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Scattering of relativistic electron beams by magnetic field errors and beam-induced waves

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Relativistic electron beams propagating in long plasma columns must be well focused to cause efficient plasma heating. Expansion of the beam area due to scattering lowers efficiency. We calculate the beam spreading expected from errors in the ambient magnetic field. We then include scattering from both electrostatic and magnetic waves generated by the beam itself. All these effects can be important in contemplated experimental regimes. However, it may prove possible to "tune" beam-plasma heating processes to avoid significant beam spreading.

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I. INTRODUCTION

Recently, attention has focused on using intense relativistic electron beams to heat long columns of plasma for fusion applications.[1] The beam and plasma would be confined by a solenoidal magnetic field and the beam energy deposited through a variety of beam-plasma instabilities.

The efficiency of such a device depends on focusing the beam to relatively small diameters (~1 cm). A natural question is whether the electron beam launched at one end of a long column (~100 m) will spread across the magnetic field lines, due to scattering by electric and magnetic fields in the plasma. If beam focusing erodes significantly as it propagates down the column, expensive countermeasures, such as increasing the ambient magnetic field, may be necessary.

This paper calculates the beam scattering from (1) errors in the ambient magnetic field, due to faulty positioning of magnets or fringing fields, (2) electrostatic beam-driven streaming instabilities, and (3) magnetic modes, such as the Alfven waves. Our scattering formalism is simpler than the early work of Hall and Sturrock,[2] from which a number of astrophysical applications have been derived.[3-6] However, since we deal with spatial diffusion transverse to B, rather than multiple reflections of particles along B (a process leading to a density gradient), the extensively developed astrophysical results do not carry over directly. For the most part, we have derived approximate forms which are useful in light of the fact that we never know in great detail the spectrum of either field errors or beam-excited waves.

II. SCATTERING FROM GUIDE-FIELD ERRORS

We begin with the quasilinear expression for the perturbed beam distribution f_1, due to scattering by random electric δE and magnetic $\delta\mathrm{B}$ fields, in terms of the equilibrium distribution $f_0(\mathbf{x},\mathbf{p})$, where q is the particle charge, velocity is $\mathbf{v}$, and momentum is $\mathbf{p}$. The beam moves along a guide field B_0, along the z axis.

$$\frac{\partial f_0}{\partial f}+\mathbf{v}\cdot\nabla f_0+\frac{q}{c}(\mathbf{v}\times\mathbf{B}_0)\cdot\frac{\partial}{\partial\mathbf{p}}f_0 =q^2\left\langle\left(\delta\mathbf{E}+\frac{\mathbf{v}}{c}\times\delta\mathbf{B}\right)\cdot\frac{\partial}{\partial\mathbf{p}}\int_{t_0}^{t}\left(\delta\mathbf{E}+\frac{\mathbf{v}}{c}\times\delta\mathbf{B}\right)\cdot\frac{\partial}{\partial\mathbf{p}}f_0\,dt'\right\rangle. \quad (1)$$

Here the integration over t' follows the zero-order helical orbits of the beam particles. In Eq. (1), we have discarded the "ballistic" propagation forward of initial perturbations. Particles begin their orbits at time t_0. Then, $t-t_0$ must be small compared to the time required to perturb the particles from their zero-orbit trajectories, and, in order to simplify Eq. (1) further, we must assume the field-error fluctuations, as seen by the particles, last a short time compared to $t-t_0$. This means particles diffuse in a stochastic "bath" of $\delta\mathrm{B}$, passing by the field fluctuations quickly compared with a diffusive time scale. Obviously, if the field errors are systematic and lengthy, they will be highly correlated along the particle orbit and this assumption will fail. If $t\gg t_0$, we can set $t_0\to-\infty$ in the integral and recover the standard (relativistic) quasilinear expression. In general, the fast electron gyromotion will contribute terms involving the gyroangle and, using cylindrical coordinates, will yield a sum over Bessel functions. The complete form appears in Eq. (12). However, to clarify matters, for the moment we anticipate that in practice we shall not know the fluctuation spectrum in enough detail to justify retaining such detail in the particle dynamics. Thus, we assume that the zero-order distribution f_0 is a slowly varying function of the guiding center orbit. Then we shall estimate how the change in particle pitch angles influences cross-field diffusion. We write the fluctuation power spectrum as a tensor

$$S_{ij}(\mathbf{k})=\int d^3\eta\,\langle\delta\mathbf{B}_i(\mathbf{x})\delta\mathbf{B}_j(\mathbf{x}+\eta)\rangle\exp(i\mathbf{k}\cdot\eta) \quad (2)$$

