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Limits on the composite structure of the \( \tau \) lepton and quarks from anomalous-magnetic-moment measurements in \( e^+e^- \) annihilation

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One of the best tests for compositeness of leptons or quarks is the presence of an anomalous magnetic moment \( F_2 \). We determine limits on \( F_2 \) for the \( \tau \), from \( e^+e^-\rightarrow\tau^+\tau^- \) at PETRA, of \( F_2^\tau \leq 0.02 \). We also present limits on the \( F_2 \) of the quarks. We suggest lower-energy experiments on the angular distribution to give an independent measurement of \( F_2^q \). Future experiments near the \( Z^0 \) peak are discussed.

There have been many recent papers in which leptons and quarks are considered to be composites of more fundamental constituents. There are some very strong constraints on such models, in particular, the phenomenal agreement between experiment and present theory (electroweak plus hadronic) for the anomalous magnetic moments \( F_2 \) of the electron (to a part in \( 10^{10} \)) and the muon (to a part in \( 10^9 \)). The corrections to \( F_2 \) for all fermions of mass \( m \) from a composite structure are \( a \) in general \( O(m/m_c) \) where \( m_c \) is the "constituent mass" and \( b \) for certain situations \( O(m m/n m^2) \) for \( m_c << m \) where \( f \) and \( B \) are the constituent fermion and boson (spin-0 or spin-1) particles. As we obtain below, present limits on \( F_2 \) for the \( \tau \), from total-cross-section measurements of \( e^+e^-\rightarrow\tau^+\tau^- \) at PETRA,\(^3\) are \( F_2^\tau \leq 0.02 \). We also obtain similar limits on quark anomalous magnetic moments from PETRA limits\(^1\) on the deviation of \( R_{\text{had}} \) from QCD. We suggest low-energy experiments on the angular distribution of \( e^+e^-\rightarrow\tau^+\tau^- \) to give an independent measurement of \( F_2^q \) of comparable or somewhat better accuracy. Future experiments at LEP near the \( Z^0 \) peak are discussed for the \( \tau \) and can be applied similarly to determine quark anomalous magnetic moments. Although the above very strong limits on composite contributions to the electron or muon moments exist, there are models\(^5\) in which the electron family \( e, \nu_e, u, d \) and the muon family \( \mu, \nu_\mu, s, c \) are elementary, whereas the \( \tau \) family \( \tau, \nu_\tau, b, t \) is composite. If a new composite mass scale exists at a few hundred GeV, for example, it could give an anomalous magnetic moment to the \( \tau \) or \( b \) quark of a few percent. Thus it is useful to stress the importance of obtaining separate although much less accurate limits for the \( F_2^q \)’s of the \( \tau \) and the quarks in addition to those for the electron and muon.

The \( F_2 \) moment is present at any \( |q^2| < \Lambda_c^2 \) where \( \Lambda_c \) is the composite mass scale and \( F_2^q \) may be observed by its effect on the total cross section or in the angular dependence of the produced particles in \( e^+e^- \) annihilation. At high \( q^2 \) the \( F_2 \) contribution grows faster with \( q^2 \) than the usual \( F_1^q \) or \( (g_q^2 + g_q^2) \) contribution and is of the order \( (q^2/m^2)F_2^q - q^2/\Lambda_c^2 \) with respect to \( F_1^q \). Thus the \( F_2 \) contribution for \( q^2 << \Lambda_c^2 \) is of the same order as the form factor \( (1 + q^2/\Lambda_c^2)^{-1} \) corrections and it may be detected or bounded simultaneously by comparison with the experimental data corrected for QED and QCD effects. The \( F_2 \) contribution gives a different angular dependence than \( F_1^q \) or \( (g_q^2 + g_q^2) \) and thus provides a complementary test for compositeness to that with the form factor. In this paper we present the details of the effects of \( F_2 \) on the total and differential cross sections in \( e^+e^-\rightarrow\tau^+\tau^- \) or \( q\bar{q} \) via photons at low or high \( q^2 \) and at the \( Z^0 \) peak. We also analyze at low \( q^2 \) the directional dependence of the charged particle coming from the two- or three-body decay of the \( \tau \).

