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N.K. Glendenning

June 1989

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Neutron Stars, Fast Pulsars, Supernovae and the Equation of State of Dense Matter[†]

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**Invited Paper
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The Nuclear Equation of State
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Abstract

We discuss the prospects for obtaining constraints on the equation of state from astrophysical sources. Neutron star masses although few are known at present, provide a very direct constraint in as much as the connection to the equation of state involves only the assumption that Einstein's general theory of relativity is correct at the macroscopic scale. If the millisecond pulses briefly observed in the remnant of SN1987A can be attributed to uniform rotation of a pulsar, then a very severe constraint is placed on the equation of state. The theory again is very secure. The precise nature of the constraint is not yet understood, but it appears that the equation of state must be neither too soft nor stiff, and it may be that there is information not only on the stiffness of the equation of state but on its shape. Supernovae simulations involve such a plethora of physical processes including those involved in the evolution of the precollapse configuration, not all of them known or understood, that they provide no constraint at the present time. Not even the broad category of mechanism for the explosion is agreed upon (prompt shock, delayed shock, or nuclear explosion). In connection with very fast pulsars, we include some speculations on pure quark matter stars, and on possible scenarios for understanding the disappearance of the fast pulsar in SN1987A.

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1 Preliminaries

Astrophysical constraints on the equation of state are imposed by three sources, (1) masses of neutron stars, (2) rotational frequencies of very fast pulsars (neutron stars), and (3) supernovae. Of the three, the theoretical and observational status of the first is most secure. For the second, a half millisecond pulse over an eight hour period was reported in January but not seen again two weeks later at the next turn at the telescope, nor at any time since. If substantiated it would be the fastest known pulsar. If it can be interpreted as neutron star rotation, then theory seems to indicate that it provides very stringent conditions on the equation of state when combined with the mass constraint. The physics of supernova involves so many factors of comparable importance but high uncertainty, that they cannot be said to provide any constraint at the present time. Indeed the consensus is that it is not known whether supernovae can explode by the prompt bounce mechanism [1,2]. no simulations whose physics has been agreed upon have been successful and earlier claims to success of the prompt explosion of $1.35M_{\odot}$ iron core progenitors have been later withdrawn [3,4]. The delayed explosion mechanism produces only small explosions so far, and may be the means by which some stars explode, but not evidently the one by which all type II stars explode.

I shall talk mostly about slow neutron stars, ones for which the rotational energy is very small compared to the binding energy and can be ignored. This is the case for almost all pulsars, but I will report the current situation with respect to fast pulsars.

Nuclear and astrophysics are connected through Einstein's theory of general relativity. The field equations can be obtained through a variation of a generalized action involving the metric of space-time and the matter fields. They are,

$$G_{\mu\nu} = -8\pi T_{\mu\nu}$$

$$\frac{\delta \mathcal{L}_m}{\delta \phi} - \partial_\mu \frac{\delta \mathcal{L}_m}{\delta (\partial_\mu \phi)} = 0, \quad \text{each matter field } \phi \quad (1)$$

Here $G_{\mu\nu}$ is Einstein's curvature tensor, a function of the space-time metric functions $g_{\mu\nu}$ and $T_{\mu\nu}$ is the matter stress-energy tensor, derivable from the matter Lagrangian $\mathcal{L}_m$. We don't usually think of the matter Lagrangian and field equations as depending on the metric because we usually ignore gravity and write $g_{\mu\nu} = (1, -1, -1, -1)\delta_{\mu\nu}$. In strong gravitational fields this is not true, and we need to justify it for nuclear astrophysics. This we now do. In the process we become familiar with gravitational units, and get a qualitative description of neutron stars.

In empty space outside a static spherical star of radius R and mass M , Schwarzschild showed that the solution of Einstein's equations have a simple form. All but the diagonal components of the metric vanish, and they are simple. The line element is

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \quad (r > R) \quad (2)$$

I want to show you that the metric functions, g_{tt}, g_{rr} in front of dt^2 and dr^2 change by an infinitesimal amount over the distance between nucleons in a star that is near the limit of collapse to a black hole. We will see that such a star is what is referred to as a neutron star. To do this I need to show you how to compute in gravitational units $G = 1 = c$.

$$1 = c = 3 \times 10^{10} \text{ cm/s}$$

$$1 = G = 6.7 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2} \quad (3)$$

These can be treated as equations so that for example,

$$1 \text{ s} = 3 \times 10^{10} \text{ cm}$$

$$1 \text{ g} = 7.4 \times 10^{-29} \text{ cm}$$

$$1 \text{ s}^{-2} = 1.5 \times 10^7 \text{ g/cm}^3$$

$$1 \text{ erg} = 1 \text{ g cm}^2 \text{ s}^{-2} = 8.2 \times 10^{-50} \text{ cm} \quad (4)$$

Next we estimate the mass and radius of a star near the limit. Notice that the metric becomes singular at $r = 2M$. For actual stars, this radius is interior to the

star itself where the Schwarzschild solution does not hold, but in the special case where the star lies within the “gravitational radius”, it must be a black hole. Let us estimate the properties of a star near the limit, $R = 2M$. Assume that gravity packs nucleons up to their hard cores, say $r_0 \approx 0.5 \times 10^{-13}$ cm. Then

$$R \approx r_0 A^{1/3}, \quad M \approx Am \quad (5)$$

where A is the number of baryons in the star and m is their mass

$$m = 940 \text{ MeV} = 1.7 \times 10^{-24} \text{ g} = 1.3 \times 10^{-52} \text{ cm} \quad (6)$$

Hence substituting eq.(5) into the equation, $R = 2M$ we find,

$$A^{2/3} = r_0/(2m) = 1.9 \times 10^{38} \quad (7)$$

Putting this answer back into the expressions for radius and mass we have

$$\begin{aligned} A &= 2.6 \times 10^{57} \\ R &= r_0 A^{1/3} = 7 \text{ km} \\ M &= R/2 = 3.5 \text{ km} = 2.3 M_\odot \end{aligned} \quad (8)$$

where I used the solar mass,

$$M_\odot = 2 \times 10^{33} \text{ g} = 1.5 \text{ km} \quad (9)$$

So here we have an estimate of the baryon number, radius and mass of a star at the limit. We expect a slightly smaller mass and larger radius than the values given by the Schwarzschild relation, say,

$$M \approx 2M_\odot, \quad R \approx 10 \text{ km} \quad (10)$$

The average density of such an object is

$$\bar{\rho} = 9.7 \times 10^{14} \text{ g/cm}^3 \approx 4\rho_0 \quad (11)$$

where $\rho_0 = 2.4 \times 10^{14} \text{ g/cm}^3$ is the density of symmetric matter at saturation. Since the density of the star near the limit is supernuclear and since it must be charge neutral else the repulsive Coulomb force will overwhelm gravity, it will be dominated by neutrons and is called a neutron star. Putting the mass and radius into the metric we have,

$$\frac{g_{rr}(R)}{g_{rr}(0)} = \left(1 - \frac{2M}{R}\right)^{-1} = \left(1 - \frac{6}{10}\right)^{-1} = \frac{1.6}{1} \quad (12)$$

So the metric changes by roughly 60 percent over the dimension of the star. It changes by $2r_0/R = 2A^{-1/3} \approx 10^{-19}$ of this over the spacing of nucleons in the star. Later we shall also be interested in pulsars with very high angular velocity. Since the gravitational attraction must exceed the centrifugal repulsion else the star would

fly apart, we are assured that the curvature of space-time due to rotation must be even less than that due to the mass, which we just saw is completely negligible in any local frame in a region spanning the distance between many nucleons. So in solving the field equations for matter, we make negligible error for neutron stars by solving them in the absence of gravity and then using the resulting stress-energy tensor, which is diagonal in a co-moving frame,

$$T_{\mu\nu} = (\epsilon, p, p, p)\delta_{\mu\nu} \quad (13)$$

in Einstein's field equation to find how matter is compacted under the influence of gravity.