and take $\delta\mathbf{E}=0$. Here (1, 2, 3) corresponds to (z, θ, r). Then, Eq. (1) becomes

$$\frac{\partial f_0}{\partial t}+\mathbf{v}\cdot\nabla f_0+\frac{q}{c}(\mathbf{v}\times\mathbf{B}_0)\cdot\frac{\partial}{\partial p}f_0 =\frac{v}{p^2}\mathbf{v}\cdot\left[\frac{\partial}{\partial\mathbf{p}}\times\frac{\Omega^2}{2\pi B_0^2}\int_{-\infty}^{\infty}d\mathbf{k}\,S_{ij}(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r}) \times\int_{-\infty}^{t}\left(\mathbf{v}\times\frac{\partial}{\partial\mathbf{p}}f_0\right)_j\exp(-i\mathbf{k}\cdot\mathbf{r})\,dt', \right. \quad (3)$$

where

$$\Omega=eB_0/mc\gamma. \quad (4)$$

Now we make the important assumption that the power

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spectrum S_{ij} can be adequately represented by the spectrum in k_z. This means the field errors perpendicular to B_0 are distributed in the same manner as those along B_0. This assumption vastly simplifies the analysis. To see why it may be valid, consider that the most strongly affected particles are resonant with some portion of the power spectrum, $k_z = \Omega / v_z$. For pitch angle ψ, this means $k_z r_L = \tan\psi$, with r_L the Larmor radius, $v_\perp / \Omega$. Diffusion is usually most important for $k_\perp r_L \sim 1$. Thus, if $\tan\psi \sim 1$ for the bulk of the distribution, the power spectrum in k_z can represent that in $k_\perp$ reasonably well. Thus, we take

$$S_{ij}(\mathbf{k}) = S_\parallel(k_z)\delta_{1i}\delta_{j1}\delta(k_2)\delta(k_3). \tag{5}$$

If $f_0(\mathbf{x}, \mathbf{p})$ is independent of x inside the beam to a good approximation, we can neglect the spatial gradients in Eq. (3). Writing $\mu = \cos\psi$ and neglecting any beam density gradients along z, Eq. (3) becomes

$$\frac{\partial f_0}{\partial t} = \frac{\Omega^2}{4\pi B_0^2} \frac{\partial}{\partial \mu}\left(\frac{1-\mu^2}{|\mu|}\int_{-\infty}^{\infty} dk_z\, S_\parallel(k_z) \times \frac{\sin\{[k_z - (\Omega/\mu v)]\mu v t\}}{[k_z - (\Omega/\mu v)]} \frac{\partial f_0}{\partial \mu}\right). \tag{6}$$

The sine term in Eq. (6) will force the k_z integration to zero if the coherence length l^* of $S_\parallel$ is comparable to the distance a particle travels, $v\mu t$. We are studying expected field errors which are not correlated over distances exceeding a few r_L, so $v\mu t \gg l^*$. Then, the sine function becomes a δ function and we find, after integrating over the beam cross section,

$$\frac{dn_0(r)}{dt} = \frac{\Omega^2}{4B_0^2}\frac{\partial}{\partial\mu}\left[\frac{1-\mu^2}{v|\mu|} S_\parallel\left(k_z = \frac{\Omega}{\mu v}\right)\frac{\partial n_0}{\partial \mu}\right] \equiv \frac{\partial}{\partial\mu} D_\mu (1-\mu^2)\frac{\partial n_0}{\partial\mu}. \tag{7}$$

This is a diffusion equation for the density which must be integrated over μ and v for the beam; all particles presumably begin at the same axial position. An average scattering time τ obtained from Eq. (7) describes a diffusion in pitch angle of $\Delta\psi \sim 1$, wherein particles steadily rearrange themselves with respect to B_0. They step sidewise a distance r_L whenever the diffusive scattering changes μ appreciably, so that the transverse spatial diffusion coefficient $D_\perp$ obeys

$$D_\perp \approx r_L^2/\tau. \tag{8}$$

Traversing a distance L along z in a time $L/v\mu$, a beam particle diffuses. Averaging over the beam velocities and pitch angles, the increase in beam area is

$$\langle(\Delta x)^2\rangle \simeq \langle \tan^2\psi\rangle \langle S_\parallel(k_z = \Omega/\mu v)\rangle L / 4B_0^2. \tag{9}$$

We can visualize $\langle S_\parallel(k_z = \Omega/\mu v)\rangle$ as the fluctuation strength $\langle(\delta B)^2\rangle$ times a characteristic dimension of the field error, l_c. Then denoting beam radius by a,

$$\frac{\langle(\Delta x)^2\rangle}{a^2} \simeq \langle\tan^2\psi\rangle \left\langle\left(\frac{\delta B_k}{B_0}\right)\right\rangle^2 \frac{L l_c}{4a^2}, \tag{10}$$

and l_c represents an average over the field-error "spectrum".