We begin with the production of \( \tau^+\tau^- \) or \( q\bar{q} \) of mass \( m \) and charge \( Q \) in \( e^+e^- \) via the electromagnetic current. The unpolarized differential cross section with \( \beta = (1 - 4m^2/q^2)^{1/2} \) is

\[
d\sigma/d\cos\theta = Q^2(2\pi\alpha^2/3q^4)\beta \times \left\{ G_0(q^2) + P_2(cos\theta)G_2(q^2) \right\},
\]

where

\[
G_0(q^2) = F_1^q(1 + 2m^2/q^2) + 3F_1F_2^q + F_2^q(q^2/8m^2 + 1),
\]

\[
G_2(q^2) = \frac{1}{2}(1 - 4m^2/q^2)(F_1^q - F_2^q q^2/4m^2).
\]

At \( q^2 >> 4m^2 \) or \( \beta - 1 \) the total cross section is

\[
\sigma = Q^2(4\pi\alpha^2/3q^4)(F_1^q + 3F_1F_2^q + 2F_2^q q^2/8m^2).
\]

Even though \( F_2 \) is expected to be smaller than \( F_1 \), the contribution of \( F_2 \) is enhanced by the factor \( q^2/8m^2 \) compared to \( F_1^q \).

The preliminary results at PETRA\(^3\) at \( q^2 \leq 37 \text{ GeV}^2 \) show that \( \tau \) production agrees with the pointlike result \( 4\pi\alpha^2/3q^4 \) to 10% at two standard deviations (2 SD 95% C.L.) or 5% at 1 SD. Using Eq. (2) with \( F_1 = 1 \) and \( q^2 = 1350 \text{ GeV}^2 \) gives a 2-SD
bound $F_1 \lesssim 0.023$, or at 1 SD $F_1 \lesssim 0.014 = \frac{1}{50}$. The dominant term containing $F_1$ here is the 3$F_1F_2$ term.

We may also set limits on the anomalous magnetic moments of quarks at high $q^2$ using the agreement of the PETRA value$^4$ of $R_{	ext{had}}$ with that expected from pointlike quarks$^3$ as compared to that given by Eq. (2) for quarks with anomalous magnetic moments at high $q^2$. The smallest acceptable normalization errors are those of PLUTO$^4$ and TASSO$^4$ which are 5% at 1 SD. We use Eq. (4) to compare with the data at $q^2 = (37 \text{ GeV})^2$ using constituent quark masses $m_u = m_d = 0.3 \text{ GeV}$, $m_c = 0.45 \text{ GeV}$, $m_t = 1.8 \text{ GeV}$, and $m_b = 4.5 \text{ GeV}$. With a 5% limit on $F_1$ and $F_2$, we get the bounds at 1 SD,

$$F_1 \lesssim 0.008, \quad F_2 \lesssim 0.017, \quad F_3 \lesssim 0.025.$$  

$$F_1 \lesssim 0.030, \quad F_2 \lesssim 0.13.$$  

(5)

Use of the current-algebra or bag-model masses of $m_u, m_d \sim 10 \text{ MeV}$ would reduce the limits on $F_1$ and $F_2$ by a factor of 30 to $F_1 \lesssim 0.0003$ and $F_2 \lesssim 0.0006$.

The anomalous magnetic moment is included in the electromagnetic-current part of the $Z^0$ coupling and shares the enhancement factor of $\sim 5000$ at the $Z^0$ peak in $e^+e^-$. The differential cross section in the $Z^0$ peak is given by

$$\frac{d\sigma}{d\cos\theta} \approx \left( g^2 m_e^2 / 12\pi \right) \left[ (q^2 - m_e^2)^2 + \Gamma^2 m_e^2 \right]^{-1} \left[ (g_f)^2 + (g_d)^2 \right] \times \left[ g_{\tau}^2 + g_{\gamma}^2 + 3 g_{V} F_1^2 + F_2^2 q^2 / 8 m^2 \right] \frac{2}{2} P_2(\cos\theta) \left( g_{\nu}^2 + g_{\mu}^2 - F_1^2 q^2 / 4 m^2 \right) + 6 \cos\theta \left( g_\nu F_1 + g_\mu F_2 \right) g_{\tau} r_\tau.$$  

