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# Kinetic Theory of Quantum Plasma and Radiation in an External Magnetic Field*

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The random-phase approximation and the Bogoliubov (adiabatic) assumption are used to obtain long-time expressions for electron and photon operators in the presence of a constant, external magnetic field. These operators are then used to obtain a kinetic equation describing the approach to equilibrium of the system. The results include the dynamical screening of the Coulomb interaction and the "dressing" of the photons. Expressions are also obtained for the density and electric-field autocorrelation functions. The structure of the asymptotic ($t\rightarrow\infty$) operators is studied, and it is shown that operators can be obtained that correspond to the creation and annihilation operators for the collective modes of the system.

## I. INTRODUCTION

In a previous paper[1] we obtained coupled kinetic equations for the particle and photon distribution functions in the case of a homogeneous, isotropic, multicomponent system of charged particles and radiation with no external fields present. In the present paper we treat the problem of the quantum electron gas coupled to the radiation field in the presence of a uniform (in space and time) external magnetic field.

This problem has been considered previously by Osborn and Klevans[2,3] and by Dreicer.[4] In the work of Osborn and Klevans[2] the kinetic equations were obtained by the use of the repeated random-phase assumption and no many-body effects were included. However, Klevans[3] later improved this method by using a modified scattering method to treat the dispersive nature of the system. Dreicer[4] used a strictly Boltzmann approach to obtain a relativistic kinetic equation. His results contain no many-body effects.

The problem of a quantum electron gas with a magnetic field, but without taking radiation into account, was considered by Ron,[5] who used Dupree's[6] technique to obtain a modified Balescu-Lenard equation.

In this paper we obtain a kinetic equation that includes the effects of dynamical screening and the effects of the dispersive nature of the plasma. This is accomplished by generalizing a method developed by Wyld and Fried[7] that makes use of the random-phase approximation (RPA) and the Bogoliubov (adiabatic) assumption concerning the time behavior of the correlation functions as compared to the time behavior of the one-particle distribution functions. This is a different method from that used in the previous paper.[1] The present method has the advantage that the "dressing" of the photons (which is very important in the magnetic field case) is accomplished in a very straightforward manner. On the other hand, we do not include the effects of electron-photon scattering and double emission-absorption processes.

The idea of the method is to obtain asymptotic ($t\rightarrow\infty$) expressions for various operators. These expressions are then used to calculate various correlation functions. We are also able to calculate two-time correlation functions, and we obtain explicit expressions for the creation and annihilation operators for the collective modes. This method has also been applied to the electron-phonon gas.[8]

## II. EQUATIONS OF MOTION

We consider a system of $N$ electrons in a neutralizing positive background contained in a volume $V$ to be interacting with each other and with the radiation field. The uniform magnetic field is chosen in the $z$ direction and the coupling to the spins is ignored. Since we will think of the volume as eventually becoming infinite, we will neglect all effects that vanish in this limit. The particle operators $\psi_s(\mathbf{r})$ and $\psi_s^\dagger(\mathbf{r})$ obey the usual anticommutation relations

$$\{\psi_s(\mathbf{r}),\psi_s^\dagger(\mathbf{r}')\}=\delta_{s,s'}\delta(\mathbf{r}-\mathbf{r}')\,,$$
and
$$\{\psi_s(\mathbf{r}),\psi_{s'}(\mathbf{r}')\}=0\,. \tag{1}$$

If we work in the Landau gauge, the external field is represented by the vector potential

$$\mathbf{A}_0(\mathbf{r})=(0,xB,0)\,, \tag{2}$$

and the total vector potential is written as

$$\mathbf{A}(\mathbf{r})=\mathbf{A}_0(\mathbf{r})+\mathbf{A}_1(\mathbf{r})\,, \tag{3}$$

where $\mathbf{A}_1(\mathbf{r})$ is the quantized internal vector potential.

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[1] W. R. Chappell and W. E. Brittin, Phys. Rev. **146**, 75 (1966).
[2] R. K. Osborn and E. H. Klevans, Ann. Phys. (N.Y.) **15**, 105 (1961).
[3] E. H. Klevans, Doctoral thesis, University of Michigan, 1962 (unpublished).
[4] H. Dreicer, Phys. Fluids **7**, 735 (1964).
[5] A. Ron, Phys. Rev. **134**, A70 (1964).
[6] T. H. Dupree, Phys. Fluids **4**, 696 (1961).
[7] H. W. Wyld, Jr. and B. D. Fried, Ann. Phys. (N.Y.) **23**, 374 (1963).
[8] W. R. Chappell, J. Math. Phys. **7**, 1153 (1966).

The Hamiltonian for the system is given by

$$H=(8\pi c^2)^{-1}\int d\mathbf{r}\left(\left|c^{-1}\frac{\partial}{\partial t}\mathbf{A}_1(\mathbf{r})\right|^2+|\nabla\times\mathbf{A}_1(\mathbf{r})|^2\right)$$
$$+(2m)^{-1}\sum_s\int d\mathbf{r}\,\psi_s^\dagger(\mathbf{r})(-\hbar^2\nabla^2)\psi_s(\mathbf{r})$$
$$+\frac{1}{2}\int\int d\mathbf{r}d\mathbf{r}'\sum_{s,s'}\psi_s^\dagger(\mathbf{r})\psi_{s'}^\dagger(\mathbf{r}')\phi(\mathbf{r}-\mathbf{r}')\psi_{s'}(\mathbf{r}')\psi_s(\mathbf{r})$$
$$+\frac{ie\hbar}{2mc}\int d\mathbf{r}\sum_s[\psi_s^\dagger(\mathbf{r})\nabla\psi_s(\mathbf{r})-(\nabla\psi_s^\dagger(\mathbf{r}))\psi_s(\mathbf{r})]\cdot\mathbf{A}(\mathbf{r})$$
$$+\frac{e^2}{2mc^2}\int d\mathbf{r}\sum_s\psi_s^\dagger(\mathbf{r})\psi_s(\mathbf{r})|\mathbf{A}(\mathbf{r})|^2,\quad(4)$$

where

$$\phi(\mathbf{r})=e^2/|\mathbf{r}|.$$

The motion of the particles caused by the external magnetic field can be incorporated by the use of the eigenfunctions $\phi_\alpha(\mathbf{r})$ for the problem of a single electron in a uniform magnetic field. That is, $\phi_\alpha(\mathbf{r})$ is a solution of the equation

$$\frac{1}{2m}\left[\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}_0(\mathbf{r})\right]^2\phi_\alpha(\mathbf{r})=\hbar E_\alpha\phi_\alpha(\mathbf{r}).\quad(5)$$

The eigenfunctions $\phi_\alpha(\mathbf{r})$ are given by[9]

$$\phi_\alpha(\mathbf{r})=V^{-1/3}e^{i(pz+qy)}U_n\left(x-\frac{\hbar q}{m\omega_c}\right).\quad(6)$$

The function $U_n(x)$ is the normalized harmonic-oscillator wave function corresponding to the eigenvalue $\hbar\omega_c\times(n+\frac{1}{2})$, where $\omega_c=eB/mc$. The energy eigenvalues $\hbar E_\alpha$ are given by

$$\hbar E_\alpha=\hbar\omega_c(n+\tfrac{1}{2})+\hbar^2p^2/2m,\quad(7)$$

and $p$ and $q$ are given by

$$p=\frac{2\pi n_1}{V^{1/3}},\quad q=\frac{2\pi n_2}{V^{1/3}},$$

where

$$n_{1,2}=0,\pm1,\pm2,\pm\cdots.$$

We note that $E_\alpha$ is independent of $q$.

