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Journal
Adaptive Optics for Extremely Large Telescopes 4 – Conference Proceedings, 1(1)

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Publication Date
2015

DOI
10.20353/K3T4CP1131567

Filtering the interaction matrix in an adaptive optics system

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ABSTRACT

We present a method to reduce the noise in the interaction matrix by calibration of the adaptive optics system. The method utilizes a matching between the actuators on the deformable mirror and the sub-apertures on the wavefront sensor to define a filter matrix. Then, the filter matrix is applied to the sparse interaction matrix to remove the elements that should be zero. This method is useful for high-order systems and/or noise calibration issues. The latter case is illustrated in the problem of on-sky calibration of an adaptive secondary system on a telescope, with a natural guide star, in which the noise in the interaction matrix is increased by the effects of the turbulent atmosphere.

Keywords: Adaptive Optics, Interaction Matrix, New Solar Telescope, Atmosphere, Filter

1. INTRODUCTION

Adaptive optics (AO) are used to compensate for the optical aberrations induced by the Earth′s turbulent atmosphere. When successful, AO improves the spatial resolution and/or contrast of imaging from the ground-based telescopes. An AO system includes two crucial components, its wave-front sensor (WFS) and its deformable mirror (DM), where the WFS measures atmospheric aberrations with a high frame rate and the shape of the DM is correspondingly adjusted compensate the measured wavefront aberrations.

In order to make the WFS and DM work properly, the transfer function from the DM to the WFS in the AO system has to be measured. This measurement process is called calibration, and the transfer function is called the interaction matrix. To measure the interaction matrix, different shapes (injection matrix) must be applied to the DM and then the corresponding pattern must be measured on the WFS.

There are many factors that may add noise to the interaction matrix during calibration. These include, but are not limited to, the read-out noise (RON) of the CCD, the photon noise,[1] the static aberrations in the system, the vibrations within the instrument, and mis-conjugation between the DM and the WFS.[2] If one calibrates the adaptive system with a natural guide star as a reference, which would be the case for an adaptive secondary AO system at a telescope without a focal plane before the secondary mirror, the atmospheric turbulence noise would be added into the interaction matrix.

We also employ several methods to reduce the influence of noise and, thereby, improve the quality of an AO system. The Hadamard matrix[3] or Sine matrix[4] can be used to improve the efficiency of the injection matrix. The push-pull method[2] can deal with aberrations induced by static and low-frequency dynamical vibrations. After calibration, the truncated control modes can be used to improve the quality of the control matrix. The AO loop also can be optimized for different atmospheric conditions.[1] In this paper we focus on filtering the noise so as to yield zero elements in the interaction matrix where appropriate. This method can be used as an additional step to improve the quality of the interaction matrix, so the mentioned methods also can be used for the same AO system.

In Section 2 of this paper we briefly describes the calibration procedure. The interaction matrix filtering method is described in section 3. As an example of this method, the results from the AO-308 system, on what is currently the world′s largest aperture solar telescope,[5] are given in section 4. The conclusions are drawn in section 5.

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2. CALIBRATION OF ADAPTIVE OPTICS SYSTEMS

Let us suppose that an AO system has an n-actuator DM, and the WFS has can make m slope measurements. Then the n by n injection matrix V, which has a n linearly independent actuation patterns formed, to be applied to the DM. The corresponding WFS measurements for each actuator pattern forms an m by n matrix W. The DM and its corresponding WFS measurements are described in equation 1:

$$W = IV + N \tag{1}$$

where the m by n matrix I is the interaction matrix, and the m by n matrix N is the noise measurement matrix.

Since the injection matrix V and the corresponding measurement W are known, the interaction matrix I can be calculated from equation 2 in which the unknown measurement noise N is not well developed.

$$I = (W - N)V^{-1} \approx WV^{-1}, \tag{2}$$

where V^{-1} is the inverse of V.

Because the noise, N, is included in W during calibration, the noise is introduced into the interaction matrix I. Mathematically, the noise in the interaction matrix can be written as equation 3. But in reality, it is very difficult to specify, and therefore remove the noise.

$$I_{noise} = NV^{-1}. \tag{3}$$

The pseudo-inverse of I is used for the zonal control system. For modal control, the command/reconstruction matrix C can be calculated in the following way: for example, in the AO-308 system, as seen in equation 4, basis functions, B, such as Zernike[7] or K-L modes,[6] are multiplied by I, then a pseudo-inverse is performed. For either zonal or modal control, the interaction matrix I is critical for the AO system,

$$C = (BI)^{+} \tag{4}$$

Where $^{+}$ means pseudo-inverse, which is the inverse of a non-square matrix.

