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The effects of lake precipitation and evaporation on reservoir modeling

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Abstract

This paper explores the effects of lake evaporation and precipitation, reliability levels, and streamflow probability distribution on reservoir capacity and average release. A series of reservoir-analysis deterministic and stochastic models of increasing complexity are employed to examine these effects, which are illustrated using a mid-sized reservoir in Santa Barbara County, California. The deterministic reservoir models show evaporation dominating reservoir water balance, and, for the most part determining optimal reservoir capacity and annual releases, whereby comparatively larger reservoirs are required to compensate for the water lost to evaporation. The more complex stochastic models use chance constraints and bootstrapped reservoir inflow probability distributions, and suggest that optimal capacity and annual releases are dominated by reliability levels, with very large optimal reservoir capacities and annual releases required when the reliability levels approach one.

Introduction

The determination of reservoir storage capacity, with its desired yield and associated performance reliability, is an issue of longstanding interest in water resources management. Deterministic (black-box) and stochastic (statistical) approaches to the reservoir design and operation problem have been studied in detail [Loáiciga, 1988; Loáiciga 2002]. Stochastic inflow is a key consideration in reservoir modeling, and the recognition and use of gamma-distributed inflows (of particular interest in this article) is well explored [Loáiciga, 2005]. Many studies utilize Monte Carlo or similar simulations to create the stochastic inflow dataset. This work expands on previous reservoir-design research, especially on the papers by Loáiciga

[2002, 2005], and introduces an innovative use of reservoir-inflow bootstrapping to obtain the closed-form deterministic equivalents of chance constraints.

Two primary reservoir functions are flow regulation (e.g. flood control) and water supply The former necessitates ample storage to receive and hold floodwaters, whereas the latter calls for large available storage to deliver a reliable supply of water for agricultural and/or municipal use. The desire to satisfy both objectives (and other plausible ones) simultaneously creates an optimization problem whose solution seeks an acceptable fulfillment of diverse functions. Loáiciga [1988] explored the relationships between mathematically feasible solutions, physically feasible solutions, and preferable solutions, which do not necessarily provide the same results, as discussed in Hashimoto et al. [1982].Evaporation plays a significant role in the water cycle throughout the semiarid western United States, where annual potential evaporation exceeds smaller and more highly variable annual rainfall and streamflow [see, for example, Loáiciga and Renehan, 1997]. In this region, lake hydrology (precipitation onto and evaporation from a lake's surface) has been shown to exert substantial influence on optimal reservoir capacity and average annual water release [Loáiciga, 2002, although limited to deterministic treatment of reservoir design]. In addition, the large variability intrinsic to annual precipitation and streamflow (compounded by sustained periods of drought to El-Niño related flooding) complicates the water-balance analysis of reservoirs for the purpose of constructing reservoir-operation models, as we shall see.

In synthesis, the objective of this research is to develop and implement a series of deterministic and stochastic reservoir-design models in which lake evaporation and precipitation are accounted for, and in which chance constraints (in the stochastic cases) are converted into closed-form deterministic equivalents using the bootstrapping of cumulative reservoir inflow. The novel application of the bootstrapping technique in resolving chance constraints analytically yields the probability distribution and quantiles of the cumulative distribution of streamflow, which are essential in the implementation of the stochastic reservoir models considered in this paper. Lastly, this paper contains an example of the methodology herein presented to demonstrate its practicality.

Methodology

Expanding research by Loáiciga [2002, 2005], four reservoir optimization models are entertained in this work. Models I and II treat reservoir inflows deterministically. Models III and IV treat inflows stochastically. Models I and III do not consider lake evaporation and precipitation in the reservoir water-balance equation, whereas models II and IV do. The combination of scenarios encompassed in models I-IV provides the basis to compare the effects of stochasticity and lake hydrology on reservoir size and average release. The four reservoir models are described mathematically in this section, leading to their respective linear programming formulations. In this article we consider a single reservoir and an annual time step i ($i = 1, 2, \ldots, n$).

The water-balance equation and the objective function. The basic water balance equation without lake evaporation and precipitation, for either deterministic or stochastic inflows is:

$$S_i = S_{i-1} + r_i - D_i - w_i \tag{1}$$

The basic water balance equation with lake evaporation and precipitation, for either deterministic or stochastic inflows is:

$$S_i = S_{i-1} + r_i - D_i - w_i + P_i - E_i \tag{2}$$

in which S_i = reservoir storage (10^6 m^3); r_i = reservoir inflow (10^6 m^3); D_i = water diversions from the reservoir (10^6 m^3); w_i = water releases (10^6 m^3); P_i = precipitation (rainfall) onto the reservoir surface area, expressed as a volume (10^6 m^3); E_i = evaporation from the reservoir, expressed as a volume (10^6 m^3).