We can work in terms of the Landau states by writing the electron operators as

$$\psi_s(\mathbf{r})=\sum_\alpha\phi_\alpha(\mathbf{r})c_{\alpha s},$$

and

$$\psi_s^\dagger(\mathbf{r})=\sum_\alpha\phi_\alpha^*(\mathbf{r})c_{\alpha s}^\dagger.\quad(8)$$

[9] L. Landau and E. Lifshitz, *Quantum Mechanics, Non-relativistic Theory* (Pergamon Press, Ltd., London, 1958).

The operators $c_{\alpha s}^\dagger$ and $c_{\alpha s}$ are the creation and annihilation operators in the Landau representation and obey the anticommutation relations

$$\{c_{\alpha s},c_{\alpha's'}^\dagger\}=\delta_{\alpha,\alpha'}\delta_{s,s'},$$

and

$$\{c_{\alpha s},c_{\alpha's'}\}=0.\quad(9)$$

It is also convenient to expand the quantized vector potential in a Fourier series as

$$\mathbf{A}_1(\mathbf{r})=\sum_\mathbf{k}\left(\frac{4\pi c^2}{V}\right)^{1/2}e^{i\mathbf{k}\cdot\mathbf{r}}\mathbf{q}_\mathbf{k}.\quad(10)$$

The canonical momentum operators $\mathbf{p}_\mathbf{k}$ are introduced in the customary manner. The field operators obey the usual commutation relations

$$[\mathbf{q}_\mathbf{k},\mathbf{p}_{\mathbf{k}'}]=i\hbar\delta_{\mathbf{k},\mathbf{k}'}(\mathsf{I}-\hat{k}\hat{k}),$$

and

$$[\mathbf{q}_\mathbf{k},\mathbf{q}_{\mathbf{k}'}]=\mathsf{O},\quad(11)$$

where $\mathsf{I}$ is the unit dyadic, $\mathsf{O}$ is the zero dyadic, and $\hat{k}=\mathbf{k}/k$. We also note that

$$\mathbf{q}_\mathbf{k}^\dagger=\mathbf{q}_{-\mathbf{k}}.\quad(12)$$

In this paper we will neglect the effects of electron-photon scattering and double emission-absorption processes that arise from the $|\mathbf{A}_1|^2$ term in the Hamiltonian. There is a contribution from the $|\mathbf{A}_1|^2$ term, however, that gives a shift in the photon frequencies from $\Omega_k=kc$ to[1]

$$\Omega_k^2=k^2c^2+\omega_p^2,$$

where $\omega_p^2=4\pi Ne^2/mV$. The latter contribution will be retained. The Hamiltonian for this problem can then be written as

$$H=\sum_\alpha\hbar E_\alpha c_{\alpha s}^\dagger c_{\alpha s}$$
$$+\tfrac{1}{2}\sum_\mathbf{k}(\mathbf{p}_\mathbf{k}^\dagger\mathbf{p}_\mathbf{k}+\Omega_k^2\mathbf{q}_\mathbf{k}^\dagger\mathbf{q}_\mathbf{k})+\tfrac{1}{2}\sum_\mathbf{k}\phi(k)\rho(\mathbf{k})\rho(-\mathbf{k})$$
$$-e\left(\frac{4\pi}{V}\right)^{1/2}\sum_\mathbf{k}\sum_{\alpha,\alpha',s}(\alpha|\mathbf{V}(\mathbf{k})|\alpha')b_s(\alpha,\alpha')\cdot\mathbf{q}_\mathbf{k},\quad(13)$$

where

$$\Omega_k^2=k^2c^2+\omega_p^2,\quad(14)$$
$$\phi(k)=4\pi e^2/k^2,\quad(15)$$
$$b_s(\alpha,\alpha')=c_{\alpha s}^\dagger c_{\alpha' s},\quad(16)$$
$$\rho(\mathbf{k})=\sum_{\alpha,\alpha',s}(\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha')b_s(\alpha,\alpha'),\quad(17)$$

and

$$(\alpha|\mathbf{V}(\mathbf{k})|\alpha')=m^{-1}\left(\alpha\left|e^{i\mathbf{k}\cdot\mathbf{r}}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}_0+\frac{\hbar\mathbf{k}}{2}\right)\right|\alpha'\right).\quad(18)$$

The operator $\rho(\mathbf{k})$ is the Fourier transform of the density operator. The Fourier transform of the current density operator is given by

$$\mathbf{j}(\mathbf{k})=e\sum_{\alpha,\alpha',s}(\alpha|\,\mathbf{V}(\mathbf{k})\,|\alpha')b_s(\alpha,\alpha')-\frac{\omega_p{}^2}{4\pi c}\mathbf{A}_\mathbf{k}, \quad (19)$$

where

$$\mathbf{A}_\mathbf{k}=(4\pi c^2V)^{1/2}\mathbf{q}_\mathbf{k}. \quad (20)$$

The simplified form of the last term in Eq. (19) results from the approximation made on the $|\mathbf{A}|^2$ term in the Hamiltonian.[1]

We choose to work in the Heisenberg representation. Hence, the equation of motion for any operator $O$ is given by $i\hbar\dot{O}=[O,H]$. The equations of motion for the operators $\mathbf{A}_\mathbf{k}$ and $b_s(\alpha,\alpha')$ are given by

$$\frac{\partial^2\mathbf{A}_\mathbf{k}}{\partial t^2}+\Omega_k{}^2\mathbf{A}_\mathbf{k}=4\pi ec\sum_{\alpha,\alpha',s}(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')\cdot(\mathbf{I}-\hat{k}\hat{k})b_s(\alpha,\alpha'), \quad (21)$$

and

$$\frac{\partial b_s(\alpha,\alpha')}{\partial t}=-i\Omega(\alpha,\alpha')b_s(\alpha,\alpha')-\frac{ie}{\hbar cV}\sum_{\beta\mathbf{k}}[(\beta|\,\mathbf{V}(\mathbf{k})\,|\alpha)b_s(\beta,\alpha')-(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\beta)b_s(\alpha,\beta)]\cdot\mathbf{A}_\mathbf{k}$$
$$+\frac{i}{\hbar V}\sum_\beta\sum_\mathbf{k}\phi(k)[(\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\beta)^*b_s(\beta,\alpha')-(\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\beta)b_s(\alpha,\beta)]\rho(\mathbf{k}), \quad (22)$$

where

$$\Omega(\alpha,\alpha')=E_{\alpha'}-E_\alpha. \quad (23)$$

From Eq. (22) we can obtain the exact equation of motion (neglecting electron-photon scattering and double emission absorption) for the one-particle distribution function

$$f_{\alpha s}=\langle c_{\alpha s}{}^\dagger c_{\alpha s}\rangle.$$

We will assume in this paper that $f_{\alpha s}$ is independent of the quantum number $q$.

