Conceptual recoding of new ideas during and after solution of an insight problem

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Abstract

Despite progress in understanding the sources of difficulty in solving insight problems, how new ideas are discovered, implemented, and learned is poorly understood. We report an experiment testing a theory of how individuals use failed attempts to discover new ideas. We compared performance on the nine-dot problem with a variant requiring solution using three lines rather than four. Results supported predictions that the three-line variant is easier than the four-line, and that transfer of solution knowledge from the three- to the four-line version is facilitative, but not vice-versa. Additionally, varying the spacing between dots facilitated discovery and transfer of solutions in both variants. Our theory specifies a priority order for seeking new ideas that offers a partial solution to the frame problem. Individuals first seek ideas from the problem statement and attempts they make. Only when these sources fail do they resort to searching memory or the external task environment.

Keywords: Insight; problem-solving; conceptual recoding

Introduction

One of the features of problems that are often described as requiring insight to solve is that, although they are usually easy to state and lie within the competence of those tasked with solving them, solution rates are typically very low. One example of such a problem is the nine-dot problem (Maier, 1930). The problem, illustrated in Fig. 1, is to cancel each of nine dots arranged in a three-by-three square-shaped grid, by drawing four continuous straight lines without retracing lines or lifting the pen from the page between lines. The problem can be described as knowledge-lean; its instructions provide all the information necessary for problem-solving. The number of operators made available in the problem statement is small: draw lines that cancel dots. Solvers rarely execute more esoteric solution strategies (e.g., tearing up the paper on which the problem array is drawn).

Despite its simple statement and restricted range of possible operations, empirical studies have found very low solution rates, typically less than 5% after 10 minutes of solving (e.g., Chein, Weisberg, Streeter, & Kwok, 2010; Chronicle, Ormerod, & MacGregor, 2001; Lung & Dominowski, 1985; Kershaw and Ohlsson, 2003; MacGregor, Ormerod & Chronicle, 2001; Öllinger, Jones & Knoblich, 2014; Scheerer, 1963; Weisberg & Alba, 1981a, 1981b).

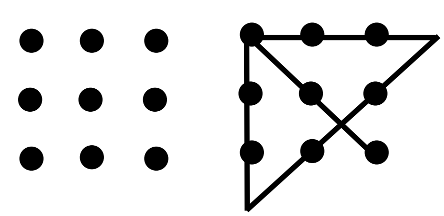

Figure 1: The nine-dot problem and its solution.

A Gestalt explanation of the problem's difficulty (e.g., Wertheimer, 1954/1959) is that the dot array forms an implicit boundary around the dots, which solvers are reluctant to violate. Indeed, it has been suggested that the problem contributes the phrase "think outside the box" to popular culture (Adair, 2007). Weisberg & Alba (1981a, 1981b) showed that the Gestalt explanation of the problem's difficulty cannot be sufficient since an instruction to draw lines outside the array did not greatly facilitate solution. Despite this evidence, the idea that the problem presentation imposes perceptual constraints on the moves that solvers will sample persists (e.g., Öllinger, Jones & Knoblich, 2014). It may, after all, be the case that a constraint imposed by an unconscious perceptual process cannot be relaxed by a verbal instruction to draw lines outside the perceived square. That said, in Weisberg and Alba's studies, participants did draw such lines and yet were often unable to solve.

A different view was taken by MacGregor, Ormerod, and Chronicle (2001). They showed that solution difficulty across a range of nine-dot problem variants can be accounted for by a simple model of search for lines that maximise apparent progress. Moves are selected if they cancel enough dots to meet a criterion of satisfactory progress consisting of the number of remaining dots divided by the number of remaining lines. No other sources of constraint on move selection are proposed in their account. When solvers fail to find moves that make sufficient progress, criterion failure occurs, and new moves are sought under an expanded problem space.

