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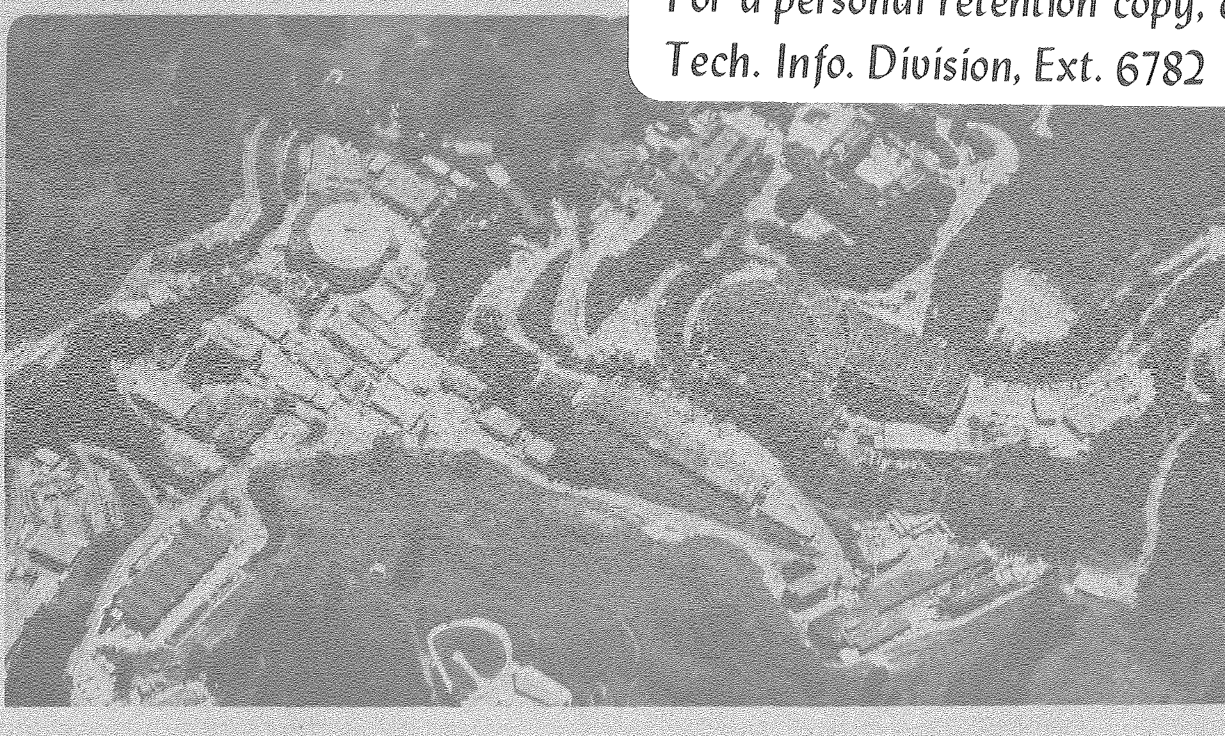

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LARGE CORRECTIONS TO HIGH p_T HADRON-HADRON SCATTERING IN QCD[*]

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ABSTRACT

We have computed the first non-trivial QCD corrections to the quark-quark scattering process which contributes to the production of hadrons at large p_T in hadron-hadron collisions. Using quark distribution functions defined in deep inelastic scattering and fragmentation functions defined in one particle inclusive e^+e^- annihilation, we find that the corrections are large. This implies that QCD perturbation theory may **not** be reliable for large p_T hadron physics.

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1. INTRODUCTION

The demonstration that the gauge theory of colored quarks and gluons is the correct theory of the strong interactions requires extensive calculation of radiative corrections. Quark and gluon radiation can induce qualitatively new phenomena. In addition, it is only by computing the higher order corrections to a given process that one can test the validity of the lower order approximation. At asymptotic energies we are assured by the asymptotic freedom of the theory[1] that the coupling constant $\alpha_s(Q^2)$ becomes logarithmically small with Q^2, the scale size of the hard interaction. However, at present sub-asymptotic energies these radiative corrections could be numerically important.