The equation of motion for $f_{\alpha s}$ is given by

$$\frac{\partial f_{\alpha s}}{\partial t}=\frac{2e}{\hbar cV}\sum_{\alpha',\mathbf{k}}\mathrm{Im}[(\alpha|\,\mathbf{V}(\mathbf{k})\,|\alpha')\cdot\langle b_s(\alpha,\alpha')\mathbf{A}_\mathbf{k}\rangle]+\frac{2}{\hbar V}\sum_{\alpha',\mathbf{k}}\phi(k)\,\mathrm{Im}[(\alpha|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha')\langle b_s(\alpha,\alpha')\rho(\mathbf{k})\rangle]. \quad (24)$$

In order to obtain a kinetic equation we need to find long-time expressions for the correlation functions that appear on the right-hand side of Eq. (24). We will accomplish this by using the RPA to linearize Eq. (22) and then by solving the resulting equations for the operators $b_s(\alpha,\alpha')$, $\rho(\mathbf{k})$, and $\mathbf{A}_\mathbf{k}$ by using Laplace transforms. We then obtain asymptotic $(t\to\infty)$ expressions for the operators. These asymptotic operators can then be used to calculate the desired correlation functions. This technique was first used by Wyld and Fried for the quantum electron gas.[7] It has been recently applied to the electron-phonon gas.[8]

The RPA equation is obtained from Eq. (22) by making the following approximations[10]:

$$b_s(\beta,\alpha')\mathbf{A}_\mathbf{k}\to\delta_{\beta,\alpha'}f_{\alpha s}\mathbf{A}_\mathbf{k},$$

and

$$b_s(\beta,\alpha')\rho(\mathbf{k})\to\delta_{\beta,\alpha'}f_{\alpha' s}\rho(\mathbf{k}).$$

The RPA is equivalent to a truncation of the hierarchy of equations for the correlation functions, the neglect of close encounters, and the neglect of exchange terms.[1,7]

The resulting equation for $b_s(\alpha,\alpha')$ is

$$\frac{\partial b_s(\alpha,\alpha')}{\partial t}=-i\Omega(\alpha,\alpha')b_s(\alpha,\alpha')-\left(\frac{i}{\hbar V}\right)\Delta_s(\alpha,\alpha')\sum_\mathbf{k}[(e/c)(\alpha'|\mathbf{V}(k)|\alpha)\cdot\mathbf{A}_\mathbf{k}+\phi(k)(\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha)\rho(\mathbf{k})], \quad (25)$$

where

$$\Delta_s(\alpha,\alpha')=f_{\alpha' s}-f_{\alpha s}. \quad (26)$$

The next assumption of importance is the Bogoliubov (adiabatic) assumption. We presuppose that by the time the correlation functions have assumed their long-time (of the order of the relaxation time) form, the one-particle distribution function has not changed appreciably. Thus, we calculate the asymptotic $(t\to\infty)$ expressions for the correlation functions by assuming $f_{\alpha s}$ is constant in time.

Thus in Eq. (25) we consider $\Delta_s(\alpha,\alpha')$ to be independent of time. Consequently, Eqs. (21) and (25) are simply the Maxwell and linearized Vlasov equations for the operator $b_s(\alpha,\alpha')$. These equations are coupled, linear equations. We solve them by introducing the one-sided Fourier transformation

$$O(\omega)=\int_0^\infty e^{i\omega t}O(t)dt,\qquad O(t)=(2\pi)^{-1}\int e^{-i\omega t}O(\omega)d\omega, \quad (27)$$

where $\mathrm{Im}\omega>0$. With the use of the above transformation, we obtain from Eqs. (21) and (25)

$$-(\omega^2-\Omega_k{}^2)\mathbf{A}_\mathbf{k}(\omega)=i\omega\mathbf{A}_\mathbf{k}(0)+\frac{\partial\mathbf{A}_\mathbf{k}(0)}{\partial t}+4\pi ec\sum_{\alpha,\alpha',s}(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')\cdot(\mathbf{I}-\hat{k}\hat{k})b_s(\alpha,\alpha',\omega), \quad (28)$$

and

$$-i[\omega-\Omega(\alpha,\alpha')]b_s(\alpha,\alpha',\omega)=b_s(\alpha,\alpha',0)-\left(\frac{i}{\hbar V}\right)\Delta_s(\alpha,\alpha')\sum_\mathbf{k}[(e/c)(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\alpha)\cdot\mathbf{A}_\mathbf{k}(\omega)+\phi(k)(\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha)\rho(\mathbf{k},\omega)]. \quad (29)$$

[10] D. Bohm and D. Pines, Phys. Rev. **92**, 609 (1953).

It is convenient to solve these equations for the quantity

$$\mathbf{E}_{\mathbf{k}}(\omega)=-\frac{i\mathbf{k}}{e}\phi(k)\rho(\mathbf{k},\omega)+\frac{i\omega}{c}\mathbf{A}_{\mathbf{k}}(\omega). \tag{30}$$

The above operator, except for a term involving the initial value of the transformed vector potential, is the transform of the electric field.

We solve these equations by first solving Eq. (29) for the operator $\rho(\mathbf{k},\omega)$ in terms of $b_s(\alpha,\alpha',0)$ and $\mathbf{E}_{\mathbf{k}}(\omega)$, using the fact that we assumed $f_{\alpha s}$ to be independent of the quantum number $q$ and the fact that[5]

$$\sum_{qq'}(\alpha|e^{i\mathbf{l}\cdot\mathbf{r}}|\alpha')(\alpha'|\mathbf{V}(-\mathbf{k})|\alpha)\propto\delta_{\mathbf{l},\mathbf{k}},$$

and

$$\sum_{qq'}(\alpha|e^{i\mathbf{l}\cdot\mathbf{r}}|\alpha')(\alpha'|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha)\propto\delta_{\mathbf{l},\mathbf{k}}, \tag{31}$$

and Eqs. (A5) and (A7). We then obtain from Eq. (29) an expression for the operator

$$\sum_{\alpha,\alpha',s}(\alpha|\mathbf{V}(-\mathbf{k})|\alpha')b_s(\alpha,\alpha',\omega)$$

in terms of $b_s(\alpha,\alpha',0)$, and $\mathbf{E}_{\mathbf{k}}(\omega)$ by using the fact that

$$\sum_{qq'}(\alpha|\mathbf{V}(\mathbf{l})|\alpha')(\alpha'|\mathbf{V}(-\mathbf{k})|\alpha)\propto\delta_{\mathbf{l},\mathbf{k}}, \tag{32}$$

