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ABSTRACT

Polynomial expressions of nucleon-nucleon elastic- and inelastic-scattering cross sections are presented for energies up to 6.2 Bev.

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A large body of experimental data is available on nucleon-nucleon scattering at various energies.[1,2] This information has been used to calculate the effective cross sections for collisions between nucleons in nuclear matter, on the assumption that these data can be directly applied to such process inside the nucleus.[3] For convenience in machine computation we have expressed the experimental data in the form of power-series functions of the energy and the cosine of the center-of-mass scattering angle. We present these expressions in the expectation that they will be useful to others. Only nuclear scattering is included. The contribution of Coulomb scattering has been subtracted.

The values of n-p scattering cross sections below 10 Mev were taken from the compilation by Hughes and Schwartz.[1] The values for nuclear p-p scattering below 10 Mev were calculated with the aid of scattering theory.[4] Nucleon-nucleon scattering data above 10 Mev were taken from a review article by Hess and from references listed there.[2]

Elastic differential data for n-p collisions are given there only to 580 Mev. We, therefore, made the usual assumption that the n-p interaction consists of equal contributions of states with total isotopic spin $T = 0$ and $T = 1$. The contribution of the state with $T = 1$, as obtained from differential p-p scattering, was subtracted. The resulting values for $T = 0$ were extrapolated to higher energies with the aid of total elastic n-p and p-p cross sections from Hess.[2] The elastic n-p differential cross sections at these higher energies were taken to be the average of the $T = 0$ and $T = 1$ data, the latter from p-p scattering experiments.

The elastic differential cross sections were expressed as power series in $\cos \Theta$, the scattering angle in the center-of-mass system. The coefficients of the terms in these expressions were in turn fitted by the method of least squares to a power series of the energy, E. In order to do this conveniently, the data were divided into several energy intervals up to 6.2 Bev. These expressions, given in Table I to IV, agree with nearly all the experimental values within the quoted experimental errors.

The inelastic data were treated in a different fashion because of their relative complexity. We fitted only the values of the total inelastic cross sections and the meson multiplicities by power series in E, again by the method of least squares; see Tables V to VII. The production of four or more mesons was considered to be negligible at 6.2 Bev and below.

The comparison of the expressions given in Table I to VII with the available data revealed only one discrepancy, a value of 35.3 mb for the elastic n-p cross section at 175 Mev instead of approximately 46.2 mb from several experiments in this energy region. At both 150 Mev and 200 Mev the experimental and calculated cross sections agreed within the experimental uncertainty.

We made only a rough evaluation of the angular distribution of the interaction products of inelastic collisions. For E less than 0.6 m_oc^2 we took $d\sigma/d\Omega$ of the pion to be proportional to $1 + 3\cos^2\theta$. For all other inelastic cases this expression was used for the nucleon differential cross section.

The expressions in the following tables were used to calculate effective nucleon-nucleon collision cross sections in nuclear matter.[3] In these tables E is in units of m_oc^2 (939 Mev). The cross sections (σ) are in millibarns and the differential cross sections ($d\sigma/d\Omega$) in millibarns per steradian.

The calculated values of the elastic and total cross sections (from these expressions) are shown in Figs. 1 and 2 as solid lines. The experimental values of the elastic cross sections are shown as solid circles and those for the total cross sections as solid squares. Some recent data, not used in the evaluation of Tables I to VII, are also included in Figs. 1 and 2. The n-n values (open circles in Fig. 1) are the difference between n-d and n-p total cross sections.[2] The p-n values (open circles in Fig. 2) are the difference between p-d and p-p total cross sections.[2]

Table I

Cross sections of elastic p-p collisions that are isotropic in the center-of-mass system, $\sigma = 4\pi\, d\sigma/d\Omega$

Range of E (m_oc^2)	Equation for differential cross section (mb/sterad)	
$< 2 \times 10^{-4}$	$d\sigma/d\Omega = 2804 - 1578 \times 10^{4}\, E + 3.80 \times 10^{10}\, E^2$	(1)
2×10^{-4} to 0.0150	$d\sigma/d\Omega = -4.3 + 0.3776\, E^{-1} - 2.961 \times 10^{-5}\, E^{-2}$	(2)
0.0150 to 0.135	$d\sigma/d\Omega = 0.63 + 0.1705\, E^{-1} + 1.878 \times 10^{-3}\, E^{-2}$	(3)