The objective function in all of the four models' formulations is to minimize the cost of building the optimal reservoir capacity minus the present value of water releases (i.e. revenue gained from these water releases) with respect to reservoir capacity (C, 10^6 m^3) and annual releases (w_i):

$$\text{Minimize} \quad K \cdot C - \sum_{i=1}^{n} \left[\frac{1}{(1+s)^i} \cdot H_i \cdot w_i \right] \tag{3}$$

w.r.t. C, w_i

subject to the constraints:

$S_{\min} \le S_i \le C$ (deterministic constraint on reservoir storage, models I, II) (4)

$P(S_{\min} \le S_i \le C) \ge \alpha$ (stochastic constraint on reservoir storage, models III, IV) (5)

$F \le w_i \le k \cdot \bar{r}$ (constraints on annual releases) (6)

$C, w_i \ge 0$ (non-negativity constraints) (7)

where k = multiple of average annual reservoir inflow (k = 4 herein); K = unit cost of reservoir capacity (\$ $10^6/10^6$ m^3); $\bar{r}$ = average annual reservoir inflow (10^6 m^3); s = discount rate; S_{min} = dead storage at reservoir site (10^6 m^3); H_i = unit value of release (\$ $10^6/10^6$ m^3); F = downstream fisheries requirement (10^6 m^3); α = reliability level assigned to a chance constraint (a probability, typically between 0.75 and 1).

The effect of lake evaporation and precipitation on reservoir storage. The models (II and IV) that take into account lake hydrology incorporate the effect of lake area (A, in 10^6 m^2), in this case average annual lake area, on precipitation (P) and evaporation (E). Define:

$$P_i = p_i \left(\frac{A_{i-1} + A_i}{2} \right) \tag{8}$$

$$E_i = e_i \left(\frac{A_{i-1} + A_i}{2} \right) \tag{9}$$

in which:

p_i = measured precipitation (in rain gage near the reservoir, in m)

e_i = measured evaporation (adjusted pan evaporation at the reservoir, in m)

$A_j = a + b \cdot S_j$, for $j = i, i\text{-}1$ (10)

a, b regression coefficients relating lake area to storage.

Equation (10) is used in equations (8) and (9), which, in turn, are substituted into equation (2). The modified equation (2) becomes:

$$S_i = K_i \cdot S_{i-1} + T_i \cdot r_i - T_i \cdot D_i - T_i \cdot w_i + G_i \qquad (11)$$

in which the constants G_i, K_i, and T_i are:

$$G_i = \frac{a \cdot (p_i - e_i)}{1 - \frac{b}{2} \cdot (p_i - e_i)} \qquad (11a)$$

$$K_i = \left[\frac{1 + \frac{b}{2} \cdot (p_i - e_i)}{1 - \frac{b}{2} \cdot (p_i - e_i)} \right] \qquad (11b)$$

$$T_i = \frac{1}{1 - \frac{b}{2} \cdot (p_i - e_i)} \qquad (11c)$$

The modified equation (11) is used in constraints (4) and (5) to produce the deterministic and stochastic constraints on storage associated with models II and IV, respectively.

Chance constraints. A key contribution of this research is the development of a methodology to cope with stochastic reservoir inflows (model III, which does not include lake evaporation and precipitation, and model IV, which does). The primary challenge lies in converting the probabilistic storage constraints into their deterministic equivalents. Model III minimum and maximum storage constraints can be written as follows (see Appendix A for a derivation):

$$P(S_i \geq S_{\min}) \geq 1 - q \Rightarrow -g \cdot C + \sum_{j=1}^{i} w_j \leq Q_q^{(i)} - B_i - S_{\min} \qquad (12)$$

$$P(S_i \leq C) \geq p \Rightarrow C \cdot (1 - g) + \sum_{j=1}^{i} w_j \geq Q_p^{(i)} - B_i \qquad (13)$$

in which g = fraction of reservoir capacity defining initial storage, $S_0 = g \cdot C$. Likewise, Model IV minimum and maximum storage constraints can be written as follows (see Appendix B for a derivation):

$$P(S_i \geq S_{\min}) \geq 1 - q \Rightarrow -\psi_i \cdot g \cdot C + \sum_{k=1}^{i} w_k \cdot T_k \cdot \phi_k \leq R_q^{(i)} - D_i^* + G_i^* - S_{\min} \qquad (14)$$

$$P(S_i \leq C) \geq p \Rightarrow C \cdot (1 - \psi_i \cdot g) + \sum_{k=1}^{i} w_k \cdot T_k \cdot \phi_k \geq R_p^{(i)} - D_i^* + G_i^* \qquad (15)$$

Constraint equations (12) – (15) contain the decision variables (C, w_i) on their left-hand sides. The constants $B_i, D_i^*, G_i^*, \phi_k, \psi_k$ that appear in equations (12)–(15) are defined in Appendices A and B, and depend on the geometry of the reservoir and on lake evaporation and precipitation in an involved fashion. In chance constraints (12) and (13) $Q_q^{(i)}$ and $Q_p^{(i)}$ are the q-th and p-th quantiles, respectively, of the probability distribution function of the sum of reservoir inflows (Q_i), where:

$$Q_i = \sum_{j=1}^{i} r_j \tag{16}$$

Likewise, in equations (14) and (15), $R_q^{(i)}$ and $R_p^{(i)}$ are the q-th and p-th quantiles, respectively, of the probability distribution function of the sum of weighted reservoir inflows (R_i), where:

$$R_i = \sum_{k=1}^{i} r_k \cdot T_k \cdot \phi_k \tag{17}$$

The determination of the quantiles of Q_i and R_i is explained next.

