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Multiparticle long-range rapidity correlations from fluctuation of the fireball longitudinal shape

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We calculate the genuine long-range multiparticle rapidity correlation functions, $C_n(y_1,\ldots,y_n)$ for $n = 2,3,4,5,6$, originating from fluctuations of the fireball longitudinal shape. In these correlation functions any contribution from the short-range two-particle correlations, and in general up to particle $(n-1)$ in C_n, is suppressed. The information about the fluctuating fireball shape in rapidity is encoded in the cumulants of coefficients of the orthogonal polynomial expansion of particle distributions in rapidity.

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I. INTRODUCTION

Fluctuations in the longitudinal structure of the fireball produced in heavy-ion collisions has drawn noticeable interest in recent years; see, e.g., Refs. [1–13]. These fluctuations result in new phenomena and modify known correlations in rapidity and azimuthal angle.

In Ref. [3] it was argued that fluctuations of the fireball longitudinal shape result in the specific long-range two-particle rapidity correlations that depend not only on the rapidity difference, $y_1 - y_2$, but also on the rapidity sum, $y_1 + y_2$. In analogy to the long-range azimuthal correlations originating from fluctuating shape of the fireball in the transverse direction [14,15], it was proposed to expand the single-particle rapidity distribution in terms of the orthogonal polynomials T_i [3]:

$$\rho^{\{a\}}(y;a_0,a_1,\ldots) = \rho(y)\Big[1 + \sum\nolimits_{i=0} a_i T_i(y)\Big], \qquad (1.1)$$

where $\rho(y)$ is the measured single-particle distribution. Here and in the following the superscript $\{a\}$ means that the corresponding distribution is at a given $a_0,a_1,\ldots$. Throughout the paper we use $y = \frac{\eta}{Y}$, where η is rapidity or pseudorapidity in the range $[-Y,Y]$. For $T_k(y)$ we choose $T_k(y) = (k+\frac{1}{2})^{1/2} P_k(\frac{\eta}{Y})$ [16], with the orthogonalization condition $\int_{-1}^{1} dy T_i(y) T_k(y) = \delta_{ik}$, where P_k are the Legendre polynomials.[1]

In Eq. (1.1) a_0 represents rapidity-independent multiplicity fluctuation of the fireball as a whole, a_1 is an event-by-event rapidity asymmetric component,[2] etc.; see Ref. [3] for more details. Averaging Eq. (1.1) over $a_0,a_1,\ldots$ with the probability distribution $P(a_0,a_1,\ldots)$ we obtain $\langle a_i \rangle = 0$.

The two-particle rapidity distribution at a given $a_0,a_1,\ldots$ is

$$\rho_2^{\{a\}}(y_1,y_2;a_0,a_1,\ldots) = \rho^{\{a\}}(y_1;a_0,a_1,\ldots)\rho^{\{a\}}(y_2;a_0,a_1,\ldots). \qquad (1.2)$$

Taking an average over a_i and subtracting $\rho(y_1)\rho(y_2)$, we obtain the final two-particle rapidity correlation function[3] [3]

$$C_2(y_1,y_2) = \rho_2(y_1,y_2) - \rho(y_1)\rho(y_2) = \rho(y_1)\rho(y_2)\Big[\sum\nolimits_{i,k} \langle a_i a_k \rangle T_i(y_1) T_k(y_2)\Big], \qquad (1.3)$$

where $\rho_2(y_1,y_2)$ is the measured two-particle density. Using Eq. (1.3) and the orthogonalization condition for T_i we obtain

$$\langle a_i a_k \rangle = \int_{-1}^{1} dy_1 dy_2 \frac{C_2(y_1,y_2) T_i(y_1) T_k(y_2)}{\rho(y_1)\rho(y_2)}. \qquad (1.4)$$

Obviously any correlation function $C_2(y_1,y_2)$ can be decomposed into a series of orthogonal polynomials, so the coefficients $\langle a_i a_k \rangle$ contain information not only about the fluctuating long-range rapidity shape of the fireball but also, e.g., about resonance decays or jets. It is essential to remove this unwanted background, and this is the subject of the paper.

We propose to measure the cumulants of the genuine multiparticle rapidity correlation functions in analogy to the multiparticle flow cumulants [19,20], which proved to be effective in removing non-flow effects from correlations in azimuthal angle [21].