In the present paper we make a first approach to the problem of the calculation of one hadron inclusive hadron-hadron scattering beyond the leading order. In the Born approximation this process has been considered by several authors, and an extensive phenomenology has been developed and compared with data[2]. The order α_s corrections to the Born diagrams which have been calculated in the leading logarithmic approximation[3] give results in agreement with the factorization theorem for the mass singularities[4]. In this paper we calculate contributions of order α_s^3 to the parton-parton cross section, retaining both the logarithmic pieces containing mass singularities and the non-logarithmic pieces. We consider only the quark-quark scattering contribution to the one-hadron inclusive cross-section. In a particular kinematic regime, gluons would not contribute significantly because of the softness of their hadronization properties. In principle, this would allow us to separate the problem of the size of the corrections to the quark-quark scattering cross-section from the problem of the definition of the gluon distribution beyond the leading order. It also dramatically reduces the number of diagrams which we have to calculate. The radiative corrections to this process are interesting for several reasons. The radiative corrections to the Drell-Yan process have been found to be large at present energies[5]. This result shifts the burden of proof; the parton modeller is now obliged to show that higher order corrections are small if he wishes to use the lowest order parton formula. One hadron inclusive hadron-hadron scattering is also interesting because unlike the processes previously considered it involves more than one large scale which characterizes the hard scattering. By calculating the higher order corrections we can determine which choice of scale minimizes the higher order corrections.

Our operating procedure is as follows. Following refs. 5, 6 we define in section 2 quark distribution functions beyond the leading order in terms of F_2, the structure function of deep inelastic scattering. We define quark fragmentation functions beyond the leading order in terms of the transverse part of one particle inclusive e^+e^- annihilation. Thus armed with these definitions the problems of analyzing the finite corrections to the one hadron inclusive cross section is well posed (apart from the definition of the gluon distribution function which we consider later). In section 3 we make use of this formalism in the analysis of one hadron inclusive hadron-hadron scattering. The actual perturbation calculation is discussed in section 4A (elastic graphs) and section 4B (inelastic graphs). In sections 5 and 6, we display the result of our calculations and discuss its consequences. Certain details of the calculation are given in the appendices.

2. FORMALISM

In the simple parton model[7] the inclusive cross section for one hadron production at large transverse momentum

$$H_1(P_1) + H_2(P_2) \rightarrow H_3(P_3) + \text{all}$$

is given by the formula[8]:

$$E_3 \frac{d^3\sigma}{d^3P_3} = \sum_{i,j,k} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 \frac{dx_3}{x_3^2} \mathscr{D}_{ok}^{H_3}(x_3) \left(p_3^o \frac{d^3\sigma_{ij}^k}{d^3p_3} \right) \Bigg|_{\substack{p_1 = x_1P_1 \\ p_2 = x_2P_2 \\ p_3 = P_3/x_3}} \mathscr{F}_{oi}^{H_1}(x_1) \mathscr{F}_{oj}^{H_2}(x_2) \tag{1}$$

This equation, illustrated in Fig. 1, describes the production of a hadron at large transverse momentum in terms of distribution functions $\mathscr{F}_{oi}^{H_1}(x_1)$ of the i^{th} type of parton inside the hadron H_1 and the fragmentation function $\mathscr{D}_{ok}^{H_3}(x_3)$ giving the distribution of a hadron H_3 in the decay products of the k^{th} type of parton. The indices i,j,k run over gluons (G) and f flavors of quarks and antiquarks (1...2f). The hadronic cross-section is thus expressed in terms of the rescaled parton cross-sections for all the combinations of incoming and outgoing partons. For example in the Born approximation to the hard scattering cross-section, the contributing graphs are given in Fig. 2.

The calculation of the parton cross-sections beyond the leading order in α_s yields large logarithms associated with mass singularities. These large logarithms, which at first sight appear to destroy the convergence of the perturbation series, are universal properties of the parton legs independent of the particular process and hence can be factored out of the parton cross-section and into the distribution functions.[4] After factorization of the mass-singularities, no small mass scales remain in the parton cross-section. The strength of the interaction is therefore controlled by the running coupling constant evaluated at a large scale associated with the hard interaction which we denote by Q^2. The asymptotic freedom of the theory assures us that, at infinite energy, the process is dominated by the Born approxiamtion.

