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Currents generated by lower hybrid waves

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Electron currents can be driven in a linear plasma by the absorption of lower-hybrid waves excited primarily in one direction. Current-drive has been demonstrated both for collisional and resonant-electron absorption. The magnitude of the excited current is compared with the predictions from an electron kinetic equation with a Lorentz collision operator in the regime $k_{\parallel}v_{te}/\omega \ll 1$.

I. INTRODUCTION

Damping of traveling radio-frequency waves has been proposed as a mechanism for driving steady-state toroidal currents in a tokamak plasma. Lower-hybrid waves are a good possibility¹ because the waves can be damped strongly by the plasma. Also, their relatively low frequency makes possible the use of high-power sources which are presently available.

As far back as 1952, Thonemann, Cowhig, and Davenport² used Alfvén waves to drive currents. Several others have demonstrated current generation through use of electromagnetic waves³⁻¹⁰ and ion cyclotron waves.¹¹ Theoretical aspects have been discussed by others,¹²⁻¹⁶ at least one of whom¹⁴ felt the concept not as viable as using ion beams for current drive. Starting about two years ago, Fisch,^{1,17-19} Bers,¹⁸ and Karney¹⁹ made convincing arguments that driving toroidal currents via rf waves was feasible. Work by Klíma²⁰ supports the conclusions of these recent arguments. Decker and Hirschfield²¹ and, more recently, Wong²² have performed experiments demonstrating that nearly electrostatic lower-hybrid waves can couple by collisionless absorption to electron currents in beam-supported plasmas. McWilliams *et al.*,²³ showed preliminary results indicating that the electrostatic lower-hybrid wave could also drive currents via collisional absorption in isotropic plasmas. There is now evidence²⁴⁻²⁶ that lower-hybrid waves can drive currents in tori. The theory for current drive has now progressed to specific design considerations for reactor grade plasmas.²⁷

This paper discusses currents driven by lower-hybrid waves not supported by beams. Experimental results show the current generated over a range of plasma and wave parameters, which include regimes where the waves are damped collisionally or damped by collisionless means. The theory presented here includes a prediction from kinetic theory of the current generated in the plasma when the lower hybrid waves are collisionally damped.

The outline of the paper is as follows: The theory is discussed in Sec. II, including a brief outline of lower hybrid wave propagation, and calculations of the current generated under conditions where the wave is damped by collisional or collisionless mechanisms.

Section III describes the experimental apparatus, plasma parameters, and experimental techniques employed. Section IV presents experimental results of electron currents driven by collisional wave damping and currents generated via collisionless damping. Section V contains a discussion and conclusions.

II. THEORY

A. Wave propagation

Let us model the waves and plasma in a slab geometry with the confining magnetic field in the z direction and the density increasing in the minus x direction away from the antenna. The plasma is homogeneous in the z and y directions. The frequencies are ordered $\omega_{ci} \ll \omega \ll \omega_{ce}$, ω_{pe} , where ω_{ci} , ω_{ce} are the ion and electron gyro-frequencies, ω is the wave frequency, and ω_{pe} is the electron plasma frequency. Hence, the ions are essentially unmagnetized and the electrons are strongly magnetized. The region at the edge of the plasma where $\omega > \omega_{pe}$ is assumed to be very thin. Proper matching across this thin layer can be done, and the reader is referred to Brambilla's work²⁸ or that done by Chan and Chiu²⁹ or Wilson and Wong.³⁰ For a slowly varying plasma density, the dispersion relation for the slow waves has been solved by Briggs and Parker³¹ and Bellan and Porkolab.³² The local dispersion relation is found to be

$$\omega^2 = \omega_{ih}^2 \left(1 + \frac{m_i}{m_e} \frac{k_{\parallel}^2}{k_{\perp}^2} \right), \quad (1)$$

where $\omega_{ih}^2 = \omega_{pi}^2 / (1 + \omega_{pe}^2 / \omega_{ce}^2)$, ω_{pi} is the ion plasma frequency, and $k_{\parallel}$ and $k_{\perp}$ are the wavenumbers parallel and perpendicular to the confining magnetic field, respectively. If we further restrict ω so that $\omega \gg \omega_{pi}$ everywhere in the plasma and $\omega_{pe}^2 \ll \omega_{ce}^2$, then

$$\omega \approx (k_{\parallel} / k_{\perp}) \omega_{pe}, \quad (2)$$

and it is easily seen that the wave propagates in the well-known resonance cones. Additionally, there are some restrictions on the wave for accessibility to a given plasma density. These restrictions have been discussed by Golant³³ and Stix³⁴ and studied experimentally by Motley *et al.*³⁵ Kinetic theory³⁶ yields a complex dispersion relation with a temporal damping rate for the waves given by

$$\gamma \approx \nu + \pi^{1/2} \omega \alpha^3 \exp(-\alpha^2), \quad (3)$$

where

$$\alpha \equiv \frac{\omega}{k_{\parallel} (2k T_e / m_e)^{1/2}}.$$

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The first contribution to (3) is ν , which represents the collisional damping, e.g., electron-ion and electron-neutral collisions, while the second term is the Landau damping contribution. For a wave launched at a given location, this damping can also be viewed as a damping of each $k_{\parallel}$ component of the wave in space as it propagates away from the antenna. Probes for this experiment were able to measure the square of the electric field, and, hence, it is of interest to calculate the expected $E^2(k_{\parallel})$ as a function of distance from the antenna.

Control of the wave spectrum launched by the antenna is important for driving currents. In particular, an asymmetric spectrum must be obtained in order to produce a unidirectional current. Figure 1 shows the calculated spectrum for an antenna of principal wavelength 8 cm excited at 70 MHz. A ring antenna similar to that used by Bellan and Porkolab³⁷ was used in the experiment. The spectrum calculated here was found by assuming a 90° phase difference between rings (equal amplitude to all rings) and taking the Fourier spectrum of the resulting vacuum electric field at the antenna. A more complete description of the particular antennas used, including the effects of different phasings to the rings, may be found later in the paper. Figure 1(a) shows the calculated spectrum at the antenna, while Fig. 1(b) shows the effect of Landau damping on the wave as it travels away from the antenna. A temperature of $T_e = 11.5$ eV was used for the calculations because in a later section we observe that an 11.5 eV temperature predicts curves which fit well with the experimental data obtained when a Langmuir probe indicated $T_e \approx 10.6$ eV. As expected, the shorter wavelengths damp rather quickly, while the longer wavelengths excited by the antenna are less affected.