In the next section we derive formulas for the genuine three-, four-, five- and six-particle correlation functions originating from the fluctuating longitudinal shape of the fireball. We discuss our results in Sec. III.

II. MULTIPARTICLE CORRELATIONS

In this section we discuss multiparticle correlations originating from the fluctuating fireball shape in rapidity. At this point it is useful to comment on the experimental way of estimating the integrals over the multiparticle distributions. In the experimental analysis [1], estimates of $\langle a_i a_k \rangle$ are obtained

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[1] In Ref. [3] we expanded the distribution (1.1) in the Chebyshev polynomials but other choices are certainly possible.

[2] In the wounded-nucleon model [17,18] and for symmetric $A + A$ collisions, a_1 corresponds to the difference between left- and right-going wounded nucleons, $a_1 \propto w_L - w_R$.

[3] Throughout the paper we use the simplified notation $\sum_{i,k,\ldots,m}$ for $\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\cdots\sum_{m=0}^{\infty}$.

from the integration of the two-particle correlation function. It is challenging to apply this method to the higher-order cumulants. However, the standard procedure of summing over n tuples, e.g.,

$$\begin{aligned}\langle T_i T_j T_k T_l\rangle &\equiv \int_{-1}^{1} dy_1 dy_2 dy_3 dy_4 \\ &\quad \times \frac{\rho_4(y_1,y_2,y_3,y_4)T_i(y_1)T_j(y_2)T_k(y_3)T_l(y_4)}{\rho(y_1)\rho(y_2)\rho(y_3)\rho(y_4)} \\ &= \left\langle \sum_{a,b,c,d} \frac{T_i(y_a)}{\rho(y_a)}\frac{T_j(y_b)}{\rho(y_b)}\frac{T_k(y_c)}{\rho(y_c)}\frac{T_l(y_d)}{\rho(y_d)}\right\rangle, \end{aligned} \tag{2.1}$$

can be applied for samples with sufficient statistics [22]. $\rho_4(y_1,y_2,y_3,y_4)$ is the measured four-particle rapidity density. In the last line of the above expression, the sum runs over four different particles in a given event and the average is over all events.

By definition,

$$\langle T_i\rangle = \int dy T_i(y) = \sqrt{2}\delta_{i,0} \tag{2.2}$$

for the chosen normalization of $T_i(y)$.

A. Three-particle correlations

The three-particle distribution at a given $a_0,a_1,\ldots$ is

$$\rho_3^{\{a\}}(y_1,y_2,y_3;a_0,a_1,\ldots) = \rho^{\{a\}}(y_1;a_0,a_1,\ldots)\rho^{\{a\}}(y_2;a_0,a_1,\ldots)\rho^{\{a\}}(y_3;a_0,a_1,\ldots). \tag{2.3}$$

Expanding on the orthogonal basis and taking an average over a_i we obtain

$$\frac{\rho_3(y_1,y_2,y_3)}{\rho(y_1)\rho(y_2)\rho(y_3)} = 1 + \sum\nolimits_{i,k} \langle a_i a_k\rangle[T_i(y_1)T_k(y_2) + T_i(y_1)T_k(y_3) + T_i(y_2)T_k(y_3)] + \sum\nolimits_{i,k,m} \langle a_i a_k a_m\rangle T_i(y_1)T_k(y_2)T_m(y_3). \tag{2.4}$$

We are interested in extracting information about the genuine three-particle correlations,[4] $C_3(y_1,y_2,y_3)$, defined as

$$\rho_3(y_1,y_2,y_3) = \rho(y_1)\rho(y_2)\rho(y_3) + \rho(y_1)C_2(y_2,y_3) + \rho(y_2)C_2(y_1,y_3) + \rho(y_3)C_2(y_1,y_2) + C_3(y_1,y_2,y_3), \tag{2.5}$$

where $\rho_3(y_1,y_2,y_3)$ is the three-particle rapidity density.