In the special case of a static star Einstein's equations take a special form first written down by Oppenheimer and Volkoff.

$$4\pi r^2 dp(r) = -\frac{GM(r)dM(r)}{r^2} \left(1 + \frac{p(r)}{\epsilon(r)}\right) \left(1 + \frac{4\pi r^3 p(r)}{M(r)}\right) \left(1 - \frac{2GM(r)}{r}\right)^{-1} \quad (14)$$

$$dM(r) = 4\pi r^2 \epsilon(r) dr \quad (15)$$

The interpretation is very simple. Think of a shell of matter in the star of radius r and thickness dr . The second equation gives the mass energy in this shell. The pressure of matter exterior to the shell is $p(r)$ and interior to it $p(r) + dp(r)$. The left side of the first equation is the net force acting outward on the surface of the shell by the pressure, and the first term on the right side is the attractive force of gravity acting on the shell by the mass interior to it in Newton's theory. The remaining three factors are the exact corrections for general relativity. So these equations express the balance of internal pressure and gravity. The equation of state $p = p(\epsilon)$ is the manner in which matter enters the equations of star structure. Otherwise they are completely specified and their correctness is confirmed by the observational tests of general relativity.

They can be integrated from the origin with the initial conditions that $M(0) = 0$ and an arbitrary value for the central energy density $\epsilon(0)$, until the pressure, $p(r)$, becomes zero. That point, R , defines the radius of the star, and $M(R)$ its mass. For the given equation of state, there is a unique relationship between the mass and central density, $\epsilon(0)$. So for each possible equation of state there is a unique family of stars, parameterized by, say, the central density. Several such families are shown in Fig.1 for different values of the nuclear compression. It will be noted that each family has a maximum mass star, called the limiting mass and that the central density of the limiting mass star is higher the softer the equation of state. The part of the curve for which the slope is positive corresponds to stable configurations. For negative slope, one can readily verify that the star is unstable to radial perturbations. In fact those beyond the maximum are unstable to collapse to black holes. It is in the limiting mass that a constraint on the equation of state arises. Obviously an acceptable equation of state must have a limiting mass at least as large as the largest observed mass.

2 Why pulsars are neutron stars

About 400 pulsars have been found since the first discovery in 1967 in the pulsed signals of a radio-telescope. The period of the pulses range from milliseconds to seconds, and is interpreted as the period of a rotation. Why? Ordinary stars have magnetic fields (~ 100 gauss) and rotate. When they collapse from a radius of 10^6 km to 10 km, both the rotation frequency and field are scaled up by the conservation laws of angular momentum and magnetic flux. The field is typically scaled to 10^{12} gauss. There is other evidence of such strong fields. The remnant of the crab pulsar is still accelerating with an apparent energy input of $\sim 10^{38}$ ergs/s, and the most likely source of input energy is the absorption of magnetic dipole radiation from the fast pulsar within it. Energy balance implies about the same strength for the magnetic field as quoted above,

$$10^{38} \text{ ergs/s} = -\frac{dE}{dt} = -\frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = \frac{1}{6}R^6 B^2 \omega^4 \sin^2 \alpha \quad (16)$$

Given the observed period and rate of change of period,

$$T \sim \frac{1}{30} \text{ s}, \quad \dot{T} \sim 4 \times 10^{-13} \text{ s/s} \quad (17)$$

we find (taking $\sin \alpha = 1$),

$$\begin{aligned} \dot{B} &\sim 4 \times 10^{12} \text{ gauss}, & I &\sim 2 \times 10^{44} \text{ g cm}^2 \\ E_{\text{rot}} &\sim \frac{1}{2}I\omega^2 \sim 4 \times 10^{48} \text{ ergs.} \end{aligned} \quad (18)$$

where I use 3.5×10^{24} gauss cm = 1 in gravitational units. The field will in general be oriented in a different direction than the rotation axis, say by an angle α . It is believed, but not understood, that radiation over a broad band of frequencies is emitted within some angular spread along the magnetic axis. Given the rotation, one has a beacon which we see as pulses as the star rotates.

We can use the period of rotation to estimate an average density of a millisecond pulsar. For the star to hold together under the opposing forces of gravity and centrifuge, we must have,

$$\frac{GmM}{R^2} > m\omega^2 R \quad (19)$$

Hence for the average density,

$$\bar{\rho} = M/\left(\frac{4\pi}{3}R^3\right) > \frac{3\pi}{T^2} = 1.4 \times 10^{14} \text{ g/cm}^3 \quad (20)$$

where the last equality holds for a millisecond pulsar. Since $\rho_0 \approx 2.4 \times 10^{14} \text{ g/cm}^3$, we learn that the average density of a pulsar has to be about as large or larger than nuclear density and so pulsars must be neutron stars. In actual models that

I have studied, $\bar{\rho}/\rho_0$ ranges from 3 for stiff equations of state to 5 for the standard compression found a decade ago in the analysis of the giant monopole resonance in nuclei ($K = 210$ MeV) to 6 or more for softer equations of state. Clearly the stiffer the equation of state the more sensitive the neutron star limiting mass will be to properties near saturation.

3 Nuclear and neutron star matter

The idealized matter of the interior of nuclei and the matter of neutron stars have similarities and differences which need to be understood. The similarities include the fact that they are composed of hadrons and the densities are the same within an order of magnitude. The differences arise from two facts. (1) Nuclei are bound by the charge symmetric nuclear force, but neutron stars are bound by gravity. Hence in nuclei, $N \approx Z$. However since the repulsive Coulomb force is so much stronger than the gravitational force that binds stars, the net charge in a star must be very small,

$$\frac{(Z_{net}e)e}{R^2} < \frac{G(Am)m}{R^2} \longrightarrow Z_{net} = Z_p + Z_e < 10^{-36} A \quad (21)$$

(where we use $\hbar c/e^2 = 137$ and $\hbar = 1.1 \times 10^{-27}$ g cm² s⁻¹ = 2.6×10^{-66} cm².) Although Z is not nearly so small as Z_{net} , it is considerably smaller than $A/2$. So nuclei are symmetric and neutron stars are asymmetric. (2) There is another and profound difference arising from the weak interaction time scale $\tau_w \sim 10^{-10}$ seconds. Because of the high density of neutron stars and the fact that baryons obey the Pauli principle, it is energetically favorable for nucleons at the top of the Fermi sea to convert to other baryons, including strange ones (hyperons). This is possible because strangeness is conserved only on the strong interaction time scale, not on the weak. Even the time scale of supernova is long compared to the weak. So strangeness is not conserved in astrophysical objects. It wouldn't be conserved in stable nuclei either, but energetically it is not favorable to have hyperons in the ground state. Nuclear reactions on the other hand are so fast that strangeness is conserved. So the matter studied in nuclei or their reactions has zero net strangeness, whereas neutron stars can and almost certainly do contain hyperons.

These are the differences. Of course the properties of such systems as the hot symmetric non-strange matter produced in relativistic nuclear collisions and the cold asymmetric charge neutral and strangeness carrying matter of neutron stars are related in any comprehensive theory of matter. It is through relativistic nuclear field theory that I shall make the connection between them. This theory can be generalized to incorporate nucleons and higher mass baryon states, interacting through exchange of mesons[5]. Its coupling constants can be fixed by properties of symmetric nuclear matter. It describes numerous properties of finite nuclei[6]. It can be extended to finite temperature[7]. It can be extrapolated to hot dense

matter and its composition (nucleons, hyperons, deltas, mesons)[5]. *One and the same theory* with fixed coupling constants describes:

1. Symmetric nuclear matter and the matter produced in high energy collisions when the field equations are solved subject to the constraints of *isospin symmetry and strangeness conservation*.
2. Neutron star matter when the field equations are solved subject to the constraints of *charge neutrality and generalized equilibrium*.

Therefore we are able to conveniently characterize the neutron star matter equation of state by the compression modulus of the corresponding symmetric matter, and shall always do so.