At sub-asymptotic energies a calculation beyond the leading logarithmic approximation can supply answers to the following questions:

1. Which combination of the hadronic variables is the best choice for the scale Q^2, controlling the fall-off of the running coupling constant in the hadronic interaction? The best choice is deemed to be that choice which minimizes the correction term. Such a choice would presumably lead to a more rapidly convergent perturbation series.

2. Which combination of the hadronic variables is the best choice for the factorization scale M^2 (effectively the scale size at which the parton distribution and decay functions are evaluated?)

3. Are the corrections large or small for all choices of Q^2 and M^2?

It is clear from Eq. (1) that, even in the Born approximation, we are dealing with a matrix problem of some complexity. In the next order the number of graphs to be evaluated escalates, and each graph contributes to at least two different inclusive cross-sections both of which have to be calculated.

The present investigation limits itself to a (gauge invariant) subset of these graphs. We consider only the radiative corrections to the parton process

$$q_i + q_j \to q_k + \text{anything} \tag{2}$$

(that is collisions of two quarks of distinct flavors $i \neq j$, where either $k = i$ or $k = j$). Whilst it is in principle possible to choose a kinematic configuration in which this subprocess gives the dominant contribution because of the harder hadronization properties of valence quarks, our main interest in the present paper is the investigation of the magnitude of the corrections.

Expediency is not the only rationale for only considering quark-quark scattering graphs. Experience from the Drell-Yan process indicates that large corrections arise because of the emission of soft radiation from a subprocess occurring in the Born cross-section. Moreover, as will be explained in more detail below, our relative ignorance of the gluon distribution vitiates any attempt to interpret the results of a calculation involving initial gluons.

To illustrate the factorization algorithm and to introduce our definition of the distribution and fragmentation functions beyond the leading order, we turn now to deep inelastic scattering. The process

$$\gamma^*(q) + H_1(P) \to \text{anything}$$

is described by a series of structure functions $F_i(x,t)$, where

$$x = \frac{Q^2}{2P \cdot q}, \quad Q^2 = |q|^2, \quad t = \ln Q^2/\mu^2 .$$

The Bjorken scaling variable is denoted by x and μ is an arbitrary scale of mass. In the simple parton model, the structure function F_2 is given by,

$$F_2(x)/x = \sum_{i=1..2f} e_i^2 \mathscr{F}^H_{oi}(x) \qquad (3)$$

where the sum runs over all flavors of quarks and antiquarks.

When corrections of order α_s are included, the naive parton model formula is modified as follows

$$F_2^H(x,t)/x = \int_o^1 dy \int_o^1 dz\ \delta(zy - x)$$

$$\times \sum_{i=1..2f} e_i^2 \sum_{j=G,1..2f} \left[\delta_{ij}\delta(z-1) + \frac{\alpha_s}{2\pi} t P_{ij}(z) + \alpha_s f_{ij}(z)\right] \mathscr{F}^H_{oj}(y) \tag{4}$$

where the $P_{ij}(z)$ are the Altarelli-Parisi functions[9].

Our definition of the parton densities beyond the leading order is the requirement that in terms of the "renormalized" scale dependent parton densities $\mathscr{F}^H_j(x,t)$, the form of Eq. (3) is preserved with no corrections in order α_s

$$F_2(x,t)/x = \sum_{i=1..2f} e_i^2 \mathscr{F}^H_i(x,t) \quad . \tag{5}$$

Equation (5) implies that, in this order in perturbation theory, the relationship between bare and "renormalized" quark densities is given by (i=(1...2f))

$$\mathscr{F}^H_i(x,t) = \mathscr{F}^H_{oi}(x,t) + \int_x^1 \frac{dy}{y} \left\{ \sum_{j=1..2f} \left[\frac{\alpha_s}{2\pi} t \delta_{ij} P_{qq}(\frac{x}{y}) + \alpha_s \delta_{ij} f_{qq}(\frac{x}{y})\right] \mathscr{F}^H_{oj}(y) \right.$$

$$\left. + \left[\frac{\alpha_s}{2\pi} P_{iG}(\frac{x}{y}) + \alpha_s f_{iG}(\frac{x}{y})\right] \mathscr{F}^H_{oG}(y) \right\} \tag{6}$$

where for clarity we have separated the contributions of the quarks and gluons.