We have

$$C_3(y_1,y_2,y_3) = \rho(y_1)\rho(y_2)\rho(y_3)\Big[\sum\nolimits_{i,k,m} \langle a_i a_k a_m\rangle T_i(y_1)T_k(y_2)T_m(y_3)\Big], \tag{2.6}$$

and

$$\langle a_i a_k a_m\rangle_{[3]} = \int dy_1 dy_2 dy_3 \frac{C_3(y_1,y_2,y_3)T_i(y_1)T_k(y_2)T_m(y_3)}{\rho(y_1)\rho(y_2)\rho(y_3)}, \tag{2.7}$$

where $\langle\cdots\rangle_{[3]}$ denotes that $\langle a_i a_k a_m\rangle$ is sensitive to C_3 but it does not depend on the lower-order correlation function C_2. For C_3 we have $\langle a_i a_k a_m\rangle_{[3]} = \langle a_i a_k a_m\rangle$, which is not the case for higher-order correlation functions.

Using Eq. (2.5) we can relate $\langle a_i a_k a_m\rangle_{[3]}$ through integrals of the two- and three-particle densities ρ_n and finally through sums over n tuples; see Eq. (2.1):

$$\begin{aligned}\langle a_i a_k a_m\rangle_{[3]} &= \langle T_i T_k T_m\rangle - \langle T_i\rangle\langle T_k T_m\rangle - \langle T_k\rangle\langle T_i T_m\rangle - \langle T_m\rangle\langle T_i T_k\rangle + 2\langle T_i\rangle\langle T_k\rangle\langle T_m\rangle \\ &\equiv \langle T_i T_k T_m\rangle_{[3]}. \end{aligned} \tag{2.8}$$

Perhaps C_3 is not particularly useful because $\langle a_i^3\rangle$ is expected to be rather small, if not zero (by definition $\langle a_i\rangle = 0$). Specific effects of correlations resulting from bias of event multiplicity on rapidity distribution, e.g., stronger longitudinal expansion in events with higher fireball density, can be tested by using mixed correlations

$$\left\langle a_0 a_k^2\right\rangle_{[3]} = \langle T_0 T_k T_k\rangle - \langle T_0\rangle\langle T_k T_k\rangle, \tag{2.9}$$

where $\langle T_0\rangle = \sqrt{2}$ for the chosen normalization and $k > 0$.

[4]In general, the genuine n-particle correlation function $C_n(y_1,\ldots,y_n)$ is nonzero only if there is a physical mechanism directly correlating n or more particles.

B. Four-particle correlation

Here we discuss the more interesting case of the four-particle correlation function. Performing analogous calculations we obtain

$$\begin{aligned}\frac{\rho_4(y_1,y_2,y_3,y_4)}{\rho(y_1)\rho(y_2)\rho(y_3)\rho(y_4)} = 1 &+ \sum\nolimits_{i,k}\langle a_i a_k\rangle[T_i(y_1)T_k(y_2)+T_i(y_1)T_k(y_3)+T_i(y_1)T_k(y_4)\\ &+T_i(y_2)T_k(y_3)+T_i(y_2)T_k(y_4)+T_i(y_3)T_k(y_4)]\\ &+\sum\nolimits_{i,k,m}\langle a_i a_k a_m\rangle[T_i(y_1)T_k(y_2)T_m(y_3)+T_i(y_1)T_k(y_2)T_m(y_4)\\ &+T_i(y_1)T_k(y_3)T_m(y_4)+T_i(y_2)T_k(y_3)T_m(y_4)]\\ &+\sum\nolimits_{i,k,m,n}\langle a_i a_k a_m a_n\rangle T_i(y_1)T_k(y_2)T_m(y_3)T_n(y_4). \end{aligned} \tag{2.10}$$

We are interested in the genuine four-particle correlation function, $C_4(y_1,y_2,y_3,y_4)$, defined as

$$\begin{aligned}\rho_4(y_1,y_2,y_3,y_4) = {}&\rho(y_1)\rho(y_2)\rho(y_3)\rho(y_4)+\rho(y_1)\rho(y_2)C_2(y_3,y_4)+\rho(y_1)\rho(y_3)C_2(y_2,y_4)\\ &+\rho(y_1)\rho(y_4)C_2(y_2,y_3)+\rho(y_2)\rho(y_3)C_2(y_1,y_4)+\rho(y_2)\rho(y_4)C_2(y_1,y_3)\\ &+\rho(y_3)\rho(y_4)C_2(y_1,y_2)+\rho(y_1)C_3(y_2,y_3,y_4)+\rho(y_2)C_3(y_1,y_3,y_4)\\ &+\rho(y_3)C_3(y_1,y_2,y_4)+\rho(y_4)C_3(y_1,y_2,y_3)+C_2(y_1,y_2)C_2(y_3,y_4)\\ &+C_2(y_1,y_3)C_2(y_2,y_4)+C_2(y_1,y_4)C_2(y_2,y_3)+C_4(y_1,y_2,y_3,y_4).\end{aligned} \tag{2.11}$$