Figure 1: Families of neutron stars corresponding to equations of state with different compression modulus.

Figure 2: Relative populations in neutron star matter as a function of density [8].

Neutron stars are not pure in neutron as their name implies, and as they were first thought of. Charge neutrality is automatically respected by pure neutron matter, but this is not the lowest energy state of dense neutral matter. The reason is that as the density of neutron matter is increased, the Fermi energy of the neutron would soon equal the energy of a proton, electron and neutrino. At this point beta decay occurs. The neutrino and any gamma ray leaks out of the star, thus lowering its energy. As the density is further increased, other thresholds are reached. In a Fermi gas model the thresholds are found from the masses of the particles. In general they depend on the interactions as well. The isospin symmetry energy arising from the coupling of baryon isospin to the neutral rho meson is very important in this

respect. Obviously it favors conversion of neutrons to baryons of opposite isospin projection, consistent of course with charge neutrality. For these reasons neutron star matter is very complex and the Lagrangian used in nuclear field theory has to be generalized to include these complications[9]. Fig.2 shows the result of such a general calculation for the populations of neutron star matter. For low density, the charge neutral uniform matter is pure in neutron, but with increasing density the proton and electron in equal numbers are populated; when the electron Fermi energy increases to the muon mass then the muon as well will be populated. The pion, as we discuss later, may also condense and then at densities beginning at about three times nuclear, hyperon thresholds are reached, and with further increase in density become important components of neutron star matter. The Lagrangian employed was,

$$\begin{aligned}
\mathcal{L} = & \sum_B \bar{\psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_{\omega B} \gamma_\mu \omega^\mu - \frac{1}{2} g_{\rho B} \gamma_\mu \tau_3 \rho_3^\mu) \psi_B \\
& + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\
& - \frac{1}{4} \rho_{\mu\nu} \cdot \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_\mu \cdot \rho^\mu - \frac{1}{3} b m_n (g_\sigma \sigma_0)^3 - \frac{1}{4} c (g_\sigma \sigma_0)^4 + \dots \\
& + \sum_\lambda \bar{\psi}_\lambda (i\gamma_\mu \partial^\mu - m_\lambda) \psi_\lambda
\end{aligned} \tag{22}$$

The first line is the sum of baryon Lagrangians and the interactions with the scalar, vector and vector-isovector mesons (σ, ω, ρ). The second line contains the Lagrangians of the scalar and vector mesons, whose interactions with the baryons give rise respectively to attraction and short range repulsion. The third line contains the Lagrangian for the isovector meson which couples to the isospin of baryons and gives rise to the charge symmetry energy. This line also contains self-interaction terms of the scalar field. The last line contains the Lagrangians for the leptons (electrons and muons) which are important agents in the charge neutrality of neutron star matter. The sum over baryons is over the charge states of nucleons, deltas and hyperons. It is of course essential to keep track of the individual charge states so that charge neutrality can be enforced. The five coupling constants appearing in the Lagrangian are fixed by properties at saturation, namely the binding energy $B = 16.3$ MeV, density, $\rho_0 = 0.153 \text{ fm}^{-3}$, symmetry energy coefficient $a_{sym} = 32.5$ MeV, the effective mass at saturation, $m_{eff} \approx 0.8m$ and $K = 200 - 300$ MeV. The last quantity seems to be the main uncertainty.

I do not write down the field equations. This has been done in detail elsewhere[9]. Instead I describe what has to be done. A description of neutron star matter is obtained as the self-consistent solution to a system of coupled non-linear equations in 7+N unknowns, as follows:

- (1) three field equations for the meson fields (σ, ω, ρ)
- (2) equation for electrical neutrality (μ_e)
- (3) equation for baryon density (μ_n)

- (4) two equations for lepton Fermi momenta, (k_e, k_μ)
- (5) N equations for the Fermi momenta of N baryon species in chemical equilibrium $(k_n, k_p, k_\Lambda, \dots, k_\Xi, \dots)$.

Such a system of non-linear equations is generally difficult to solve, which probably accounts for the fact that most applications of equations of state to neutron star structure approximate the star as either pure in neutron, or else as involving beta equilibrium in the restricted sense of equilibrium among only neutrons, protons and electrons. This is not really very useful in connection with the mass constraint on the equation of state, because both are gross approximations. Two early studies, one by Pandharipande [10], and one by Bethe and Johnson [11] did include hyperons. These works are non-relativistic so the equation of state explicitly violates causality at high density, and of course must be wrong even at a lower density than the point where causality is first violated. The work of Bethe and Johnson suggests a smaller fraction of hyperons than in the work of Pandharipande. However it suffers from the fact that no control was placed on the compression and the symmetry energy, even though neutron stars are highly compressed and isospin asymmetric. The hyperon fraction that I find is of similar importance as found by Pandharipande. The advantage of the present approach is that it is relativistically covariant, and the coupling constants of the theory are related to nuclear matter properties in a way that allows one to investigate the dependance of neutron star structure on nuclear matter properties. Is there such a dependance? Although the density at the center of a neutron star may be quite high it would be incorrect to assume that properties near saturation density are not important. This is so because the center contributes little to the mass on account of the volume element. I find that the average density is only three to five or so times nuclear density, depending on how stiff or soft the equation of state is. (See Fig.3). Moreover the equation of state at higher density is connected to that near saturation through continuity and causality.

In terms of the solution for the field amplitudes, chemical potentials and Fermi momenta, the equation of state is given by,

$$\begin{aligned}
p &= -\frac{1}{3}bm_n(g_\sigma\sigma_0)^3 - \frac{1}{4}c(g_\sigma\sigma_0)^4 - \frac{1}{2}m_{\sigma_0}^2\sigma_0^2 + \frac{1}{2}m_{\omega_0}^2\omega_0^2 + \frac{1}{2}m_{\rho_{03}}^2\rho_{03}^2 \\
&\quad + \frac{1}{3}\sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_B} \frac{p^4}{\sqrt{k^2 + (m_B - g_{\sigma B}\sigma_0)^2}} dp \\
&\quad + \frac{1}{3}\sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda} \frac{p^4}{\sqrt{k^2 + m_\lambda^2}} dp \\
\epsilon &= \frac{1}{3}bm_n(g_\sigma\sigma_0)^3 + \frac{1}{4}c(g_\sigma\sigma_0)^4 + \frac{1}{2}m_{\sigma_0}^2\sigma_0^2 + \frac{1}{2}m_{\omega_0}^2\omega_0^2 + \frac{1}{2}m_{\rho_{03}}^2\rho_{03}^2 \\
&\quad + \sum_B \frac{2J_B + 1}{2\pi^2} \int_0^{k_B} \sqrt{k^2 + (m_B - g_{\sigma B}\sigma_0)^2} p^2 dp \\
&\quad + \sum_\lambda \frac{1}{\pi^2} \int_0^{k_\lambda} \sqrt{k^2 + m_\lambda^2} p^2 dp
\end{aligned} \tag{23}$$

Figure 3: Fraction of mass of neutron star resident in matter at densities greater than ρ .

Figure 4: Compares equation of state of neutron star matter with and without hyperons included in the equilibrium composition.

We illustrate in Fig.4 the equation of state in the form of $E/A \equiv \epsilon/\rho$ vs ρ for two cases, one in which only beta equilibrium between neutrons, protons and electrons is taken into account, and one in which full equilibrium between all particle species to convergence is taken into account. The latter is considerably softer than the former, for the reason that the Fermi pressure of neutrons and protons near the top of the Fermi sea is relieved by allowing them to hyperonize. The corresponding results for neutron star masses is shown in Fig.5 and we see that gravity very effectively exploits the softening at higher density introduced by hyperonization. A similar reduction in limiting mass can be found between pure neutron matter and n+p matter.