and Eq. (A7). This expression we substitute into Eq. (28), which gives an equation for $\mathbf{E}_{\mathbf{k}}(\omega)$ in terms of the initial values of the operators and $\mathbf{E}_{\mathbf{k}}(\omega)$. We finally obtain the result

$$\mathbf{Y}(\mathbf{k},\omega)\cdot\mathbf{E}_{\mathbf{k}}(\omega)=-c^{-1}\left[\mathbf{A}_{\mathbf{k}}(0)+\frac{i}{\omega}\frac{\partial\mathbf{A}_{\mathbf{k}}(0)}{\partial t}\right]+\mathbf{k}e^{-1}\phi(k)\sum_{\alpha,\alpha',s}\frac{(\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha')b_s(\alpha,\alpha',0)}{\omega-\Omega(\alpha,\alpha')}+4\pi e\omega^{-1}\sum_{\alpha,\alpha',s}\frac{(\mathbf{I}-\hat{k}\hat{k})\cdot(\alpha|\mathbf{V}(-\mathbf{k})|\alpha')b_s(\alpha,\alpha',0)}{\omega-\Omega(\alpha,\alpha')}. \tag{33}$$

$$\mathbf{Y}(\mathbf{k},\omega)=\mathbf{I}\left(1-\frac{\omega_p^2}{\omega^2}\right)-\frac{k^2c^2}{\omega^2}(\mathbf{I}-\hat{k}\hat{k})+\frac{m\omega_p^2}{N\hbar\omega^2}\sum_{\alpha,\alpha',s}\frac{(\alpha|\mathbf{V}(-\mathbf{k})|\alpha')(\alpha'|\mathbf{V}(\mathbf{k})|\alpha)\Delta_s(\alpha,\alpha')}{\omega-\Omega(\alpha,\alpha')}. \tag{33'}$$

## III. ASYMPTOTIC OPERATORS

Since we are interested in the asymptotic $(t\rightarrow\infty)$ behavior of the electric field, we must obtain the inverse Fourier transform and then let $t$ become large. This necessitates the study of the singularities in the $\omega$ plane. In particular, singularities can arise in two ways. There are the simple poles associated with the factors $[\omega-\Omega(\alpha,\alpha')]^{-1}$, and the determinant of $\mathbf{Y}(\mathbf{k},\omega)$ can have zeros as a function of $\omega$.

The quantity $\mathbf{Y}(\mathbf{k},\omega)$ plays an analogous role to that played by the dielectric function in the electron gas.[7,8] That is, the zeros of the determinant of $\mathbf{Y}$ correspond to the propagating modes in the system. These collective modes consist of photons, plasmons, and various mixtures of transverse and longitudinal waves. The type of mode possible depends upon the magnitude of $\mathbf{k}$ and its orientation with respect to the magnetic field. We will not be concerned here with the actual details of these modes. We assume that the system is stable, i.e., $f_{\alpha s}$ is such that all modes decay in time.

We note that $\mathbf{Y}(\mathbf{k},\omega)$ is related to the conductivity tensor $\boldsymbol{\sigma}(\mathbf{k},\omega)$ by the relation[11]

$$\mathbf{Y}(\mathbf{k},\omega)=\mathbf{I}-\frac{k^2c^2}{\omega^2}(\mathbf{I}-\hat{k}\hat{k})-4\pi i\omega^{-1}\boldsymbol{\sigma}(\mathbf{k},\omega). \tag{34}$$

It is also related to the dielectric tensor $\boldsymbol{\varepsilon}(\mathbf{k},\omega)$ by the relation[5]

$$\mathbf{Y}(\mathbf{k},\omega)=\boldsymbol{\varepsilon}(\mathbf{k},\omega)-\frac{k^2c^2}{\omega^2}(\mathbf{I}-\hat{k}\hat{k}). \tag{35}$$

Since we have limited ourselves to the stable case where all of the poles arising from $\mathbf{Y}(\mathbf{k},\omega)^{-1}$ lie in the lower half of the $\omega$ plane, we will assume that the corresponding contributions represent transients that can be ignored. In the unstable case, of course, all of these poles cannot be ignored. Consequently, the remaining singularities are those arising from the simple poles at $\omega=\Omega(\alpha,\alpha')$, which are easily integrated to give the asymptotic expression for $\mathbf{E}_{\mathbf{k}}$

$$\mathbf{E}_{\mathbf{k}}(t)\sim-i\sum_{\alpha,\alpha',s}\Omega(\alpha,\alpha')^{-1}\mathbf{Y}_{+}[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}\cdot(\alpha|\mathbf{V}(-\mathbf{k})|\alpha')b_s(\alpha,\alpha',0)e^{-i\Omega(\alpha,\alpha')t}, \tag{36}$$

where

$$\mathbf{Y}_{+}(\mathbf{k},\omega)=\mathbf{Y}(\mathbf{k},\omega+i\eta). \tag{37}$$

The quantity $\eta$ is the positive infinitesimal. We will suppress the subscript + henceforth. In order to obtain Eq. (36) we have used Eq. (A1) to combine terms.

We can also obtain long-time expressions for $\rho(\mathbf{k},t)$ and $\mathbf{A}_k(t)$. With the use of Eq. (30), we find

$$\rho(\mathbf{k},t)\sim\mathbf{k}\cdot\sum_{\alpha,\alpha',s}\mathbf{Y}[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}\cdot(\alpha|\mathbf{V}(-\mathbf{k})|\alpha')\times\Omega(\alpha,\alpha')^{-1}b_s(\alpha,\alpha',0)e^{-i\Omega(\alpha,\alpha')t}, \tag{38}$$

and

$$\mathbf{A}_{\mathbf{k}}(t)\sim-4\pi ec(\mathbf{I}-\hat{k}\hat{k})\cdot\sum_{\alpha,\alpha',s}\Omega(\alpha,\alpha')^{-2}\mathbf{Y}[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}\cdot(\alpha|\mathbf{V}(-\mathbf{k})|\alpha')b_s(\alpha,\alpha',0)e^{-i\Omega(\alpha,\alpha')t}. \tag{39}$$

Another expression of interest is the Fourier transform of the transverse current operator

$$\mathbf{j}_T(\mathbf{k},t)=(\mathbf{I}-\hat{k}\hat{k})\cdot\mathbf{j}(\mathbf{k},t). \tag{40}$$

[11] J. J. Quinn and S. Rodriguez, Phys. Rev. 128, 2487 (1962).

