51,58,65} = -\frac{1}{2} \ , & s_{51,59,63} = -\frac{1}{2} \ , & s_{51,60,62} = \frac{1}{2} \ , \\
s_{51,61,64} = \frac{1}{2} \ , & s_{51,70,77} = -\frac{\sqrt{3}}{2} \ , & s_{52,53,78} = -\frac{1}{2} \ , & s_{52,54,69} = -\frac{1}{2} \ , \\
s_{52,55,65} = \frac{1}{2} \ , & s_{52,56,68} = -\frac{1}{2} \ , & s_{52,57,63} = -\frac{1}{2} \ , & s_{52,58,67} = \frac{1}{2} \ , \\
s_{52,59,66} = -\frac{1}{2} \ , & s_{52,60,64} = -\frac{1}{2} \ , & s_{52,61,62} = \frac{1}{2} \ , & s_{52,70,78} = -\frac{\sqrt{3}}{2} \ .
\end{array}
$$

C The matrices.

Here we write the matrices c_i, $i = 53, \ldots, 78$.

$$
c_{53} = \begin{pmatrix}
-\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & -\frac{i}{2\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{i}{2\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\
-\frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,0,0,0,0,0,0,0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \qquad (C.1)
$$

$$
c_{54} = \begin{pmatrix}
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\
0 & -i\sqrt{\frac{2}{3}} & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{2i}{\sqrt{3}} & 0,0,0,0,0,0,0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \qquad (C.2)
$$

$$
c_{55} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{\sqrt{6}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{\sqrt{3}} \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -i\sqrt{\frac{2}{3}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -\frac{2i}{\sqrt{3}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.3}
$$

$$
c_{56} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{\sqrt{6}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{\sqrt{3}} \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0 \\
0, & 0, & 0, & -i\sqrt{\frac{2}{3}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0 & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & -\frac{2i}{\sqrt{3}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.4}
$$

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c_{57} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -i\sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.5}
$$

$$
c_{58} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -i\sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.6}
$$

$$
c_{59} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -i\sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.7}
$$

$$
c_{60} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -i\sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.8}
$$

$$
c_{61}=\left(\begin{array}{c}
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{i}{\sqrt{6}},0,0,0,0,0,0,0,0,-\frac{i}{\sqrt{3}}\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{i}{2},0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{i}{2},0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{i}{2},0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{i}{2},0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,-i\sqrt{\frac{2}{3}},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{i}{2},0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,-\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,-\frac{2i}{\sqrt{3}},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
\end{array}\right) \tag{C.9}
$$

$$
c_{62}=\left(\begin{array}{c}
0,0,0,0,0,0,0,0,0,-\frac{i}{\sqrt{2}},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-\frac{i}{2},0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2},0\\
-\frac{i}{2\sqrt{2}},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,\frac{i}{2\sqrt{6}},0,0,0,0,0,0,0,0,-\frac{i}{\sqrt{3}}\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,\frac{i}{\sqrt{6}},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,-\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,\frac{i}{2},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\\
0,0,0,0,0,0,0,0,0,-\frac{2i}{\sqrt{3}},0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
\end{array}\right) \tag{C.10}
$$

$$
c_{63} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.11}
$$

$$
c_{64} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.12}
$$

$$
c_{65} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i\frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \quad \text{(C.13)}
$$

$$
c_{66} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \quad \text{(C.14)}
$$

$$
c_{67} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
-\frac{i}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2\sqrt{6}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{\sqrt{3}} \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{\sqrt{6}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{2i}{\sqrt{3}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.15}
$$

$$
c_{68} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
-\frac{i}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2\sqrt{6}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{\sqrt{3}} \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{\sqrt{6}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{2i}{\sqrt{3}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0
\end{pmatrix} \tag{C.16}
$$

$$
c_{69} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.17}
$$

$$
c_{70} = \begin{pmatrix}
-\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} \\
0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\kappa}{2} & 0 \\
\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.18}
$$

where $\kappa = \frac{i}{\sqrt{3}}$,

$$
c_{71} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.19}
$$

$$
c_{72} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.20}
$$

$$
c_{73} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.21}
$$

$$
c_{74} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{C.22}
$$

$$
c_{75} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{\sqrt{2}}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
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\frac{i}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2\sqrt{6}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{\sqrt{3}} \\
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0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{2i}{\sqrt{3}}, & 0, & 0, & 0, & 0
\end{pmatrix} \qquad \text{(C.23)}
$$

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c_{76} = \begin{pmatrix}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{\sqrt{2}}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{2}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
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\frac{i}{2\sqrt{2}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & \frac{i}{2\sqrt{6}}, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{i}{\sqrt{3}} \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & -\frac{2i}{\sqrt{3}}, & 0, & 0, & 0
\end{pmatrix} \qquad \text{(C.24)}
$$

$$
c_{77} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0 & 0
\end{pmatrix} \tag{C.25}
$$

$$
c_{78} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{2}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{i}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{\sqrt{6}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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\frac{i}{2\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{i}{2\sqrt{6}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{i}{\sqrt{3}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2i}{\sqrt{3}} & 0
\end{pmatrix} \tag{C.26}
$$

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