Defining the moments of the quark distribution functions

$$\mathscr{F}_i^{(n)}(t) = \int_o^1 dx\, x^{n-1} \mathscr{F}_i(x,t) \qquad (7)$$

we may write the generalization of eq. (6) which contains all terms in the expansion in $(\alpha_s t)$ $(i = 1..2f)$.

$$\mathscr{F}_i^{(n)}(t) = \sum_{j,k=G,1..2f} (\delta_{ij} + \alpha_s f_{ij}^{(n)}) \left[T \exp \int_{\alpha_s(o)}^{\alpha_s(t)} d\alpha \, \frac{\gamma_{jk}^{(n)}(\alpha)}{\beta(\alpha)} \right] \mathscr{F}_{ok}^{(n)} \quad . \qquad (8)$$

In eq. (8), γ is the lowest order expansion of the standard anomalous dimensions (moments of the fuctions P). Thus the quark distribution function is completely specified.

For our limited purposes the only function which we need to know is the one relevant for "non-singlet" quark distributions. We choose to regulate the mass singularities in the relevant diagrams by continuing the number of space-time dimensions n [10,11]. In this notation, the function f_{qq} (see eq. 6) is given by:

$$\alpha_s f_{qq}(z) = \frac{\alpha_s}{2\pi} C_F \left\{ (1 + z^2)\left(\frac{\ell n(1-z)}{1-z}\right)_+ - \frac{3}{2}\frac{1}{(1-z)_+} - \frac{1+z^2}{1-z}\ell n\, z \right.$$

$$\left. + 3 + 2z - \left(\frac{9}{2} + \frac{\pi^2}{3}\right)\delta(1-z) \right\} + \frac{\alpha_s}{2\pi} P_{qq}(z)\left(-\frac{1}{\varepsilon} + \gamma_E - \ell n 4\pi\right) \qquad (9)$$

where

$$P_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right] \qquad (10)$$

In these equations, C_F is the quadratic Casimir operator for the fundamental (quark) representation of the SU(N) color group. For the specific case of SU(3), $C_F = \frac{4}{3}$ (in general, $C_F = \frac{N^2-1}{2N}$). The expansion factor of the dimensional regularization is $\varepsilon = \frac{4-n}{2}$, and γ_E is Euler's constant. The distributions denoted by the plus subscript are discussed in Appendix A.*

The definition of fragmentation functions beyond the leading order follows an entirely analogous procedure[6]. We define the fragmentation functions beyond the leading order in terms of the transverse part of one hadron inclusive e^+e^- annihilation.

In the one photon approximation, (neglecting for simplicity effects due to weak interactions), the cross-section differential in angle and energy for the reaction ($e^+e^- \to H + \text{all}$) with unpolarized beams can be written in the general form:

$$\sigma^H(z,\cos\theta,t) = \frac{3}{8}(1+\cos^2\theta)\sigma_T^H(z,t) + \frac{3}{4}(1-\cos^2\theta)\sigma_L^H(z,t) \qquad (11)$$

---

* Using dimensional regularization of the mass singularities, we obtain:

$$\frac{\alpha_s}{2\pi}tP_{qq}(x) + \alpha_s f_{qq}(z) = -\frac{1}{\varepsilon}\frac{\alpha_s}{2\pi}P_{qq}(z)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$+ \text{ terms regular as } \varepsilon \to 0.$$

The factor $\mu^{2\varepsilon}$ comes from taking $g\mu^{\varepsilon}$ as the QCD coupling constant when $n \neq 4$. Expanding about $\varepsilon = 0$ and using $t=\ell n(Q^2/\mu^2)$ leads to eq.(9) above.