B. Simple current generation model

Oscillating electromagnetic fields can cause steady-state forces (i.e., time averages are nonzero) to act on plasmas. Examples of this are the ponderomotive force or other mechanisms which can make $\mathbf{j} \cdot \mathbf{E}$ nonzero, such as Landau damping or collisions. We wish to determine the effect on electron motion when collisions are the mechanism for energy loss in a wave.

A simple model²³ (also obtained by Schmidt³⁸) is developed for order-of-magnitude estimates and to help in visualizing the underlying physical processes involved. The next section will present a kinetic theory applicable to our experiment so that a more precise estimate of the currents generated in a plasma can be obtained.

The plasma is considered to be confined by a uniform magnetic field, $\mathbf{B}_0 = B_0 \hat{z}$. The ions are cold and the plasma parameters are such that $\omega_{ce} \gg \omega_{pe} \gg \omega > \omega_{pi}$. The motion of an electron is then governed by the Lorentz force and a collisional term

$$\frac{dv}{dt} = -\frac{eE}{m} \sin(k_{\parallel}z - \omega t) - \nu v, \quad (4)$$

for an rf traveling wave. The experimental conditions were such that

$$\delta \equiv \nu/\omega \sim k_{\parallel}v_{te}/\omega \ll 1. \quad (5)$$

We desire to know the long term effects of the wave on the particles and hence divide the velocity and position of the electron into a slow and fast part

$$v = \langle v \rangle + \delta v, \quad z = \langle z \rangle + \delta z, \quad (6)$$

where $\langle \rangle$ denotes a time average over the wave period. Carrying terms to $O(E^2)$ and expanding about $\langle z \rangle$ gives

$$\begin{aligned} \frac{d\delta v}{dt} &\approx -\frac{eE}{m} \sin(k_{\parallel}\langle z \rangle - \omega t) \\ &\quad - \frac{eE}{m} k_{\parallel}\delta z \cos(k_{\parallel}\langle z \rangle - \omega t) - \nu\delta v. \end{aligned} \quad (7)$$

FIG. 1. Theoretical slow wave antenna spectra. (a) Calculated spectrum for a slow wave antenna of the type used in this experiment. Eight cm principal wavelength. (b) Calculated spectrum as a function of distance from the antenna, when Landau damping is significant. $f = 70$ MHz, $T_e = 11.5$ eV.

We desire to find the induced current in the electrons and so look for solutions of δv with one component in phase with E and one component out of phase with E ; i.e.,

$$\delta v = A \sin(k_{\parallel} z) - \omega t + B \cos(k_{\parallel} z) - \omega t. \quad (8)$$

Substituting this into Eq. (7) we get the following restraints on A and B :

$$-A\omega = -(eE/m)k_{\parallel}\delta z - B\nu, \quad (9)$$

$$B\omega = -eE/m - A\nu.$$

Since δz is probably of order E , Eq. (9) can be solved to order E , giving A and B . δv can now be integrated to yield:

$$\begin{aligned} \delta z \approx & \frac{-\nu}{\omega(\nu^2 + \omega^2)} \frac{eE}{m} \cos(k_{\parallel} z) - \omega t \\ & + \frac{1}{(\nu^2 + \omega^2)} \frac{eE}{m} \sin(k_{\parallel} z) - \omega t. \end{aligned} \quad (10)$$

The time evolution of $\langle v \rangle$ can now be written

$$\begin{aligned} \frac{d\langle v \rangle}{dt} \approx & -\frac{eE}{m} \left\langle \sin(k_{\parallel} z) - \omega t \right. \\ & \left. + k_{\parallel} \delta z \cos(k_{\parallel} z) - \omega t \right\rangle - \nu \langle v \rangle \\ \approx & \frac{\nu}{\omega(\nu^2 + \omega^2)} \frac{e^2 E^2}{m^2} \frac{k_{\parallel}}{2} - \nu \langle v \rangle. \end{aligned} \quad (11)$$

Thus,

$$\langle v \rangle = C e^{-\nu t} + \frac{1}{2} \frac{e^2 E^2}{m^2} \frac{k_{\parallel}}{\omega(\nu^2 + \omega^2)}, \quad (12)$$

where C is determined by the initial conditions. For the condition $\delta \ll 1$, there results a current density of

$$\langle j \rangle = -e\omega_{pe}^2 k_{\parallel} E^2 / 8\pi m \omega^3. \quad (13)$$

Hence, for electrons interacting with an rf traveling wave in the presence of a mechanism which can change the electron response from being purely reactive, a net electron flow in the direction of propagation of the wave will occur on time scales $t \gg 1/\omega$.

C. Full collisional current calculation

The previous section shows that one may expect a damped traveling wave to generate a current in a plasma. Let us now develop a model with more detail and apply the results to lower-hybrid waves specifically. Consider a cylindrical plasma of length L in a uniform magnetic field $B_0 = B_0 \hat{z}$ with cold ions and electron-ion collisions the dominant collisional process. This model approximates one of the conditions in which the experiments were performed. The electron motion may be approximately described by an electron kinetic equation with the Lorentz collision operator³⁹:

$$\begin{aligned} \frac{\partial f_e}{\partial t} + v_z \frac{\partial f_e}{\partial z} - \frac{e}{m} E_z \left(\frac{1-\mu^2}{v} \frac{\partial}{\partial \mu} + \mu \frac{\partial}{\partial v} \right) f_e \\ = \frac{v_z^3}{v^3} \frac{\partial}{\partial \mu} (1-\mu^2) \frac{\partial f_e}{\partial \mu}, \end{aligned} \quad (14)$$

where f_e is the electron distribution function

$$v_z = \left(\frac{2kT_e}{m_e} \right)^{1/2}, \quad \nu = \frac{2\pi n_e e^4 \ln \Lambda}{m_e^2 v_z^3},$$

and $\mu = \cos \theta$, with θ the angle of v with respect to the main magnetic field. As in the previous section (and also for part of the experiment) the problem may be restricted to the regime

$$k_{\parallel} v_z / \omega \sim \nu / \omega \equiv \delta \ll 1, \quad (15)$$

with ω the wave frequency and $k_{\parallel}$ the wavenumber parallel to B_0 . It will prove convenient to divide the distribution function into slow and fast components $f_e = F + f$, such that

$$\langle f \rangle \equiv \int_0^{2\pi/\omega} f dt = 0,$$

and also to write

$$E_z = \frac{1}{2} [E \exp(ik_{\parallel} z - i\omega t) + c.c.].$$

Expanding Eq. (14) to varying order in δ allows f to be found. If we write $f = f^0 + f^1 + f^2 + \dots$, where the ordering is $f^n \sim \delta^n$, then we find