For a 1-cm-radius beam traversing a 100-m system, with $\langle\tan^2\psi\rangle = 1$, the beam area doubles when

$$\left\langle\left(\frac{\delta B_k}{B_0}\right)^2\right\rangle \frac{l_c}{a} \sim 4\times 10^{-4}. \tag{11}$$

Designs for fusion systems use $B_0 \approx 50$ kG, so errors of the order of 1 kG can be tolerated if the average l_c is small, as seems probable.

A more general treatment, modifying the work of Hall and Sturrock,[2] gives

$$\frac{\langle(\Delta x)^2\rangle}{a^2} = \frac{B_0^{-2}L}{4a^2}\sum_{n=-\infty}^{\infty}\int f_0(v_\perp, v_z)\, d\mathbf{v}\left[\tan^2\psi J_{n+1}^2\left(\frac{k_\perp v_\perp}{\Omega}\right) \times \left\langle S_\parallel\left(k_z = \frac{n\Omega}{v_z}\right)\right\rangle + J_n^2\left(\frac{k_\perp v_\perp}{\Omega}\right)\left\langle S_{33}\left(k_z = \frac{n\Omega}{v_z}\right)\right\rangle\right]. \tag{12}$$

If the field-error spectrum is well known in $k_\perp$ and k_z, one can assign average correlation lengths for each direction and carry out the sum. Note that Eq. (12) reduces to Eq. (9) for $S_{33} = 0$ and $n = -1$, if $J_0^2(k_\perp r_L) \approx 1$.

Equation (12) is a quite accurate representation of the essential physics. The approximations involved in the simpler form, Eq. (11), probably make it accurate to within a factor of 2.

We have treated resonant diffusion because, in the context of quasilinear theory, nonresonant diffusion is "fake" diffusion, i.e., memory of initial orbits is not lost (Ref. 7). It is possible to generalize quasilinear theory by including resonance broadening.[8] However, this demands knowledge of the statistical properties of the fluctuations, which, in general, we do not have.

Also, nonresonant contributions are largest for very short fluctuations ($k \gg \Omega/\mu v$) (because short fluctuations are sensed as quick "collisions", whereas long fluctuations are adiabatic in the particle frame, and thus yield no diffusion.) For systems with magnets spaced at intervals exceeding 10 cm,[9] the errors will probably be 10 cm or longer, whereas the resonant $\lambda_z = 2\pi\mu v/\Omega \sim 1$ cm for $\gamma = 10$, $B = 50$ kG, $\mu \sim 0.5$, and $v \approx c$. Thus, there should be very little $S_\parallel(k \gg \Omega/\mu v)$, and nonresonant scattering will be unimportant for this application.

III. SCATTERING FROM BEAM-INDUCED WAVES

A. Electrostatic modes

Resonant scattering of a beam occurs most easily when the beam itself produces waves in the background plasma which are very nearly resonant with the beam velocity; i.e., those which correspond to space-charge oscillations on the beam.

An obvious candidate for such a wave is the familiar streaming instability. The electrostatic instability is usually dominant over the electromagnetic form,[10] and Eq. (12) can be modified easily to include electrostatic wave scattering by adding to S_{11} a term $(c/\mu v)^2 S_{11}^E$, where

$$S_{11}^E(\mathbf{k}) = \int d\eta\, \langle\delta E_z(\mathbf{x})\delta E_z(\mathbf{x}+\eta)\rangle \exp(i\mathbf{k}\cdot\eta). \tag{13}$$

and a similar form for S_{33}. Then a nonlinear theory (for example, Ref. 10) for the saturated value of the electric fields and their spectral range can be used to calculate the diffusion, as in the previous discussion.