4 Pion and Kaon condensation

Pion condensation was a subject of much investigation a few years ago, especially for symmetric nuclear matter and nuclei. Pions are more likely to condense in neutron star matter. Condensation occurs in nuclear matter if the pion energy becomes degenerate with the normal state, which might happen if the interaction is attractive and strong enough. However in neutron stars, charge neutrality favors pion condensation. This is so because as a function of increasing density, neutrons at the top of the Fermi sea will decay to proton plus electron. The electrons are fermions and their Fermi level increases with further increase in density. When the electron chemical potential (Fermi energy) becomes equal to the *effective* pion mass

in the medium, it will be favorable thereafter for negative pions to play the role that the electrons had in preserving charge neutrality because they are bosons and can all condense in the lowest state. Thus while $\mu_e = \mu_n - \mu_p$ is essentially zero in symmetric matter, it is positive in neutron star matter, thus favoring the π^- since $\mu_{\pi^-} = \mu_e$. On the other hand the other charge states of the pion are excluded since $\mu_{\pi^+} = -\mu_e$ and $\mu_{\pi^0} = 0$.

When pions condense the growth of the electron chemical potential with further increase in density is arrested at a value equal to the effective pion mass. This means that the electron chemical potential cannot approach the mass of the negative kaon, which is greater than that of the pion, and so they cannot condense. Condensation of the other kaon states is even less likely for the same reason as given above for the other charge states of the pion.

Since the hyperons have charges of both signs and carry the conserved baryon charge, charge neutrality can be achieved mainly among baryons at sufficiently high density. This means that the electron chemical will initially be an increasing function of density, will saturate if and when it becomes equal to the pion effective mass, will remain essentially saturated through a range of density, and then will decrease as the hyperon populations grow. The pions at this point will be reabsorbed. So pion condensation is an intermediate density phenomenon in neutron stars. This can be seen in Fig.2. I do not find that pion condensation has a large effect on neutron star structure, an example of which can be seen in Fig.6. In this calculation the effective pion mass was taken to be the vacuum mass. This probably overestimates the role of the pion. In neutron star matter, the pion experiences a repulsive s-wave interaction which would increase its effective mass. It experiences also an attractive p-wave interaction which however it must pay for by having a finite momentum. Our earlier estimate of the effective mass due to the latter is $\sim 200 \text{ MeV}$ [12].

5 Limiting neutron star mass and the equation of state

I have shown you that neutron stars are complex in composition. Models that treat them as pure in neutron or treat only beta equilibrium among neutrons, protons and electrons are inadequate since they overestimate the limiting mass by as much as a half a solar mass. This is very large in comparison with the range in which the limiting mass can lie. For a non-interacting neutron gas, which provides a lower limit because there is no repulsive force, the limiting mass is $\sim .75 M_\odot$. At the other extreme the hardest equations of state without full equilibrium among species give $\sim 2.75 M_\odot$. So the effective range in which it can fall is about 2 solar masses, and taking into account more realistic models is less than this.

Do we know the limiting mass in nature? We do know of the existence of about 400 pulsars, but we know the masses of only seven. This is because mass measurements are possible only for binaries. Of these the most massive is 4U0900-

Figure 5: Neutron star masses compared with and without inclusion of hyperons in the equilibrium composition.

Figure 6: Neutron star masses compared with and without pion condensation.

40 for which $M = 1.85 \pm 0.3 M_{\odot}$. The most accurately measured is PSR1913+16 for which $M = 1.442 \pm .003$. With so few known masses it is unlikely that we know the most massive one. Therefore we can know only a lower limit on the maximum mass which translates to a lower limit on the stiffness of the equation of state. In Fig.7 I show calculated limiting masses from the above described theory as a function of compression K . There is some uncertainty owing to the fact that one of the saturation properties used to fix the coupling constants is the nucleon effective mass at saturation. While this is apparently known in a narrow range, there remains some uncertainty. To account for PSR1913+16 we need $K > 200$ MeV. To account for 4U0900-40 at the most probable mass we need $K > 300$ MeV or at the lower limit $K > 200$ MeV.

Of course it is possible to criticize the above results as being model dependent. Nonetheless the model contains a lot of physics, much more than parameterizations, or models based on potential interactions of nucleons which become acausal at high density, and which omit the large effect of equilibrium in the star. Recall that our theory is constrained at saturation by five nuclear properties and at all densities, but most importantly as a constraint at high densities, it is causal. These are strong constraints and should be contrasted with theories or parameterizations with fewer constraints. To emphasize this we contrast results for the linear and non-linear versions of nuclear field theory which allow respectively three and five properties to be constrained at saturation (two and four respectively for the symmetric matter equation of state). We show in Fig.8 that in the first case the theory computed in mean field approximation can be made to agree at saturation, but no where

else, with the theory which includes vacuum renormalization. In contrast, with the additional constraints (K and m^*), the mean field theory agrees within three percent up to ten times nuclear density, although the constraints apply only at saturation density! [8].

Figure 7: Limiting neutron star masses as a function of K for several choices of the value of m^* at saturation.

Figure 8: Equation of state with vacuum polarization (RHA, solid line) approximated in mean field theory with 2 parameter constraint at saturation, (B and ρ_0), and with four, (K and m^* in addition) [8].

Let us compare results with a recent work that purports to explain neutron star masses with much softer equations of state[13]. In Fig.9, results from that work are shown for the limiting mass of pure neutron stars and stars with only neutrons, protons and electrons. Essentially I agree with these results as far as they go. However the authors claim on the basis of their calculations that neutron stars can be accounted for with K as low as 120 MeV. What there results clearly show is that there is a substantial reduction in limiting mass in going from pure neutron to beta equilibrium. This is well known. There is a further reduction due to hyperonization as I have frequently emphasized which is shown by my calculation by the solid line. When all of this is taken account of, K has to be much larger than the low value they quote. This illustration is rather typical of calculations that are incomplete with respect to the space of baryon types or which are constrained by only several saturation properties.

Nonetheless no matter that we constrain the theory as best as can be presently done, both at saturation and at high density by causality, it is a fact that the connection between nuclear matter and neutron star matter can be made only through

theory. Moreover the use of K as a characterization of the stiffness of the equation of state at high density is model dependent. Within nuclear field theory K characterizes the stiffness at high density if the effective mass of the nucleon at saturation is well known, since the latter uniquely specifies the coupling constant of the vector meson which dominates the energy at high density. To facilitate the comparison of different models and parameterizations we propose reference to a standard form for the equation of state. Define a standard stiffness by an effective K^{eff} of symmetric matter which is chosen so that the standard form equals the binding energy per nucleon of the equation of state in question at $4\rho_0$. This measures the stiffness of the compression energy from saturation to $4\rho_0$ relative to the standard form,

$$E(\rho) = \frac{K^{eff}}{9\rho_0(\rho + \rho_0)}(\rho - \rho_0)^2 - 16 \text{ MeV} \quad (24)$$

This will be recognized as the average of two commonly employed parameteri-

Figure 9: Limiting neutron star mass as function of K for pure neutron matter and matter with neutron and proton from [13] compared to results when hyperons are taken account of (this work).

Figure 10: Comparison of various equations of state for symmetric nuclear matter. See Table I for further identification.

zations, one of which is asymptotically linear in ρ and the other quadratic. The suggested form here is asymptotically linear and hence causal at high density. At $4\rho_0$ it has the value

$$E(4\rho_0) = K^{eff}/5 - 16 \text{ MeV} \quad (25)$$

which by equating to any equation of state at $4\rho_0$ is solved for the effective stiffness. In Table I, we list the effective stiffness for several equations of state, and in Fig.10

we plot several of them. It should be emphasized that this effective stiffness is not any longer related to the curvature at saturation, but is rather a more global curvature spanning the density range up to $4\rho_0$. By this measure we can see that

Table 1: Effective stiffness for several equations of state

Description	K^{eff} (MeV)
This work $K = 500$ MeV, $m^*/m = 0.8$	540
This work $K = 300$ MeV, $m^*/m = 0.8$	440
This work $K = 200$ MeV, $m^*/m = 0.8$	290
This work $K = 300$ MeV, $m^*/m = 0.75$	500
This work $K = 300$ MeV, $m^*/m = 0.85$	340
Friedman and Pandharipande [14]	330
Ainsworth et al. [15]	130
ter Haar and Malfliet [16]	650
BCK [17,18] $K = 180$ MeV $\gamma = 2.5$	154

the equation of state Ainsworth et al. and of BCK (the one that is referred to in [19] as the preferred one) are very soft, and that of ter Harr and Malfliet, calculated in relativistic Dirac-Brueckner approach is very stiff.