$$f^0 = \frac{ie}{m\omega} E \mu \frac{\partial F}{\partial v}, \quad (16)$$

$$f^1 = \frac{eE}{m\omega} \frac{\partial F}{\partial v} \left[i \frac{kv}{\omega} \mu^2 + 2 \left(\frac{v_z}{v} \right)^3 \frac{\nu}{\omega} \mu \right], \quad (17)$$

$$\begin{aligned} f^2 = \frac{eE}{m\omega} \frac{\partial F}{\partial v} \left\{ 2 \frac{kv}{\omega} \frac{\nu}{\omega} \left(\frac{v_z}{v} \right)^3 (4\mu^2 - 1) \right. \\ \left. + i \left[\frac{k^2 v^2}{\omega^2} \mu^3 - 4 \left(\frac{v_z}{v} \right)^6 \frac{\nu^2}{\omega^2} \mu \right] \right\}. \end{aligned} \quad (18)$$

f^2 was obtained assuming $\int_0^L v_z (\partial f^0 / \partial z) dz = 0$, since it is reasonable to assume that there is no net potential drop along the plasma column.

When the $\langle \rangle$ operator is applied to Eq. (14) along with the assumption used to get Eq. (18), we find that

$$\frac{-e}{m} \left\langle E_z \frac{\partial f}{\partial v_z} \right\rangle = \frac{v_z^3 \nu}{v^3} \frac{\partial}{\partial \mu} (1 - \mu^2) \frac{\partial F}{\partial \mu} \quad (19)$$

describes the steady-state solution for the distribution function. This equation amounts to neglecting two terms from the time average of Eq. (14). We find that this approximation is good only for very modest powers in our experiment (i.e., powers less than a watt). At higher powers these terms come into play and act to inhibit the current flow. (For a further discussion see the Appendix.) The right-hand side of Eq. (19) is almost Legendre's differential equation and hence suggests looking for solutions to F by expanding F in a Legendre series. After performing the algebra one arrives at

$$F = F_0 + \frac{e^2 k_{\parallel}^2 E^2}{m^2 \omega^3} \mu \left(v (4\mu^2 - 1) \frac{\partial^2 F_0}{\partial v^2} + (10 - 16\mu^2) \frac{\partial F_0}{\partial v} \right), \quad (20)$$

where we have taken F_0 to be a Maxwellian. Figure 2 shows a plot of $f_1 \equiv F - F_0$ versus the perpendicular and parallel electron velocities normalized to v_z . As expected, $F - F_0$ is symmetric for $v_{\perp} \rightarrow -v_{\perp}$, since for an applied rf field oscillating in the z direction there is no preferred perpendicular reaction. On the other hand,

FIG. 2. Plot of the perturbed distribution function, $F - F_0$ vs parallel and perpendicular velocity, as described by Eq. (20).

$F - F_0$ is anti-symmetric for $v_{\parallel} \rightarrow -v_{\parallel}$. This is also to be expected, since examination of the right-hand side of Eq. (14) shows that this operator acts to make functions which are even in μ lead to no net contribution to a current source. The figure also shows that the largest perturbations to the distribution function occur around v_t . One would expect that very fast particles will not collide and, hence, would not contribute to a current. The particular collision model chosen here breaks down for the very slow particles ($|v| \sim v_{t,i}$), but since there are not many electrons meeting that criterion, we suspect they do not substantially change the above picture. Electrons in the region $v_{t,i} < |v| \ll v_t$ collide sufficiently quickly with the ions never to contribute momentum to the current (and the ions do not contribute since for a given momentum their contribution to a current is down by the electron to ion mass ratio).

We may now calculate the time-averaged current density induced by the collisionally damped waves, and get

$$\begin{aligned} \langle j \rangle &= -2\pi e \hat{z} \int_0^\infty dv v^3 \int_{-1}^1 d\mu \mu F \\ &= \frac{-13}{10\pi} \frac{e}{m_e} \frac{\omega_{pe}^2}{L} \int_0^L dz \frac{k_{\parallel} |E_{\perp}|^2}{\omega^3} \hat{z}. \end{aligned} \quad (21)$$

For a lower-hybrid wave launched from an antenna and damped before reaching the end of the column, the resulting total current may be found to be

$$\langle I \rangle = -10.4 \frac{e}{m} \frac{P \tau_e}{L f \lambda_{\parallel}}, \quad (22)$$

where P is the power input to the lower hybrid wave, $\lambda_{\parallel} = 2\pi/k_{\parallel}$,

$$\tau_e = 3.5 \times 10^5 T_e^{3/2} / (\lambda n),$$

$$\lambda = 23.4 - 1.15 \log_{10} n_e + 3.45 \log_{10} T_e$$

(see Ref. 40, T_e in eV), and $2\pi f = \omega$. The minus sign in Eq. (22) indicates that electrons are being driven in the direction of the wave propagation parallel to the magnetic field. The numerical factor in Eq. (22) re-

sults from integrals over the velocity distribution and from geometric factors.

D. Landau damping currents

In the last section it was shown that a collisionally damped traveling wave can act as a source for generating an electron current in a plasma. Of more interest to thermonuclear research is the question of driving currents when the waves are damped by means of resonant particle interaction with the wave rather than via collisional processes. Fisch,^{1,17-19} Bers,¹⁸ and Karney¹⁹ have proposed that a current of electrons may be generated in a tokamak via resonant interaction with the wave and that the current would then be dissipated by collisions with the bulk electrons which would then allow transfer of the momentum to the ions. The question of producing an electron current via resonant interaction can be addressed on the H-1 device.⁴¹ This is a linear machine; when currents were generated via resonant interaction it was found that end-loss was the major current sink rather than collisional dissipation.