Table II

Cross sections of anisotropic elastic p-p collisions, $\sigma = 2\pi \int_0^{\pi} (d\sigma/d\Omega) \sin\Theta\, d\Theta$

Range of E (m_oc^2)	Equation for differential cross section (mb/sterad)	
0.135 to 0.600	$d\sigma/d\Omega = a + b \cos^4\Theta$	(4)
	$a = 1.949 - 0.327\, E$	(5)
	$b = 9.1\, E^5$	(6)
0.600 to 1.065	$d\sigma/d\Omega = a + b \cos^4\Theta$	(7)
	$a = -20.97 + 90.52\, E - 117.25\, E^2 + 48.02\, E^3$	(8)
	$b = 150.03 - 578.33\, E + 734.67\, E^2 - 299.46\, E^3$	(9)
1.065 to 6.55	$d\sigma/d\Omega = c \cos^N \Theta$	(10)
	$c = -29.22 + 53.99\, E - 21.765\, E^2 + 3.537\, E^3 - 0.19672\, E^4$	(11)
	$"N" = -9.68 + 16.89\, E - 4.348\, E^2 + 0.4469\, E^3$	(12)
	(Take N = integer nearest the value of "N" calculated. Also take $\lvert\cos^N\Theta\rvert$ for N an odd integer.)	
> 6.55	$d\sigma/d\Omega = 23.1 \cos^{50}\Theta$	(13)

Table III

Cross sections of elastic n-p collisions that are isotropic in the center-of-mass system, $\sigma = 4\pi\, d\sigma/d\Omega$

Range of E ($m_o c^2$)	Equation for differential cross section (mb/sterad)	
$< 2 \times 10^{-4}$	$d\sigma/d\Omega = 1620 - 79.2 \times 10^{5} E + 1.89 \times 10^{10} E^{2}$	(14)
2×10^{-4} to 0.0150	$d\sigma/d\Omega = -3400 E + 78.93 + 0.3757 E^{-1} - 9.977 \times 10^{-5} E^{-2} + 1.0608 \times 10^{-8} E^{-3}$	(15)

Table IV

Cross sections of anisotropic elastic n-p collisions,

$$\sigma = 2\pi \int_0^{\pi} (d\sigma/d\Omega)\,\sin\theta\, d\theta \quad = 2\pi \int_1^{-1} \frac{d\sigma}{d\Omega}(-d\cos\theta)$$

Range of E ($m_o c^2$)	Equation for differential cross section (mb/sterad)	
0.0150 to 0.200	$d\sigma/d\Omega = t + u\cos^2\theta + v\cos^{14}\theta$, for $-1.0 < \cos\theta < 0$	(16)
	$d\sigma/d\Omega = p + q\cos^4\theta$, for $0 < \cos\theta < 1.0$	(17)
	$t = 2.10 - 0.1624\,E^{-1} + 0.04011\,E^{-2} - 4.0 \times 10^{-4}\,E^{-3}$	(18)
	$u = 1.57 + 251.7\,E - 5021\,E^2 + 41192\,E^3 - 1.162 \times 10^5\,E^4$	(19)
	(when u is negative, at $E \to 0.200$, set $u = 0$)	
	$v = -41.0\,E + 2978\,E^2 - 33953\,E^3 + 108909\,E^4$	(20)
	(when v is negative, at $E \to 0.0150$, set $v = 0$)	
	$p = -0.90 + 0.377\,E^{-1} + 0.0184\,E^{-2} - 1.77 \times 10^{-4}\,E^{-3}$	(21)
	$q = 182.9\,E - 1408.8\,E^2 + 3066.2\,E^3$	(22)
0.200 to 0.620	$d\sigma/d\Omega = w + x\cos^4\theta + y\cos^{14}\theta + z\cos^{100}\theta$, for $-1.0 < \cos\theta < 0$	(23)
	$d\sigma/d\Omega = r + s\cos^2\theta$, for $0 < \cos\theta < 1.0$	(24)
	$w = -0.50 + 18.26\,E - 41.64\,E^2 + 25.08\,E^3$	(25)
	$x = 37.42 - 227.38\,E + 512.62\,E^2 - 380.37\,E^3$	(26)
	$y = -4.42 + 35.43\,E - 78.62\,E^2 + 59.52\,E^3$	(27)
	$z = 4.0$	(28)
	$r = 1.32 + 3.95\,E - 6.71\,E^2$	(29)
	$s = 4.19 + 3.34\,E - 44.86\,E^2 + 62.18\,E^3$	(30)
0.620 to 1.065	$d\sigma/d\Omega = a + b\cos^4\theta$	(31)
	$a = -20.84 + 88.88\,E - 114.82\,E^2 + 47.08\,E^3$	(32)
	$b = 149.74 - 562.82\,E + 704.13\,E^2 - 285.13\,E^3$	(33)
1.065 to 6.55	$d\sigma/d\Omega = c\cos^N\theta$	(34)
	$c = -11.37 + 26.603\,E - 11.6091\,E^2 + 2.01675\,E^3 - 0.119009\,E^4$	(35)
	$"N" = -9.68 + 16.89\,E - 4.348\,E^2 + 0.4469\,E^3$	(36)
	(Take N = integer nearest value of "N" calculated. Also take $\lvert\cos^N\theta\rvert$ for N an odd integer.)	
> 6.55	$d\sigma/d\Omega = 13.0\cos^{50}\theta$	(37)