Although sources of difficulty with the nine-dot problem have been explored extensively, less attention has been paid to the question of where individuals can find ideas that do lead to solution. Arguably, this issue pervades insight problem-solving research: Little is known about how individuals know where to seek new ideas when they are stuck. Knoblich, Ohlsson, Haider, and Reinhus's (1999) Representational Change Theory (RCT) suggests that the experience of failure leads individuals to relax the constraints on move selection imposed by perception or by prior knowledge, which allows for new solution ideas to be found. However, the theory remains underspecified in terms of *where* a solver should look next. This issue is important, since the range of new ideas that might be applied to any problem is, in principle, infinite. MacGregor et al.'s (2001) Criterion of Satisfactory Progress (CSP) theory is similarly lacking in providing an account of how and where individuals expand the problem space to seek new solution ideas.

A new theory of idea discovery in insight

We propose a new account of how individuals discover new solution ideas in attempting to solve the nine-dot problem, with the focus of idea discovery being on processing previous failed solution attempts to discover concepts that might allow new move attempts to be made.

Our contention is that solvers are highly conservative in the ideas they are prepared to consider: They will always try to minimise the space of possible ideas that must be explored. The knowledge-lean nature of the nine-dot problem means that the problem statement offers a narrow range of possible sources of ideas. The theory suggests that, once ideas inherent in the problem statement are exhausted, the next place that individuals look is in their failed attempts. This contrasts with theories such as RCT, in which the source of new ideas is the activation of prior knowledge that is made possible once constraints on accessing that knowledge are relaxed.

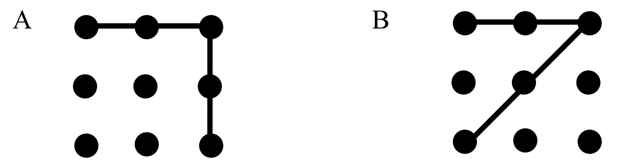

Figure 2. Alternative attempts at drawing the first two lines in solving the standard nine-dot problem

Consider, for example, the two attempts shown in Figure 2, each consisting of the first two lines drawn in trying to solve the standard problem. Neither attempt lies on the solution path, but comparison of the two attempts yields new information: Both cancel 5 dots and use two lines, but they differ in that the angle between lines is different (90 vs 45 degrees) and the second attempt has line lengths that are different to those in the first attempt ($\sqrt{2}$ between dots for the diagonal second line in Fig 2B, vs 1 unit between each cancelled dot for all other lines). Our theory suggests that, once impasse is reached (i.e., solvers fail to find any solutions based on their initial representation of the problem) they seek new solution ideas by comparing (either consciously or unconsciously) the properties of their previous attempts to extract properties that can be varied in subsequent attempts. Once discovered, these properties are conceptually recoded as variables that can be applied to the discovery of new moves. In the example shown in Figure 2, the properties of varying angle and line length can be applied by the solver to discover new move attempts. For example, knowing that lines can be longer than the one-unit gap between dots horizontally and vertically sets up the possibility that lines can extend along these axes independently of the dots themselves.

Our theory generates predictions regarding factors that affect solutions to variants of the nine-dot problem. Essentially, the more problem variants differ in the properties that can be extracted from their initial solution attempts, the more ideas can be found, so the easier they will be to solve. Similar effects of the positive impacts of problem presentation variability on solution rates are reported by Ross and Vallée-Tourangeau (2021).

Another area of interest in insight problem-solving concerns transfer of knowledge between analogous insight problems. For example, Knoblich et al. (1999) point out that, in the case of matchstick algebra problems, once an insight has been achieved, transfer of that insight to new matchstick algebra problems will likely be entirely facilitative, because the gain of new knowledge is germane to problems that have both superficial and conceptual similarity. This level of success works because of the presence of the key components of superficial and conceptual similarity. In the absence of either or both, as Barnett, and Ceci (2002) point out, transfer between problems and domains is very rare. One way in which transfer can be facilitated is through manipulations that impact both knowledge and strategy. For example, Ormerod and MacGregor (2017) found that, when participants were given an explicit strategic cue to the reasons for failure of progress-maximising moves in solving the nine-ball problem, transfer was facilitated for distant analogs. Because of its focus on discovery of new solution ideas, our theory allows predictions regarding transfer between variants. Problems that allow attempts having properties relevant to the solution of alternative variants will promote positive transfer.