In the center of mass frame, θ is the angle of the hadron H with respect to the beam direction and z is the energy of the hadron H (expressed as a fraction of the beam energy). In terms of P and q, (the hadron and current momenta), we have:

$$q^2 = Q^2, \quad t = \ln Q^2/\mu^2, \quad z = \frac{2P\cdot q}{Q^2}$$

After integration over cos θ we obtain:

$$\sigma^H(z,t) = \sigma_T^H(z,t) + \sigma_L^H(z,t) \tag{12}$$

In the naive parton model σ_T is given by:

$$\sigma_T^H(z) = 3\,\sigma_o \sum_{a=1}^{f} e_a^2 \left[\mathscr{D}^H_{oq_a}(z) + \mathscr{D}^H_{o\bar{q}_a}(z)\right] \tag{13}$$

where

$$\sigma_o = \frac{4\pi\alpha^2}{3Q^2}$$

is the point like cross-section for $e^+e^- \to \mu^+\mu^-$. In Eq. (13) the factor 3 is from color and the index a runs over the various flavors of quarks whose charge (in units of the proton charge) is given by e_a. $\mathscr{D}^H_{oq_a}(z)$ is the number density of hadrons of type H which carry a fraction z of the energy of the quark q_a.

Calculating beyond the leading order, the simple parton model formula is modified as follows

$$\sigma_T^H(z,t) = 3\sigma_o \int_z^1 \frac{dy}{y} \sum_{i=1..2f} e_i^2 \sum_{j=G,1..2f} \mathscr{D}_{oj}^H(y)$$

$$\times \left[\delta_{ji}\delta\left(\frac{z}{y}-1\right) + \frac{\alpha s}{2\pi} P_{ji}\left(\frac{z}{y}\right)t + \alpha_s d_{ji}\left(\frac{z}{y}\right)\right] . \qquad (14)$$

As before we impose the condition that in terms of the renormalized scale dependent parton distributions, the form of Eq. (13) is unchanged

$$\sigma_T^H(z,t) = 3\sigma_o \sum_{i=1..2f} e_i^2 \mathscr{D}_i^H(z,t) \qquad (15)$$

For our purposes all that we require is the quantity

$$\alpha_s d_{qq}(z) = \frac{\alpha_s}{2\pi} C_F \left\{ (1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + 2\,\frac{1+z^2}{1-z}\ln z - \frac{3}{2}\frac{1}{(1-z)_+}\right.$$

$$\left. + \frac{3}{2}(1-z) + \left(\frac{2\pi^2}{3} - \frac{9}{2}\right)\delta(z-1)\right\} + \frac{\alpha_s}{2\pi} P_{qq}(z)\left(-\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi\right)$$

$$(16)$$

In order to extract the quark-quark scattering cross-section beyond the leading order we also need information on the definition of the gluon distribution beyond the leading order. In the context of the parton model, we must impose the condition of total momentum conservation

$$\sum_{i=G,1\cdots 2f} \int dx\, x\, \mathscr{F}_i(x,t) = 1$$

This in turn leads to the following constraints in order α_s on the functions f_{ij}

$$\sum_{i=G,1..2f} \int_0^1 dx\, x \left[f_{ij}(x)\right] = 0 \quad (j = 1 \cdot\cdot 2f) \tag{17}$$

$$\sum_{i=G,1..2f} \int_0^1 dx\, x \left[f_{iG}(x)\right] = 0 \tag{18}$$

These conditions on the second moment are not sufficient to completely determine the gluon distribution function beyond the leading order.

In our calculation, we will need to know f_{Gq}. This is true (as will be explained in Section 3) even though we are neglecting initial and fragmenting gluons by considering process (2) alone. A complete definition of f_{Gq} cannot be obtained from one-loop calculations of current induced processes because there is no direct photon-gluon coupling. Thus, apart from the constraint of eq. 17, we have an uncertainty in our quark-quark scattering calculation.*

To determine how sensitive our final results are to the choice of f_{Gq}, we tried several different forms. One possible choice is:

$$\alpha_s f_{Gq}(z) = -\frac{\alpha_s}{2\pi} C_F \frac{1}{6}\delta(1-z) + \frac{\alpha_s}{2\pi} P_{Gq}(z)\left(-\frac{1}{\varepsilon} + \gamma_E - \ell n 4\pi\right) \tag{19}$$

---

* This ambiguity would not be present in two-particle inclusive hadron-hadron scattering if we require that the two hadrons each come from the two fragmenting quarks produced at large p_T.

where

$$P_{Gq}(z) = C_F \left[\frac{1 + (1 - z)^2}{z} \right] \qquad (20)$$

This particular form is an extremal choice, since in practice we would not expect to encounter $\delta(1 - z)$ terms in any process measuring the off-diagonal $f_{Gq}(z)$.