6 Rapidly rotating neutron stars

Pulsed signals from the location of the supernova explosion of 1987 were reported in January of this year. A period of 1/2 millisecond was measured and the pulses were frequency modulated with a seven hour period, that is presumed to be due to the doppler shift caused by orbital motion of the pulsar with a companion of about a Jupiter mass. The signals, which have been attributed to a pulsar, were not seen two weeks later at next search, nor have they been seen since, either by the reporting group nor any other [20,21]. Nonetheless, because of the high quality of the data it cannot be lightly dismissed. In Fig.11 the amplitude of the frequency modulation is shown over the eight hours of observation in the laboratory frame. A skeptic who asserts that the signal is due to instrumental error, or to a signal in the observatory, would be very hard pressed to explain how it is that by taking into account the earth's rotation about its axis and its orbital motion about the sun, the laboratory data is transformed into a sinusoidal curve as would be characteristic of an emitting pulsar in a seven hour orbit. If the pulses represent rotation, it would be the fastest known pulsar. Friedman, Ipser and Parker[22] have studied the fast rotation of neutron stars based on a number of equations of state from various models, assuming uniform rotation. Very rapidly rotating stars provide an additional constraint. This can be understood from eq.(19), which says that the

maximum frequency of a star of mass M and radius R is

$$\omega \approx \sqrt{\frac{M}{R^3}} = 3.7 \times 10^5 \sqrt{\frac{M/M_\odot}{(R/km)^3}} \text{ s}^{-1} \quad (26)$$

Of course this is only a rough indication of the way the mass and radius determine the limiting frequency. The relativistic equations have to be solved for the rapidly rotating star, and these equations are much more complicated than those for the static stars that we have studied up to now. Friedman et al.[23] find from their numerical studies that as a rough rule of thumb, the above formula gives the limiting frequency of the limiting mass star when the factor 3.7 in the above formulae is replaced by 2.4. and M, R are those of the non-rotating star.

Figure 11: Shows frequency modulation of the new pulsar in SN1987A as seen in the laboratory frame and when corrected for the earth's motion (barycentric frame) [20]

The limiting mass of a rapidly rotating star is somewhat ($<20\%$) larger than for a static star, because of the additional resistance to collapse provided by the centrifugal force. If a star near the mass limit at high frequency slows down with time due to radiation damping which it will do, it will collapse to a black hole. Because of the very short period of the pulsar, so much data was collected in the eight hours of observation that a measurement of the rate of change of the period was possible. It appears that $dT/dt = 3.3 \times 10^{-15} \text{ s/s}$. At this very slow rate of change the pulsar will not collapse in our lifetime. ($1 \text{ yr} = 3 \times 10^7 \text{ s}$)

Friedman et al.[23] have studied rotating stars based on an old compilation of equations of state [24] that range from soft to stiff in the sense of how large the

Figure 12: Maximum angular velocity, Ω_K , vs M computed for a number of equation of state. The final dot in each case represents the maximum mass and angular velocity consistent with stability. [23] The angular velocity of the 1/2 millisecond pulsar is shown by the dashed line.

limiting mass star is for the eos. Their findings are summarized in Fig.12. Most models fail by a large margin to support the very high spin implied by the signals seen in January. There are two cases (B and G) that do support a high spin, but the same equations of state would not support slowly rotating pulsars at their observed mass, not even PSR1913+16, let alone the more massive star I referred to earlier and so are unsatisfactory. There are several cases for which the star barely reaches the observed frequency. Stars near the limiting frequency are expected to be unstable to non-axial deformations which would damp the star's rotation by gravitational radiation causing it to spin down rapidly. So these too, for a different reason are unsatisfactory. It may be remarked that most of the models that come close to supporting the alleged spin, violate causality at densities not much above the termination point. All of those that come close or exceed the spin of the new pulsar have central densities of 15 to 20 times ρ_0 . They are therefore being sampled in a density region where a description in terms of hadrons is no longer possible, since they would be strongly overlapping.

So as it stands, if the observed signals in January are those of a rapidly spinning star, it appears that a very stringent condition may be placed on the equation of state. However none of the models included in the study by Friedman et al are satisfactory. It also appears that none of my models will account for the high spin, though only one has been tested so far. It remains to be found what attributes of the equation of state will allow it at one and the same time to support the observed masses of slowly rotating stars and the high spin of this fast one. One possibility is discussed next.

6.1 Pure quark matter star

I mention here that I have computed a pure quark matter star consisting of u,d,s massless quarks, and a bag constant $B^{1/4} = 170$ MeV, which is half way between the value used in the MIT bag model of baryon resonances and the value which yields a phase transition in hot matter with zero baryon chemical potential at the temperature favored by lattice QCD simulations. This star has zero charge with no need for leptons. According to Witten, such a star may be absolutely stable [25]. Bethe et al. argue to the contrary[26]. Both arguments, pro and con are schematic since only perturbative models with unsatisfactory convergence are available. Maybe fast pulsars can give a hint. Using the rule of thumb quoted above for fast pulsars, the quark star has a limiting value of $\omega = 1.3 \times 10^4 \text{ s}^{-1}$ compared to the observed value of 1.237 in the above units so that it appears to be able to support slightly higher spin than that seen in the fast pulsar. It can also support masses observed for slowly rotating neutron stars, since its limiting mass is $1.5M_{\odot}$. The window for which the bag constant can produce a star of sufficiently high mass but small radius so that the double constraint can be satisfied is very small. For example $B^{1/4} = 145$ MeV produces a star with mass and radius of the limiting star of $M \approx 2M_{\odot}, R \approx 11$ km which does not satisfy the rule of thumb for high spin. On the otherhand $B^{1/4} = 200$ MeV produces a limiting star of $M \approx 1.1M_{\odot}$ so that it cannot satisfy the mass constraint. It remains to be seen whether with the obvious refinements of the model and when tested in the relativistic fast rotating star calculation, it really does support the observed spin. Also of great importance, it is not known at this time how unique this solution is to the double constraint of high spin of the fast pulsar and the observed masses of a slow pulsars.

There is an altogether different possibility however, namely that a pure quark matter star is stable against mass loss at any frequency of rotation. However it might still be unstable to non-axisymmetric modes which would damp the spin rapidly until the axisymmetric oblate deformation of the star becomes stable. Any halo of hadronic matter may have already been ejected by the rotation in the case of a fast pulsar. Stability against mass loss at high spin would then depend upon whether or how fast quarks at the surface could recombine to form hadrons. Cooling could occur by means of the fission of chromo-electric flux tubes formed as quarks occasionally cross the surface of the star in the course of their random motion [27,28]. This preferentially leads to pion emission, which however must be very slow even at temperatures of a few MeV because the escape probability is appreciable only for leading quarks with momentum greater than about 1 GeV. [27]. Emission of baryons by this process is much further inhibited by the necessity of double pair-creation of quarks in the color field. The loss of baryon number of a high-spin pure quark star is likely to be very slow therefore.

In any case it is interesting to contrast a pure quark star with a neutron star. In the latter, the hadrons are bound only by gravity and for low mass neutrons stars, near the lower limit of stability, the outer regions of the star become very diffuse because of the small gravitational attraction of the low mass. As the limiting mass is

Figure 13: Radius vs mass for neutron stars (top three curves) and pure quark matter stars (bottom three) labeled by the compression and bag constant respectively.

