The Dynamics of Simple Prediction: Judging Reachability

Iris van Rooij (irisvr@uvic.ca)
Cognitive Psychology, P.O. Box 3050, Victoria BC, V8W 3P5, Canada
Raoul M. Bongers (bongers@psych.kun.nl)
Developmental Psychology, Montessorilaan 3, 6525 HR Nijmegen, The Netherlands
W.F.G. (Pim) Haselager (pimh@nici.kun.nl)
NICI/Cognitive Science, Montessorilaan 3, 6525 HR Nijmegen, The Netherlands

Abstract

This study addresses the dynamical nature of a 'representation hungry' cognitive task. Participants were asked to judge whether or not they thought they could reach a distant object with a hand-held rod. The dynamical effects observed in this study support a two-attractor model designed by Tuller, Case, Ding, & Kelso (1994). The results suggest that predictive judgments regarding the (im)possibility of an action may be better understood in terms of dynamically evolving basins of attraction rather than as depending on stable representational structures.

The ability to think about the outcome of a yet to be performed action seems to necessitate a representational explanation. How else to explain this ability except by assuming that the system constructs a model of the situation, represents the imagined action, and concludes on the basis of the ensuing representational structure whether the goal can be achieved by means of the action or not? In this paper we aim to question this representational presupposition by investigating the potential of dynamical systems theory (DST) to model simple prediction.

Within DST, the behavior of a system is analyzed as an emergent property of the interactions between its subsystems. During the last decade the tools of DST have proven to be valuable assets for understanding behavior emerging out of multiple interacting components (Beek, Peper, & Stegeman, 1995; Haken, Kelso, & Bunz, 1985; Schmidt, Carello, & Turvey, 1990; Vallacher & Nowak, 1994). However, most of the behavioral phenomena that are currently described with models developed in DST are not regarded as clear cases of *cognitive* behavior. DST has been challenged to try to deal with more 'representation-hungry' domains (Clark, 1997, p. 166-170; see also Clark & Toribio, 1994). One such domain, according to Clark, involves the class of cases that "include thoughts about temporally or spatially distant events and *thoughts about the potential outcome of imagined actions*" (Clark, 1997, p. 167; our emphasis). In the present paper we take a first, exploratory attempt towards answering this challenge by exploring whether participants' verbal reports on the (im)possibility of an imagined action can be understood from within a DST framework. In our task participants have to indicate whether they think they can reach for an object on a distant table with a rod. This task can be seen as a simple example of a situation in which one has to predict the possible outcome of an imagined action. By systematically manipulating rodlength we set out to study the dynamical aspects of this prediction behavior.

Model description

Within the DST approach many different models have been developed to account for global patterns in behavior. Given that the task we studied involved discerning which rods enabled successful reaching and which did not, we used a dynamical model particularly designed to account for behavior with two attractor states. Tuller and colleagues (Tuller, Case, Ding, & Kelso, 1994; see also Case, Tuller, Ding, & Kelso, 1995) applied such a model to speech categorization phenomena. Following the example of Tuller et al. (1994) we use equation 1 to model our data.

$$V(x) = kx - \tfrac{1}{2}x^2 + \tfrac{1}{4}x^4 \qquad (1)$$

V(x) is a potential function with two minima which are assumed to correspond to two stable conceptual states, viz. 'No' (i.e., the participant indicates the belief or judgment that it is not possible to reach the object with the rod) and 'Yes' (the participant indicates the belief or judgment that it is possible to reach the object with the rod) respectively. The judgment regarding the imagined action is qualitatively denoted by x and k is the control parameter specifying the direction and the degree of tilt of the potential function (c.f. Tuller et al., 1994). As can be seen in Figure 1, for $k = -1$ only one stable state exists in the system (i.e., 'No'). Increasing k forces the function to tilt. Although the initial stable state persists, the attractor becomes more shallow. When the control parameter reaches the critical value $-k_c$ an additional attractor appears ('Yes'). From this point on, until k reaches the second critical value $+k_c$, the two stable states coexist (Both 'No' and 'Yes' are possible responses). At $+k_c$, however, the attractor corresponding to 'No' ceases to exist. Increasing k further only deepens the remaining attractor.

