Submitted to
Physics Letters B

LBL-4308
Preprint

MODIFIED THOMAS-FERMI APPROXIMATION FOR MOMENTUM-DEPENDENT INTERACTIONS

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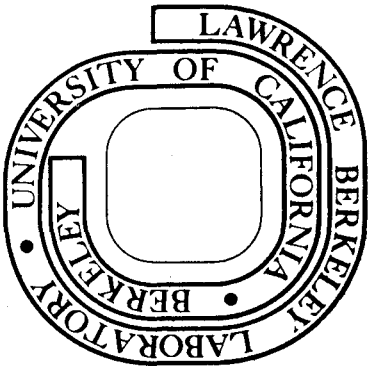
September 1975

Prepared for the U.S. Energy Research and
Development Administration under Contract W-7405-ENG-48



DISCLAIMER

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MODIFIED THOMAS-FERMI APPROXIMATION FOR MOMENTUM-DEPENDENT INTERACTIONS*

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ABSTRACT

A modification of the standard Thomas-Fermi approximation is suggested for the treatment of many-particle models with interactions having a quadratic momentum dependence. A numerical illustration is made for the nuclear surface.

In this letter is suggested a modification of the standard Thomas-Fermi approximation for the treatment of interactions having a quadratic momentum dependence. While of general theoretical nature this subject has particular interest to nuclear physics, where numerical studies are carried out with phenomenological momentum-dependent interactions.

Suppose, in a many-particle system, a two-body interaction is present which depends quadratically on the relative momentum $\vec{p}_{12}$. Let the spatial dependence be given by the function $g(\vec{r}_{12})$, where $\vec{r}_{12}$ is the particle separation. For the following discussion the momentum-independent part of the interaction can be disregarded. Such interaction may then have the form $\frac{1}{2}(p^2 g + g p^2)$, or $\vec{p} g \vec{p}$, or a linear combination of these two forms. This group constitutes the class of interactions to which the suggested modification applies.

The standard Thomas-Fermi treatment is briefly recalled. Suppose that the local Fermi momentum $P_F(\vec{r})$ is a known function of position. At each point the particles are assumed to be distributed uniformly within the available momentum sphere. This implies that the local density $\rho(\vec{r})$ is given by

$$\rho(\vec{r}) = \frac{t}{h^3}\int^{P_F(\vec{r})} d^3\vec{p} = \frac{4\pi}{3}\, t\, \frac{P_F(\vec{r})^3}{h^3}$$

where t denotes the degeneracy in each single-particle orbit. The (only) kinetic density is given by

$$\gamma(\vec{r}) = \frac{t}{h^3}\int^{P_F(\vec{r})} p^2 d^3\vec{p} = \frac{3}{5}\, P_F(\vec{r})^2\, \rho(\vec{r})$$

Any two-body interaction of the form $\vec{p}^2 g(\vec{r})$ gives rise to the same effective static potential $V = p^2 \mathcal{R}(\vec{r}) + \mathcal{G}(\vec{r})$ irrespective of the particlar algebraic

form of the interaction. Here the functions $\mathcal{R}$ and $\mathcal{G}$ are obtained by folding ρ and γ by the spatial function g. Furthermore, the associated interaction energy density is given by $v = \frac{1}{2}(\gamma\mathcal{R} + \rho\mathcal{G})$.

Thus, in the standard Thomas-Fermi treatment, no distinction is made between the *different* interactions mentioned above. However, at least in the nuclear case, different algebraic forms may lead to significantly different results for the self-consistent many-particle solution. Therefore, it is desirable to have available, within the simple Thomas-Fermi framework, a tool for the general analysis of such models.

The modification suggested may be formulated as follows. Any of the momentum-dependent interactions considered can be rewritten in the general form

$$W_{12}(\eta) = \tfrac{1}{2}\{\vec{p}_{12}, \tfrac{1}{2}\{\vec{p}_{12}, g_{12}\}\} - \eta\,\tfrac{1}{2}[\vec{p}_{12}, \tfrac{1}{2}[\vec{p}_{12}, g_{12}]]$$

where $g_{12} = g(\vec{r}_{12})$. The square brackets denote the commutator and the curly brackets the anti-commutator. The parameter η is a real number characterizing a given particular interaction. For example, the first of the two forms mentioned above corresponds to $\eta = -1$ and the second one corresponds to $\eta = 1$.