Figure 14: Simulation of pulses plotted modulo the period for a central slice of the period and stacked by pulse number, for $e = -0.2$, $\Omega/n = 100$, $\gamma = 0.05\pi$. Observer's orientation given by polar angle $\theta = -.35\pi$. [29]

approached neutron stars become very compact, and the outer crust and atmosphere become thin. A quark matter star on the other hand is first and foremost bound by the QCD confinement, which in the simple model is represented by the bag pressure. Gravity is what makes it dense, and therefore what possibly transforms a neutron star into a quark matter star. As a consequence the behavior of the radius with mass is very different for neutron stars and pure quark stars, as shown in Fig.13.

7 The Disappearance of the fast pulsar

We noted above the high quality of the pulsed data coming from the center of the supernova 1987A, and in particular how the transformation to the barycentric frame brought the data on the frequency modulation into sinusoidal form as would be the case for modulation due to orbital motion with a companion. However the pulsed signals were not seen again two weeks later at the time of the next search, nor have they been seen since on fourteen subsequent searches reported as of this date. The disappearance can be interpreted in a number of ways:

1. The signals were of a different origin, instrumental or other earth bound source. This seems to be refuted by the order brought into the frequency modulation by the transformation to the barycentric frame.

2. The signals are from a pulsar which has temporarily ‘disappeared’. There are several possibilities:
 - (a) The pulsar has been obscured by debris from the supernova. The debris would likely share the same angular momentum of the progenitor star as the fast pulsar, and clouds of it could have moved into the line of sight following the discovery. However with time it should grow optically thinner, and the longer the elapsed time the less likely this explanation.
 - (b) The pulsar may have a slight non-axial symmetric deformation which would cause precession, possibly rotating the magnet axis in such a way that the cone of radiation emitted around its direction no longer subtends our line of sight. We examine details of such a possibility[29].
3. The pulsar has collapsed to a black hole. According to the measured rate of change of the period, it was slowing down extremely slowly at the time of observation, and spin down to the point of loss of the additional stability provided by the high spin is unlikely in such a short time scale. We will examine another scenario which leads to rapid spindown and collapse to a black hole[30].

7.1 Cyclic appearances and disappearances of pulsars

We discuss how a very small eccentricity whose symmetry axis is inclined at an angle to the direction of the angular velocity vector will slowly rotate a pulsar about that axis, causing cyclic disappearance and reappearance of radiation beamed along a magnetic axis fixed in the star [29]. Using the observed rate of change of the millisecond period as a constraint on the eccentricity, we find that the period of the cycle for disappearances could range from several hour to the age of the universe, if no subsequent adjustment of the shape occurred.

Precession in planets and stars has been discussed previously assuming that the body is neither perfectly rigid nor fluid [31,32]. In such a case the motion is described relative to a reference system in which the elastic energy vanishes[32]. Otherwise the precession is like that of a rigid body. In the following what we refer to as the body symmetry axis should be understood more generally as the symmetry axis of the above reference system, which coincide for rigid bodies, and similarly other variables should be understood as effective variables in the reference system. From the mechanics of a rigid body possessing an axis of symmetry that makes an angle β with the angular momentum axis which is fixed in space, the body will rotate about its own symmetry axis with an angular velocity of [33],

$$n = \Omega \frac{e}{1+e} \cos \beta, \quad e \equiv \frac{I_3}{I_1} - 1 \quad (27)$$

where I_1, I_2, I_3 are the principal moments of inertia and Ω is the precession frequency of the symmetry axis about the fixed direction of the angular momentum vector.

For illustration we have taken $I_1 = I_2$. For very small 'eccentricity', e , Ω is almost equal to the angular velocity, ω , of the body. The precise relation is given by,

$$\Omega \cos \beta = \omega \frac{(1+e)}{\sqrt{1+(1+e)^2 \tan^2 \beta}} \quad (28)$$

From eq.(27) we have the inequality,

$$\left| \frac{n}{\Omega} \right| < \left| \frac{e}{1+e} \right| \quad (29)$$

If the pulsar in SN1987A, in addition to the oblate deformation caused by its high spin, has an additional non-axially deformation, there will be a time varying mass quadrupole field, and the eccentricity of the latter must be very small because of the measured $\dot{T} \approx 3.3 \times 10^{-15}$ and the short period, $T \approx 5 \times 10^{-4}$ seconds. Gravitational radiation yields a rate of change of the period according to,

$$\dot{T} = (2\pi)^4 \frac{64}{25} \frac{1}{T^3} M R^2 e^2 \quad (30)$$

in gravitational units ($G = c = 1$). For a rapidly rotating neutron star near the limits of stability, we take $M = 1.7M_\odot$, $R = 11$ km [22]. We obtain the constraint $e < 10^{-7}$. According to eq.(29) this would yield a period for the rotation of the star around its own axis of $> 6 \times 10^3$ seconds. If instead we assumed a period of a week for the rotation about the symmetry axis, then we find $e \geq 8 \times 10^{-10}$, which would make gravitational radiation completely negligible at the lower bound.

We tentatively propose that the subsequent failed searches for the signals in Sn1987A have occurred during the portions of the above cycle when the magnetic axis has been rotated too far out of our line of sight by the slow rotation about the symmetry axis. If this explanation is correct, the pulsar should reappear and disappear at cyclic intervals. The duration of the appearance need not be the same as the duration of the disappearance. This depends on the angle between the symmetry axis and the angular momentum axis, on the angle between the magnetic axis and the symmetry axis, and on the beam width of the radiation about the magnetic axis. As shown above, a period of the reappearance cycle anywhere from two hours to infinity is compatible with the observed rate of change of the precession associated with the millisecond period. (An eccentricity associated with the mass of a fly (taken as 1/10 g) situated on the surface of the star would yield a period for the rotation about the symmetry axis of $\sim 10^{23}$ years, assuming a solar mass for the star and assuming no readjustment in the shape of the star. Because of the eccentricity of the earth ($e \approx -0.003$), it rotates about its polar axis with a period of a little more than 400 days.) With respect to very long disappearance cycles we should mention the caveat that the shape is likely to make adjustments on a shorter time scale that brings the star to a different precessional motion. Starquakes (with periods of months to years) are an example of an adjustment. The new motion might never bring the beamed radiation back into view.

We show in Fig.14 an example of how the frequency is modulated by a precession such as discussed above and how the signal seen by an observer fixed in space will vary in intensity. We use parameters suitable for graphical display which leads to a ratio of periods of 100 rather than 10^9 . The intensity variation is more clearly illustrated in Fig.15. The signals from the model pulsar were computed assuming a beam of radiation whose intensity is Gaussian about the direction of the magnetic axis. This axis is taken to be inclined at an angle γ from the symmetry axis. The polar angles of the magnetic axis referred to the angular momentum direction as the z-axis, with origin at the center of the star are given by,

$$\Theta(t) = \arctan\left(\frac{\sqrt{(\sin\beta + \cos\beta \tan\gamma \cos nt)^2 + \tan^2\gamma \sin^2 nt}}{\cos\beta - \sin\beta \tan\gamma \cos nt}\right) \quad (31)$$

$$\Phi(t) = \Omega t + \arctan\left(\frac{\tan\gamma \sin nt}{\sin\beta + \cos\beta \tan\gamma \cos nt}\right) \quad (32)$$

The first term of eq.(32) shows that the *precession* frequency, and not ω , is the pulsar frequency and the second term exhibits a frequency modulation.