B. Magnetic modes

Since the beam-heated plasma is "high beta" in the sense of having plasma pressure comparable to magnetic field pressure, there may be significant magnetic waves present to scatter the beam. In particular, waves transverse to B_z are most effective because they exert a steady decelerating force in the particle rest frame. If the transverse wave magnetic field $\delta\mathbf{B}$ exerts a constant force parallel to $\mathbf{B}_0$ through the Lorentz force $\mathbf{v}_\perp \times \delta\mathbf{B}$, the pitch angle of the resonant electrons scatters during the correlation time τ during which the field and particle are in resonance.

Low-frequency modes transverse to B_0 are Alfven type when[11]

$$k_z c \ll 2\omega_{pi} \ll \gamma\Omega, \tag{14}$$

where ω_{pi} is the ion plasma frequency. They are helicon type when

$$2\omega_{pi} \ll k_z \ll \gamma\Omega.$$

Alfven waves obey

$$\omega = k_z v_A \equiv k_z B(4\pi n_i m_i)^{-1/2}, \tag{15}$$

and helicons have $\omega \approx \gamma\Omega(k_z c/\omega_{pe})^2$. For resonance with the beam, we require $\Omega \approx (k_z v_z - \omega)/n$. However, for a hydrogen plasma, $\omega = k_z v_A \sim 2 10^8 k_z \ll \Omega$ for systems of interest, so $k_z \approx n\Omega/v_z$. Now $\gamma \gg 1$ ensures $\gamma\Omega \gg k_z c$, but $\omega_{pi}/k_z c \sim 1$, so the waves are in the region of the dispersion relation between simple Alfven waves and helicon modes. For simplicity, we consider the Alfven region. We take a system of interest to have $\gamma = 10$, $B_0 = 50$ kG, $\mu \approx 0.5-1.0$, $v \approx c$, and $n_p \leq 10^{16}$ cm^{-3}.

To excite Alfven waves requires[12]

$$(v_A/v\mu) < m\gamma/M, \tag{16}$$

where v_A is the Alfven velocity and M is the ion mass. For gross confinement, the beam must be stable against filamentation,[13] which requires

$$(J\gamma)^{1/2} < 10^{-3}B,$$

where J is current density in A/cm^2 and B is the field in kG. The 50-kG field satisfies this condition. Equation (16) may be written, taking $\mu = 0.5$ and the average atomic number of ions as 4,

$$3.35 < \frac{n_p}{10^{16}\ \text{cm}^{-3}} \frac{\gamma}{10} \frac{10^3}{B} \quad ;$$

so for our contemplated system of interest, this condition is satisfied by a factor of 6. These waves can be excited either by the beam current itself or by background plasma electrons which are counterdrifting to carry a return current. However, the plasma electrons will generally not drift faster than v_A, and we neglect instability due to them. Thus, we turn to solely beam-generated turbulence.

C. Alfven-wave growth rate

We consider a relativistic electron beam with beam frequency

$$\omega_b = (4\pi n_b e^2/\gamma m)^{1/2} \tag{17}$$

and distribution function $f_0(\mathbf{p}, \mu)$. A general form for the growth rate has been given by Lerche[5] for applications to cosmic rays; we adapt to our case and find the growth rate

$$\gamma_k = \frac{\pi}{8} \frac{\omega_b^2}{\Omega} \frac{v_A}{c} \int_0^\infty \int_0^\pi 2\pi p^2 dp \sin\mu \, d\mu \times p_0 \left[\left(p \frac{v_A}{c} \frac{\partial f_0}{\partial p} + \frac{\partial f_0}{\partial \mu}\right)(1-\mu^2)\right] \delta\left(p\mu - \frac{eB_z}{kc}\right), \tag{18}$$

where $\mu = p_z/p$. We expect the beam distribution f_0 will have a wide distribution in μ and will be peaked in momentum p at $p_0 = m\gamma v_b$. The integral in γ_k will then give a factor no smaller than $p_0/\Delta p$, where Δp is a width of the distribution in momentum space. We can then estimate

$$\gamma_k \gtrapprox \frac{\pi}{8} \frac{\omega_b^2}{\Omega} \frac{v_A}{c} {}^2 \frac{p_0}{\Delta p}. \tag{19}$$

This is independent of n_b, and γ and yields for our parameters

$$\gamma_k \approx 3B_0(p_0/\Delta p)10^3, \tag{20}$$

so with $B_0 = 10^5$ G, $p_0/\Delta p \sim 1$ and growth times ~ 3 nsec result. These waves will be excited throughout the plasma column and are convected at phase velocity v_A, not c, so they are virtually stationary in the beam frame After ~ 30 nsec, they should reach large amplitude and begin to resonantly scatter the beam. However, to make a useful calculation, we must estimate the saturated magnetic fields of the waves, for use in Eq. (12).