Performing straightforward calculations, we obtain

$$\frac{C_4(y_1,y_2,y_3,y_4)}{\rho(y_1)\rho(y_2)\rho(y_3)\rho(y_4)} = \sum\nolimits_{i,k,m,n}\langle a_i a_k a_m a_n\rangle_{[4]}T_i(y_1)T_k(y_2)T_m(y_3)T_n(y_4), \tag{2.12}$$

where

$$\langle a_i a_k a_m a_n\rangle_{[4]} \equiv \langle a_i a_k a_m a_n\rangle - \langle a_i a_k\rangle\langle a_m a_n\rangle - \langle a_i a_m\rangle\langle a_k a_n\rangle - \langle a_i a_n\rangle\langle a_k a_m\rangle, \tag{2.13}$$

with $\langle\cdots\rangle_{[4]}$ denoting that the object depends on C_4 but not on the lower-order correlations.

At this stage we are mostly interested in extracting the leading terms, $i=k=m=n$ (and in particular the asymmetric term a_1),

$$\begin{aligned}\langle a_i^4\rangle_{[4]} &\equiv \langle a_i^4\rangle - 3\langle a_i^2\rangle^2\\ &= \int dy_1dy_2dy_3dy_4\frac{C_4(y_1,y_2,y_3,y_4)T_i(y_1)T_i(y_2)T_i(y_3)T_i(y_4)}{\rho(y_1)\rho(y_2)\rho(y_3)\rho(y_4)}\\ &= \langle T_iT_iT_iT_i\rangle - 3\langle T_iT_i\rangle^2,\end{aligned} \tag{2.14}$$

where in the last line of the above equation ($i>0$) we show the way to calculate the cumulant; see Eq. (2.1).

Another interesting term in the expansion (2.12) is the mixed term $\langle a_0^2a_k^2\rangle$ for $k>0$. The expression for such a cumulant reads

$$\begin{aligned}\langle a_0^2a_k^2\rangle_{[4]} &\equiv \langle a_0^2a_k^2\rangle - \langle a_0^2\rangle\langle a_k^2\rangle - 2\langle a_0a_k\rangle^2\\ &= \langle T_0T_0T_kT_k\rangle - \langle T_0T_0\rangle\langle T_kT_k\rangle - 2\langle T_0T_k\rangle^2 - 2\langle T_0\rangle\langle T_0T_kT_k\rangle + 2\langle T_0\rangle^2\langle T_kT_k\rangle.\end{aligned} \tag{2.15}$$

This expression removes two- and three-particle correlations and correctly takes into account that $\langle T_0\rangle \neq 0$. In particular $\langle a_0^2a_2^2\rangle_{[4]}$ could be a measure of the genuine correlations between event multiplicity and the width of the particle distribution in rapidity.

C. Five-particle correlation

For the genuine five-particle correlation function, $C_5(y_1,\ldots,y_5)$, defined as[5]

$$\rho_5 = \rho\rho\rho\rho\rho + \underbrace{\rho C_4}_{5} + \underbrace{\rho\rho C_3}_{10} + \underbrace{\rho\rho\rho C_2}_{10} + \underbrace{\rho C_2C_2}_{15} + \underbrace{C_2C_3}_{10} + C_5, \tag{2.16}$$

we obtain

$$\frac{C_5(y_1,\ldots,y_5)}{\rho(y_1)\cdots\rho(y_5)} = \sum\nolimits_{i,k,m,n,r}\langle a_ia_ka_ma_na_r\rangle_{[5]}T_i(y_1)T_k(y_2)T_m(y_3)T_n(y_4)T_r(y_5), \tag{2.17}$$

[5]For clarity we skip the argument y_i and show only the numbers of possible combinations.

where

$$\langle a_i a_k a_m a_n a_r \rangle_{[5]} \equiv \langle a_i a_k a_m a_n a_r \rangle - \left[\underbrace{\langle a_i a_k \rangle \langle a_m a_n a_r \rangle + \cdots}_{\text{10 variations}} \right]. \tag{2.18}$$