For the slow wave which is nearly electrostatic, the wave momentum density has a component along the magnetic field of

$$\pi_{\parallel} \approx \frac{k_{\parallel}}{16\pi\omega} \mathbf{E}^* \cdot \frac{\partial}{\partial \omega} (\omega \mathbf{K}_{\parallel}) \cdot \mathbf{E}, \quad (23)$$

where $\mathbf{K}_{\parallel}$ is the Hermitian part of the dielectric tensor. For the lower-hybrid wave propagating under our experimental conditions this expression reduces to

$$\begin{aligned} \pi_{\parallel} &\approx \frac{k_{\parallel}}{16\pi\omega} \left[E_{\perp}^2 \left(1 + \frac{\omega_{pe}^2}{\omega^2} + \frac{\omega_{pe}^2}{\omega_{ce}^2} \right) + E_{\parallel}^2 \left(1 + \frac{\omega_{pe}^2}{\omega^2} \right) \right] \\ &\approx \frac{k_{\parallel} E_{\perp}^2}{8\pi\omega} \left(1 + \frac{\omega_{pe}^2}{2\omega^2} + \frac{\omega_{pe}^2}{2\omega_{pe}^2} + \frac{\omega_{pe}^2}{2\omega_{ce}^2} \right) \approx \frac{k_{\parallel} E_{\perp}^2}{8\pi\omega}. \end{aligned} \quad (24)$$

If the waves are now damped by electron Landau damping, an electron current density of

$$j \approx (-e/m_e)(k_{\parallel} E_{\perp}^2 / 8\pi\omega) \quad (25)$$

will result. In a toroidal device it is anticipated that the electrons will travel around the torus repeatedly sampling the waves until enough momentum has been gained by the resonant particles to flatten the distribution function over the width of resonant interaction in velocity space.¹ For a linear machine, the current will flow out the end of the device. This current flow entails a readjustment of the plasma sheaths at each end of the column, which can go on until the current approaches $j \approx ne(kT_e/m_i)^{1/2}$.

III. EXPERIMENTAL ARRANGEMENT

A. Plasma source and parameters

The experiments were performed on the linear H-1 device⁴¹ (see Fig. 3) at Princeton University, utilizing a helium, neon, or argon plasma approximately 10 cm in diameter and 200 cm long. The steady-state confining magnetic field, $B_0 = B_0 \hat{z}$, was typically 5–12 kG. The plasma was generated by a 155 MHz rf discharge (≈ 7 kW) pulsed twenty times per second. Measurements were made at various delay times in the after-

FIG. 3. Diagram of the H - 1 device.

glow plasma, during which the electron density varied from 2×10^{12} to $2 \times 10^{10} \text{ cm}^{-3}$, and the electron temperature fell from around 12 eV to around 1 eV, depending on the filling gas type and pressure. An example of the density and temperature dependence on delay time is shown in Fig. 4. Over most of the afterglow the ion temperature was thought to be held just above the background gas temperature by charge-exchange cooling.

These plasma parameters allowed three distinct regimes for the propagation of the lower hybrid waves, which we shall call hot, warm, and cold. In the hot regime the waves were damped via Landau damping, with the electron-ion or electron-neutral collisions at least ten times less significant than the Landau damping. The warm plasma was characterized by a temperature high enough so that electron-ion collisions were insignificant, yet low enough so that Landau effects were weak. Hence, in the warm plasma, the waves made a transit down the plasma column without depositing much energy or momentum in the particles. In the cold plasma, the waves were substantially damped by electron-ion collisions. As an example, the plasma described in Fig. 4 was warm for delay times less than about 50 μsec and cold for delay times greater than 100 μsec . As a general rule hot plasmas were obtained only at short delay times in helium discharges.

B. Diagnostics and wave source

Electron temperature was inferred from Langmuir

FIG. 4. Density and temperature versus delay time in neon afterglow plasma. Plasma described is "warm" ("cold") for delay times less than 50 μsec (greater than 100 μsec).

probe traces, the blackbody radiation near the electron cyclotron frequency,⁴² and at late delay times ($\geq 100 \mu\text{sec}$) in the afterglow, by use of an electrostatic energy analyzer (see, for example, Ref. 43). Currents generated by the waves were detected by means of four loops, two on each side of the antenna, oriented to detect $\partial B_\theta / \partial t$ and by an electrode placed at the end of the plasma column. Electron density was inferred from the phase shift of an 8.6 mm microwave interferometer and from the angle of propagation of the lower-hybrid waves with respect to B_0 .

The lower-hybrid waves were excited by application of 0.2–5.0 μsec bursts (chosen to be many wave periods long, but shorter than the time for most plasma parameters to change) of 60–120 MHz fields to eight phased loops³⁷ mounted (typically 2.5 cm apart) on the outside of a 9.4 cm diam fused quartz tube. Control of the phase to each of the loops allowed the wave to be launched with controlled directionality. For example, the wave could be launched either parallel or antiparallel to B_0 , or in both directions at once, as can be seen in Fig. 5. Initially, an antenna was used which was not shielded from the plasma. This was found to produce a pattern of divergent current flow in the plasma column when the rf was applied, since the loops then acted as electron sinks. The quartz shielding of the loops solved this problem. The metal end electrode was also found to act as an electron sink if it intercepted the rf resonance cone.

The forward and reflected powers to the antenna were measured by inserting directional couplers in the transmission lines immediately before the loops. All references to power in this article refer to the difference in the power radiated by the antenna with and without plasma in the column. Without plasma, approximately 95% of the forward power was reflected back down the line. With plasma, only about 5%–20% of the power was reflected (for the plasmas studied here). In general, the antenna coupled to both the slow and fast modes in the plasma, but since the accessibility conditions for the two waves are different, it was possible to produce plasmas which minimized the fast wave coupling while still having the right parameters for good slow wave coupling.

FIG. 5. Calculated power spectrum of the antenna for two different phasings between loops. Loop spacing 2.5 cm.

The lower-hybrid waves were detected using a single or double tip triaxial probe which could be moved in the axial and radial directions. The tips of the double probe were separated radially by less than 1 mm and the sum or difference of the signals to the two tips could be taken, enhancing the fast or slow signal, respectively, due to the relatively long wavelength of the fast wave and the relatively short wavelength of the lower-hybrid wave.