Table V

Inelastic p-p cross sections		
Range of E (m_oc^2)	Equation for inelastic cross section[a] (mb)	
< 0.35	$\sigma = 0$	(38)
0.35 to 1.00	$\sigma = 0.128 + 11.992\,(E-.35) + 133.892\,(E-.35)^2 - 130.305\,(E-.35)^3$	(39)
1.00 to 2.00	$\sigma = 32.97 - 3.70\,E$	(40)
> 2.00	$\sigma = 26.0$	(41)

[a]For both p-p and n-p inelastic collisions, we have taken,

$E < 0.600$	$(d\sigma/d\Omega)_{pion} = (1+3\cos^2\Theta)\,\sigma/8\pi$	(42)
$E > 0.600$	$(d\sigma/d\Omega)_{nucleon} = (1+3\cos^2\Theta)\,\sigma/8\pi$	(43)

Table VI

Inelastic n-p cross sections[a]		
Range of E (m_oc^2)	Equation for inelastic cross section (mb)	
< 0.35	$\sigma = 0$	(44)
0.35 to 1.60	$\sigma = 0.077 + 4.557\,(E-.35) + 104.895\,(E-.35)^2 - 153.955\,(E-.35)^3 + 65.561(E-.35)^4$	(45)
1.60 to 3.50	$\sigma = 19.55 + 10.04\,E - 2.344\,E^2$	(46)
> 3.50	$\sigma = 26.0$	(47)

[a]See footnote to Table V.

Table VII

The fraction, F, of inelastic p-p and n-p collisions resulting in the production of 1, 2, and 3 pions[a]

Range of E (m_oc^2)	Equation for F	
< 1.0	$F_1 = F\ (1\ \text{pion}) = 1.0$	(48)
> 1.0	$F_1 = 0.103 + 1.424\ E^{-1} - 0.527\ E^{-2}$	(49)
1.0 to 1.6	$F_2 = F\ (2\ \text{pions}) = 1 - F_1 = 0.897 - 1.424\ E^{-1} + 0.527\ E^{-2}$	(50)
< 1.6	$F_3 = F\ (3\ \text{pions}) = 0$	(51)
> 1.6	$F_3 = -\ 0.200 + 0.136\ E - 0.0068\ E^2$	(52)
	$F_2 = 1 - F_1 - F_3$	
	$= 1.097 - 1.424\ E^{-1} + 0.527\ E^{-2} - 0.136\ E + 0.0068\ E^2$	(53)

[a]Based on experiments at 6.2 Bev and below.