D. Nonlinear saturation of Alfven waves

Alfven waves will grow to an amplitude which is limited by their coupling to other lower-frequency modes such as the ion acoustic. The ion-acoustic spectrum may also be excited for a time by return-current instabilities. However, electron heating and nonlinear effects may stabilize the ion sound waves by the time the slower-growing Alfven modes rise to large amplitudes. We shall assume this in order to simplify the calculation.[4] Energy balance is expressed by Sagdeev and Galeev as[7]

$$\frac{\partial N_k}{\partial t} = 4\pi \sum_{k';q} |V_{k,k';q}|^2 [N_{k'}N_q - \text{sgn}(\omega_k\omega q) \times N_k N_{k'} - \text{sgn}(\omega_k\omega_{k'})N_k N_q]\delta(\omega_k - \omega_{k'} - \omega_q)\delta_{k,k'} + q, \tag{21}$$

where the initial Alfven wave k decays into another Alfven mode k' and an ion sound wave q is given by $\omega = qc_s$, with c_s the sound speed. $\omega_k N_k$ is the energy density in the kth mode, both mechanical (kinetic) and electromagnetic,

$$N_k = (\delta B_k)^2/k 8\pi\omega_k. \tag{22}$$

The transition probabilities $V_{k,k',q}$ are given by the usual time-dependent perturbation expressions.[11] If $c_s \lesssim v_A$, $\omega_k \approx \omega_{k'}$, and $\omega_q \approx \omega_k 2c_s/v_A = qc_s$, then

$$|V_{k,k'}q|^2 \approx \omega_k\omega_k'\omega_q/\rho_m c_s^2 \approx 2\omega_k^3/\rho_m v_A c_s, \tag{23}$$

where ρ is the plasma mass density, $n_p M$, with M being the ion mass. The above expression is valid for plane-polarized Alfven waves; for circularly polarized modes, another anumerical factor of the order of unity enters. The sum over k and k', using the δ function, results in a factor v_A^{-1}.

If the ion-acoustic modes are linearly stable, because the drift velocity of the plasma electrons is less the c_s (as we expect if some electron heating has occurred), we can approximate $N_q \approx 0$. Then,

$$\frac{dN_k}{dt} \approx -\frac{4\pi}{v_A} N_k N_{k-l} \left| V_{k,k-l} \right|^2, \tag{24}$$

where

$$l = 2k(c_s / v_A). \tag{25}$$

However, most of the energy transfer of this process goes into Alfven waves of lower k; so to find $N_k N_{k-l}$, we perform a Taylor expansion,

$$N_k N_{k-l} \left| V_{k,k-l} \right|^2 \approx l\left(\frac{\partial}{\partial k} N_k N_{k-l} \left| V_{k,k-l} \right|^2\right)_{l=0}. \tag{26}$$

The rate of energy loss can be expressed as a dissipative frequency ω_d,

$$\frac{dN_k}{dt} = -\omega_d N_k,$$

$$\omega_d \approx \frac{4(\delta B_k)^2 k}{\rho_m v_A} \alpha, \tag{27}$$

where

$$\frac{\partial}{\partial k} N_k N_{k-l} \left| V_{k,k-l} \right|^2 = \alpha N_k N_{k-l} \left| V_{k,k-l} \right|^2.$$

If $N_k \sim$ const in the region of interest $(k = n\Omega / c)$, then $\alpha \approx 3$. We can write

$$\omega_d \approx 16\pi \left| \delta B_k / B_0 \right|^2 \alpha \omega \tag{28}$$

near the resonance. Stabilization occurs when

$$\omega_d = \gamma_k = \frac{\pi}{8}\left(\frac{\omega_b}{\Omega}\right)^2 \Omega \left(\frac{v_A}{c}\right)^2 \frac{p_0}{\Delta_p} \tag{29}$$

or

$$\left\langle \left(\frac{\delta B_k}{B_0}\right)^2 \right\rangle \approx (128\alpha)^{-1} \left(\frac{\omega_b}{\Omega}\right)^2 \frac{v_A}{c} \frac{p_0}{\Delta p}. \tag{30}$$