Bootstrapping of linear combinations of reservoir inflows. Consider, for the sake of argument, the sum of weighed reservoir inflows R_i expressed in equation (17). The probability distribution function (pdf) of R_i is complex, especially because of inter-annual correlation among reservoir inflows. Classical statistical methods to develop such pdfs (see e.g., Loáiciga and Leipnik, 1999) are mathematically unwieldy when more than two random variables are added. The bootstrapping method has been successfully used in hydrology to approximate drought probabilities [Loáiciga et al, 1993]. In the context of this research – and keeping the random variable R_i in mind – the bootstrapping method is implemented as follows: (1) draw at random (with replacement) i values from the sample of n (n = 84 in this study) annual reservoir inflows and construct the bootstrapped weighted sum $R_i^{(k)}$ (see equation (17)) in which k = 1; (2) repeat step (1) for k = 2, 3, …, N, in which N is the sample size of bootstrapped sums $R_i^{(k)}$ (N = 200 in this study); (3) the sample of bootstrapped sums, $R_i^{(1)}$, $R_i^{(2)}$, …, $R_i^{(N)}$ is fitted with a theoretical pdf and evaluated with the χ^2 goodness-of-fit test; (4) steps (1) – (3) are implemented for i = 1, 2, …, n, thus producing the bootstrapped pdfs of the R_is. The same method is used to approximate the pdfs of the sums Q_i (see equation (16)), or for that matter, of any other function of random variables whose mathematical complexity defies standard methods.

Summary of reservoir optimization models. In all four models the objective function is given by equation (3), in which the nonnegative decision variables are reservoir capacity (C) and annual water releases (w_i). The maximum release constraint (given by equation (6)) applies in all cases. The constraint sets associated with the four alternative reservoir optimization models are:

Model I – Deterministic reservoir inflows without lake evaporation and precipitation, where the minimum constraint is given by

$$S_{\min} \leq S_i = S_{i-1} + r_i - D_i - w_i \leq C \tag{18}$$

and the maximum constraint is given by equation (6).

Model II – Deterministic reservoir inflows with lake evaporation and precipitation, where the minimum and maximum constraints are given by

$$S_{\min} \leq S_i = K_i \cdot S_{i-1} + T_i \cdot r_i - T_i \cdot D_i - T_i \cdot w_i + G_i \leq C \tag{19}$$

and in which the constants G_i, K_i, and T_i are defined by equations (A2), (A3), and (A4), respectively, in Appendix A.

Model III – Stochastic reservoir inflows without lake evaporation and precipitation, where the minimum and maximum constraints are given by equations (12) and (13), respectively.

Model IV – Stochastic reservoir inflows with lake evaporation and precipitation, where the minimum and maximum constraints are given by equations (14) and (15), respectively.

The four reservoir-optimization models constitute linear programming problems whose solutions were obtained with the Solver software included with Microsoft Excel. Data and site description of the case study are presented next.

Site description

The current research focuses on Lake Cachuma (see Figure 1), located in the Santa Ynez watershed (Santa Barbara County, California, U.S.A.), which supplies most of the urban and agricultural water for approximately 250,000 people in cities and towns within its reach and along the adjacent coast [Loáiciga, 2002].

Storage. The minimum storage, or dead storage, for Cachuma Reservoir is S_{min} = 20,000 AF (acre-feet) ≈ 25 x 10^6 m^3. To prevent overtopping, the maximum storage for each model scenario is set less than or equal to the optimal capacity as determined by the optimization process. The a and b coefficients in the lake area vs. storage equation (10) are 1.3007 and 0.05054, respectively [Loáiciga, 2002].

Releases

Diversions (D_i = D = 39.825 x 10^6 m^3 yr^{-1} [Loáiciga, 2002]), included in the annual water balance, remove water for municipal and agricultural uses. Fisheries and other ecological requirements determine the minimum annual release in all models. These are currently estimated at F = 2.932 x 10^6 m^3 [Loáiciga, 2002]. Maximum release is estimated to be four times the average reservoir inflow (for the 84 years of data). Therefore, k = 4 in the release constraint (6). This value supplies the models a reasonable range within which to find a solution yet prevents excessive releases which could produce extremely detrimental effects on downstream development, including agricultural and residential areas.

Hydrologic data. Figure 2 depicts annual reservoir inflow at Lake Cachuma for water years 1917-1918 through 2000-2001 (data available from author). High stream flow variability and frequent periods of below average stream flow are readily apparent in this figure. While there are a number of sharp spikes indicating large stream flows they are solitary and dispersed. Periods of two to four years of below average reservoir inflow are the more common observation. Figure 3 depicts annual rainfall and evaporation for water years 1917-1918 through 2000-2001 (data available from author). It is important to note that annual evaporation consistently and significantly exceeds annual rainfall. Annual evaporation also exhibits far less fluctuation than annual rainfall. To further exacerbate matters, periods of low rainfall (drought) tend to be associated with periods of high evaporation (and therefore increased water demand) and periods of high rainfall tend to be associated with periods of lower evaporation (which is therefore not present to remove the excess water), resulting in added strain on water resource management.