The double-anti-commutator algebraic form has been exhibited because this particular form tends to average out the undulations in the density distributions due to the quantum phase correlations in the single-particle wave functions. The presumption is now made that the Thomas-Fermi approximation, which completely neglects phase correlations, is most accurate for the quantities represented in terms of the double-anti-commutator form. The modification suggested thus amounts to adding to the standard Thomas-Fermi formulas the contribution arising from the remaining η-dependent term.

Consequently, the effective single-particle potential V takes on the form $V_\eta = V_0 + \eta\delta V$, where V_0 is the effective potential given by the standard Thomas-Fermi approximation. The modification potential (which is momentum independent) is given by

$$\delta V(\vec{r}) = \hbar^2 \Delta \mathcal{R}(\vec{r})$$

where the function $\mathcal{R}$ is obtained from the density distribution ρ by folding with the spatial part of the interaction, g.

Correspondingly, the interaction-energy density v takes on the form $v_\eta = v_0 + \eta\delta v$, when v_0 is the standard Thomas-Fermi result and the modification term is given by

$$\delta v = \tfrac{1}{2}\hbar^2 \tfrac{1}{4}(\rho\, \Delta\mathcal{R} - 2\vec{\nabla}\rho \cdot \vec{\nabla}\mathcal{R} + \Delta\rho\, \mathcal{R})$$

It is noted that in regions of constant ρ and $\mathcal{R}$ there is no modification.

As an illustration, the case of the nuclear surface is considered. For this, a Seyler-Blanchard [1] type model is used,

$$W_{12}^{SB} = -C\, g\left(\frac{r_{12}}{a}\right)\left(1 - \frac{p_{12}^2}{b^2}\right)$$

where $g(r) = \exp(-r)/r$. Following Myers and Swiatecki [2], the three parameters entering are taken as a = 0.62567 fm, b = 372.48 MeV/c, and C = 328.61 MeV. Numerical calculations have been carried out for the family of interactions corresponding to the above form.

The self-consistent solution was calculated by use of modified Thomas-Fermi approximation, as a function of the parameter η. Figures 1 and 2 display the results for the surface diffuseness and the surface energy. Furthermore, "exact" values were obtained by use of the

Hartree approximation, which maintains the independence of the particles while treating the quantum mechanics in an exact manner.

It is seen that the general η dependence of the results, as given by the Hartree values, is well reproduced in the modified Thomas-Fermi approximation. The systematic underestimation by the Thomas-Fermi approximation can be mainly accounted for by the lack of a density tail and the neglect of phase correlations near the surface.

When judging the value of the modification it should be borne in mind that without the modification all the different Hartree results would be approximated by the Thomas-Fermi results obtained for $\eta = 0$.

The suggested modified Thomas-Fermi approximation is currently being employed in a study of the nuclear surface [3].

The author is indebted to W. J. Swiatecki for many inspiring discussions on the subject.

FOOTNOTES AND REFERENCES

*Work performed under the auspices of the U. S. Energy Research and Development Administration

†On leave from the University of Aarhus, Aarhus, Denmark.

[1] R. G. Seyler and C. H. Blanchard, Phys. Rev. 124 (1961) 227; 131 (1963) 355.

[2] W. D. Myers and W. J. Swiatecki, Ann. Phys. 55 (1969) 395.

[3] J. Randrup, Lawrence Berkeley Laboratory Preprint LBL-4302, to be published.

FIGURE CAPTIONS

Fig. 1. Surface diffuseness. As a function of the parameter η is plotted the surface diffuseness for the self-consistent solution obtained with the modified Thomas-Fermi approximation. The quantities displayed are the 10-90% diffuseness, t_{10-90}, and the second surface moment b. The crosses indicate results obtained in the Hartree approximation.