The asymptotic expression for this operator is obtained by use of Eq. (28). We obtain

$$\mathbf{j}_T(\mathbf{k},t)\sim e(\mathbf{I}-\hat{k}\hat{k})\cdot\sum_{\alpha,\alpha',s}[\Omega^2(\alpha,\alpha')-k^2c^2]\times\Omega(\alpha,\alpha')^{-2}\mathbf{Y}[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}\cdot(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')\times b_s(\alpha,\alpha',0)e^{-i\Omega(\alpha,\alpha')t}. \quad (41)$$

The expressions for $\rho(\mathbf{k},t)$ and $\mathbf{A}_\mathbf{k}(t)$ can now be used to obtain the asymptotic expression for the operator $b_s(\alpha,\alpha',t)$. We substitute the expressions given by Eqs. (38) and (39) into Eq. (25) and then solve the resulting equation subject to the self-consistency conditions that

$$\rho(\mathbf{k},t)=\sum_{\alpha,\alpha',s}b_s(\alpha,\alpha',t), \quad (42)$$

and

$$\mathbf{j}_T(\mathbf{k},t)=e(\mathbf{I}-\hat{k}\hat{k})\cdot\sum_{\alpha,\alpha',s}(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')b_s(\alpha,\alpha',t)-\frac{\omega_p^2}{4\pi c}\mathbf{A}_\mathbf{k}(t). \quad (43)$$

The result of this calculation is

$$b_s(\alpha,\alpha',t)\sim b_s(\alpha,\alpha',0)e^{-i\Omega(\alpha,\alpha')t}-\frac{m\omega_p^2}{\hbar N}\Delta_s(\alpha,\alpha')\sum_\mathbf{k}\Bigg\{(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\alpha)\cdot(\mathbf{I}-\hat{k}\hat{k})\cdot\sum_{\beta,\beta',s'}\frac{\mathbf{Y}[\mathbf{k},\Omega(\beta,\beta')]^{-1}}{\Omega^2(\beta,\beta')[\Omega(\beta,\beta')-\Omega(\alpha,\alpha')+i\eta]}\cdot(\beta|\,\mathbf{V}(-\mathbf{k})\,|\beta')b_s(\beta,\beta',0)e^{-i\Omega(\beta,\beta')t}$$
$$+k^{-1}(\alpha'|\,e^{i\mathbf{k}\cdot\mathbf{r}}\,|\alpha)\hat{k}\cdot\sum_{\beta,\beta',s'}\frac{\mathbf{Y}^{-1}[\mathbf{k},\Omega(\beta,\beta')]}{\Omega(\beta,\beta')[\Omega(\beta,\beta')-\Omega(\alpha,\alpha')+i\eta]}\cdot(\beta|\,\mathbf{V}(-\mathbf{k})\,|\beta')b_s(\beta,\beta',0)e^{-i\Omega(\beta,\beta')t}\Bigg\}. \quad (44)$$

We can now calculate asymptotic ($t\rightarrow\infty$) expressions for all of the correlation functions of interest. Before we proceed to the calculation of the kinetic equation, it is convenient and interesting to consider certain autocorrelation functions. With the use of Eqs. (36) and (38), we obtain for the density autocorrelation function and the electric-field autocorrelation functions the expressions

$$\langle\rho(\mathbf{k},t)\rho(-\mathbf{k},t')\rangle=\sum_{\alpha,\alpha',s}|\,\mathbf{k}\cdot\mathbf{Y}[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}\cdot(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')\,|^2\times\Omega(\alpha,\alpha')^{-2}f_{\alpha s}(1-f_{\alpha' s})e^{-i\Omega(\alpha,\alpha')(t-t')}, \quad (45)$$

and

$$\langle\mathbf{E}_\mathbf{k}(t)\mathbf{E}_{-\mathbf{k}}(t')\rangle=\sum_{\alpha,\alpha',s}\Omega^{-2}(\alpha,\alpha')\mathbf{Y}[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}\cdot(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')\times\mathbf{Y}^*[k,\Omega(\alpha,\alpha')]^{-1}\cdot(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')^*\times f_{\alpha s}(1-f_{\alpha' s})e^{-i\Omega(\alpha,\alpha')(t-t')}. \quad (46)$$

In obtaining these expressions, we have used the identity

$$\langle b_s(\alpha,\alpha',0)b_{s'}(\beta,\beta',0)\rangle=\delta_{\alpha,\beta'}\delta_{\alpha',\beta}\delta_{s,s'}f_{\alpha s}(1-f_{\alpha' s})+\delta_{\alpha,\alpha'}\delta_{\beta,\beta'}f_{\alpha s}f_{\beta s'}+g_{ss'}(\alpha,\alpha';\beta,\beta',0). \quad (47)$$

The quantity $g_{ss'}(\alpha,\alpha';\beta,\beta',0)$ is the initial two-particle correlation function that will be neglected.

If the propagation vector $\mathbf{k}$ is parallel to the external magnetic field, it is easy to show that

$$\langle\rho(\mathbf{k},t)\rho(-\mathbf{k},t')\rangle=\sum_{\alpha,\alpha',s}|\,(\alpha|\,e^{-i\mathbf{k}\cdot\mathbf{r}}\,|\alpha')D[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}\,|^2\times f_{\alpha s}(1-f_{\alpha' s})e^{-i\Omega(\alpha,\alpha')(t-t')}, \quad (48)$$

where $D(\mathbf{k},\omega)$ is the longitudinal dielectric constant

$$D(\mathbf{k},\omega)=1+\frac{m\omega_p^2}{\hbar Nk^2}\sum_{\alpha,\alpha',s}\frac{|\,(\alpha|\,e^{-i\mathbf{k}\cdot\mathbf{r}}\,|\alpha')\,|^2\Delta_s(\alpha,\alpha')}{\omega-\Omega(\alpha,a')}. \quad (49)$$

Equation (48) is the result obtained by Ron,[5] who neglected the transverse fields.

## IV. KINETIC EQUATION

In order to obtain a kinetic equation we must obtain asymptotic expressions for the correlation functions $\langle b_s(\alpha,\alpha',t)\mathbf{A}_\mathbf{k}(t)\rangle$ and $\langle b_s(\alpha,\alpha',t)\rho(\mathbf{k},t)\rangle$. This is done by use of the expressions for $\rho(\mathbf{k},t)$, $\mathbf{A}_\mathbf{k}(t)$, and $b_s(\alpha,\alpha',t)$ given by Eqs. (38), (39), and (44), respectively, and by the use of Eq. (47). Again, we neglect the effect of initial correlations, which are assumed to be taken out by phase mixing. After some combining of terms and the use of the identity given in Eq. (A1), we obtain

$$\frac{\partial f_{\alpha s}}{\partial t}=\frac{8\pi e^2}{\hbar V}\sum_k\mathrm{Im}\Bigg\{(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')\cdot\mathbf{Y}[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}\cdot(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\alpha)\Omega(\alpha,\alpha')^{-2}f_{\alpha s}(1-f_{\alpha\, s})$$
$$-\frac{m\omega_p^2}{\hbar N}\Delta_s(\alpha,\alpha')\sum_{\beta,\beta',s'}|\,(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\alpha)\cdot\mathbf{Y}[\mathbf{k},\Omega(\beta,\beta')]\cdot(\beta|\,\mathbf{V}(-\mathbf{k})\,|\beta')\,|^2\frac{f_{\beta s}\,(1-f_{\beta' s'})}{\Omega^4(\beta,\beta')[\Omega(\beta,\beta')-\Omega(\alpha,\alpha')+i\eta]}\Bigg\}. \quad (50)$$