Figure 1 illustrates the tendency of dynamic systems to cling to the state they reside in. For each value of the control parameter the state in which the systems has settled is

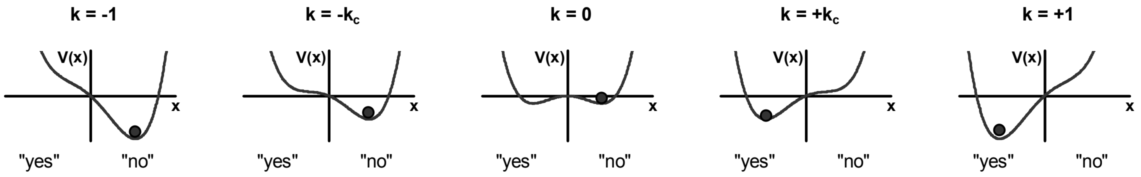

Figure 1: Potential landscape defined by equation 1 for different values of k (after Tuller, Case, Ding, & Kelso, 1994)

indicated by the black dot. Ideally, the black dot will remain in the attractor it is in for as long as the attractor is relatively stable. This means that when multistability exists the location of the black dot on the potential function depends on whether the control parameter increased from –1 to +1 or decreased from +1 to –1. As can be seen in Figure 1 this can lead to an observable effect classically associated with dynamical system's behavior, namely *hysteresis*. That is, the switch from 'No' to 'Yes' occurs at a higher value of the control parameter than the switch back from 'Yes' to 'No'.

As was said, this holds for the ideal case, in which the system is not perturbed in any way. Switches between states within the multistable region can occur, however, as a consequence of random disturbances. In a cognitive task like the one we studied, random disturbances may be assumed to correspond to psychological factors, such as fatigue, attention, boredom, and so on (c.f., Tuller et al., 1994).

To capture participants' behavior in our task the relationship between the control parameter k and the independent variable has to be specified. Following Tuller et al. (1994) we assume that this relationship is not a one-to-one correspondence. Instead k is a function of (1) rodlength, (2) the number of repetitions of the categorical judgments[1], and (3) perceptual and cognitive characteristics of the participant. The relationship between the control parameter and rodlength can be symbolized by the following equation,[2]

$$k = \lambda + (N_{no}-N_{yes})S, \qquad (2)$$

in which k specifies the value of the control parameter, λ is linearly proportional to the length of the rod, N_{no} and N_{yes} are growing functions of the number of accumulative repetitions of 'No' and 'Yes' respectively, $S \geq 0$ and represents relevant characteristics of the participant that may fluctuate during the time course of the experiment. Given that S represents uncontrolled factors influencing task behavior, we cannot know the exact value of S. Therefore, we take a qualitative approach to the combined influences of $(N_{no}-N_{yes})$ and S on the dynamics of the behavior of the participants. In equation 2 if $(N_{no}-N_{yes})S=0$ then $k=\lambda$. So when either $N_{no}=N_{yes}$ or $S=0$, then there is a one-to-one correspondence between k and λ. However, for $S>0$, k will be larger than λ when $N_{no}>N_{yes}$ and k will be smaller than λ when $N_{no}<N_{yes}$. The rationale of Equation 2 is illustrated by Figure 2 for a coupled sequential run, in which rodlength first systematically increases and subsequently systematically decreases (abbreviated ID-run). For the sake of clarity we hold S constant and only look at the effect of the accumulative repetitions of a response.

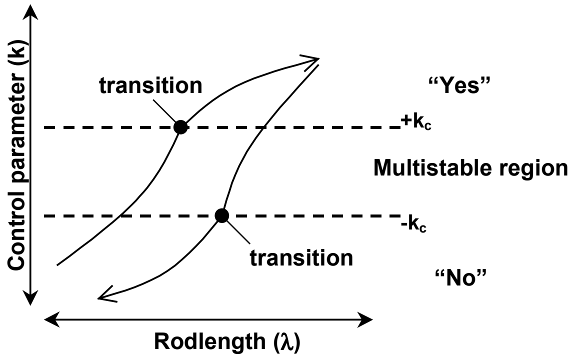

Figure 2: Illustration of the relationship between the control parameter and rodlength, for fixed S>0, in a coupled sequential run in which rodlength first increases and subsequently decreases (see text for details).