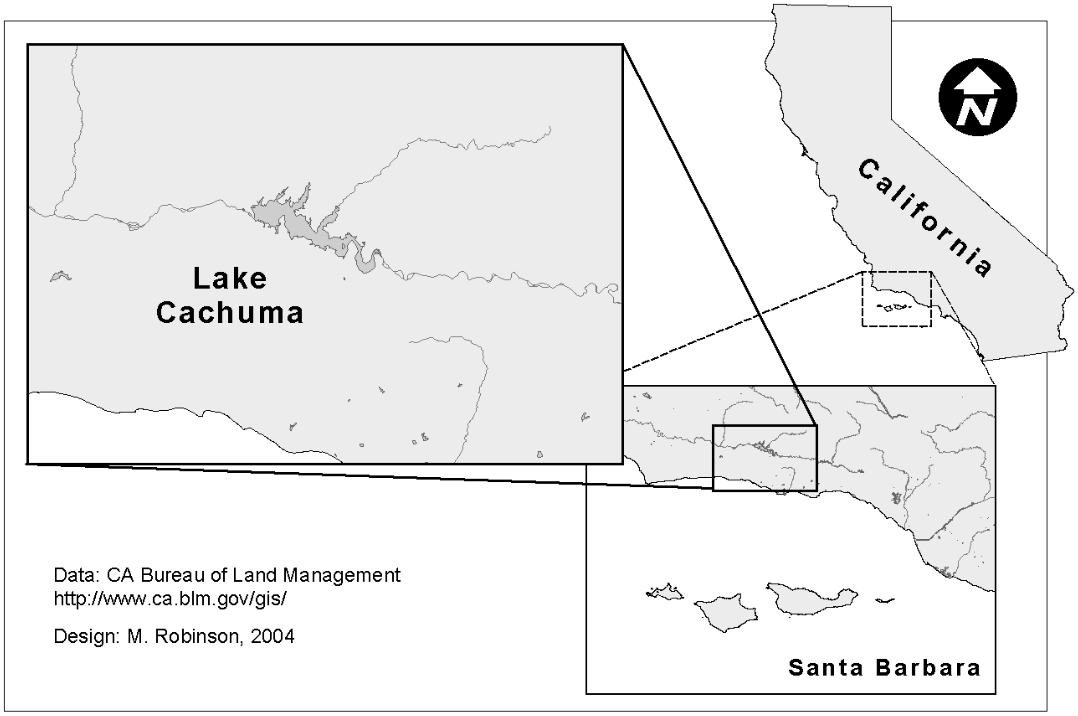

Figure 1. Lake Cachuma, Santa Barbara County, California.

Results and discussion

In each of the four alternative reservoir optimization models the initial storage, S_0, is a fraction, g ($0.5 \leq g \leq 1$), of the optimal reservoir capacity. The four reservoir optimization models were formulated in Microsoft Excel spreadsheets and solved using Solver, Excel's built-in mathematical programming package. Each formulation of the reservoir optimization model necessitated the solution of a linear programming problem for the range of initial storages (g = 0.5, 0.6, 0.7, 0.8, 0.9, 1.0). The stochastic components also include a range of reliability levels ($p = 1 - q = 0.75$, 0.80, 0.85, 0.90, 0.95, 0.99) for each range of initial storages.

Bootstrapping results. Bootstrapping was used to model reservoir inflow for Models III and IV, the stochastic cases. The distribution of each i-th sample is fit to a theoretical distribution and the χ^2 goodness-of-fit test is run on each distribution (in which the null-hypothesis statistic at a 5% significance level is, $\chi_4^2(0.05) = 9.488$). A gamma distribution was used in most cases, though a lognormal distribution was used in cases where the gamma distribution did not produce a good fit. In total, 80 of the 84 distributions (approximately 95%) fit within acceptable limits, with a general trend of closer approximation to gamma distribution exhibited by larger i values. Reliability levels (quantiles) ranging from 0.75 (below which reliability is assumed unacceptable for management purposes) to 0.99 (above which solution is assumed impractical for management purposes) are determined from these bootstrapped distributions and used in the probabilistic models.

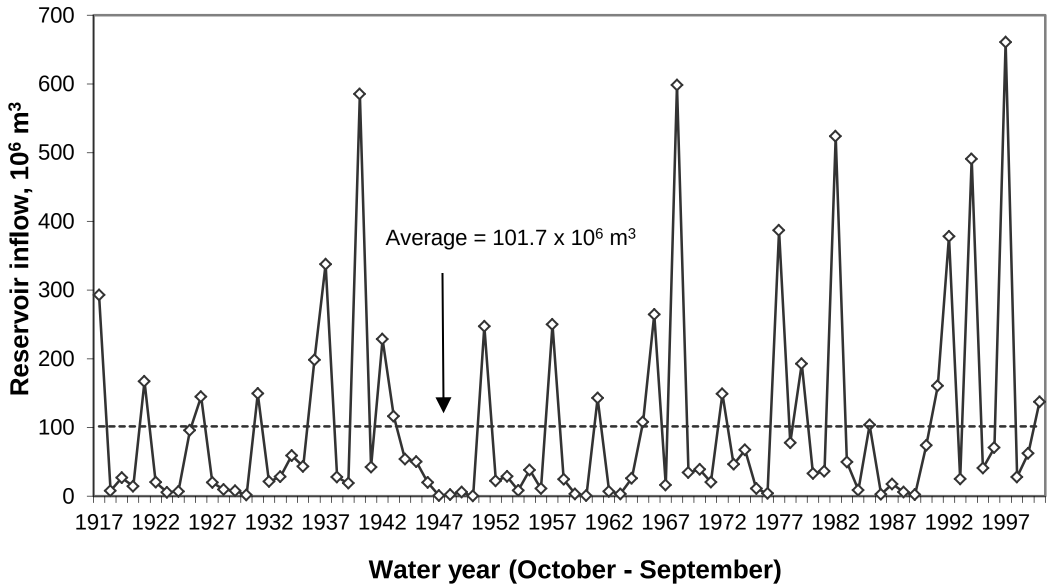

Figure 2. Annual stream flow from Santa Ynez River and Santa Cruz Creek into the Cachuma reservoir, 1917-1918 through 2000-2001. (Source: United States Geological Survey; Loáiciga, 2002.).