For $\alpha = 3$, $B = 100$ kG,

$$\langle \delta B_k / B_0 \rangle \sim (p_0 / \Delta p) 10^{-5}. \tag{31}$$

This implies that very monoenergetic beams $(\Delta p \ll p_0)$ are more effectively scattered by their self-generated waves than "hot" beams, since they produce a stronger spectrum. This will be so as long as the growth rate, Eq. (19), is not so small that the Alfven waves never reach saturation in a beam pulse time. Equation (31) can be used in the general formalism of Eq. (12), though Eq. (31) is not precise enough to justify doing a detailed sum and integration. Using Eq. (31) in our more approximate form, Eq. (10), we find for $L = 10^5$ cm.,

$$\left\langle \left(\frac{\Delta x}{a}\right)^2 \right\rangle \sim \frac{p_0}{\Delta p} \frac{l_c^*}{40}, \tag{32}$$

where l_c^* is the average length over which beam particles and the beam-generated fields are correlated. A crude estimate of this is

$$l_c^* \sim 2\pi (v_z / \Delta\omega) = 2\pi v_z [\Delta(k_z v_z - n\Omega)]^{-1}, \tag{33}$$

since the wave-particle phase relation depends on the "sharpness" of $\omega - k_z v_z - n\Omega$, and $\omega \ll k_z v_z$. Assuming $\Delta k_z = 0$, i.e., a single standing wave in the plasma column,

$$l_c^* \approx \frac{2\pi v_z}{k_z \Delta v_z} = \frac{2\pi}{n\Omega} \frac{(v\mu)^2}{\Delta(v\mu)} = \frac{2\pi}{n\Omega} \frac{1}{\Delta(1/v\mu)}. \tag{34}$$

From Eq. (32), estimating $\Delta p = p_0 \Delta\mu$ where $\Delta\mu$ is the spread in μ, and using $B_0 = 10^5$ G again,

$$\left\langle \left(\frac{\Delta x}{a}\right)^2 \right\rangle \sim \frac{1}{400} \left(\frac{\mu}{\Delta\mu}\right)^2 \left(\frac{v}{c}\right)^2; \tag{35}$$

so only if $\Delta\mu < 0.1\mu$ is the wave scattering significant. However, only more detailed study of the nonlinear estimates made here will provide an estimate reliable to better than an order of magnitude. For example, if the ion-acoustic spectrum is active because of return-current instabilities, $\langle (\delta B_k / B_0)^2 \rangle$ may be larger. In this case, estimates of saturated field amplitudes have been given[12] and can be substituted in our formalism, Eq. (10) or (12), directly.

IV. CONCLUSIONS

We have found that expansion of a relativistic electron beam may come from several sources of field fluctuations. The errors in the ambient field B_0 may scatter quite effectively; Eq. (10) gives an approximate form, and Eq. (12) gives a more general result.

Scattering by beam-induced instabilities poses a larger number of unknowns, since we must first know the nonlinear saturated spectrum of waves. Modification of Eq. (12) for electrostatic modes is simple [see Eq. (13)], and some existing electrostatic wave theories[10] may give reliable results for the fields.

Magnetic modes, however, can be as effective as the electrostatic waves, particularly in the cases where plasma pressure is comparable to ambient magnetic field pressure. We have studied the scattering from beam-driven Alfven modes, attempting an approximate treatment of the nonlinear saturation of these waves by mode coupling to ion-acoustic waves. Our crude estimates, summarized in Eq. (32), suggest that scattering from beam-induced modes may not be as significant as scattering from errors in the ambient field. Calculations for particular regimes of interest may yield somewhat different estimates, however.

Fusion systems using intense electron beams cannot tolerate very much spreading of the beam, since thermonuclear ignition is the paramount problem. Thus, spreading of the heated cylinder reduces effectiveness so much that a doubling of the beam radius (i.e., decrease of maximum attainable temperature by a factor of 4) is probably the tolerable upper limit on beam diffusion. (This is deduced from Refs. 1 and 9.)

Equation (35) is cause for optimism. It shows that a reasonably "hot" beam, with significant spread in pitch angles μ, does not produce a magnetic field spectrum large enough to yield significant beam diffusion.

Equations (11) and (12) suggest that, if field errors on a scale less than 1 cm can be avoided, no large diffusion occurs.

However, there remain two sources of fluctuations which depend on precisely how beam-plasma heating proceeds: (i) scattering by electrostatic waves [Eq. (13)] and (ii) scattering by ion waves caused by return current flow, as remarked below Eq. (35). Given a particular heating scheme, these sources of fluctuations can be tailored for maximum efficiency, including beam diffusion. Thus, the outlook for rapid heating by electron beams is rather favorable.

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