The above expression can be further simplified by using the fact that

$$(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')\cdot \mathbf{Y}^*(\mathbf{k},\omega)^{-1}\cdot(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\alpha)=(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')\cdot \mathbf{Y}^*(\mathbf{k},\omega)^{-1}\cdot[(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\alpha)\cdot \mathbf{Y}(\mathbf{k},\omega)^{-1}\cdot \mathbf{Y}(\mathbf{k},\omega)]. \quad (51)$$

We then find that

$$\mathrm{Im}[(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')\cdot \mathbf{Y}^*(\mathbf{k},\omega)^{-1}\cdot(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\alpha)]=\frac{1}{2i}(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')\cdot \mathbf{Y}^*(\mathbf{k},\omega)^{-1} \cdot\{(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\alpha)\cdot \mathbf{Y}^{-1}(\mathbf{k},\omega)\cdot[\mathbf{Y}(\mathbf{k},\omega)-\mathbf{Y}^\dagger(\mathbf{k},\omega)]\}\,, \quad (52)$$

where $\mathbf{Y}^\dagger(\mathbf{k},\omega)$ is the Hermitian conjugate of $\mathbf{Y}(\mathbf{k},\omega)$. With the use of the formula

$$\frac{1}{X-i\eta}=P\frac{1}{X}+i\pi\delta(X)\,, \quad (53)$$

we obtain

$$\frac{1}{2i}[\mathbf{Y}(\mathbf{k},\omega)-\mathbf{Y}^\dagger(\mathbf{k},\omega)]=-\frac{\pi m\omega_p^2}{N\hbar\omega^2}\sum_{\alpha,\alpha',s}(\alpha|\,\mathbf{V}(-\mathbf{k})\,|\alpha')(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\alpha)\Delta_s(\alpha,\alpha')\delta[\omega-\Omega(\alpha,\alpha')]. \quad (54)$$

If Eq. (54) is substituted into Eq. (50), we obtain for the kinetic equation

$$\frac{\partial f_{\alpha s}}{\partial t}=\frac{2\pi m^2\omega_p^2}{\hbar^2N^2}\sum_{\alpha'}\sum_{\beta,\beta',s'}\sum_{\mathbf{k}}|(\alpha'|\,\mathbf{V}(\mathbf{k})\,|\alpha)\cdot\mathbf{Y}[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}\cdot(\beta'|\,\mathbf{V}(-\mathbf{k})\,|\beta)|^2\Omega(\alpha,\alpha')^{-4}[f_{\alpha's}f_{\beta's'}(1-f_{\alpha s})(1-f_{\beta s'}) -f_{\alpha s}f_{\beta s'}(1-f_{\alpha' s})(1-f_{\beta' s'})]\delta[E_\alpha+E_\beta-E_{\alpha'}-E_{\beta'}]. \quad (55)$$

It is quite simple to show that the above equation leads to an $H$ theorem. The form of Eq. (55) is similar to the form of the quantum-mechanical Balescu-Lenard equation.

## V. QUASIPHOTONS AND PLASMONS

The zeros of the determinant of $\mathbf{Y}(\mathbf{k},\omega)$ correspond to the collective modes in the system. Several authors have obtained approximate expressions for the operators corresponding to the collective modes for various systems.[10,12–15] The presence of the quantity $\mathbf{Y}(\mathbf{k},\omega)^{-1}$ in the asymptotic expressions for $\mathbf{E}_\mathbf{k}(t)$, $\rho_\mathbf{k}(t)$, and $\mathbf{A}_\mathbf{k}(t)$ [Eqs. (36), (38), (39)] suggests that collective operators can be obtained by considering the contributions from the collective poles in det$[\mathbf{Y}(\mathbf{k},\omega)^{-1}]$.

We will exhibit the results of this procedure for a few special cases that are easy to calculate.

We begin by considering the case where $\hat{k}$ is parallel to the external field. It is then easy to show that

$$\rho(\mathbf{k},t)=\sum_{\alpha,\alpha',s}(\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha')D[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}\times b_s(\alpha,\alpha',0)e^{-i\Omega(\alpha,\alpha')t}. \quad (56)$$

We then assume $D(\mathbf{k},\omega)^{-1}$ to be dominated by its plasmon poles and write[16]

$$\begin{aligned} D(\mathbf{k},\omega)&\approx(2/\omega_k)(\omega-\omega_k+i\gamma_k), && \omega\approx\omega_k \\ &\approx-(2/\omega_k)(\omega+\omega_k+i\gamma_k), && \omega\approx-\omega_k\,, \end{aligned} \quad (57)$$

where $\omega_k$ is the plasmon frequency and

$$\gamma_k\approx(\omega_k/2)\,\mathrm{Im}D(k,\omega_k)\,. \quad (58)$$

We have assumed that $\gamma_k/\omega_k\ll 1$.

We then obtain for the collective part of $\rho(\mathbf{k},t)$ the expression

$$\rho_c(\mathbf{k},t)=\frac{\omega_k}{2}\sum_{\alpha,\alpha',s}\left[\frac{e^{-i\omega_kt}}{\Omega(\alpha,\alpha')-\omega_k+i\gamma_k}-\frac{e^{i\omega_kt}}{\Omega(\alpha,\alpha')+\omega_k+i\gamma_k}\right](\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha')b_s(\alpha,\alpha',0)\,. \quad (59)$$

We can then define a plasmon annihilation operator $B_\mathbf{k}$ as

$$B_\mathbf{k}(t)=\alpha_k\sum_{\alpha,\alpha',s}\frac{(\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha')b_s(\alpha,\alpha',0)e^{-i\omega_kt}}{\Omega(\alpha,\alpha')-\omega_k+i\gamma_k}. \quad (60)$$

The normalizing factor $\alpha_k$ is determined by the requirement that

$$[B_\mathbf{k},B_\mathbf{k}^\dagger]_{\mathrm{RPA}}=1\,, \quad (61)$$

where $[\ ,\ ]_{\mathrm{RPA}}$ means that we calculate the commutator within the RPA. We obtain

$$\alpha_k=(m\omega_p^2\omega_k/N\hbar k^2)^{1/2}\,, \quad (62)$$

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where we have used the approximation

$$\frac{\gamma_k}{(\omega-\omega_k)^2+\gamma_k^2}\approx\pi\delta(\omega-\omega_k). \tag{63}$$

The form obtained here for the plasmon operator is similar to that obtained by other authors for the quantum electron gas, and for the electron-phonon gas.[13–16] The usual way of introducing these collective mode operators involves making a guess of their general form. In the procedure used here, the collective operators appear in a natural manner as the contributions arising from the resonances corresponding to the collective modes.