(6)

where $r_\tau = g_{\tau} g_{\tau}^*/[\left( (g_f)^2 + (g_d)^2 \right]$, $F_2 = 2 \sin^2\theta_{\mu} F_2$, and $\theta$ is the angle between the $\tau^-$ and $e^-$ directions. For the $\tau^-$, $F_2^2$ will only give a small asymmetry but the symmetrical angular dependence at the $Z^0$ peak is

$$(1 + \cos^2\theta) + 2620 \sin^2\theta (F_2^2)^2.$$  

So a 10% limit on the $\sin^2\theta$ coefficient will set a limit

$F_2^2 < 0.006$. For the $b$ quark the angular dependence is

$$(1 + \cos^2\theta) + 284 \sin^2\theta (F_2^2)^2$$  

and a 10% limit would give $F_2^2 < 0.02$. The total $\tau$ cross section at the $Z^0$ peak from Eq. (6) is proportional to $(1 + 213 F_2^2)$. A total $\tau$-production measurement accurate to $1 \pm \sigma$ will give a limit on $F_2$ of $F_2 < \sqrt{\sigma}/14.6$. For 10% accuracy this gives $F_2^2 (m_\tau^2) \approx 0.02$.

Finally, we present here the method of measuring or bounding the $\tau$ anomalous moment to an accuracy of the order of 1% at relatively low $E_{\text{c.m.}} \lesssim 10 \text{ GeV}$ ($q^2 = E_{\text{c.m.}}^2$) by a high-statistics measurement of the angular distribution of $\tau$ decay products. In the preceding high-$E_{\text{c.m.}}$ analysis, the $\tau$'s direction was essentially that of the charged particle in its decay. At lower $E_{\text{c.m.}}$, the $\tau$ direction is not precisely known. However, since the nature of the two- and three-body decays of the $\tau$ are known, namely, $\tau^- \to \mu^- \nu_{\mu}$ (22%), $\tau^- \to \nu_{\tau} \pi^-$ (8%), $\tau^- \to \nu_{\tau} \bar{\nu}_{\mu}$ (18%), and $\tau^- \to \nu_{\tau} \bar{\nu}_{e}$ (17%), we can integrate over the unobserved neutrino directions and predict the angular distribution for the charged particle including the effect of the $F_2^2$ form factor.

For the two-body decays of $\tau^- \to \nu_{\tau} \pi^-$ and $\tau^- \to \nu_{\tau} \mu^-$ we denote the final charged particle's mass as $m$, and its scaled momentum by $z = P_z / E$ where $E = E_{\text{c.m.}}/2$ is the $\tau$'s energy and $\beta = (1 - m^2 / E^2)^{1/2}$. The angle between the final charged particle's direction and the $\tau$ direction is given by

$$\cos\phi = \frac{2 E (z^2 E^2 + m^2)^{1/2} - m^2 - m_z^2}{2 \beta z^2}.$$  

(7)

The limits on $z$ are given by

$$z_{\text{max}} = [1 + \beta - (1 - \beta) m_z^2 / m^2] / 2$$  

and

$$z_{\text{min}} = [1 - \beta + (1 + \beta) m_z^2 / m^2] / 2.$$  

(8)

For the two-body decays the differential cross section of the charged $\pi$ or $\rho$ with angle $\cos\theta$, relative to the $e^-e^+$ beam axis and branching ratio $B(c)$ is given by

$$\frac{d\sigma}{dz d\cos\theta} = \frac{2 \pi a^2}{3 q^2} B(c) \frac{1}{(1 - m_z^2 / m^2)}$$  

$$\times (1 + 4 m_z^2 / z^2 q^2)^{-1/2} \Gamma,$$  

(9)

where

$$\Gamma = G_0 (q^2) + P_2 (\cos\theta) P_3 (\cos\theta) G_3 (q^2).$$  

A high-statistics study of the angular and $z$ distribution (as contained in $\cos\phi$) can isolate the ratio of the coefficients of the orthogonal angular terms giving a value or limit for $F_2^2$.