The frequency modulation exhibited in Fig.14 raises the question of whether the observed seven hour modulation of the millisecond pulses could be related to precession. The frequency modulation in the second term of the equation for $\Phi(t)$ is sinusoidal to high accuracy under the condition $\tan\gamma \ll \tan\beta$,

$$\Phi \approx \Omega + \left\{1 + \left(\frac{\tan\gamma \sin nt}{\sin\beta}\right)^2\right\}^{-1} n \frac{\tan\gamma}{\sin\beta} \cos nt \quad (33)$$

as for a pulsar whose signal is modulated by circular orbital motion. We check the constraints imposed on such an hypothesis for the pulsar discovered in the remnant of SN1987A. The seven hour modulation is closely sinusoidal and its amplitude is $\Delta\Omega/\Omega \approx 7.5 \times 10^{-7}$. The seven hour period requires $e > 10^{-7}/6$, which together with the constraint calculated above for the rate of change of the short period due to gravitational radiation, imposes the compatible conditions, $\frac{1}{6} < e/10^{-7} < 1$. However from eq.(7), the amplitude of the frequency modulation is given by,

$$\frac{\Delta\Omega}{\Omega} \ll \left|\frac{e}{1+e}\right| < 1 \times 10^{-7} \quad (34)$$

whereas the observed amplitude of the frequency modulation is 7 times larger than the right side. Although the general form of the modulation is difficult to analyze, it seems unlikely that the observed modulation is due to precession.

The proposition seems viable that the disappearance of the pulsar is due to the slow rotation of the magnetic axis, fixed in the star, about an axis established by a very small eccentricity offset from the angular momentum axis. It can be tested if the slow period is not too long by making observations over a sequence of nights that would not be synchronous with a periodic phenomenon. Of course the disappearance could have been occasioned by debris moving into the line of

Figure 15: For same case as previous figure, intensity of individual pulses is plotted for a hundred of the fast pulses over one period of the rotation of the star about the axis containing the non-axial deformation. [29]

sight. However this hypothesis becomes less tenable as time passes, because such supernova debris is expanding. Elsewhere we have discussed the possible extinction of the pulsar by collapse to a black hole [30].

We mention that the above model of an axially deformed pulsar gains some support from the fact that it can qualitatively simulate several features observed in the signals of many other pulsars, namely subpulse drifting, nulling, and mode switching, which is the subject of another manuscript [34].

7.2 Possible gravitational collapse into a black hole

We shall explore a scenario in which a rapidly spinning neutron star will subside into a black hole within the two week interval between first sighting and next search, due to the recapture of a small mass object ejected at the early stage of formation of the pulsar and the consequent braking of the rotation to a sub-critical value due to the gravitational radiation caused by the aspherical transport of the captured object [30]. The constraints found for the object are that it is a black hole of sub-Jupiter mass up to $\frac{1}{20}M_{Jupiter}$. Interestingly enough, since this suggestion was first made the authors of the discovery paper have improved their analysis of the data and find evidence of a second companion of mass $\approx \frac{1}{60}M_{Jupiter}$, which is close enough for our purpose[21].

The key to this scenario is the observation that a rapidly spinning neutron star can be stabilized at a mass that is greater than the limiting mass of a star with the same baryon number but lower frequency [22]. This is almost certainly the case for this millisecond pulsar as the study of Friedman, Ipser and Parker[22] suggests. Assume that following the main supernova explosion the collapsing core had angular velocity that exceeded the limit for its mass and that it shed matter in the equatorial plane, of which the observed Jupiter-mass companion is one result.

This is plausible since the companion is unlikely to have survived the explosion had it been formed earlier. Because the seven hour frequency modulation of the millisecond signals from the pulsar is rather closely sinusoidal, with only small deviations, the Jupiter-mass companion must form the bulk of material in orbit with the pulsar but it is unlikely that all of the matter expelled was so coherent as to form a single companion because of the turbulent conditions. We suppose that one additional small object, its orbit damped by gravitational radiation and perturbed by the other companion and the pulsar, has fallen back onto the surface of the neutron star near the equatorial plane from which it was first ejected, creating an aspherical transport of matter in the neutron star. Since the neutron star will have cooled substantially in the intervening two years since its birth to ~ 100 KeV temperature the dense matter is highly degenerate and the viscosity is expected to be high. The captured object is expected to remain localized for tens of years from the estimate of the viscous damping time of neutron star matter by Comins [35] as interpreted by Friedman et al. [22]. In this event the pulsar now has a time-varying mass quadrupole moment which will produce gravitational waves and damp the pulsar's rotation. We calculate how massive the recaptured object must have been so that the resulting gravitational radiation will have damped the rotation sufficiently in two weeks to bring it below the critical angular velocity that provides the marginal stability against collapse to a black hole.

For a spherical rotating star of radius R and mass M with a lump of mass m attached at its equator, the angular velocity, ω , is damped by gravity waves according to

$$\dot{\omega} = -16 \frac{m^2}{M} R^2 \omega^5 \equiv -A \omega^5 \quad (35)$$

This equation integrates to

$$\frac{1}{\omega^4} - \frac{1}{\omega_0^4} = 4A(t - t_0) \quad (36)$$

From this we find the period doubling time,

$$t_{(2)} = \frac{15}{4A} \frac{1}{\omega_0^4} \approx 2.7 \times 10^{10} \frac{M}{M_\odot} \left(\frac{M_\odot}{m} \right)^2 \frac{T_s^4}{R_{10}^2} \text{ s} \quad (37)$$

where T_s is the period of the pulsar in seconds and R_{10} is its radius in units of 10 km. Assuming that the pulsar's observed frequency is near the limit for its mass we estimate from the tables of Friedman, Ipser and Parker [22] that the mass is $\approx 1.66 M_\odot$ with a radius of $R \approx 11$ km. We find from Eq.(37) that the mass of the object must have been $m \approx 4.4 \times 10^{-5} M_\odot \approx \frac{1}{20} M_{\text{Jupiter}}$ so as to double the period in two weeks. This appears not to be unreasonably large, its mass being constrained from above qualitatively by the small degree of perturbation of the pulsar's orbit as inferred from its frequency modulation. The effects of period doubling of pulsars near the limiting frequency is drastic as can be inferred from Fig. 1 of Ref.[22] so

that the above mass is *more* than sufficient to destabilize the pulsar. We used the period doubling only as a rough but conservative estimate of the destabilization effects of spindown.

For the proposed scenario to work, the captured object must satisfy the two mass constraints stated above (sufficiently large to destabilize the pulsar within two weeks but sufficiently small compared to the Jupiter-companion as not to appreciably alter the sinusoidal frequency modulation). Also it must not be torn apart by tidal forces and it must be small compared to the neutron star so that its mass is localized after capture. There are no known astrophysical objects of $M \approx \frac{1}{20} M_{\text{Jupiter}}$ which are small compared to expected neutron star dimensions ($R = 10 - 15$ km) excepting for a small black hole. Planets and white dwarfs are much larger in size, and in any case would be destroyed by tidal forces and accreted as dust at a rate consistent with the Eddington limit. Exotic stars have been conjectured (see refs. [36,37,38] and cited references) that can fall in the desired size and mass range, but they are more likely to have been created in the early universe, if any of them exist, than in the collapse of a star. Black holes accrete background radiation (3°K) or evaporate according to whether their mass is greater or less than $\sim 10^{-7} M_\odot$, so that we have no concern that the conjectured small black hole originating in the turbulence of the supernova will have evaporated in the intervening two years.

Of course the most plausible reason for a temporary disappearance of the pulsar is that it has been obscured by a cloud of debris from the supernova which meanwhile has moved into the line of sight because of its angular velocity. This conjecture becomes less plausible with time because if nothing else happens to the pulsar itself, then it will reappear as the cloud becomes less opaque due to its expansion or as the cloud is carried away by its angular velocity. In any case the scenario proposed above shows that whether or not the newly born pulsar has subsided into a black hole by now, it is in peril of doing so promptly following the aspherical capture of a object having mass of the above order.

Whether "last gasp" signals emitted just prior to disappearance of a neutron star into a black hole could be observed depends to a large degree on the brevity of the final collapse, since a concentrated pulse of neutrinos or gravitational radiation is more easily detected than a long one carrying the same energy.

If this scenario does indeed describe the fate of the young pulsar so fleetingly glimpsed in January, then we have witnessed a remarkable sequence of events that is not likely to occur again for many generations, a spectacular supernova display whose equal has not occurred in this part of the universe since the Crab supernova 900 years ago, the birth of a neutron star at its center, followed soon after by its disappearance into a black hole.