Another possibility is:

$$f_{Gq}(1 - z) = f_{qq}(z), \quad z < 1 \qquad (21)$$

Note that eq. 17 is automatically satisfied. This is reminiscent of the procedure for obtaining $P_{Gq}(z)$ from $P_{qq}(z)$ which relies on the probabilistic interpretation of $P_{qq}(z)$ for $z < 1$. However, it is important to emphasize that one cannot interpret $f_{qq}(z)$ as a probability (e.g. $f_{qq}(z)$ is not positive definite for $z < 1$).

Using eq. 9, we see that $f_{Gq}(z)$ defined by eq. 21 contains singularities at $z = 0$ and $z = 1$. As a third possible choice which contrasts with eqs. 19 and 21, we choose:

$$\alpha_s f_{Gq}(z) = \frac{-\alpha_s}{2\pi} C_F \frac{(1 - z)}{3z} + \frac{\alpha_s}{2\pi} P_{Gq}(z)\left(-\frac{1}{\varepsilon} + \gamma_E - \ell n\, 4\pi\right) \qquad (22)$$

In section 5, we will show how sensitive our final results are to the choice of f_{Gq}.

3. ONE HADRON INCLUSIVE HADRON-HADRON SCATTERING

In this section we display the implications of the factorization theorem for one hadron inclusive scattering. This allows us to separate the pieces of the perturbative calculation which ultimately are absorbed into the distribution and fragmentation functions from the genuine higher order corrections. We choose to express the invariant hadronic cross section in terms of the variables S, V, and W defined below:

$$W = \frac{-U}{S + T} \tag{23}$$

$$V = 1 + \frac{T}{S} \tag{24}$$

where $S = (P_1 + P_2)^2$, $T = (P_1 - P_3)^2$, and $U = (P_2 - P_3)^2$. Similarly, we define corresponding kinematical variables s, v, and w for the parton process. We will always use lower case letters for parton variables and upper case letters for hadron variables. Using $p_1 = x_1 P_1$, $p_2 = x_2 P_2$, and $p_3 = P_3/x_3$, we obtain*:

$$v = \frac{x_2 x_3 - 1 + V}{x_2 x_3} \tag{25}$$

---

* Note that $v = \dfrac{p_1 \cdot (p_2 - p_3)}{s}$ and $w = \dfrac{p_2 \cdot p_3}{p_1 \cdot (p_2 - p_3)}$ with analogous relations for V and W.

$$w = \frac{x_2 VW}{x_1(x_2 x_3 - 1 + V)} \tag{26}$$

$$s = x_1 x_2 S \tag{27}$$

In terms of these variables, we may write the hadronic cross section:

$$\begin{aligned}\frac{1}{SV}\frac{d\sigma}{dVdW} = &\int_o^1 dx_1 dx_2 \frac{dx_3}{sx_3^2}\mathscr{F}_i(x_1,M^2)\mathscr{F}_j(x_2,M^2)\mathscr{D}_k(x_3,M^2)\\ &\times\left[\frac{1}{v}\frac{d\sigma^{qq}}{dv}(s,v)\delta(1-w) + \theta(1-w)\frac{\alpha_s^3}{2\pi}\mathscr{K}(s,v,w)\right]\\ &+\int_o^1 dx_1 dx_2 \frac{dx_3}{sx_3^2}\mathscr{F}_i(x_1,M^2)\mathscr{F}_G(x_2,M^2)\mathscr{D}_k(x_3,M^2)\\ &\times\frac{1}{v}\frac{d\sigma^{Gq}}{dv}(s,v)\delta(1-w)\end{aligned} \tag{28}$$

where M^2 is the scale at which we measure the distribution and fragmentation functions, and $\mathscr{K}$ is the correction term* which we compute in this paper. The expressions for the quark-quark (fig. 2a) and the quark-gluon (fig. 2b, c, d) cross sections in the Born approximation for arbitrary N and ε are given below:

$$\frac{d\sigma^{qq}}{dv}(s,v) = \frac{\pi\alpha_s^2\mu^{2\varepsilon}C_F}{Ns\Gamma(1-\varepsilon)}\left[\frac{4\pi\mu^2}{sv(1-v)}\right]^{\varepsilon}\left[\frac{1+v^2-\varepsilon(1-v)^2}{(1-v)^2}\right] \tag{29}$$