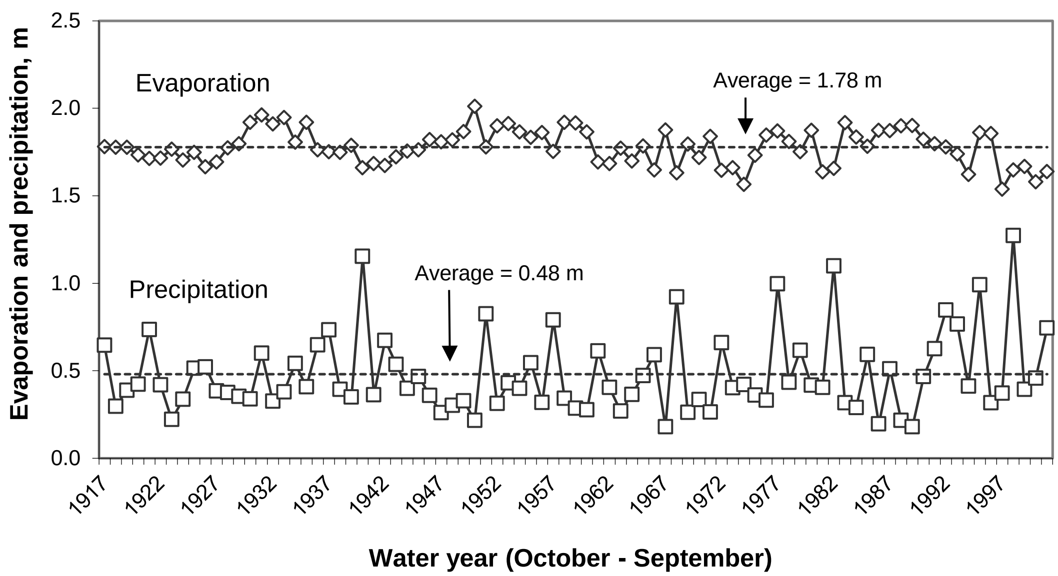

Figure 3. Annual Lake Cachuma rainfall and evaporation data, 1917-1918 through 2000-2001. (Source: United States Bureau of Reclamation [evaporation], 2002; Santa Barbara County Water Resources [rainfall], 2002.)

Optimal Capacity. Optimal capacity varies greatly between the models (see Figures 4 and 5). Model II optimal capacity (303 x 10^6 m^3) remains above model I optimal capacity (239 x 10^6 m^3) regardless of the value of initial storage. The larger optimal capacity of model II is most likely required to compensate for the comparatively large amount of water lost to evaporation. Models III and IV initially exhibit a similar relationship, though with much larger optimal capacities. In the latter two models, optimal capacity is viewed in terms of reliability. It is interesting to note that in model III optimal capacity is constant for initial storages only to around 95% of capacity. After this the optimal capacity increases rapidly, producing infeasible solutions when initial storage equals 100% of capacity. A look at annual average releases helps explain this finding. As the fraction of initial storage approaches 100% capacity the average annual releases approach their maximum, four times the average reservoir inflow for the 84 years of data. It appears that when initial storage equals 100% capacity the reservoir is required to release more than the model constraints allow and, thus, is not able to release enough water to produce a feasible solution. In this case, evaporation is not present to remove the water necessary to produce a feasible solution. This situation does not occur in model IV, presumably due to the effect of evaporation. In both cases optimal capacity increases first gradually then rapidly as larger reliability values are approached. At lower reliability values model IV requires larger optimal capacities than model III, again most likely due to the loss of water to evaporation. However, as reliability increases the two models approach similar optimal capacities, with model III slightly surpassing model IV at very high reliability values (99%). This seems to indicate that at very high reliabilities (associated with very large capacities) the effects of lake hydrology diminish relative to the reliability level.