We tested these general predictions in an experiment that compared performance on the standard four-line version of the nine-dot problem with a variant in which the problem must be solved using only three straight lines (see Fig.3). The three-line variant has properties that differentiate it from the four-line version. First, adopting the notion of criterion failure from MacGregor et al.'s (2001) CSP theory, criterion failure is encountered after the first line drawn that adheres to a direct line between dots. Criterion failure does not necessarily occur in the four-line version until three lines have been drawn. Criterion failure triggers the search for alternative solution ideas, which in our theory occurs by comparing the properties of previous attempts. Thus, the three-line version ought to be solved more readily than the four-line version because this comparison process is initiated earlier.

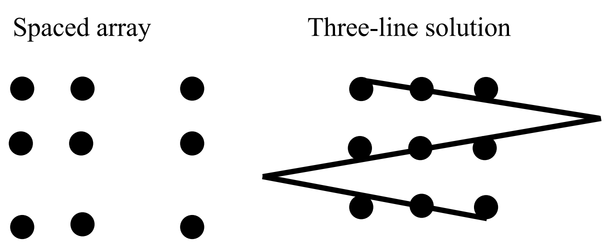

Figure 3. Spaced problem array and three-line solution

Second, the three-line solution has properties that are relevant to the solution of the four-line problem, notably lines of greater length than the problem array and acute angles between lines. These properties are more likely to be discovered in attempts to solve the three-line variant because criterion failure happens earlier. Thus, we predict more transfer from a three-line to a four-line problem than vice versa.

A further prediction is that, if individuals seek solution ideas from the problem at hand rather than relying on prior knowledge or the external environment, then manipulating the problem array to make move properties more salient should increase solution rates and transfer. To test this prediction, we varied the spacing between dots in the array. This manipulation makes salient the property that line lengths vary, which should facilitate discovery of lines that extend more than one unit (where the space between horizontal and vertical lines of dots is one unit). Thus, solution rates to spaced problems should be higher than for standard spaced problems, and experience with spaced source problems should yield higher solution rates on transfer problems.

To summarise, the experiment reported below tested the following hypotheses:

1. The three-line variant should be solved more often than the four-line variant as a source problem, because criterion failure (i.e., a failure to find moves that cancel enough dots given the number of remaining lines) occurs earlier in the former than the latter, triggering a search for new ideas earlier.

2. Because criterion failure occurs earlier with the three-line than the four-line source problem, there are more opportunities to discover move properties in the former than the latter. Thus, there should be more positive transfer from a three-line source to a four-line transfer problem than vice versa.

3. The addition of extra spaces between dots in the source problem arrays should facilitate solution of both three-line and four-line source problems, and transfer between variants, because the extra space serves as a source of information concerning the variability of line lengths between dots.

Experiment

Method

Participants A total of 136 adults attending a series of student recruitment lectures at the University of Sussex took part on a voluntary basis, comprising 50 males (M_{age} = 26.1 years, SD_{age} = 1.9) and 86 females (M_{age} = 25.5 years, SD_{age} = 2.1).

Materials and design Participants were assigned to either 3-line or 4-line groups, and within each group were further assigned to regular or spaced array variants. The spaced source problem and the three-line solution are illustrated in Fig.1. The array for each problem was presented 10 times on a sheet of paper to allow multiple solution attempts. Participants who received the 3-line source received a regular 4-line transfer problem, and participants receiving the 4-line source received a regular 3-line transfer problem.