3. FILTERING THE INTERACTION MATRIX

In most AO systems, the influence function for DM actuators,[8] which is the area and shape change of the DM caused by applying a control matrix to the actuators, is considered to be a truncated Gaussian function. The actuator function only has a limited influence area on the AO system and its WFS. The local influence function decreases rapidly as the distance between the actuator and its corresponding sub-apertures. In other words, for each actuator, only a limited number of sub-apertures in the interaction matrix should be measured, because their significant effects are only on nearest neighbors. The technique proposed in this paper is limited to those AO systems behaving like AO-308 . For illustration, the influence function of the center actuator of the AO-308 system DM from Xinetics is shown in figure 1. Since the interaction matrix records the relation between the WFS and DM, the interaction matrix is a sparse matrix.

From the DM and WFS match map, and the influence function of the DM actuators, we can know which DM actuators can/cannot be seen by which WFS sub-apertures. In other words, the DM actuators and WFS sub-apertures have a certain fixed matching relationships. So the nonzero values in the interaction matrix in each row means a DM actuator can be seen on the specified WFS sub-apertures; and each column tells us which DM actuators a WFS can see. For example, the DM and WFS match map on the AO-308 system[9] is shown in figure 2, the corresponding interaction matrix is shown in figure 3.

The DM and WFS map relation can be used to filter out much of the noise in the interaction matrix. First, a DM and WFS relation window filter, which has the same dimensions as the interaction matrix, can be made in the following way: If a DM actuator can be seen by a sub-actuator, the corresponding element is 1, otherwise is 0. Then the post-filtering interaction matrix can be achieved by multiplying the relation window matrix and

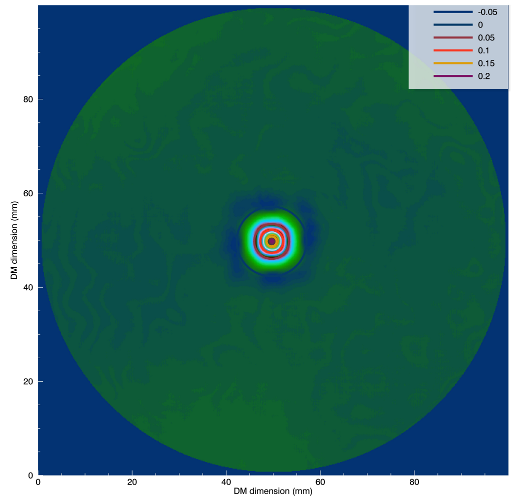

Figure 1. The influence function of the central actuator of the AO-308 Xinetics DM as measured by the interferometer. Other actuators in the DM have similar influence functions shape. The DM is 100 mm in diameter. The contour in the image is the 2-dimensional Gaussian fit. The influence function shows the actuator has its influence on a circle with about a 25 mm-diameter. That is, it corresponds to about 6 by 6 sub-apertures on its corresponding WFS sub-aperture. The DM actuators and WFS sub-aperture relation is shown in figure 2.

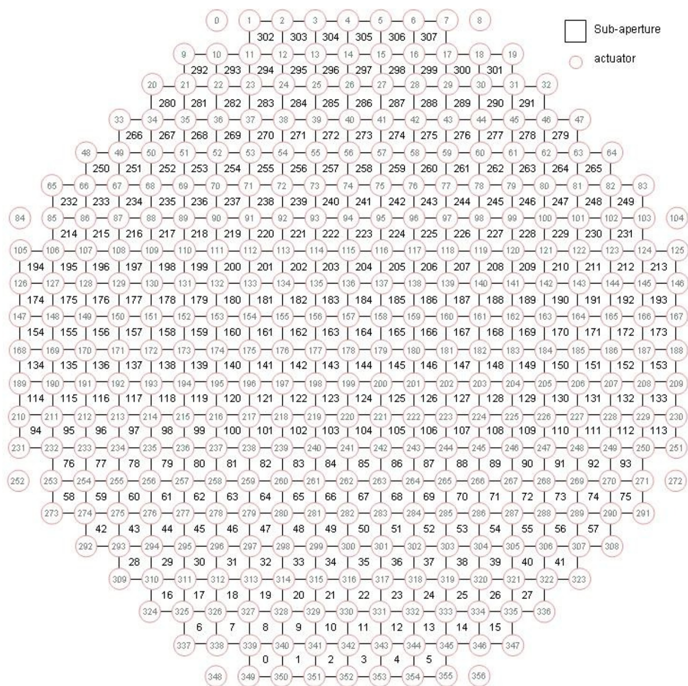

Figure 2. The DM actuators and WFS sub-aperture match map for the AO-308 system on the New Solar Telescope at Big Bear Solar Observatory. The DM actuator and WFS sub-aperture indexes are used in the subsequent figures.