We can also obtain operators corresponding to transverse photons. For simplicity, we consider those wave vectors such that $k_z\ll k_\perp$. In this case there is one transverse direction that is uncoupled to the other directions.[11] Let us call this the $\lambda=1$ polarization and denote its direction by the unit vector $\mathbf{e}_{\mathbf{k}1}$. We then have

$$[\mathbf{Y}(\mathbf{k},\omega)^{-1}]_{11}=Y_{11}(\mathbf{k},\omega)^{-1}. \tag{64}$$

Furthermore, if the transverse mode is long-lived, $\gamma_k/\omega_k\ll1$, we have for $kc\gg\omega_p$

$$\begin{aligned}Y_{11}(\mathbf{k},\omega)&\approx(2/\omega_k)(\omega-\omega_k+i\gamma_k) & \omega\approx\omega_k\\ &\approx-(2/\omega_k)(\omega+\omega_k+i\gamma_k), & \omega\approx-\omega_k\end{aligned} \tag{65}$$

where

$$\gamma_k=-\frac{\pi m\omega_p^2}{2N\hbar\omega_k}\sum_{\alpha,\alpha',s}\mathbf{e}_{\mathbf{k}1}\cdot(\alpha|\mathbf{V}(-\mathbf{k})|\alpha')\mathbf{e}_{\mathbf{k}1}\cdot(\alpha'|\mathbf{V}(\mathbf{k})|\alpha)\times\Delta_s(\alpha,\alpha')\delta[\omega_k-\Omega(\alpha,\alpha')]. \tag{66}$$

Let us consider the asymptotic operator

$$\hat{e}_{\mathbf{k}1}\cdot\mathbf{q}_k=-(4\pi/V)^{1/2}e\sum_{\alpha,\alpha',s}\mathbf{e}_{\mathbf{k}1}\cdot(\alpha|\mathbf{V}(-\mathbf{k})|\alpha')\Omega(\alpha,\alpha')^{-2}\times Y_{11}[\mathbf{k},\Omega(\alpha,\alpha')]^{-1}b_s(\alpha,\alpha',0)e^{-i\Omega(\alpha,\alpha')t}. \tag{67}$$

If we approximate $Y_{11}^{-1}$, as shown in Eq. (65), we obtain

$$q_{\mathbf{k}1}=-e(\pi/\omega_k^2V)^{1/2}\sum_{\alpha,\alpha',s}\mathbf{e}_{\mathbf{k}1}\cdot(\alpha|\mathbf{V}(-\mathbf{k})|\alpha')\times b_s(\alpha,\alpha',0)\left[\frac{e^{-i\omega_kt}}{\Omega(\alpha,\alpha')-\omega_k+i\gamma_k}-\frac{e^{i\omega_kt}}{\Omega(\alpha,\alpha')+\omega_k+i\gamma_k}\right]. \tag{68}$$

In the case of a free field the relation between $q_{\mathbf{k}1}$ and the creation and annihilation operators is

$$q_{\mathbf{k}1}=(\hbar/2kc)^{1/2}(a_{\mathbf{k}1}+a_{-\mathbf{k}1}^\dagger). \tag{69}$$

Thus we define the dressed photon or quasiphoton annihilation operator as

$$A_{\mathbf{k}1}=e(2\pi/\hbar V\omega_k)^{1/2}\sum_{\alpha,\alpha',s}\mathbf{e}_{\mathbf{k}1}\cdot(\alpha'|\mathbf{V}(\mathbf{k})|\alpha')\times\frac{b_s(\alpha,\alpha',0)e^{i\omega_kt}}{\omega_k-\Omega(\alpha,\alpha')+i\gamma_k}. \tag{70}$$

With the use of the RPA and Eq. (63) we obtain the commutation relations

$$[A_{\mathbf{k}1},A_{\mathbf{k}'1}]_{\mathrm{RPA}}=0, \tag{71}$$

and

$$[A_{\mathbf{k}1},A_{\mathbf{k}'1}^\dagger]_{\mathrm{RPA}}=\delta_{\mathbf{k},\mathbf{k}'}.$$

The average number of dressed photons with momentum $\mathbf{k}$ is given by [with the use of Eq. (63)]

$$n_{\mathbf{k}1}=\langle A_{\mathbf{k}1}^\dagger A_{\mathbf{k}1}\rangle=\frac{\pi m\omega_p^2}{2\hbar N\gamma_k\omega_k}\sum_{\alpha,\alpha',s}|\mathbf{e}_{\mathbf{k}1}\cdot(\alpha'|\mathbf{V}(k)|\alpha)|^2f_{\alpha's}(1-f_{\alpha s})\times\delta[\omega_k-\Omega(\alpha,\alpha')]. \tag{72}$$

If the electrons are described by the Fermi-Dirac distribution, then

$$n_{\mathbf{k}1}=(e^{\beta\hbar\omega_k}-1)^{-1}. \tag{73}$$

We can obtain an explicit expression for the contribution of these photons to the kinetic equation given by Eq. (55). The sum over the wave vector is first broken into a part such that $k_z\ll k_\perp$ plus the remainder. We then separate out that part due to the $\mathbf{e}_{\mathbf{k}1}$ direction. We have for this part the expression

$$\begin{aligned}\left.\frac{\partial f}{\partial t}\right|_{\mathrm{ph}}=&\left(\frac{2\pi m^2\omega_p^2}{\hbar^2N^2}\right)\sum_{\alpha'}\sum_{\beta,\beta',s'}\\ &\times\sum_{\substack{\Delta\mathbf{k}\\ k_z\ll k_\perp}}|(\alpha'|\mathbf{V}(\mathbf{k})|\alpha)\cdot\mathbf{e}_{\mathbf{k}1}|^2|(\beta'|\mathbf{V}(-\mathbf{k})|\beta)\cdot\mathbf{e}_{\mathbf{k}1}|^2\\ &\times\Omega(\alpha,\alpha')^{-4}|Y_{11}[\mathbf{k},\Omega(\alpha,\alpha')]|^{-2}\\ &\times[f_{\alpha's}f_{\beta's'}(1-f_{\alpha s})(1-f_{\beta s'})\\ &-f_{\alpha s}f_{\beta s'}(1-f_{\alpha's})(1-f_{\beta's'})]\\ &\times\delta[E_\alpha+E_\beta-E_{\alpha'}-E_{\beta'}].\end{aligned} \tag{74}$$

With the use of Eq. (65) we can write

$$|Y_{11}(\mathbf{k},\omega)|^{-2}\approx\left(\frac{\pi\omega_k^2}{4\gamma_k}\right)\delta(\omega-\omega_k)+\left(\frac{\pi\omega_k^2}{4\gamma_k}\right)\delta(\omega+\omega_k). \tag{75}$$