Fig. 2. Surface energy. As a function of the parameter η is plotted the surface energy coefficient a_s for the self-consistent solution obtained with the modified Thomas-Fermi approximation. The crosses indicate results obtained in the Hartree approximation.

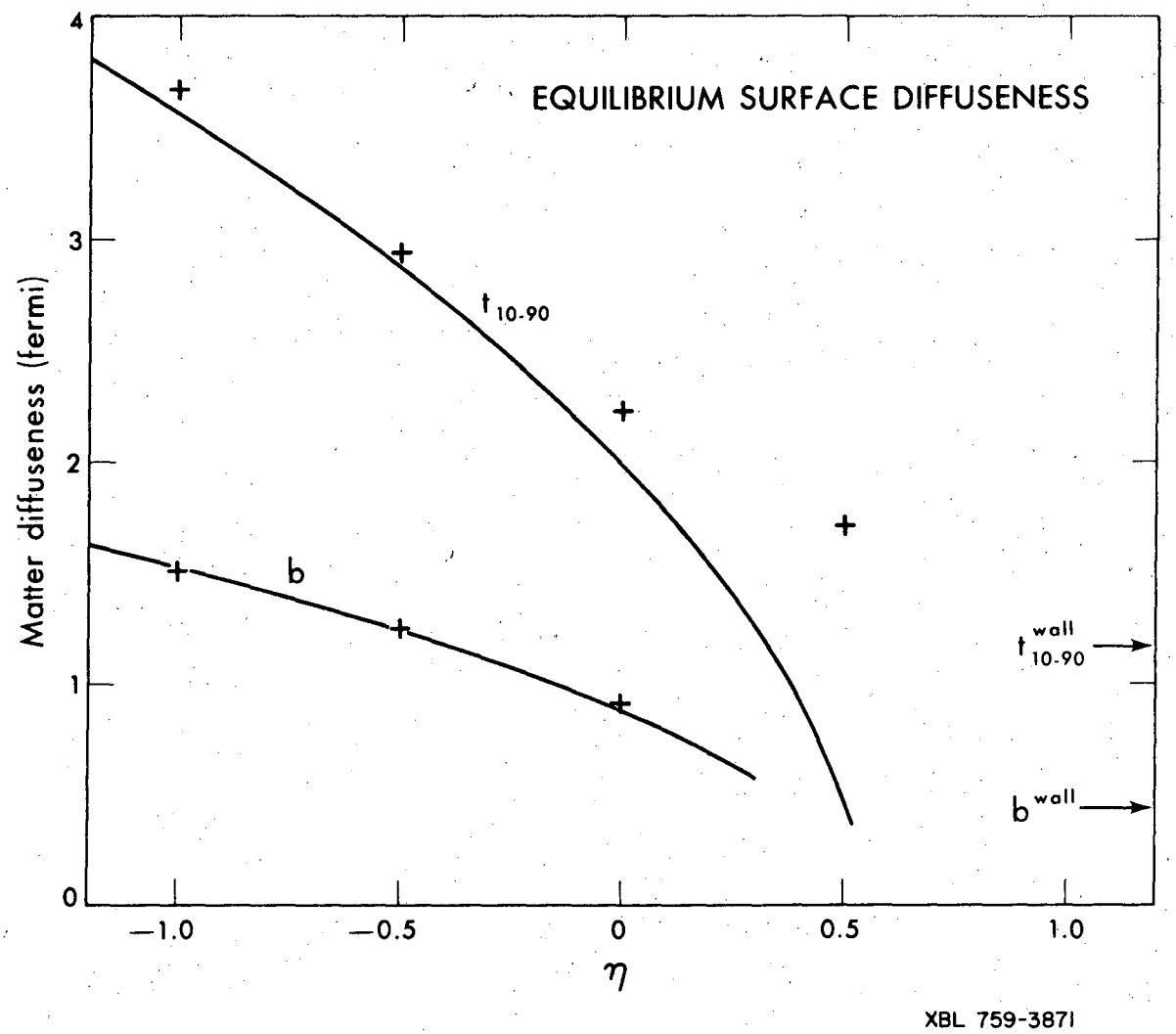

Fig. 1

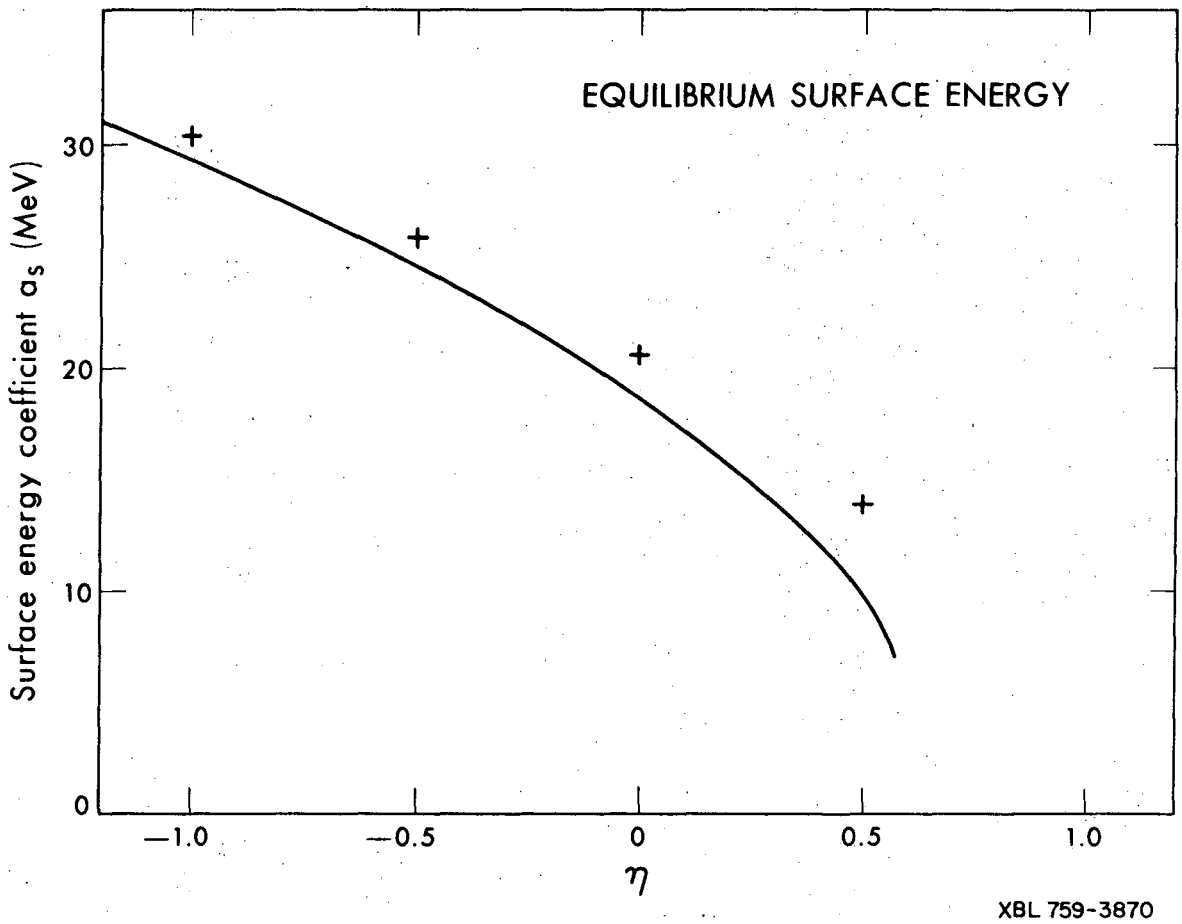
EQUILIBRIUM SURFACE ENERGY
Surface energy coefficient a_s (MeV)
30
20
10
0
−1.0
−0.5
0
0.5
1.0
η
XBL 759-3870

Fig. 2

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