the interaction matrix, as shown in equation 5. The corresponding filtered noise can be calculated by equation 6 and/or 7.

$$I_f = I.I_{window} \tag{5}$$

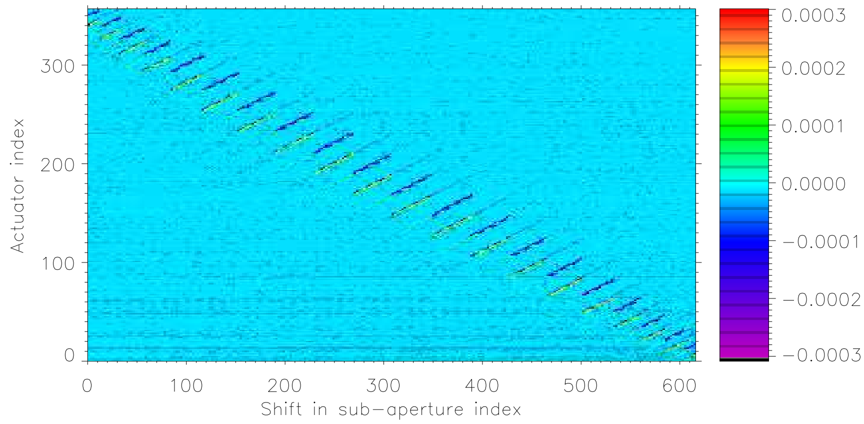

Figure 3. The interaction matrix after the calibration of the AO-308 system. The column are the x and y shifts calculated from the sub-apertures, which is similar to the slopes for a point target . The DM actuators and WFS sub-apertures relationship and sub-aperture indices are shown in figure 2.

$$I_n = I - I_f \tag{6}$$

$$I_n = I.I_{\bar{window}} \tag{7}$$

Where $I_{\bar{window}}$ means an interchange of $0'$s and $1'$s in I_{window}.

To define a suitable filtering window size, firstly the influence function of the DM should be measured. Then, each sub-aperture size on the DM of an AO system can be known. Finally, the number of sub-apertures on the WFS should be influenced by an actuator can be calculated, which is filtering window size.

4. WAVEFRONT VARIANCE CAUSED BY NOISE IN THE INTERACTION MATRIX

The wavefront variance, σ^2, of an AO system can be written as equation 8.

$$\sigma^2 =< a^T a >=< (Cg)^T (Cg) >=< g^T (C^T C) g > \tag{8}$$

where g represents the WFS slopes/shifts, a is the modal coefficient vector for modal control or actuator command vector for the zonal control, $<>$ stands for ensemble averaging.

We use zonal control as an example (B in equation 4 is an identity matrix), the $(C^T C)$ part in equation 8 can be expressed as equation 9, by combining equations 4, 5 and 7.

$$
\begin{aligned}
C^TC &= [(I_f + I_n)^{-1}]^T (I_f + I_n)^{-1} \\
&= [(I_f + I_n)^T]^{-1} (I_f + I_n)^{-1} \\
&= [(I_f + I_n)(I_f + I_n)^T]^{-1} \\
&= [(I_f + I_n)(I_f^T + I_n^T)]^{-1} \\
&= (I_f I_f^T + I_f I_n^T + I_n I_f^T + I_n I_n^T)^{-1}
\end{aligned} \tag{9}
$$

Since they are independent, the noise and filtered interaction matrices are orthogonal to each other, then $I_f I_n^T$ and $I_f I_f^T$ are zero matrices. Let us suppose the actuators on the DM are independent of each other, same as the noise of each column/row in I_n , then we have equation 10 and 11. So equation 9 becomes 12.