In an ID-run the participant is presented at first with the smallest rod (bottom left in Figure 2). For short rods the participants start with no-responses and N_{no} will become increasingly larger than N_{yes} (which will remain zero) with every next trial. Due to the fact that N_{no} grows increasingly larger than N_{yes}, k will increase faster than λ increases. When k reaches the value of $+k_c$ a transition occurs and the participant switches to yes-responses. With every next trial N_{yes} will grow, whereas N_{no} will not. Hence the slope of the function k will decrease. Because the increase sequence is followed by a decrease sequence N_{yes} will start to outnumber N_{no}. This will cause k to decrease faster than λ. When $-k_c$ is reached a transition occurs and the participant will switch to yo-responses. Figure 2 thus illustrates that for sufficiently large S the transition from 'Yes' to 'No' occurs at a larger rodlength than the transition from 'No' to 'Yes'. This is an example of the *enhanced contrast* effect. One can imagine that for a certain settings of the parameters one may find that the first and second transition occur at exactly the same rodlength, i.e. a *critical boundary*. However, the number of parameter settings that result in critical boundary

---

[1] See also Parducci's (1965; Parducci & Wedell, 1986) range-frequency theory and Helson's (1964) adaptation-level theory.

[2] See Tuller et al. (1994) for the original, more explicitly specified relationship between the control parameter k and the experimental variable λ. The simplification in the form of equation 2 is sufficient for our purposes.

is much smaller than the number of settings that result in either hysteresis or enhanced contrast.

The interrelationship between Equation 1 and 2 as described above leads to the following predictions for our experiment: (1) There is a tendency in the dynamic system to remain in the state it resides in. This means that participants will tend to give the same response as on preceding trials. (2) Accumulative repetitions of ‘yes’ will cause the multistable region to shift towards the upper end of the rodlength continuum. Conversely, accumulative repetitions of ‘No’ will cause the multistable region to shift towards the lower end of the rodlength continuum. (3) The higher the number of repetitions of ‘Yes’ in a run where rodlength increases and subsequently decreases the greater the chance of observing enhanced contrast and the smaller the chance of observing hysteresis. Conversely, the higher the number of repetitions of ‘No’ in a run where rodlength decreases and subsequently increases the greater the chance of observing enhanced contrast and the smaller the chance of observing hysteresis. Observations of critical boundary will overall be very limited. (4) Within the multistable region switches in perception can occur as a consequence of random disturbances. The narrower the multistable region (e.g., due to repetitions of a certain response – see figure 2) the smaller the chance of observing perceptual switches.

Method

Participants

Fourteen participants, 5 male and 9 female, participated in the experiment. All but two female participants were right-handed. The age of thirteen participants ranged from 22 to 28 years. One male participant was significantly older than the rest, viz. 56 years of age. The height of participants ranged from 1.56 to 1.88 meters, with an average of 1.76 meters. All participants, except one who volunteered, were paid for their participation or participated as a means to fulfilling a course requirement.

Material

Rods with a diameter of 1.25 cm were used, ranging in length from 57.0 to 91.5 cm, in 1.5 cm increments.[3] The twenty-four rods were constructed from wood (density 0.67 g/cm^3). Attached to each rod was a handle of identical material with the length of 11.5 cm and a diameter of 1.25 cm. A small disc divided the handle from the rod.

A PVC cylinder (diameter 5 cm, height 6 cm) was placed on a table (25x25 cm). The height of the table was adjusted to the participant’s wrist height with the arm at the side. The back of the cylinder was placed against a barrier of 12.5 cm height and the front of the cylinder was aligned with the front edge of the table.