IV. EXPERIMENTAL OBSERVATIONS

A. Wave identification

For the parameter range studied here the lower-hybrid group and phase velocities are

$$\mathbf{v}_g = -\frac{\omega}{k_\perp} \hat{x} + \frac{\omega}{k_\parallel} \hat{z}, \quad \mathbf{v}_\phi = \frac{\omega k_\perp}{(k_\perp^2 + k_\parallel^2)} \hat{x} + \frac{\omega k_\parallel}{(k_\perp^2 + k_\parallel^2)} \hat{z}.$$

The dispersion relation shows that the wave energy will propagate at an angle $\theta \approx \omega/\omega_{pe}$ with respect to B_0 . Hence, a wave generated by a slow wave structure two wavelengths long in the z direction should propagate into the plasma at the angle θ and should be approximately two wavelengths long in the axial direction and two wavelengths wide in the radial direction with $2\lambda_\perp$

$$\approx 2\omega\lambda_\parallel/\omega_{pe}.$$

Figure 6(a) shows radial scans of the signal detected by the rf axial probe of the lower-hybrid wave at two different axial positions. The center of the antenna was located at $z=0$, so the figure shows the wave propagating into the plasma in a well-defined resonance cone as it moves down the column. The plasma density was between $4-5 \times 10^{10} \text{ cm}^{-3}$, and for waves at 70 MHz, $\theta \approx 3.8 \times 10^{-2}$. From the figure, $\theta \approx \Delta r/\Delta z \approx 3.7 \times 10^{-2}$, 4.4×10^{-2} for the cone regions on the left and right side of the figure, respectively. The antenna impressed a wavelength $\lambda_\parallel = 10 \text{ cm}$ and thus $2\lambda_\perp \approx 2\theta\lambda_\parallel \approx 7.6 \times 10^{-1} \text{ cm}$ should be the width of the cone in the radial direction. The figure yields a cone width of about $8.2 \times 10^{-1} \text{ cm}$. Also, as seen in Fig. 6(b) an antenna two wavelengths long in z produces waves which are about two wavelengths wide radially.

The backward nature of the lower hybrid wave is demonstrated in Fig. 7. As seen in Fig. 6(a) the wave fields propagate inward radially. Figure 7 shows that the perpendicular phase velocity is radially outward at a speed of $v_\phi \approx 4.1 \times 10^7 \text{ cm/sec}$. For a plasma density of $2.5 \times 10^{10} \text{ cm}^{-3}$, $f=70 \text{ MHz}$, and $\lambda_\parallel = 10 \text{ cm}$, the dispersion relation yields $v_\phi \approx 3.5 \times 10^7 \text{ cm/sec}$.

The experiment also confirmed $\mathbf{v}_\phi \cdot \mathbf{v}_g \approx 0$ for these waves. Using an 8 cm antenna we measured $v_{g\parallel}$, as described in the next paragraph, and $v_{\phi\perp}$ from which k can be inferred. The angle of propagation of the energy was measured as in Fig. 6(a). We found (for $f=70 \text{ MHz}$, $n_e = 3 \times 10^{10} \text{ cm}^{-3}$) that $\mathbf{v}_g$ had an angle of 4.5×10^{-2} with respect to B_0 and $\mathbf{v}_\phi$ had an angle of $\pi/2 - 5.03 \times 10^{-2}$ with respect to B_0 , and, hence, $\mathbf{v}_g \cdot \mathbf{v}_\phi \approx 0$.

Measuring the group velocity is another method of wave identification. This was done by measuring the arrival time of a wave packet at different axial positions in the column. Initially, these time-of-flight

FIG. 6. Wave amplitude and interferogram. (a) Signal detected by the axial probe of the lower-hybrid wave for radial scans at two axial positions. $z=0$ corresponds to the z location of the center of the antenna. $n_e \approx 4.5 \times 10^{10} \text{ cm}^{-3}$, $f=70 \text{ MHz}$, $\lambda_\parallel = 10 \text{ cm}$. (b) Interferogram of the same signal.

FIG. 7. Radial motion of the wave phase in time. Note that the phase is moving outward with increasing time. $n_e \approx 2.5 \times 10^{10} \text{ cm}^{-3}$, $f=70 \text{ MHz}$, $\lambda_\parallel = 10 \text{ cm}$.

FIG. 8. Group velocity parallel to B_0 . Dots are experimental measurement divided by theoretical value for principal wavelength excited by the antenna. Solid dots, principal wavelength 7.5 cm; open dots, principal wavelength 8.0 cm.

measurements gave results indicating a group velocity of roughly twice what we expected. It was then found that sufficient power at the second harmonic (i.e., $2f$) was generated by the amplifier to cause the leading edge of the wave front to travel at $|v_g| \approx 2f\lambda_{th}$. With a filter in the transmission line blocking the second harmonic, the wave packet propagated down the column at roughly the expected velocity, as indicated in Fig. 8, which shows the measured group velocity as a function of wave frequency.

B. Collisional currents

Section II C showed a calculation predicting an electron current, I_{th} , when the lower-hybrid wave is collisionally damped on the plasma column. For waves incompletely damped on the column, account must be taken for the fraction of the wave lost out the end of the column. The current generated under circumstances of incomplete damping may be approximated by

$$I = I_{th} [1 - \exp(-s/l_c)],$$

where s is the distance from the antenna to the end of the plasma column and l_c is the momentum damping distance.

Figure 9 shows the current generated as a function of delay time in a neon afterglow plasma. The plasma parameters are those described in Fig. 4. At small delay times where T_e was relatively large (several eV, but not large enough for Landau damping), the current was reduced. The current was also reduced at large delay times where the density was small. Incomplete damping of the waves was observed at small and large delay times and this is consistent with theoretical calculations of the damping. Hence, as expected, smaller currents resulted if the waves were only partially absorbed during transit down the plasma column. As an example, consider the current generated at a delay time of 807 μsec as indicated in Fig. 9. The plasma parameters are given in Fig. 4 and give $I = 0.47I_{th}$ as a result of incomplete damping. The current in Fig. 9 for complete damping is about 30 mA. The predicted value of 807 μsec is thus 14.5 mA. The observed value

FIG. 9. Wave generated current versus delay time in neon afterglow plasma. Solid and open dots represent waves launched parallel or antiparallel to B_0 , respectively. 70 MHz pulse duration of 2 μsec , T_e and n_e as in Fig. 4.

is 18 mA.

Control of the directionality of the current will prove to be important for useful applications, such as driving the toroidal current in a tokamak. Waves driven parallel (antiparallel) to B_0 should cause electron currents driven parallel (antiparallel) to B_0 . This dependence was observed and is displayed in Fig. 9. Apparently, control of the current direction in future uses will thus depend on the ability to specify the phasing on the wave launching antenna.