The leading term is

$$\langle a_i^5 \rangle_{[5]} \equiv \langle a_i^5 \rangle - 10 \langle a_i^2 \rangle \langle a_i^3 \rangle = \int dy_1 \cdots dy_5 \frac{C_5(y_1,\ldots,y_5) T_i(y_1)\cdots T_i(y_5)}{\rho(y_1)\cdots\rho(y_5)} = \langle T_i T_i T_i T_i T_i \rangle - 10 \langle T_i T_i \rangle \langle T_i T_i T_i \rangle, \tag{2.19}$$

where in the last line of the above equation we assume $i > 0$.

D. Six-particle correlation

Finally, for the six-particle correlation function $C_6(y_1,\ldots,y_6)$, defined as

$$\rho_6 = \rho\rho\rho\rho\rho\rho + \underbrace{\rho C_5}_{6} + \underbrace{\rho\rho C_4}_{15} + \underbrace{\rho\rho\rho C_3}_{20} + \underbrace{\rho\rho\rho\rho C_2}_{15} + \underbrace{\rho C_2 C_3}_{60} + \underbrace{\rho\rho C_2 C_2}_{45} + \underbrace{C_2 C_4}_{15} + \underbrace{C_3 C_3}_{10} + \underbrace{C_2 C_2 C_2}_{15} + C_6, \tag{2.20}$$

we obtain

$$\frac{C_6(y_1,\ldots,y_6)}{\rho(y_1)\cdots\rho(y_6)} = \sum\nolimits_{i,k,m,n,r,s} \langle a_i a_k a_m a_n a_r a_s \rangle_{[6]} T_i(y_1) T_k(y_2) T_m(y_3) T_n(y_4) T_r(y_5) T_s(y_6), \tag{2.21}$$

where

$$\begin{aligned}
\langle a_i a_k a_m a_n a_r a_s \rangle_{[6]} \equiv{}& \langle a_i a_k a_m a_n a_r a_s \rangle - \left[\underbrace{\langle a_i a_k \rangle \langle a_m a_n a_r a_s \rangle + \cdots}_{\text{15 variations}} \right] - \left[\underbrace{\langle a_i a_k a_m \rangle \langle a_n a_r a_s \rangle + \cdots}_{\text{10 variations}} \right] \\
&+ 2 \left[\underbrace{\langle a_i a_k \rangle \langle a_m a_n \rangle \langle a_r a_s \rangle + \cdots}_{\text{15 variations}} \right].
\end{aligned} \tag{2.22}$$

The leading term is

$$\begin{aligned}
\langle a_i^6 \rangle_{[6]} &\equiv \langle a_i^6 \rangle - 15 \langle a_i^2 \rangle \langle a_i^4 \rangle - 10 \langle a_i^3 \rangle^2 + 30 \langle a_i^2 \rangle^3 \\
&= \int dy_1 \cdots dy_6 \frac{C_6(y_1,\ldots,y_6) T_i(y_1)\cdots T_i(y_6)}{\rho(y_1)\cdots\rho(y_6)} \\
&= \langle T_i T_i T_i T_i T_i T_i \rangle - 15 \langle T_i T_i \rangle \langle T_i T_i T_i T_i \rangle - 10 \langle T_i T_i T_i \rangle^2 + 30 \langle T_i T_i \rangle^3,
\end{aligned} \tag{2.23}$$

where in the last line we assume $i > 0$.

III. COMMENTS AND CONCLUSIONS

We propose to measure higher-order cumulants of event-by-event fluctuations of rapidity distribution. The multiparticle rapidity distributions $\rho_n(y_1,\ldots,y_n)$ can be expanded in the basis of orthogonal polynomials $T_{i_1}(y_1)\cdots T_{i_n}(y_n)$ with coefficients $\langle a_{i_1}\cdots a_{i_n}\rangle$. The coefficients have contributions from the long-range rapidity shape fluctuations of the distribution function, that we are after, and from other sources, e.g., the short-range correlations from resonance decays and jets. Calculating higher cumulants of such averages reduces the contribution from these short-range correlations. For example, in the fourth-order cumulant $\langle T_i T_j T_k T_l \rangle_{[4]}$ the contribution from two- and three-particle correlations is removed and, if the fourth-order background correlations are neglected, it can be directly compared to the fourth cumulant of the expansion coefficients $\langle a_i a_j a_k a_l \rangle_{[4]}$.