Procedure

A participant was asked to bend forward, with his/her preferred arm stretched as far as possible (i.e., bending forward while maintaining enough balance to stay flat on the feet). The distance between the feet and the hand in this position was measured. This measure was used to determine the distance to the table at which each participant was to be positioned during the experimental session (i.e., maximum distance reachable without rod + 75 cm[4]). Participants were subsequently asked to take this position and stayed there during the entire experiment. While standing at this distance it was explained to the participant that the goal was to displace the cylinder positioned on the table. The participant was subsequently handed a rod and was instructed to hold the rod so that it made an angle of approximately 45 degrees upwards with the horizontal. The participant stood upright with the rod in one hand and judged whether he was able to reach the cylinder with the rod from that position while keeping the two feet flat on the floor. After a participant had given his categorical judgment he returned the rod to the experimenter and was handed a new rod for which the participant again made a judgment. No feedback regarding accuracy was given.

Design

Each participant performed this judgment under several conditions. There were three kinds of *sequences* in which rods were given to the participant, namely (1) increase sequences (I): rodlength increased from minimum to maximum in 1.5 cm increments; (2) decrease sequences (D): rodlength decreased from maximum to minimum in 1.5 cm increments; (3) random sequences (R): the rods ranging in length from the minimum to maximum were randomly assigned to the task. The two sequential condition I and D were always coupled, resulting in two kinds of coupled sequential runs: increase-decrease (ID) runs and decrease-increase (DI) runs. Coupled sequential runs were always followed by a random sequence, resulting in two possible blocks of runs, namely increase-decrease-random (IDR) blocks and decrease-increase-random (DIR) blocks. The random sequence served as a kind of buffer between the coupled sequential run preceding it and the coupled sequential run of the next block, and as a control condition in the analyses of the data.

Two different *ranges* were used in the experiment, namely range1 of 57.0 - 85.5 cm and range2 of 63.0 - 91.5 cm.

---

[3] Psychophysics studies (e.g. Morgan & Watt, 1989; Watt, 1984) suggest that the Weber fraction ($\Delta I/I$) for length discrimination is approximately 0.05. In our experiment the fraction between the increment and rodlength ranged from 0.026 to 0.016. This means that in increase and decrease sequences the direction of change in rodlength is not perceivable for participants from one trial to another. On most occasions the fact that the hand-held rod is longer or shorter than a preceding rod does not become apparent within less than three or more trials.

[4] We added 75 cm to the personal maximum reaching distance, because in the range of rodlength used in this experiment 12 rods ≤ 75 cm and 12 rods > 75 cm. Hence, for all participants exactly half of the rods used in the experiment would enable reaching, and half would not.

Thus, there were two possible minima and maxima for the three sequences described above. Within a given block the minimum and maximum for the three constitutive sequences (I, D and R) were the same. The four possible combinations of block and range in the experiment were thus, in shorthand, IDR-1, IDR-2, DIR-1 and DIR-2. Each of these combinations occurred twice in one experimental session, resulting in a total of 480 trials (2 ranges x 2 blocks x 3 sequences x 20 rods x 2 repeated measures) per participant. The block-range combinations were randomized within an experimental session, with the constraint that each block-range combination appeared as often in the first half of a session as in the second half.

Results

Most participants showed a transition in judged possibility in all sequences (increase-, decrease- and random-sequences). Two of the fourteen participants, however, overestimated the distance reachable so much that the lower-end of the range (57.0 cm) was still too high to evoke a perceptual transition. For this reason these two participants were excluded from the analyses. Plotting the average response against rodlength for the remaining twelve participants for the three types of sequences resulted in the cumulative distributions as depicted in Figure 3.

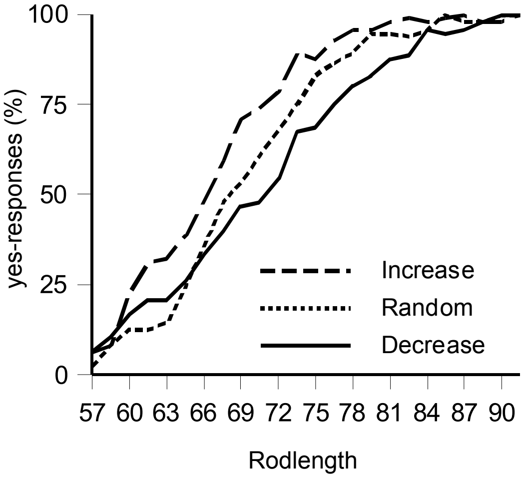

Figure 3: Percentage of ‘Yes’ responses, averaged over subjects, per rodlength for increase, random and decrease sequences separately.