$$
I_f I_f^T = \sigma_f^2 S \tag{10}
$$

where S is the identity matrix, $\sigma_f^2 = \overline{\sum_{j=1}^{n} I_{f\,ij}^{\,2}}$ $(i = 0, 1, 2....m)$, where the bar in the equations implies an average.

$$
I_n I_n^T = \sigma_n^2 S \tag{11}
$$

where $\sigma_n^2 = \overline{\sum_{j=1}^{n} I_{n\,ij}^{\,2}}$ $(i = 0, 1, 2....m)$

$$
C^TC = \frac{1}{(\sigma_f^2 + \sigma_n^2)} S \tag{12}
$$

With equation 12, equation 8 can be re-written as equation 13.

$$
\sigma^2 = \frac{1}{\sigma_f^2 + \sigma_n^2} < g^T g > \tag{13}
$$

Thus, the measured wavefront error introduced by the noise in the interaction matrix is shown in equation 14.

$$
\begin{aligned}
\sigma_{ne}^2 &= (\frac{1}{\sigma_f^2 + \sigma_n^2} - \frac{1}{\sigma_f^2}) < g^T g > \\
&= \frac{\sigma_n^2}{\sigma_f^2 + \sigma_n^2} < g^T g >
\end{aligned} \tag{14}
$$

As seen from equation 14, the σ_{ne}^2 is proportional to the percentage of noise in the interaction matrix and the atmospheric variance. Since the interaction matrix is a sparse matrix, when the noise level is constant, σ_n^2 increases faster than σ_f^2. So the influence of noise increases as number of actuators increases, which is the case for a larger aperture telescope. In short, for a high-order AO and/or the noisy calibration environment, the noise, I_n, filtered out by this method can be large.

5. TEST CASE: AO-308

The AO-308 system works on the world largest aperture solar telescope, named the New Solar Telescope, at the Big Bear Solar Observatory in Big Bear, California, USA.[9] The AO-308 has a 357 actuators DM with 308 sub-apertures on its WFS, hence the name AO-308. The DM actuator and WFS sub-aperture match map is shown in figure 2. The digital signal processors (DSPs) are used for real-time control. The system can provide diffraction limited images even at visible wavelengths.

An experiment was done with the AO-308 system to test the suggested method. The interaction matrix was obtained under good calibration conditions (sufficient photons and high contrast images). But there were

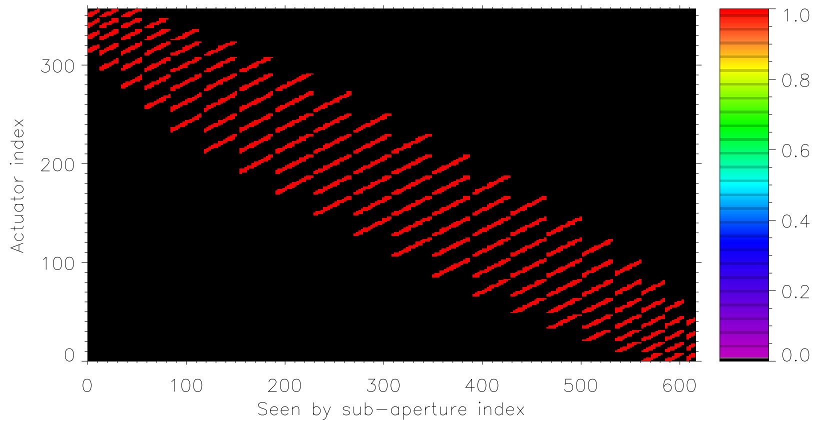

Figure 4. The window matrix for filtering the noise in the AO-308 system. The relationship between the DM actuator and WFS sub-aperture map is shown in figure 1. For each actuator, the nearest 6 by 6 sub-apertures are considered as being seen on the WFS.

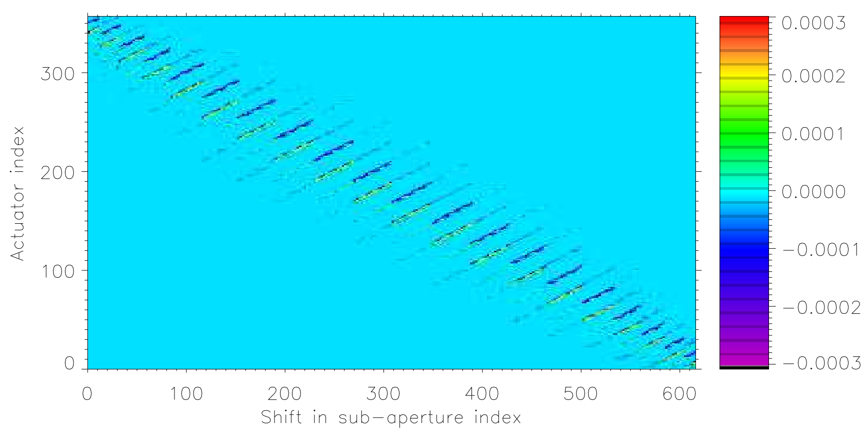

Figure 5. The interaction matrix of AO-308 system after being filtered.