In the three-body $\tau$ decays into $\mu$ or $e$ plus two neutrinos, a result similar to the two-body decay is obtained, only now involving an integration over the invariant-mass squared of the two neutrinos $m_{\nu}^2$ or $y = m_{\nu}^2 / m_z^2$. Neglecting $m_{\nu}^2 / m_z^2$, we have the angle between the final charged lepton and the $\tau$ given by

$$\cos\phi = 1 - (1 - \beta) (1 - z - y) / 2$$  

and limits $z_{\text{max}} = (1 + \beta)/2$ and $z_{\text{min}} = (1 - \beta)/2$. The differential cross section for the charged final lepton is
TABLE I. Coefficients \( b(E_{\text{c.m.}}) \) for Eq. (11) and statistical limits on \( F_{2}^{\text{joint}} \) obtainable at various \( E_{\text{c.m.}} \) from a simplified two-bin analysis of the \( \tau \) angular distribution using Eq. (12) with \( (L_{31} Y A) = 1 \).

<table>
<thead>
<tr>
<th>( E_{\text{c.m.}} ) (GeV)</th>
<th>( b^{\text{1-2}, \mu, \tau} )</th>
<th>( b^{\rho} )</th>
<th>( F_{2}^{\text{joint}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.056</td>
<td>0.081</td>
<td>0.014</td>
</tr>
<tr>
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<td>0.11</td>
<td>0.18</td>
<td>0.0074</td>
</tr>
<tr>
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<td>0.30</td>
<td>0.0053</td>
</tr>
<tr>
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<td>0.37</td>
<td>0.0049</td>
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<tr>
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<td>0.41</td>
<td>0.0051</td>
</tr>
<tr>
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<td>0.41</td>
<td>0.47</td>
<td>0.0068</td>
</tr>
<tr>
<td>30</td>
<td>0.45</td>
<td>0.47</td>
<td>0.0096</td>
</tr>
</tbody>
</table>

where \( L_{31} \) is the luminosity in units of \( 10^{21} \text{cm}^{-2} \text{sec}^{-1} \), \( Y \) is effective running time in years, and \( A \) is the acceptance. The limits that can be obtained at various energies from the \( e, \mu, \tau \) lumped together with total branching ratio \( b \) of 43% (since they have the same \( b^{s} \)) and for the \( \rho \) are nearly equal. In Table I we show the weighted limit that can be obtained by combining both of these limits. The table shows that a limit \( F_{2}^{\text{joint}} \leq 0.02 \) is obtainable statistically and the favored energy range is \( 7 \leq E_{\text{c.m.}} \leq 20 \text{ GeV} \).

In conclusion we have shown that including the anomalous-magnetic-moment effects of quarks or leptons is important in determining whether they have a structure. We have used PETRA data on the \( e^{+}e^{-}\rightarrow \tau^{+}\tau^{-} \) total cross section to set a \( 1 \sigma \) limit on the anomalous magnetic moment of the \( \tau \) of \( F_{2}^{\text{joint}} < \frac{1}{90} \), corresponding to a limit on a composite scale of \( \Lambda_{c} > 100 \text{ GeV} \). We have also set similar limits on the anomalous magnetic moment of the quarks. We have shown how an improved limit can be set by observing the \( F_{2}^{\text{joint}} \) effect from the quark or lepton electromagnetic-current part of the neutral current in \( Z^{0} \) production at the next generation of \( e^{+}e^{-} \) colliders. Finally we have shown the feasibility at present colliding rings of independently setting a limit on anomalous magnetic moments from the angular dependence of the \( \tau \) 's charged decay products or analogously the leading particle in a quark jet.

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5See, e.g., the SO\textsubscript{14} unified gauge model of G. L. Shaw and F. Daghianian, Phys. Rev. D 26, 1798 (1982).

6Since the limits on the deviation of the $F_1(q^2)$ form factor from 1 are less than a few percent, we also expect the $F_2$-form-factor composite contribution to be undiminished from its $q^2=0$ value.

7The QCD corrections to $R_{\text{had}}$ start with $(R_{\text{had}} \alpha_s / \pi)$. The value of $\alpha_s$ at these $q^2$ is obtained from different experiments [P. Duinker, Rev. Mod. Phys. 54, 325 (1982), gives $\alpha_s = 0.17 \pm 0.04$] and the error in $\alpha_s$ only affects $R_{\text{had}}$ at the 1% level.