On average participants in this experiment tended to *overestimate* their reaching distance. Given the individually defined distance to table (personal maximum reaching distance without rod + 75 cm) the *expected* 50% category boundary would be about 75 cm for all participants. The *observed* 50% category boundaries as depicted in Figure 3 are all lower than this. The finding that participants tended to overestimate the distance reachable is in correspondence with findings in other experiments on judging reachability (Heft, 1993; Rochat, & Wraga, 1997).

To test the effect of sequence, suggested in Figure 3, a measure was required for the transition point in each sequence independently. Because in 36 sequences multiple transitions were observed across the rodlength continuum, the data of these sequences were transformed so that a single ‘average’ transition-point resulted[5]. For the other 192 sequences the real transition-point was simply used as average transition-point. On the average transition-points a 3 x 2 x 2 repeated-measures ANOVA was performed with Sequence (random-, increase- and decrease-run), Block (IDR, DIR), and Range (range1, range2). A main effect of Sequence was found, $F(2, 22) = 14.68$, $p < .001$. A difference contrast, comparing the average transition-points in increase- and decrease-sequences, revealed a contrastive effect, viz. the average transition-point was significantly lower in increase sequences (66.91 cm) than in the decrease sequences (70.73 cm), $F(1, 11) = 17.19$, $p = .002$. The average transition-point in random sequences was 69.23 cm. Further, the main effect of Range was significant, $F(1, 11) = 12.39$, $p = .005$. The average transition-point was smaller for range1 (68.20 cm) than for range2 (69.70 cm). None of the other effects was significant.

To see whether local contrastive or assimilative effects were present in random sequences the conditional probability of judging each rod as belonging to the same category as the preceding rod was investigated. We found that in random sequences participants tended to give the same response as given on the previous trial, $\chi^2(1) = 58.54$, $p < .001$.

Within the multistable region perturbing influences can make one percept change into the other and vice versa. Outside the multistable region only one perceptual form is possible. Taking these theoretical assumptions into consideration the boundary of the multistable region was estimated by the *last* transition-point[6] within a given sequence. Each coupled sequential run (ID-runs and DI-runs) was coded for the type of response pattern it showed, i.e. either hysteresis, critical boundary or enhanced contrast. In 66 of the 91 coupled sequential runs[7] (72.8%) an enhanced contrast effect occurred and in 20 runs (20.7%) a hysteresis effect. Critical boundary occurred in only 6 coupled sequential runs (6.5%), and no more than once per participant.

---

[5] This transformation involved re-ordering of the no- and yes-responses within a given sequence so that a single transition-point resulted. The total number of no-responses was projected onto the lower part of the rodlength continuum and the total number of yes-responses onto the upper part. The average transition-point was taken to be exactly between the rod receiving the last no-response and first yes-response in the transformed data.

[6] In an increase sequence this last transition-point was defined as being in between the longest rod receiving a no-response and its subsequent rod. Conversely, in a decrease sequence the last transition-point would be in between the shortest rod receiving a yes-response and its subsequent rod.

[7] Five coupled sequential runs were excluded from the analyses because no perceptual transition occurred.

Because participants overestimated the distance reachable the number of accumulative repetitions of ‘Yes’ in a ID-run were, on average, larger than the accumulative repetitions of ‘No’ in a DI-run. Hence, the dynamical model predicts that the chance of observing enhanced contrast is greater, and the chance of hysteresis is smaller, in ID-runs than in DI-runs. An analysis of the frequencies of the two response patterns confirmed this prediction. A Pearson Chi-Square test with SequenceCoupling (ID-, DI-runs), and ResponsePattern (enhanced contrast, hysteresis) indicated a significant association between SequenceCoupling and ResponsePattern, $\chi^2(1) = 4.44$, $p = .035$. Enhanced contrast occurred more frequently in ID-runs (37 times) than in DI-runs (29 times). Hysteresis on the other hand occurred more frequently in DI-runs (14 times) than in ID-runs (6 times). Critical boundary occurred as often in ID-runs (3 times) as in DI-runs (3 times).