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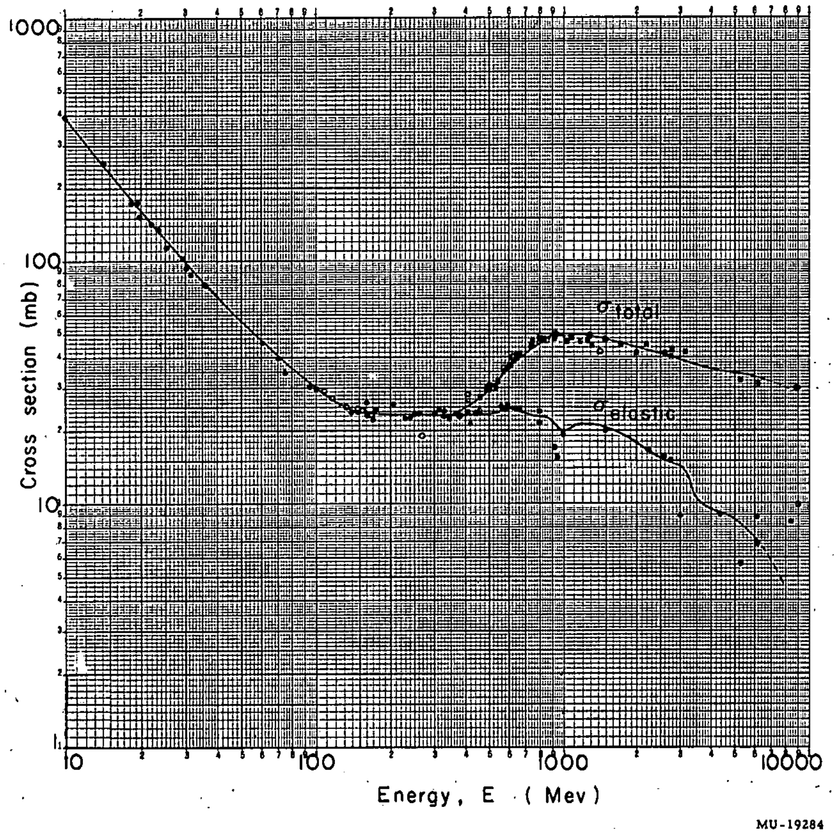

Fig. 1. The values of the elastic and total p-p and n-n cross sections. The values given by the solid line were calculated from the expressions given in Tables I, II, and V. The experimental values of the elastic p-p cross sections are shown as solid circles and those for the total p-p cross sections as solid squares. The n-n values, indicated by open circles, are the difference between n-d and n-p total cross sections.

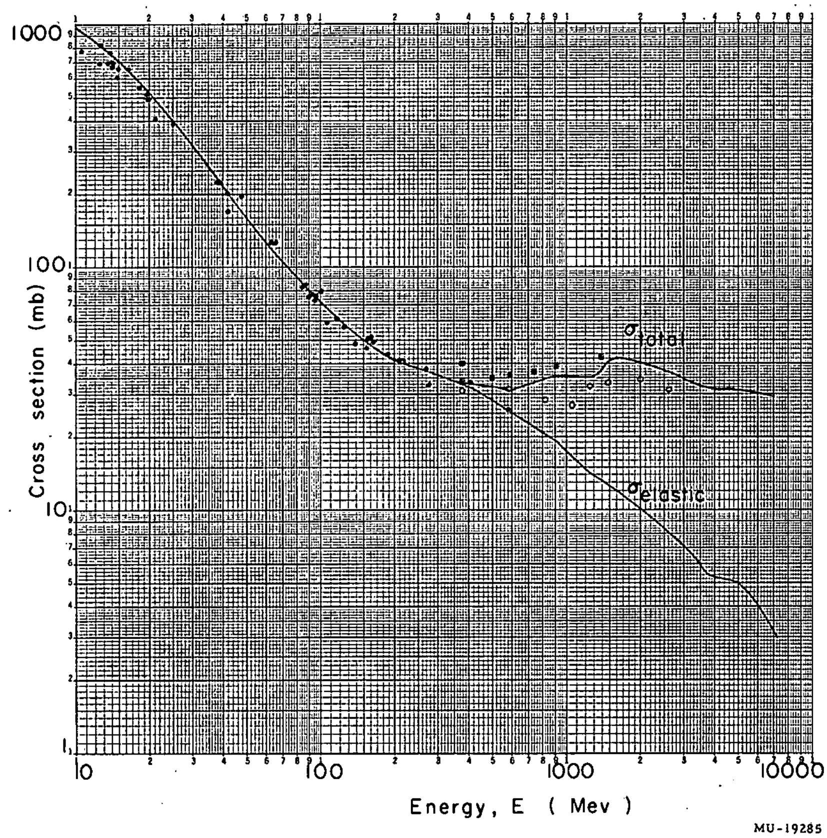

Fig. 2. The values of the elastic and total n-p and p-n cross sections. The values given by the solid line were calculated from the expressions given in Tables III, IV, and VI. The experimental values of the elastic n-p cross sections are shown as solid circles and those for the total n-p cross sections as solid squares. The p-n values, indicated by open circles, are the difference between p-d and p-p total cross sections.