Procedure Participants solved problems individually in groups of approximately 30 people. To reduce the likelihood of collusion, participants sitting adjacent to each other were assigned to different experimental groups. At the start of the lecture, participants were each given a booklet containing an ethical consent form, two nine-dot variant problems, and a study debrief sheet. Participants attempted to solve the source problem, drawing solution attempts on the displays given on the sheet. Where participants indicated they had a solution, it was checked by a researcher and, if incorrect, participants were instructed to continue solving. After 10 minutes, participants were told to turn to the next page in the booklet, where a correct solution was revealed. Participants then turned to a blank page and put the booklets aside for 30 minutes while the lecture continued. They then attempted the target problem for a further 10 minutes. Finally, participants were debriefed as to the purpose of the study.

Results

The number of correct solutions to source and target problems is shown in Table 1. To test the hypotheses concerning the relative ease of 3- and 4-line problems and effects of varying the spacing in the array, a binary logistic regression was conducted using Source (3-line, 4-line) and Array (regular, spaced) and the interaction between these factors as predictors. The analysis yielded a significant model, $\chi^2(3, N = 136) = 11.35$, $p = .01$, with Source, Wald = 7.63, $p < .006$, as a significant predictor in the model. A total of 24/69 (35%) solved the 3-line variant compared with 9/67 (12%) solving the 4-line variant. The effect of Array, Wald = 2.37, $p = .124$, and the interaction between Source and Array, Wald = 0.02, $p = .898$, were not significant.

To test the hypothesis that attempting a 3-line source would yield positive transfer to a 4-line target but not vice versa, a binary logistic regression was conducted using Problem (3-line, 4-line), Order (first or second problem attempted by participants) and Array (regular, spaced) and interactions between these factors as predictors. The analysis yielded a significant model, $\chi^2(7, \mathrm{N} = 136) = 20.36$, $p < .005$, with Order, Wald = 6.76, $p = .009$, Array, Wald = 3.99, $p = .046$, and the interaction between Problem and Order, Wald = 7.89, $p = .005$, as significant predictors in the model. For the 4-line variant, solution rates were significantly higher when it was solved as the target problem (29/69 – 42%) than as the source problem (9/67 – 12%). In contrast, solution rates for the 3-line variant were not higher when it was the target problem (22/67 – 33%) compared with the source problem (24/69 – 35%).

Table 1. Number of participants solving regular and spaced variants of 3- and 4-line source and target problems (% in brackets)

		Source	Target
3-source, 4-target	Regular	9/33 (27)	11/33 (33)
	Spaced	15/36 (41)	18/36 (50)
4-source, 3-target	Regular	3/33 (9)	10/33 (30)
	Spaced	6/34 (18)	12/34 (35)

Problems with a spaced array (51/140 – 36%) were solved more often than problems having a regular array (33/132 – 25%). The effect of Problem, Wald = 2.34, $p = .126$, and interactions between Problem and Array, Wald = 0.252, $p = .616$, Array and Order, Wald = .175, $p = .676$, and Problem, Array and Order, Wald = 0.090, $p = .675$, were not significant.

Discussion

The experiment tested predictions derived from a new theory of idea generation during insight problem-solving, concerning the initial difficulty of nine-dot variants, transfer of solution-relevant knowledge, and effects of manipulating the array to add discoverable information about line lengths. Analysis of source solution data confirmed the prediction regarding initial problem difficulty, showing that the 3-line variant is easier than the 4-line variant. This finding is consistent with the prediction that early criterion failure, which happens immediately after the 1st line in the 3-line variant but later in the 4-line variant, triggers a search for alternative solution ideas.

In terms of transfer of solution knowledge, attempting to solve the 3-line variant facilitated solution of the 4-line variant, but not vice-versa. Properties discovered during solution attempts can be used to create solution-relevant moves for the 4-line transfer problem. In contrast, a 4-line source problem provides fewer solution ideas for solving the 3-line transfer problem. Manipulating the problem array in the source problem increased solution rates overall, problems having a spaced array being solved more often than problems having a regular array.