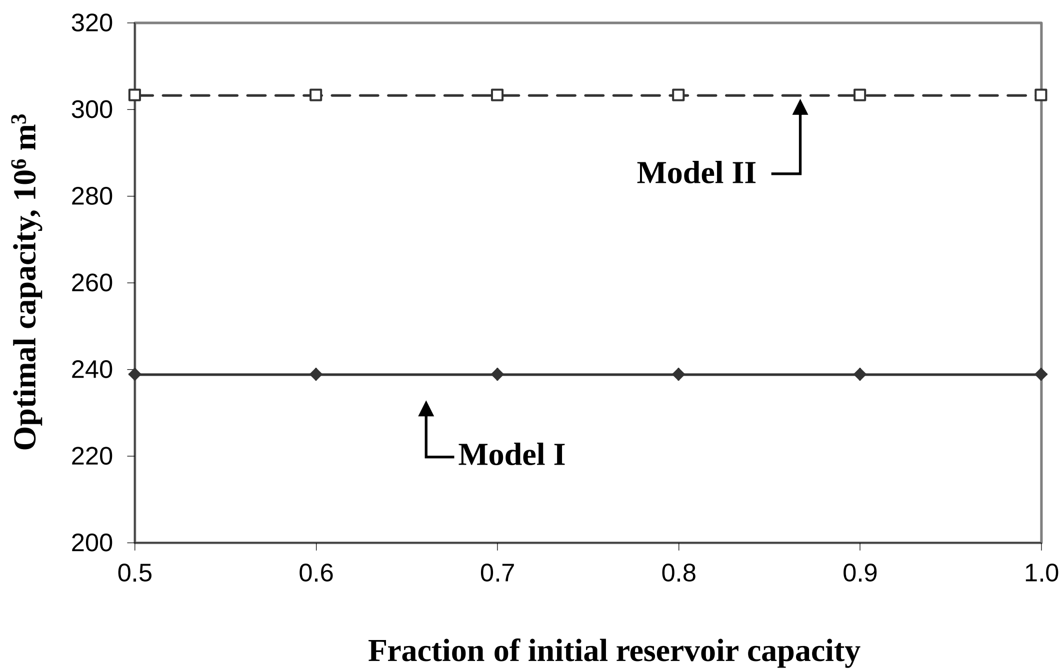

Figure 4. Comparison of optimal capacity for models I and II.

Annual Average Releases. Optimal annual average releases were determined by each of the models (see Figures 6 and 7). Model I produces larger annual average releases than model II. This can most likely be attributed to the need for model I to release the water that model II loses through evaporation. Models III and IV exhibit the opposite pattern. Model IV produces larger annual average releases than model III, though they appear to approach each other with increasing reliability. This change may be due to the very large optimal capacities required in model IV. These large capacities may generate correspondingly large annual releases. This may best be explained by understanding the reliability constraints. Very low reliabilities produce very small or essentially nonexistent reservoirs. One does not need a reservoir if water storage is not an issue. On the other hand, very high reliabilities lead to very large reservoirs. A large reservoir is needed if one does not ever want to run out of water on the one hand (in case of prolonged drought) or risk overtopping on the other (in case of substantial precipitation and reservoir inflow). Given these scenarios, low reliability and relatively smaller reservoirs requires releasing much of the potentially large reservoir inflow and precipitation. High reliability requires maintaining large capacity and retaining as much water as possible to preserve the largest feasible water supply. One must keep in mind, however, that a reservoir with very large capacity is associated with a high probability of performance rather than with a guarantee of desired water supply.

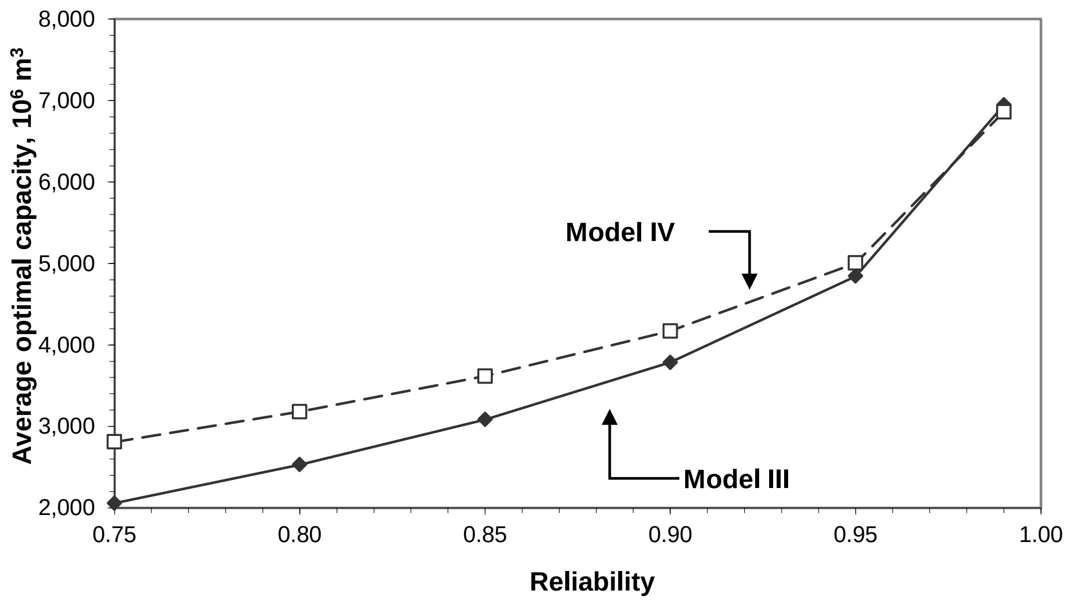

Figure 5. Comparison of optimal capacity for models III and IV.

Conclusion

The strong effect of chance constraints on reservoir storage, and the highly variable lake hydrology and reservoir inflow render the determination of optimal reservoir capacity a challenging problem. Loáiciga [2002], using deterministic modeling, concluded that lake hydrology plays a substantial role on optimal reservoir capacity and average annual water release in semiarid river basins. The current research confirms this conclusion and shows that the role is complex and significantly influenced by the reliability level of chance constraints imposed on reservoir storage. While lake evaporation seems to play a more dominant role on storage in the simpler (deterministic) models, the pronounced streamflow variability has a large impact on storage in the more complex (stochastic) reservoir models. In addition, optimal reservoir capacity was found to be very sensitive to the chance-constraint reliability level in the stochastic reservoir models. The most complex reservoir model, dealing with lake hydrology and stochastic inflows, calls for a very large and costly optimal capacity. At the same time, this model's output also requires large annual average release, which produces substantial revenue.