---

* Note that $\mathscr{K}(s,v,w)$ also depends on M^2.

$$\frac{d\sigma^{Gq}}{dv}(s,v) = \frac{\pi\alpha_s^2\mu^{2\varepsilon}(1-\varepsilon)}{Ns\Gamma(1-\varepsilon)}\left(\frac{4\pi\mu^2}{sv(1-v)}\right)^{\varepsilon}\left[1 + v^2 - \varepsilon(1-v)^2\right] \tag{30}$$

$$\times\left[\frac{C_F}{v} + \frac{N}{(1-v)^2}\right]$$

A word of explanation is needed for the presence of the second term in eq. 28 which contains the gluon distribution function $\mathscr{F}_G$. As previously discussed, we are neglecting the possibility of initial gluons or gluons which fragment into the observed hadron. However, when one calculates in perturbation theory, one of the leading logarithmic pieces in the quark-quark scattering diagram (say, $q_i + q_j \to q_k + X$ where $i = k \neq j$) comes from the region where a gluon emitted from q_j is on its mass shell. The gluon (G) then participates in the hard scattering: $G + q_i \to G' + q_k$. The resulting singularity is a contribution to the gluon distribution function beyond the leading order. Hence, the extraction of the quark-quark scattering correction term $\mathscr{K}$ requires knowledge of the gluon distribution function in order α_s. Note that to the order in which we work q_i cannot emit an on-shell gluon (which subsequently participates in a hard scattering), because we require q_k to be emitted at large p_T.

We next propose to use those functions $\mathscr{F}$ and $\mathscr{D}$ as measured in deep inelastic scattering and one-hadron inclusive e^+e^- annihilation respectively. That is, we substitute into eq. 28 the following expressions :

$$\mathscr{F}_i(x,M^2) = \int_x^1 \frac{dy}{y}\left[\delta(1-\frac{x}{y}) + \frac{\alpha_s}{2\pi}\, t\, P_{qq}(\frac{x}{y}) + \alpha_s f_{qq}(\frac{x}{y})\right]\mathscr{F}_{oi}(y) \tag{31}$$

$$\mathscr{D}_k(x,M^2) = \int_x^1 \frac{dy}{y}\left[\delta(1-\frac{x}{y}) + \frac{\alpha_s}{2\pi}\, t\, P_{qq}(\frac{x}{y}) + \alpha_s d_{qq}(\frac{x}{y})\right]\mathscr{D}_{ok}(y) \tag{32}$$

where $t = \ln(M^2/\mu^2)$ (cf. eqs. 4 and 14). Note that in neglecting incoming and fragmenting gluons, we have set $\mathscr{F}_{oG} = \mathscr{D}_{oG} = 0$. In addition $\mathscr{F}_G$ is non-zero at order α_s:

$$\mathscr{F}_G(x,M^2) = \int_x^1 \frac{dy}{y}\left[\frac{\alpha_s}{2\pi}\, t\, P_{Gq}(\frac{x}{y}) + \alpha_s f_{Gq}(\frac{x}{y})\right]\mathscr{F}_{oj}(y) \tag{33}$$

In order to economize our notation, we will write:

$$H_{qq}(x) \equiv tP_{qq}(x) + 2\pi f_{qq}(x) \tag{34}$$

$$H_{Gq}(x) \equiv tP_{Gq}(x) + 2\pi f_{Gq}(x) \tag{35}$$

$$\tilde{H}_{qq}(x) \equiv tP_{qq}(x) + 2\pi d_{qq}(x) \tag{36}$$

Inserting eqs. 31, 32, and 33 into eq. 28 yields:

$$\frac{1}{SV}\frac{d\sigma}{dVdW} = \int dx_1 dx_2 \frac{dx_3}{sx_3^2} \mathcal{F}_{oi}(x_1)\mathcal{F}_{oj}(x_2)\mathcal{D}_{ok}(x_3)$$