8 Supernovae

Supernovae involve a number of factors of comparable importance but high uncertainty. Among the factors in this connection we note:

(1). Stars in the range $8 < M/M_{\odot} < 100$ which evolve to be the progenitors for type II supernovae, cook for 10^7 years, evolving from a hydrogen gas to layers of heavier elements and eventually an iron core. The mass and entropy of the core are crucial but highly uncertain because of the large network of nuclear reactions and their rates that must be simulated, some of them known and others highly controversial. The results of the simulation of such an evolution provide the initial conditions for the simulation of the collapse. The presupernova evolution calculations yield iron core masses in the range $\sim 1.3 - 2.5 M_{\odot}$ [39].

(2). When stability is lost at a point corresponding to approximately a Chandrasekhar mass of iron core, collapse proceeds on the millisecond time scale.

(3). When the core reaches supernuclear density a shock originates at a radius that is *interior* to the iron core at a point that includes about $1/2 M_{\odot}$. It must propagate outward through the remaining iron core and expel most of the stars mass beyond the core, $> 8 M_{\odot}$. Otherwise no supernova and neutron star; instead a black hole.

(4). The shock suffers severe energy loss as it propagates through the infalling overlaying core to the mantle that it must expel. The losses are due to the fact that the shock front dissociates nuclei. The energy loss is easy to calculate. For each $1/10 M_{\odot}$ of nuclei dissociated ($B \sim 10$ MeV/nucleon) the energy dissipated is $\sim A/10 \times B = 10^{57}/10 \times 10 \text{ MeV} = 1.6 \times 10^{51} \text{ ergs}$. This is typically of the same order as the entire energy of the supernova explosion. For example SN1987A is believed to have exploded with an energy $1 - 3 \times 10^{51} \text{ ergs}$.

(5). The lower limit on the iron core mass found in published stellar evolution calculations is $\sim 1.27 M_{\odot}$ [40]. This is marginally within the domain of possible success of the explosion. Obviously from the above points, the smaller this mass the more favorable for the success of the prompt shock mechanism. Recent work has therefore focussed on attempts to make plausible a lower core mass.

(6). Neutrino physics is as yet highly uncertain and is treated differently by the various groups doing supernova simulations. Some early claims to success were later understood to be mistaken, by failing to account for neutrino losses of the core. The excess of electron neutrinos provided an unrealistic pressure boost to the shock [1].

(7). It had been claimed by the Stony Brook - Brookhaven group that if the equation of state is sufficiently soft then supernovae with iron cores as massive as $1.35 M_{\odot}$ can be successfully simulated [18,19]. This has been widely cited as evidence that the equation of state must be soft. The claim has floundered on two important issues: (i) The equation of state that was used is too soft to support the masses of several known neutron stars [41] (Ref. [18] seems to suggest, erroneously as it turns out, that it will do so). (ii) Corrections to the handling of the neutrino transport, whose importance was emphasized by Bludman [1] is now conceded to have played such an important role in the previously claimed success for the prompt explosion of $1.35 M_{\odot}$ iron core models that it is now agreed that no matter how soft the equation of state, such models cannot be made to explode promptly [3,4].

We show in Fig.16 that the equation of state used in the Stony Brook-Brookhaven simulations is too soft to support known neutron star masses. In the one case we use a Z/A ratio of $1/3$ which is on the average appropriate for the matter of the collapsing material. Later as the neutron star is formed further neutronization occurs because it leads to a lower energy state and hence softer equation of state. Therefore we show also the neutron star masses for a Z/A ratio more appropriate to the evolved neutron star matter, and of course the limiting mass is even less. The appropriate lepton contributions to pressure and energy are included in both cases. In neither case are the observed neutron star masses supported by this equation of state. Effectively the explosion energy in the supernova simulations has been bought at the cost of neutron star mass. It might be claimed that the BCK equation of state is intended for use only at lower densities than in the core of a neutron star. However in one of the models (# 40) reported in ref. [18] the maximum central density achieved just before bounce is $12\rho_0$ so in fact it was used at very high density by its authors. It might also be claimed that it stiffens at high density for some reason or other. However new physical effects will come into play at high density only if they are energetically favorable because physical systems arrange themselves so as to find the lowest possible energy. Of course any processes that lower the energy also soften the equation of state. Examples of this are the neutronization and hyperonization whose softening effect on the equation of state were discussed earlier. I recently read another formulation of the same idea by Bethe in discussing dense matter. "The results of these considerations are that the repulsive nuclear loops saturate, but the attractive forces increase rapidly in magnitude.... These behaviors follow the general rule that repulsive interactions tend to screen themselves so as to cut down the repulsion, while attractive interactions do not." [42]

The foregoing is the story as regards supernovae and the equation of state as of the published literature to this date. Recent developments for the reasons discussed above have focussed on making a case for smaller iron cores in the lighter presupernova stars. This will reduce the energy losses of the shock wave and presumably will also relieve the need for very soft equations of state. Reaction rates, some not well known, are crucial to the presupernova evolution scenarios. Of particular importance is the $C^{12} + \alpha \rightarrow \gamma + O^{16}$ reaction which has been measured at Muenster and Cal. Tech. They are difficult experiments and they are not in complete agreement. Moreover the accelerator experiments have to be extrapolated to thermal energies, a highly uncertain matter. A particular extrapolation of Brown and Woosley [43] yields iron cores $1 - 1.1M_\odot$ down from the $\sim 1.3 - 1.4M_\odot$ of the earlier evolution scenarios [39]. If there is a class of progenitors that have such low iron core masses, they clearly become possible candidates for the prompt explosion mechanism. However it should be noted that the neutron stars that would be produced are of very low mass. Although it may be argued that neutron stars with masses as large as that of 4U0900-40 with $M = 1.85 \pm 0.3M_\odot$ may have acquired their high mass after initial formation through accretion from a companion, this cannot be claimed for the neutron star in PSR1913+16, since this system, studied over years, is very well

Figure 16: For the equation of state used in the supernova simulations neutron star masses are shown for two values of Z/A . The smaller corresponds more closely to that prevalent in neutron stars. In neither case can the BCK equation of state support stars of observed mass. [41]

understood in terms of general relativity without mass loss or accretion[44]. It is unlikely therefore that such a scenario with such light iron cores can account for anything but a subclass of supernova.

In connection with the last point we mention the late-time neutrino reheating mechanism of Wilson [45,46,47]. The shock typically stalls at ~ 100 km because of severe energy losses as discussed above and turns into an accretion shock. Wilson found that it is possible for such shocks to be revived on a long time scale by reheating due to absorption of neutrino's diffusing from the proto-neutron star as it further neutronizes. The scenario appears to work for a broad range of iron core masses and with no strong constraints on the equation of state.

Conclusions for supernovae:

1. It has not been established that supernovae provide a constraint on the equation of state as was frequently quoted. There are many other factors involved of comparable importance but high uncertainty.
2. It is not presently understood why stars explode.

9 Summary

We have reviewed the prospects for obtaining constraints on the equation of state from astrophysical sources. Neutron star masses although few are known at present, provide a very direct constraint in as much as the connection to the equation of state involves only the assumption that Einstein's general theory of relativity is correct at the macroscopic scale. This is almost as secure as Maxwell's equations. If

the millisecond pulses briefly observed in the remnant of SN1987A can be attributed to uniform rotation of a pulsar, then a very severe constraint is placed on the equation of state. The theory again is very secure. The precise nature of the constraint is not yet understood, but it appears that the equation of state must be neither too soft nor stiff, and it may be that there is information not only on the stiffness of the equation of state but on its shape. Supernovae simulations involve such a plethora of physical processes including those involved in the evolution of the precollapse configuration, not all of them known or understood, that they provide no constraint at the present time. Not even the broad category of mechanism for the explosion is agreed upon (prompt shock, delayed shock, or nuclear explosion).

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