The current as a function of input power is plotted in Fig. 10. At low powers the current is proportional to the input power (as was the spatially averaged signal level, $\int \Phi^2 dA$, of the lower-hybrid wave taken over a plasma cross section) and is roughly half that predicted by the theory. We note that the current measuring probes were calibrated using a cylindrically symmetric current source, while the observed resonance cones were not always symmetric. Additionally, the power

FIG. 10. Current versus input power. Solid line is current predicted by theory. Dashed line is $j_s A$, where A is approximate area perpendicular to z where the lower-hybrid wave existed. Neon plasma, $T_e = 0.9$ eV, $n_e \approx 2.2 \times 10^{11} \text{ cm}^{-3}$, $f = 70$ MHz, principal wavelength $\lambda_{th} = 10$ cm.

radiated by the antenna is the power quoted in the figure. What fraction of the radiated power appears in the lower-hybrid wave is difficult to determine. Hence, the use of the radiated power in Eq. (22) leads to an overestimate by the theory. Figure 10 also shows evidence of a tendency for the current to saturate at powers exceeding a few watts. It appears that the plasma has difficulty sustaining currents much in excess of $j_s \approx ne(kT_e/m_e)^{1/2}$, which would violate plasma charge neutrality. Another factor limiting the current at higher powers may be the coupling of the antenna to the lower-hybrid wave. This will be discussed later in the paper. We also noted that at powers over a few watts the current substantially terminated within 500 nsec of startup.

Figure 11 shows the spatial distribution of the electron current, determined with the aid of the moveable B_θ loops. The B_θ loops measured a signal proportional to

$$\Psi \equiv \int_0^{R_p} rj(r)dr,$$

where R_p is the radial position of the probe. By varying R_p , we obtain a plot which allows us to find $\partial\Psi/\partial r$, which is proportional to the current at the specified radius. The current was found to be localized to the radial range over which the lower-hybrid wave existed and was found to be essentially uniform in the z direction.

The theory predicts that the current is dependent on $\lambda_{||}$. This parameter was varied by changing the phasing of the antenna loops. One can see the results in Fig. 12. The data show that, at low power levels, doubling the wavelength approximately halved the current, as

FIG. 11. Resonance cones at two different axial locations from antenna (which was located radially at $r=4.7$ cm) and distribution of current versus radius. $f=70$ MHz, $\lambda_{||}=10$ cm.

FIG. 12. Current vs input power at two different $\lambda_{||}$ in argon afterglow plasma. Solid lines are least squares fit of degree 2.

predicted by theory. In the region where the current appears to saturate, the dependence on wavelength decreased.

End effects can sometimes be important in linear devices. With waves launched in just one direction in the column, asymmetric heating of the electrons toward one end may cause a net current to be generated by changing the end sheath potentials. A calculation (see the Appendix) suggests that such an effect, for the parameters of Fig. 10, would not generate more than 1 mA/W, which is much smaller than the observed currents.

C. Landau regime identification

Lower-hybrid waves can experience substantial Landau damping in a transit down the plasma column when the wave frequency, wavelength, and plasma temperature are such that the imaginary part of $k_{||}$,

$$k_{||,i} = \pi^{1/2} k_{||} \alpha^3 \exp(-\alpha^2), \quad (26)$$

with $\alpha = \omega/(k_{||}v_t)$, is large. This occurs roughly when $\alpha \lesssim 3$. Due to the exponential dependence on α^2 in Eq. (26), this onset of Landau damping is a rather sudden function of α . Figure 13 shows the imaginary part of the wavenumber in the parallel direction plotted versus wave frequency for several different parallel wavelengths in an 11.5 eV plasma. For a fixed wavelength, one sees the strong dependence of the damping on frequency. The dots on the graph represent the points at which $\alpha=3$.

An experiment was arranged with an antenna of 8 cm principal wavelength. The plasma was formed in helium with a neutral pressure of about 5.3×10^{-4} Torr and plasma density of approximately 3.7×10^{10} cm $^{-3}$. Wave pulses of 2 μ sec duration were launched during the first 2 μ sec of the afterglow, during which the Langmuir traces indicate that the electron temperature fell from 10.6–8.1 eV (an equilibrium calculation for the H-1 plasma⁴¹ yields an electron temperature immedi-

FIG. 13. Inverse damping distance vs frequency for different wavelengths. $T_e = 11.5$ eV. Dots are points where $\alpha = 3$.

ately at the turnoff of the plasma source of 13 eV). Hence, α rapidly increased with delay time in the afterglow and Landau damping was important only for the first few microseconds of the afterglow. For these plasma parameters the electron-ion collision frequency is less than $1.2 \times 10^5 \text{ sec}^{-1}$ and the electron-neutral collision frequency⁴⁴ is less than $7 \times 10^5 \text{ sec}^{-1}$. Therefore, electron-neutral and electron-ion collisions will not significantly damp the lower-hybrid wave in its transit down the plasma column.

At low powers the wave was significantly damped in its transit, as shown in Fig. 14. The figure shows the wave energy over a cross section at different axial locations measured from the end of the antenna. Also shown are the predicted wave amplitudes for different

FIG. 14. Wave power vs axial distance from the antenna. Solid lines are theory for the indicated electron temperatures. Dots are experimental data. Principal wavelength 8 cm, $f = 70$ MHz, input power $\approx 4 \times 10^{-2}$ W.

temperature plasmas. The theory uses Landau damping as the mechanism and is calculated using the appropriate power spectrum for the antenna, as shown in Sec. II A. One sees that the observed damping is compatible with Landau damping ($10 < T_e < 13$ eV).

A good check on the Landau damping hypothesis is to measure the damping length as a function of wave frequency. The results of such a test are shown in Fig. 15. Collisional damping effects are calculated and shown for comparison. Once again the calculated damping rates are shown with the experimental data and one sees that the observed damping is consistent with a plasma temperature around 11.5 eV.

D. Currents in Landau regime

In the previous section we showed that at very early delay times in a helium discharge afterglow, the plasma and wave parameters were such that electron Landau damping was the strongest damping mechanism. Under these circumstances the wave momentum in a lower-hybrid wave should be delivered to the electrons, resulting in a net current if a unidirectional wave were excited. The plasma parameters were thus arranged as described in the last section: $n_e \approx 3.7 \times 10^{10} \text{ cm}^{-3}$, $T_e \approx 10.6$ eV, $B_0 \approx 6$ kG, and neutral density $n_0 \approx 5.3 \times 10^{-4}$ Torr.

As we found in the case of the currents driven by means of collisional damping, the direction of the electron flow induced by the waves was found to be parallel (antiparallel) to B_0 when the waves were launched parallel (antiparallel) to B_0 .

Figure 16 shows a plot of the electron current induced by Landau damping of 70 MHz lower-hybrid waves. At low powers (less than 50 W) the current is proportional to the input power and has a magnitude of about 1 mA/W. Equation (25) predicts that a current equal to

FIG. 15. Inverse damping distance vs frequency for an 8 cm principal wavelength antenna. Solid lines are theory, dots are data. Dashed line is inverse damping distance calculated from collisional effects.