Additional switches (i.e., alternating yes- and no-responses on successive trials preceding the last transition-point) occurred on 88 of the 3574 trials[8] in sequential runs. We found that more additional switches occurred in range1 (61 times) than in range2 (27 times; $\chi^2=13.70$, $p < .001$). Interestingly, this effect of range was observable for both increase and decrease sequences (see Figure 4).

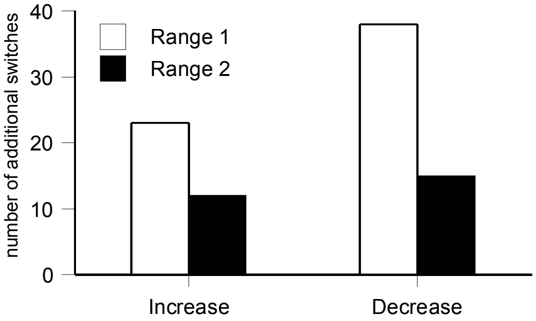

Figure 4: Number of additional switches observed in range 1 and range 2 for increase and decrease sequences separately.

The effect of range in decrease sequences can be understood as being due to the relatively large number of repetitions of ‘Yes’ in decrease sequences within range2 as compared to range1. The two-attractor model can also account for the effect of range in increase runs. As can be seen in Figure 3, even the shortest rods used in the experiment were occasionally judged to enable successful reaching. Further, the shortest rod in range2 (i.e., 63,0 cm) was judged as enabling successful reaching once by two participants, and as enabling successful reaching even 50% of the time by three participants. This means that the left boundary of the multistable region was not only on average closer to the left end of the range of rodlengths, but even outside range2 for a considerable number of subjects. Thus, for these participants, the increase sequences in range2 started well within the multistable region. Consequently there were simply fewer opportunities for switching in increase sequences in range2 as compared to range1, which explains the low frequency of additional switches in range 2 for increase sequences.

Discussion

Clark (1997) challenged DST to explain behavioral phenomena that are considered to be ‘representation hungry’ cases of cognition. We focused on the ability to predict the outcome of a to be performed action. Participants had to judge whether a rod afforded displacing an object from a certain distance. In our interpretation of Clark (1997) such behavior can be classified as ‘representation-hungry’, that is, the task seems to require a model of the situation, a representation of the imagined action, and computations based on those representations to determine whether the action will satisfy the goal.

In the present study we explored whether the judging behavior of our participants could be explained with a dynamical model. The results are in close agreement with the predictions derived from a two-attractor model (c.f., Case et al., 1995; Tuller et al., 1994). First, it was found that in random sequences participants tended to give the same categorical judgment as on preceding trials. This *assimilative* effect is in accordance with the notion that a dynamical system tends to cling to the state it resides in (*Prediction 1*).

Further, we observed that on average the transition from ‘No’ to ‘Yes’ in increase runs occurred at a shorter rodlength than the transition from ‘Yes’ to ‘No’ in decrease runs. Also we found that on average the transition occurred at a shorter rodlength in range 1 than in range2 (independent of the order of presentation of the rods). Both these effects can be interpreted as being due to the influence of accumulated repetitions of a certain response causing the multistable region to shift closer to one of the ends of the rodlength continuum (*Prediction 2*).

In coupled sequential runs (ID- and DI-runs) we observed all three effects that are predicted by the model, viz. hysteresis, critical boundary, and enhanced contrast. As expected critical boundary was the rarest of the three. Because participants overestimated their reaching distance to a large degree many more accumulative repetitions of ‘Yes’ responses occurred in coupled sequential runs in which rodlength first increased and subsequently decreased (ID-runs) than ‘No’ responses occurred in runs in which rodlength first decreased and then increased (DI-runs). As predicted, enhanced contrast occurred more often, and conversely hysteresis less often, in ID-runs as compared to DI-runs (*Prediction 3*).

Finally, more additional switches (alternating ‘No’ and ‘Yes’ responses) were observed when the multistable region

---

[8] This number indicates the total number of trials in the sequential runs of the experiment, i.e. (12 participants) x (40 trials) x (8 coupled sequential runs) = 3840, *minus* the first trial of each coupled sequential run and *minus* the trials in which a last transition-point was observed.

was expected to be relatively large, than when it was expected to be relatively small (*Prediction 4*).