The extracted value of the higher cumulants $\langle a_{i_1}\cdots a_{i_n}\rangle_{[n]}$ can be compared to predictions of models of event-by-event fluctuations of rapidity distributions (1.1). The cumulant of the coefficients a_i can be calculated in models once their event-by-event distribution is known. The proposed method to study the cumulants of the expansion coefficients does not rely on the precise model assumptions for fluctuations of rapidity distributions. It remains a subject of further studies to calculate all significant four- or six-order cumulants, in different models of energy deposition in hadronic collisions. The direct calculation of higher-order cumulants requires a very large statistics, which is easily available at the CERN Large Hadron Collider (LHC) (see, e.g., Ref. [21]), but prevented us from applying the method to realistic hydrodynamic calculations.

Finally, we would like to emphasize that our method is applicable not only to symmetric $A + A$ collisions but also to asymmetric interactions including $p + A$. It would be also interesting to perform measurement in $p + p$ collisions, where the internal quark (diquark [23]) structure of a proton should results in, e.g., nonzero asymmetric term $\langle a_1^n \rangle_{[n]}$, $n = 2,4,6$.

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[1] ATLAS Collaboration, ATLAS-CONF-2015-020; http://cds.cern.ch/record/2029370.
[2] A. Bialas, A. Bzdak, and K. Zalewski, Phys. Lett. B **710**, 332 (2012).
[3] A. Bzdak and D. Teaney, Phys. Rev. C **87**, 024906 (2013).
[4] P. Bożek, W. Broniowski, and J. Moreira, Phys. Rev. C **83**, 034911 (2011).
[5] P. Bożek, W. Broniowski, and A. Olszewski, Phys. Rev. C **91**, 054912 (2015).
[6] A. Bialas and K. Zalewski, Nucl. Phys. A **860**, 56 (2011).
[7] A. Olszewski and W. Broniowski, Phys. Rev. C **92**, 024913 (2015).
[8] Y. Cheng, Y.-L. Yan, D.-M. Zhou, X. Cai, B.-H. Sa, and L. P. Csernai, Phys. Rev. C **84**, 034911 (2011).
[9] L. P. Csernai, G. Eyyubova, and V. K. Magas, Phys. Rev. C **86**, 024912 (2012).
[10] L. Pang, Q. Wang, and X.-N. Wang, Phys. Rev. C **86**, 024911 (2012).
[11] V. Vovchenko, D. Anchishkin, and L. P. Csernai, Phys. Rev. C **88**, 014901 (2013).
[12] J. Jia and P. Huo, Phys. Rev. C **90**, 034915 (2014).
[13] L.-G. Pang, G.-Y. Qin, V. Roy, X.-N. Wang, and G.-L. Ma, Phys. Rev. C **91**, 044904 (2015).
[14] S. A. Voloshin, A. M. Poskanzer, and R. Snellings, arXiv:0809.2949 [nucl-ex].
[15] B. Alver and G. Roland, Phys. Rev. C **81**, 054905 (2010); **82**, 039903 (2010).
[16] J. Jia, S. Radhakrishnan, and M. Zhou, arXiv:1506.03496 [nucl-th].
[17] A. Bialas, M. Bleszynski, and W. Czyz, Nucl. Phys. B **111**, 461 (1976).
[18] A. Bialas and W. Czyz, Acta Phys. Pol. B **36**, 905 (2005).
[19] N. Borghini, P. M. Dinh, and J. Y. Ollitrault, Phys. Rev. C **63**, 054906 (2001).
[20] N. Borghini, P. M. Dinh, and J. Y. Ollitrault, Phys. Rev. C **64**, 054901 (2001).
[21] CMS Collaboration, V. Khachatryan *et al.*, Phys. Rev. Lett. **115**, 012301 (2015).
[22] A. Bilandzic, R. Snellings, and S. Voloshin, Phys. Rev. C **83**, 044913 (2011).
[23] A. Bialas and A. Bzdak, Phys. Rev. C **77**, 034908 (2008); Phys. Lett. B **649**, 263 (2007).