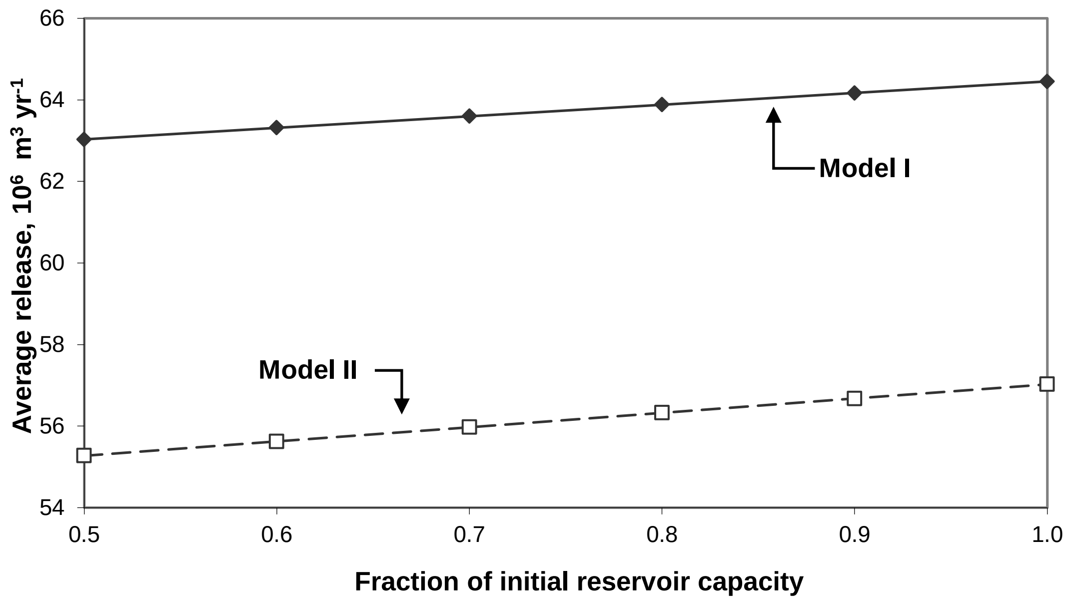

Figure 6. Comparison of optimal annual average releases for models I and II.

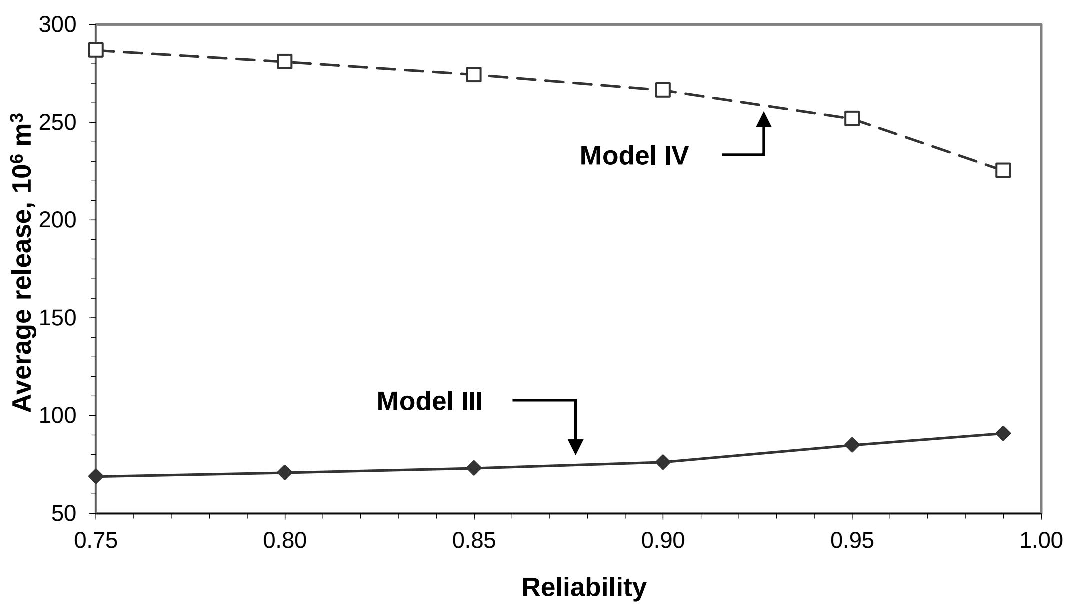

Figure 7. Comparison of optimal annual average releases for models III and IV.

Lastly, previous research has suggested that the gamma distribution is a good choice for modeling stochastic reservoir inflows. The bootstrapping method employed in this research supports this proposal, with approximately 95% of the 84 years of reservoir inflow data fitting the gamma distribution within acceptable limits.

Appendix A. Derivation of chance constraints not considering lake evaporation and precipitation (model III, stochastic reservoir inflow).