FIG. 16. Current vs input power in Landau damping regime. Solid line is current predicted by theory assuming 25% of the power coupled to the unidirectional lower-hybrid wave. Dashed line is $j_e A$ as described for Fig. 10 Helium plasma, $T_e \approx 10.6$ eV, $n_e \approx 3.7 \times 10^{10} \text{ cm}^{-3}$, $f = 70$ MHz, principal wavelength 10 cm.

$$I = (-e/m_e)(k_{\parallel}^2 P / \omega^2), \quad (27)$$

where P is the power in the wave, should be driven by the wave. For an antenna of 10 cm principal wavelength, Eq. (27) predicts a current of 3.6 mA/W. The experimental value is thus about 0.25 times the theoretical value here. The recent work by Wong,²² a similar experiment with slightly different plasma and wave parameters, cites a current of 0.35 that predicted by theory. Both experiments have the same difficulty; it is difficult to determine the absolute amount of the power radiated by the antenna which couples into the unidirectional lower-hybrid wave.

At power levels above 50 W the current plotted in Fig. 16 appears to saturate at around 85 mA. This is again consistent with the plasma being unable to sustain a current larger than $j_e A$, as in the collisional current case. This limitation arises only from the finite length of the column and should not arise in a toroidal device.

FIG. 17. Current vs wave frequency. Solid line is current predicted by assuming 25% of the power coupled to the unidirectional lower-hybrid wave and the wave was completely damped in transit down plasma column. Dashed line takes the fraction damped during transit (as reported in Fig. 15) into account. $T_e \approx 10.6$ eV, $n_e \approx 3.7 \times 10^{10} \text{ cm}^{-3}$, 8 cm principal wavelength. Input power 17.9 W.

Equation (27) predicts a $1/f^2$ dependence for the current generated by the wave when Landau damped.

Figure 17 shows that the current generated experimentally is a monotonically decreasing function of frequency. There are two reasons to expect this; (1) the current should decrease as shown in Eq. (27), and (2) the wave damping is incomplete at the higher frequencies. The figure also shows the current predicted by Eq. (27) (multiplied by the 0.25 factor discussed herein), which assumes complete damping of the wave momentum, and also a prediction taking into account the actual amount of wave momentum damped as measured and shown in Fig. 15. The actual current lies between these two lines for the higher frequencies. It seems likely that higher $k_{\parallel}$ components of the wave still attenuate at the higher frequencies and thus contribute to a small current even though the principal wavenumber component radiated by the antenna does not decay during a transit down the plasma column at these frequencies.

V. DISCUSSION

For damped lower-hybrid waves in a plasma where the ions are cold, the momentum is expected to be transferred to the electrons from the waves. A theory has been developed for the case where the waves are collisionally damped and the resultant electron current is predicted. This current is proportional to the input power (at low power levels). Linear machines can drive currents up to about j_e , the sound current, at which point it is suspected that ambipolar effects limit the total current. The induced electron flow from the wave damping is in the direction of the flow of the waves (i.e., parallel or antiparallel to the confining magnetic field).

Theory predicts that waves which can damp via resonant interaction with the electrons will induce a net electron current carried by the resonant electrons. We have observed that lower-hybrid waves subjected to electron Landau damping induce such a current. Once again, the current is proportional to the input power (at low power levels); the electrons are given a momentum component in the direction of the wave flow. Evidently, the ability to specify the antenna phasing; i.e., the ability to produce a unidirectional wave, is crucial for generating substantial amounts of net current.

Fisch and others have predicted that these currents may be used as a method for driving steady-state toroidal currents in tokamaks. The exact magnitude which can be achieved is open to speculation at the moment. It is not clear how well quasi-linear diffusion will be able to flatten the resonant region of the distribution function in the face of collisional diffusion. Also, the theoretical models usually assume that the lower-hybrid wave is present everywhere in the plasma, in contradiction to the expected resonance cone behavior. As a result, induced currents may drive instabilities in the plasma. On the positive side, since wave trapping broadens the resonant region, the maximum current which can be driven by the lower hybrid is probab-

ly not limited to the level predicted by Fisch. In quasi-linear theory, once enough power has been put into the particles to flatten the resonant region and maintain the plateau against collisional losses, no additional current may be driven even if the power is increased. Trapping would allow a larger current to be driven once the trapping width starts to exceed the inherent resonant width of the antenna. However, due to the uncertainty of the radiated antenna spectrum and poor definition of the resonance cone structure in a tokamak, it may be difficult to determine when trapping will be dominant.

Current drive by lower-hybrid wave excitation has already been demonstrated in toroidal devices. Recent work by LaHaye, *et al.*²⁵ on the General Atomic Octopole, by Wong, Horton, and Ono²⁴ on the Princeton ACT-1 device, and by Yamamoto *et al.*²⁶ on the JFT-2 tokamak have shown that unidirectional electron currents can be driven with an order of magnitude improvement in the amps/watt figure over the linear results. This is to be expected since the interacting electrons are not lost out the ends (as in the linear H-1 device), but circle around the torus and can sample the wave many times before losing the momentum gained in one transit.

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APPENDIX A: COUPLING VERSUS POWER

As mentioned earlier, a direct measure of the absolute power resident in the lower-hybrid wave is difficult to achieve. So nearly all references to power in this paper are to the power emitted by the antenna with a plasma in the column as compared with no plasma in the column. As has been shown, there is a tendency for currents driven by the wave to saturate as the power is increased. This is probably due to limitations in how much current can be driven out of a linear device without disturbing charge neutrality.

We have noticed one other effect which will cause the current to increase at less than a linear rate in power. Chan and Chiu²⁹ have predicted that ponderomotive forces near the antenna will cause a decrease in the coupling efficiency of the antenna. An rf field exerts a force on the plasma causing it to be pushed away from the antenna until the plasma pressure can balance the radiation force. This reduction of density near the antenna will result in poorer coupling to the plasma. Hence, the net energy density of the lower-hybrid wave is lower than that predicted by linear theory. Figure 18 contains a plot of the detected power in the lower-hybrid wave versus input power for a cold (0.7 eV) argon plasma. One sees that at power levels above a few watts the coupled power is dropping below the linear prediction and that at powers above around 20 W, the

FIG. 18. Power resident in the lower-hybrid waves vs power input to the antenna. 0.7 eV argon plasma with n_e at the center of plasma column of $1.1 \times 10^{11} \text{ cm}^{-3}$.

linear prediction is not even approximately correct. Stevens⁴⁵ has also observed this experimentally and a more detailed study of ponderomotive effects on lower-hybrid coupling has been done in recent work by Wilson and Wong.³⁰ A rough estimate of the density perturbation due to ponderomotive effects is

$$\frac{\Delta n}{n} \approx \frac{\bar{v}^2}{v_{th,e}^2} = \frac{e^2 E^2}{m^2 \omega^2 v_{th,e}^2}.$$

Wilson suggests⁴⁶ that for our experiment, when $\bar{v}^2/v_{th,e}^2$ is a few tenths or greater than the coupling efficiency into the lower-hybrid wave should start to decrease. This would arise for power levels of 20–40 W for the plasma parameters of Fig. 18. Decyk, Morales, and Dawson⁴⁷ have done simulations of lower-hybrid heating which show such effects for a capacitor plate antenna. While ponderomotive processes probably would not lead to a saturation of the current, they could cause the current to be smaller than that predicted by linear coupling theories.