The results are consistent with our theory and not readily explained by theories, such as RCT, that posit relaxation of knowledge constraints, or by CSP that suggests the removal of a need to maximise move value. Manipulations of source problem and array do not remove 'constraints' imposed by prior knowledge (e.g., that lines must end on dot points or that lines cannot be drawn at acute angles). Indeed, if these constraints were the source of problem difficulty, then one might expect the three-line version to be more difficult than the four-line version, since it requires both non-dot turns and highly acute angles between lines. Moreover, if the discovery or presentation of a solution to the source problem removes these constraints, then there should be no difference in solution rates for the three-line or four-line target problem. Nor do the manipulations affect the need for apparent progress towards solution: Nothing in the three-line version suggests that letting go of an impeller to maximise progress will lead to more solutions.

We suggest that solvers are not relaxing constraints or reducing progress demands; instead, they are discovering new ideas. In the case of nine-dot variants, these new ideas cannot come from prior knowledge. Indeed, attempts to retrieve a solution to the nine-dot problem from memory often fail, even when individuals who have previously solved remember that the solution requires acute angles, lines that end on non-dot points, and lines extending beyond the dot array (Ormerod, Fioratou, Chronicle, & MacGregor, 2006). Instead, the results indicate that new ideas for solving nine-dot variants come from the problem-solving experience; or in the case of analogous problems, from a recent solving episode for a simpler variant (e.g., the 3-line problem).

An alternative explanation for our results rests in the original Gestalt proposal that problem difficulty is mediated by the perceptual organisation of the problem array. It may, for example, be that the three-line variant is easier than the four-line variant because the provision of only three lines overrides the square concept, by equating the problem array as three lines of dots onto which the lines can map rather than as a square. The provision of four lines may reinforce the square concept, the lines offering a 'border' to the square. Similarly, the effect of adding spaces between some dots in the problem array may weaken the perceptual integrity of the square, thereby making the drawing of lines beyond the dots easier to execute. However, it is notable that, with three-and four-line variants in both regular and spaced array formats, moves whose lines remained within the dot array still dominated attempts to solve. This suggests that manipulating line number and dot spacing did not remove a perceptual constraint, if indeed one was ever in place.

The conceptual recoding of promising states has wider applications in problem-solving, particularly in learning and transfer (and failures thereof). Ormerod et al. (2006) analyzed attempts to re-solve the nine-dot problem from participants who reported having previously solved it. Sixteen out of 40 participants failed to re-solve within two 30s attempts, and 11 failed to solve within 10 attempts. Of the 16 participants who failed to solve within two trials, 13 made attempts going outside the dot array, suggesting they recalled the insight (i.e., they applied one of the possible conceptually recoded properties) but were unable to capitalise upon it. It appears that solving the nine-dot problem generates conceptually recoded properties, but their retrieval does not guarantee re-solution.

Ormerod et al. (2006) also compared performance on the six-coin problem (Chronicle et al., 2004) with a variant arranged in a Y shape. The latter was chosen because, in pilot testing, participants commented that the solution seemed "to close in on itself", "like a crab's claw" or "a pincer". No such verbal descriptions were given for the standard problem. Initial solution rates did not differ significantly (25% for the standard problem vs 35% for the Y shape). However, when the problems were re-presented, the Y shape (74%) was re-solved more often than the standard problem (47%), and the difference increased when problems were rotated through 90 degrees (88% vs 25%, respectively). Ormerod et al. suggested three ways in which solution knowledge can be recoded: Perceptually, procedurally or conceptually. The solution to the Y shape problem allows conceptual recoding ("like a crab's claw"), whereas the standard problem solution is amenable only to perceptual and procedural recoding, rendering solution knowledge vulnerable to changes in presentation.

Here we have examined the generation of solutions to a knowledge-lean problem. It remains to be seen whether the same ideas can apply to knowledge-rich problems, that is, problems that necessarily require accessing information either from memory or from the external task environment. Fundamental to the idea we present here is the notion of conservative or minimal expansion of the space of possible ideas. We suggest that progress in understanding how knowledge-rich problems are solved may benefit from applying a similar heuristic. Do not think outside the box: Instead, expand the box in a controlled way, based on the discovery of properties cued by previous attempts to solve that provide properties, and which enable new moves to be sampled.

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