Constraint on minimum capacity:

$$P(S_i \geq S_{\min}) \geq 1 - q, \text{ (e.g., } 1 - q = 0.95) \tag{A1}$$

Let:

$$Q_i = \sum_{j=1}^{i} r_j \tag{A2}$$

$$B_i = \sum_{j=1}^{i} D_j \tag{A3}$$

$$C_i = \sum_{j=1}^{i} w_j \tag{A4}$$

with initial storage $S_0 = g \cdot C$. Expressing storage in terms of the initial condition S_0 yields:

$$S_i = S_0 + Q_i - B_i - C_i \tag{A5}$$

The constraint (B1) is rewritten as follows:

$$P(Q_i \geq S_{\min} + B_i + C_i - S_0) \geq 1 - q \tag{A6}$$

The cumulative inflow Q_i is random, and the deterministic equivalent of equation (A6) is:

$$S_{\min} + B_i + C_i - S_0 \leq Q_q^{(i)} \tag{A7}$$

in which $Q_q^{(i)}$ is such that:

$$P\left(Q_i \geq Q_q^{(i)}\right) = 1 - q \tag{A8}$$

Notice that Q_i equals the sum of i reservoir inflows. Therefore, the determination of the quantile $Q_q^{(i)}$ necessitates the probability distribution function (pdf) of Q_i. The determination of this pdf is nontrivial, a task accomplished in this work through the use of the bootstrapping method explained in the main text of this article. Equation (A7) yields:

$$-g \cdot C + \sum_{j=1}^{i} w_j \leq Q_q^{(i)} - B_i - S_{\min} \tag{A9}$$

[see equation (12)].

Constraint on maximum capacity:

$$P(S_i \leq C) \geq p, \text{ (e.g., } p = 0.95) \tag{A10}$$

using equation (A5) in (A10) produces:

$$P(Q_i \leq C + B_i + C_i - S_0) \geq p \tag{A11}$$

which yields the deterministic equivalent:

$$C + B_i + C_i - S_0 \geq Q_p^{(i)} \tag{A12}$$

in which the quantile $Q_p^{(i)}$ is such that:

$$P\left(Q_i \le Q_p^{(i)}\right) = p \tag{A13}$$

The determination of the quantile $Q_p^{(i)}$ is approached with the bootstrapping method described in the main text. Equation (A12) yields:

$$C \cdot (1-g) + \sum_{j=1}^{i} w_j \ge Q_p^{(i)} - B_i \tag{A14}$$

[see equation (13)].

Appendix B. Derivation of chance constraints considering lake evaporation and precipitation (model IV, stochastic reservoir inflow).

Constraint on minimum capacity:

$$P(S_i \ge S_{\min}) \ge 1-q, \text{ (e.g., } 1-q = 0.95) \tag{B1}$$

Let:

$$R_i = \sum_{k=1}^{i} r_k \cdot T_k \cdot \phi_k \tag{B2}$$

$$D_i^* = \sum_{k=1}^{i} D_k \cdot T_k \cdot \phi_k \tag{B3}$$

$$W_i = \sum_{k=1}^{i} w_k \cdot T_k \cdot \phi_k \tag{B4}$$

$$G_i^* = \sum_{k=1}^{i} G_k \cdot \phi_k \tag{B5}$$

$$\psi_i = \prod_{k=1}^{i} K_k \tag{B6}$$

$$\phi_k = \prod_{r=k+1}^{i} K_r \equiv 1 \text{ if } r > i \tag{B7}$$

with initial storage $S_0 = g \cdot C$ and where G_k, K_k, T_k were defined in equations (11a), (11b), and (11c), respectively. Expressing storage in terms of the initial condition S_0 yields:

$$S_i = \psi_i S_0 + R_i - D_i^* - W_i + G_i^* \tag{B8}$$

The constraint (C1) is rewritten as follows:

$$P\left(R_i \ge S_{\min} + D_i^* + W_i - G_i^* - \psi_i S_0\right) \ge 1-q \tag{B9}$$

The cumulative weighted inflow R_i is random, and the deterministic equivalent of equation (B9) is:

$$S_{\min} + D_i^* + W_i - G_i^* - \psi_i S_0 \le R_q^{(i)} \tag{B10}$$

in which $R_q^{(i)}$ is such that:

$$P\left(R_i \ge R_q^{(i)}\right) = 1-q \tag{B11}$$

Notice that R_i is the weighted sum of i reservoir inflows. The probability distribution function (pdf) of R_i is required to determine the quantile $R_q^{(i)}$. In this work, we resort to a bootstrapping method to obtain the quantile $R_q^{(i)}$ (see main text). Equation (B10) implies that:

$$-\psi_i \cdot g \cdot C + \sum_{k=1}^{i} w_k \cdot T_k \cdot \phi_k \leq R_q^{(i)} - D_i^* + G_i^* - S_{\min} \quad \text{(B12)}$$

[see equation (14)].

<u>Constraint on maximum capacity</u>:

$$P(S_i \leq C) \geq p, \text{ (e.g., } p = 0.95) \quad \text{(B13)}$$

Using equation (B8) in equation (B13) yields:

$$P\left(R_i \leq C + D_i^* + W_i - G_i^* - \psi_i S_0\right) \geq p \quad \text{(B14)}$$

and the deterministic equivalent becomes:

$$C + D_i^* + W_i - G_i^* - \psi_i S_0 \geq R_p^{(i)} \quad \text{(B15)}$$

in which the quantile $R_p^{(i)}$ is such that:

$$P\left(R_i \leq R_p^{(i)}\right) = p \quad \text{(B16)}$$

The determination of the quantile $R_p^{(i)}$ is approached with the bootstrapping method described in the main text. Equation (B15) yields:

$$C \cdot (1 - \psi_i \cdot g) + \sum_{k=1}^{i} w_k \cdot T_k \cdot \phi_k \geq R_p^{(i)} - D_i^* + G_i^* \quad \text{(B17)}$$

[see equation (15)].

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