some vibrations and laboratory atmospheric turbulence in the optical path. The result is shown in figure 3. A corresponding filter matrix was created (for each actuator, its nearest 6 by 6 sub-apertures on the WFS are considered as 1), as shown in figure 4.

Figure 5 shows the result after applying the filter matrix to the measured interaction matrix. We can see that the filtered interaction matrix becomes cleaner. The removed noise in the measured interaction matrix is shown in figure 6.

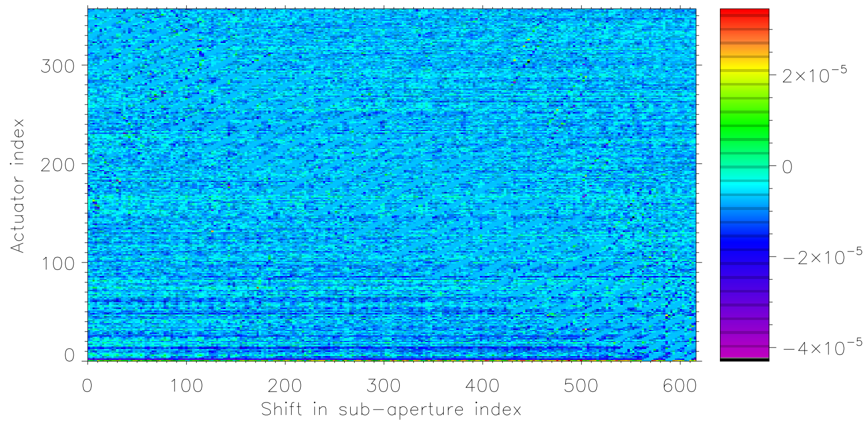

Figure 6. The filtered noise in the AO-308 system. For each actuator, the nearest 6 by 6 sub-apertures are considered as being seen on the WFS.

The noise can be clearly seen in figure 6, even the interaction matrix was calibrated under good conditions, i.e., plenty of photons, few static and dynamic aberrations, and a high contrast target. This is as opposed to a noisy environment (poorer conditions) , opposite to the aforementioned good conditions, the noise in the interaction matrix should increase drastically.

For the AO-308 system, the wavefront measurement error introduced by the noise in the interaction matrix is about 1.9%, as calculated from equation 14, which is small. But, if one increases the noise level in the interaction matrix by three times, then the wavefront measurement error introduced by the noise in the interaction matrix becomes 14.8%, so for the noisy calibration environment, this method can improve the AO performance.

6. CONCLUSIONS

The interaction matrix, which is obtained by calibration, records the control relationship between the DM and the WFS. Normally, the interaction matrix should be a sparse matrix, since each DM actuator only has a small area of influence on the WFS. But there are many factors that may add noise to the interaction matrix during calibration. This makes the zero-elements in the interaction matrix become non-zero, which then decreases the performance of the AO. This became more serious for a high-order AO system, since the number of the non-zero elements increases dramatically.

We suggest using the relationships between the DM actuators and the WFS sub-apertures to define a window filtering matrix, which can be applied to the measured interaction matrix, to remove the non-zero elements and replace them with zeroes. This method can be very useful for high-order AO and/or a noisy calibration environment, e.g., the on-sky calibration for an adaptive secondary mirror on a telescope with a natural guide star. This method can also used to check the noise level in the interaction matrix.

Acknowledgement

We gratefully acknowledge the support of the Air Force Office of Scientific Research grant FA9550-12-1-0066, the National Science Foundation grant AGS-1250818 and the National Aeronautics and Space Administration grant NNX13AG14G. The lead author sincerely thanks Philip R. Goode (BBSO)for supporting the work, and Douglas Summers, Julian Christou and Christian Veillet (all LBTO) for supporting the conference payment.

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