Dynamic systems models typically describe behavior on the level of the whole system. On this account behavior is seen as a self-organized pattern, emerging from the interaction between subsystems. Such a pattern is called the collective variable or order parameter, which in turn can 'enslave' the behavior of the components (cf. Haken & Wunderlin, 1990, p. 7; Kelso, 1995, pp. 8-9). Despite the great complexity at the level of the interacting components the behavior of the system as a whole can be described and understood in terms of the lower-dimensional order parameter dynamics.

According to Clark (1997) an explanation of cognitive capacity in representational terms is valuable if the representations are distinguishable as entities serving a role as information-carriers for behavior. But what if, as DST would have it, a behavioral pattern is best understood as an emergent property of the overall activity of the system? Clark argues that in "such cases (if there are any), the overall system would rightly be said to represent its world– but it would not do so by trading in anything we could usefully treat as internal representations" (Clark, 1997, p. 168). We submit that the effects we observed can be fruitfully interpreted as a consequence of the inter-relationship between control parameter k and the collective variable V(x) governing the system. In all, these findings suggest that predictions regarding the possible outcome of an imagined reach are better understood in terms of dynamically evolving basins of attraction rather than as depending on stable representational structures.

References

Beek, P. J., Peper, C. E., & Stegeman, D. F. (1995). Dynamical models of movement coordination. *Human Movement Science*, *14*, 573-608.

Case, P., Tuller, B., Ding, M., & Kelso, J. A. (1995). Evaluation of a dynamical model of speech perception. *Perception & Psychophysics*, *57*(7), 977-988.

Clark, A., & Toribio, J. (1994). Doing without representing? *Synthese*, *101*, 401-431.

Clark, A. (1997). *Being there: Putting brain, body and world together again*. Cambridge, Massachusetts: MIT-Press.

Haken, H., Kelso, J. A. S., & Bunz, H. (1985). A theoretical model of phase transitions in human hand movements. *Biological Cybernetics*, *51*, 347-356.

Haken, H., & Wunderlin, A. (1990). Synergetics and its paradigm of self-organization in biological systems. In H. T. A. Whiting, O. G. Meijer & P. C. W. van Wieringen (Eds.), *The natural-physical approach to movement control.* Amsterdam: University Press.

Heft, H. (1993). A methodological note on overestimates of reaching distance: Distinguishing between perceptual and analytical judgements. *Ecological Psychology*, *5*, 255-271.

Helson, H. (1964). *Adaptation-level theory: An experimental and systematic approach to behavior.* New York: Harper & Row.

Kelso, J. A. S. (1995). *Dynamic patterns: The self-organization of brain and behavior.* Cambridge, Massachusetts: MIT-Press.

Morgan, M. J., & Watt, R. J. (1989). The Weber relation for position is not an artefact of eccentricity. *Vision-Research*, *29*(10), 1457-1462.

Parducci, A. (1965). Category judgment: range-frequency model, *Psychological Review, 72*, 407-418.

Parducci, A., & Wedell, D. H. (1986). The category effect with rating scales: Number of categories, number of stimuli, and method of presentation*. Journal of Experimental Psychology: Human Perception and Performance*, *12*, 496-519.

Rochat, P., & Wraga, M. (1997). An account of the systematic error in judging what is reachable. *Journal of Experimental Psychology: Human Perception and Performance*, *23*, 199-212.

Schmidt, R. C., Carello, C., & Turvey, M. T. (1990). Phase transitions and critical fluctuations in the visual coordination of rhythmic movements between people. *Journal of Experimental Psychology: Human Perception and Performance, 16*(2), 227-247.

Tuller, B., Case, P., Ding, M., & Kelso, J. A. (1994). The nonlinear dynamics of speech categorization. *Journal of Experimental Psychology: Human Perception and Performance*, *20*(1), 3-16.

Vallacher, R. R., & Nowak, A. (1994). *Dynamical systems in social psychology.* San Diego: Academic Press.

Watt, R. J. (1984). Towards a general theory of the visual acuities for shape and spatial arrangement. *Vision-Research*, *24*(10), 1377-1386.