APPENDIX B: SOME OTHER CURRENT SOURCES IN A COLLISIONAL PLASMA

Another mechanism may possibly drive currents in a linear device where the lower-hybrid wave is collisionally damped. This current could result from the differences occurring at the two end sheaths of the column when only one end of the column is heated, as would be the case when the wave is launched in only one direction down the plasma.

For convention, a positive current is taken to be a current traveling from sheath 1 to sheath 2. For the following discussion subscripts $e, i, 1, 2$ refer to electrons, ions, sheath 1, and sheath 2, respectively. In equilibrium, with no lower-hybrid wave present, plasma flows out both ends of the column with

$$j_1 = \frac{1}{4} [nev_{e1} \exp(-eV_1/T_{e1}) - nev_{i1}], \quad (\text{B1})$$

$$j_2 = \frac{1}{4} [nev_{e2} - nev_{i2} \exp(-eV_2/T_{e2})], \quad (\text{B2})$$

where $v_i = (2kT_e/m_i)^{1/2}$ and, of course, $j_1 = j_2$. If a lower-hybrid wave is launched toward (and collisionally

damps before getting to) sheath 2, then the plasma potential at each end will respond, in addition to T_2 increasing, to the changed plasma. For small changes in temperature toward sheath 2 we may calculate the new currents out the ends to find

$$j_1 = \frac{1}{4} [nev_{e1} \exp[-e(V_1 + \Delta V_1)/T_{e1}] - nev_{i1}] \approx \frac{-nev_{i1}}{4} \frac{e\Delta V_1}{T_{e1}}, \quad (B3)$$

$$j_2 = \frac{1}{4} [nev_{i2} - nev_{e2} \exp[-e(V_2 + \Delta V_2)(T_{e2} + \Delta T_{e2})^{-1}]] \approx \frac{nev_{i2}}{4} \left(\frac{e\Delta V_2}{T_{e2}} - \frac{eV_2 \Delta T_{e2}}{T_{e2}^2} \right), \quad (B4)$$

where we have made use of the fact that in equilibrium

$$\frac{nev_e \exp(-eV/T)}{4} = \frac{nev_i}{4}. \quad (B5)$$

If the two ends of the column are at different potentials, then a current will flow, dictated by Ohm's law to be

$$j_1 \approx \frac{-nev_{i1}}{4} \frac{e\Delta V_1}{T_{e1}} = \frac{\Delta V_1 - \Delta V_2}{L\eta} \quad (B6)$$

with L the column length and η the plasma resistivity. Equation (B6) can be solved for ΔV_1 yielding

$$\Delta V_1 = \alpha \Delta V_2, \quad \alpha \equiv \left(1 + \frac{ne^2 v_{i1} L \eta}{4 T_{e1}} \right)^{-1}.$$

Evidently $\alpha < 1$, which is reasonable since if end 2 is heated, an increased electron flux would be expected out of sheath 2, which is what Eq. (B6) says if $\Delta V_1 < \Delta V_2$, i.e., $\alpha < 1$.

To maintain charge neutrality

$$j_1 = j_2$$

yielding

$$\Delta V_2 = [1/(1 + \alpha)] (V \Delta T_{e2}/T_{e2}). \quad (B7)$$

Hence, the current induced on the plasma column by heating preferentially at one end of the column is

$$j \approx [-\alpha/(1 + \alpha)] (ne^2 v_i V \Delta T/4T^2),$$

for a column initially at T and potential V . This current is in the same direction as the current calculated in the section describing the collisional current calculation and the relative magnitudes should now be compared for the experimental conditions on H-1.

For the neon plasma giving the results plotted in Fig. 10

$$ne^2 v_i L \eta / 4T \approx 7 \times 10^{-2},$$

and, hence,

$$\alpha \approx 9.25 \times 10^{-1}.$$

ΔT may be estimated from

$$\frac{3}{2} N \Delta T = P \Delta t,$$

using $\Delta t = 300$ nsec, since that was the turn-on time for the observed currents. P is the total input power and N is the total number of electrons involved in the heating. Using these estimates, the total current driven by thermal effects under these conditions is approxi-

mately

$$I \approx 4.1 \times 10^{-4} A/W,$$

which is about an order of magnitude less than that which was observed. Thus, sheath effects may contribute to the current but are considerably less than those described in the collisional current section.

At this point it is also appropriate to consider the terms dropped in order to obtain Eq. (19). The two neglected terms are $\mathbf{v} \cdot \nabla F$ and $-e\mathbf{E}/m \cdot \partial F/\partial \mathbf{v}$, where F is the slowly varying distribution function and $\mathbf{E}$ represents the slowly varying, i.e., dc electric field. Comparing these terms with the collisional term on the right-hand side of Eq. (14), we get

$$\mathbf{v} \cdot \nabla F \left(\frac{v^3}{v^3} \frac{\partial}{\partial \mu} (1 - \mu^2) \frac{\partial F}{\partial \mu} \right)^{-1} \sim \frac{vF/L}{(v^3_{th} v/v^3)F} \sim \frac{v_{th}}{vL} \sim \frac{l}{L},$$

where l is the mean-free-path and L is the plasma column length, and

$$\frac{(e\mathbf{E}/m)(\partial F/\partial \mathbf{v})}{(v^3_{th} v/v^3)F} \sim \frac{eE}{mvv}.$$

For the collisional plasmas studied here the first term is typically 0.25. The second term may be dropped for plasma conditions where the particle acceleration is substantially retarded by collisional drag forces. Evidently, as sufficient charge leaves the column this second term will become stronger and lead to conditions inhibiting further electron flow out of the column (as was observed and was discussed in the experimental observations).

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