If we use the above expression in Eq. (56), the expression for $n_{\mathbf{k}1}$ given in Eq. (72), and the following expression for $1+n_{\mathbf{k}1}$

$$1+n_{\mathbf{k}1}=\langle A_{\mathbf{k}1}A_{\mathbf{k}1}^\dagger\rangle=\frac{\pi m\omega_p^2}{2\hbar\gamma_kN\omega_k}\times\sum_{\beta,\beta',s}|\mathbf{e}_{\mathbf{k}1}\cdot(\beta'|\mathbf{V}(\mathbf{k})|\beta)|^2f_{\beta s}(1-f_{\beta's})\times\delta[\omega_k-\Omega(\beta,\beta')], \tag{76}$$

we obtain

$$\begin{aligned}\left.\frac{\partial f_{\alpha s}}{\partial t}\right|_{\mathrm{ph}}=&\left(\frac{\pi m\omega_p^2}{\hbar N\omega_k^2}\right)\sum_{\Delta\mathbf{k}}\sum_{\alpha'}|\mathbf{e}_{\mathbf{k}1}\cdot(\alpha'|\mathbf{V}(\mathbf{k})|\alpha)|^2\\ &\times\{[f_{\alpha's}(1-f_{\alpha s})(1+n_{-\mathbf{k}1})-f_{\alpha s}(1-f_{\alpha's})n_{\mathbf{k}1}]\\ &\times\delta[\omega_k-\Omega(\alpha,\alpha')]+[f_{\alpha s}(1-f_{\alpha's})(1+n_{\mathbf{k}1})\\ &-f_{\alpha's}(1-f_{\alpha s})n_{-\mathbf{k}1}]\delta[\omega_k+\Omega(\alpha,\alpha')]\}.\end{aligned} \tag{77}$$

The above equation has a familiar form. It simply describes the rate of change in $f_{\alpha s}$ as being due to simple absorption-emission processes. The effective transition rate for such a process involving a dressed photon with momentum $\hbar\mathbf{k}$ is

$$\frac{\pi m\omega_p^2}{\hbar N\omega_k^2}|\mathbf{e}_{\mathbf{k}1}\cdot(\alpha'|\mathbf{V}(k)|\alpha)|^2\delta(\omega_k-E_{\alpha'}+E_\alpha). \quad (78)$$

This differs from the "Golden Rule" answer by having $kc$ replaced by $\omega_k$.

It is clear that the same procedure can be used for more complicated collective modes. Clearly, we can never obtain terms that describe the scattering of photons by electrons since we neglected the $\mathbf{A}^2$ terms in this paper.

## VI. CONCLUSION

In this paper we have considered a quantum electron gas including radiation in the presence of a uniform magnetic field. With the use of the RPA and the Bogoliubov (adiabatic) assumption we were able to obtain long-time expressions for certain electron and field operators. These expressions were then used to calculate autocorrelation functions and to obtain a kinetic equation. The kinetic equation has the form of a generalized Balescu-Lenard equation. In the time required for this kinetic equation to become valid the photons have become dressed by the electrons, and the photon densities are functionals of the electron distribution function. This is similar to the results of Rostoker, Aamodt, and Eldridge.[17]

The asymptotic operators contain the inverse of the matrix that characterizes the collective modes of the system. If the resonances corresponding to the collective modes are very sharp, these operators can be separated into a part due to the collective modes and a part due to the individual particle motion (a similar breakup has been discussed by Carmi[13]). It is then possible to define the creation and annihilation operators for the collective modes. This has been done for two special cases that give rise to a longitudinal mode corresponding to the plasmons and a transverse mode corresponding to the dressed photon.

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## APPENDIX

The purpose of this Appendix is to list some interesting and useful identities. From the identity

$$[e^{i\mathbf{k}\cdot\mathbf{r}},H_0]=-\frac{\hbar\mathbf{k}}{m}\cdot e^{i\mathbf{k}\cdot\mathbf{r}}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}_0+\frac{\hbar\mathbf{k}}{2}\right),$$

where

$$H_0=\frac{1}{2m}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}_0\right)^2,$$

one obtains the relation

$$\Omega(\alpha,\alpha')(\alpha|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha')=-\mathbf{k}\cdot(\alpha|\mathbf{V}(\mathbf{k})|\alpha'). \quad (A1)$$

If we use the identity

$$[e^{-i\mathbf{k}\cdot\mathbf{r}},[e^{i\mathbf{k}\cdot\mathbf{r}},H_0]]=-\frac{\hbar^2k^2}{m},$$

we obtain

$$\sum_{\alpha'}[(\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha')(\alpha'|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha)+(\alpha|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha')(\alpha'|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha)]\Omega(\alpha,\alpha')=\frac{\hbar^2k^2}{m}. \quad (A2)$$

If we use Eq. (A1) we then obtain

$$\sum_{\alpha'}[(\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha')\mathbf{k}\cdot(\alpha'|\mathbf{V}(\mathbf{k})|\alpha)-(\alpha'|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha)\mathbf{k}\cdot(\alpha|\mathbf{V}(\mathbf{k})|\alpha')]=\frac{\hbar^2k^2}{m}. \quad (A3)$$

The use of Eq. (A1) results in the following relation:

$$\sum_{\alpha,\alpha',s}\frac{(\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha')\mathbf{k}\cdot(\alpha'|\mathbf{V}(\mathbf{k})|\alpha)\Delta_s(\alpha,\alpha')}{\omega-\Omega(\alpha,\alpha')}=\omega\sum_{\alpha,\alpha',s}\frac{|(\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha')|^2\Delta_s(\alpha,\alpha')}{\omega-\Omega(\alpha,\alpha')}. \quad (A4)$$

With the use of Eqs. (A1) and (A3) we can show that

$$\sum_{\alpha,\alpha',s}\frac{\mathbf{k}\cdot(\alpha|\mathbf{V}(-\mathbf{k})|\alpha')\mathbf{k}\cdot(\alpha'|\mathbf{V}(\mathbf{k})|\alpha)\Delta_s(\alpha,\alpha')}{\omega-\Omega(\alpha,\alpha')}=\omega^2\sum_{\alpha,\alpha',s}\frac{|(\alpha|e^{-i\mathbf{k}\cdot\mathbf{r}}|\alpha')|^2\Delta_s(\alpha,\alpha')}{\omega-\Omega(\alpha,\alpha')}+\frac{\hbar Nk^2}{m}. \quad (A5)$$

If we use the explicit form for the wave functions $\phi_\alpha(\mathbf{r})$ [Eq. (6)], it is easy to show that

$$\left(npq\left|\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}_0\right|npq\right)=\hbar p\frac{\mathbf{B}}{B}. \quad (A6)$$

We can then use the above equation and Eqs. (A1) and (A6) to show that

$$\sum_{\alpha,\alpha',s}\frac{\mathbf{k}\cdot(\alpha|\mathbf{V}(\mathbf{k})|\alpha')(\alpha'|\mathbf{V}(-\mathbf{k})|\alpha)\Delta_s(\alpha,\alpha')}{\omega-\Omega(\alpha,\alpha')}=-\omega\sum_{\alpha,\alpha',s}\frac{(\alpha|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha')(\alpha'|\mathbf{V}(-\mathbf{k})|\alpha)\Delta_s(\alpha,\alpha')}{\omega-\Omega(\alpha,\alpha')}-\frac{\hbar N\mathbf{k}}{m}. \quad (A7)$$

Some of the mathematical properties of the matrix element $(\alpha|e^{i\mathbf{k}\cdot\mathbf{r}}|\alpha')$, including the classical limit, are listed in Ref. 11.

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