$$\times \left\{ \frac{1}{v}\frac{d\sigma^{qq}}{dv}(s,v)\delta(1-w) \right.$$

$$+ \frac{\alpha_s}{2\pi}\theta(1-w)\left[\alpha_s^2\mathcal{K}(s,v,w) + \frac{1}{v}H_{qq}(w)\frac{d\sigma^{qq}}{dv}(ws,v)\right.$$

$$+ \frac{1}{1-vw}H_{qq}\left(\frac{1-v}{1-vw}\right)\frac{d\sigma^{qq}}{dv}\left(\left(\frac{1-v}{1-vw}\right)s,vw\right)$$

$$+ \frac{1}{1-vw}H_{Gq}\left(\frac{1-v}{1-vw}\right)\frac{d\sigma^{Gq}}{dv}\left(\left(\frac{1-v}{1-vw}\right)s,vw\right)$$

$$\left.\left. + \frac{1}{1-v+vw}\tilde{H}_{qq}(1-v+vw)\frac{d\sigma^{qq}}{dv}\left(s,\frac{vw}{1-v+vw}\right)\right]\right\} \qquad (37)$$

In order to calculate the desired correction term $\mathcal{K}(s,v,w)$, we compute the Feynman diagrams displayed in figures 3 and 4. The result can be written in terms of the variables for the perburbation theory diagrams,

$$\frac{1}{v}\frac{d\sigma}{dvdw} = \frac{1}{v}\frac{d\sigma^{qq}}{dv}(s,v)\delta(1-w) \qquad (38)$$

$$+ \frac{\alpha_s}{2\pi}\theta(1-w)k(s,v,w)$$

where $\frac{\alpha_s}{2\pi}k(s,v,w)$ represents the total order α_s^3 contribution. The hadronic cross section is then obtained by convoluting eq. 38 with the bare distribution and fragmentation functions:

$$\frac{1}{SV}\frac{d\sigma}{dVdW} = \int dx_1 dx_2 \frac{dx_3}{sx_3^2} \mathscr{F}_{oi}(x_1)\mathscr{F}_{oj}(x_2)\mathscr{D}_{ok}(x_3)$$

$$\times \left[\frac{1}{v}\frac{d\sigma^{qq}}{dv}(s,v)\delta(1-w) + \frac{\alpha_s}{2\pi}\theta(1-w)k(s,v,w)\right] \tag{39}$$

At this point, we note that both equations 37 and 39 contain terms which diverge as $\varepsilon \to 0$. However, the statement of factorization of mass singularities means that $\mathscr{K}(s,v,w)$ will be finite as $\varepsilon \to 0$. To obtain an equation for $\mathscr{K}$, we simply equate equations 37 and 39. The result is:

$$\alpha_s^2\mathscr{K}(s,v,w) = k(s,v,w) - \frac{1}{v} H_{qq}(w)\frac{d\sigma^{qq}}{dv}(ws,v)$$

$$- \frac{1}{1-vw} H_{qq}\left(\frac{1-v}{1-vw}\right)\frac{d\sigma^{qq}}{dv}\left(\left(\frac{1-v}{1-vw}\right)s, vw\right)$$

$$- \frac{1}{1-vw} H_{Gq}\left(\frac{1-v}{1-vw}\right)\frac{d\sigma^{Gq}}{dv}\left(\left(\frac{1-v}{1-vw}\right)s, vw\right)$$

$$- \frac{1}{1-v+vw}\tilde{H}_{qq}(1-v+vw)\frac{d\sigma^{qq}}{dv}\left(s, \frac{vw}{1-v+vw}\right) \tag{40}$$

We have calculated $k(s,v,w)$ and as expected, $\mathscr{K}$ is finite as $\varepsilon \to 0$. We discuss the computation of $k(s,v,w)$ in section 4 and give the explicit result for $\mathscr{K}$ in section 5. It is convenient to state the final formula for the one-hadron inclusive cross section (i.e. eq. 28) by eliminating the variables x_1 and x_2 in favor of v and w (using eqs. 25 and 26). The result is:

$$E_3 \frac{d^3\sigma}{d^3P_3} = \frac{1}{\pi S}\int_{1-V+VW}^{1} \frac{dx_3}{x_3^2} \int_{\frac{VW}{x_3}}^{1-(\frac{1-V}{x_3})} \frac{dv}{1-v} \int_{\frac{VW}{x_3 v}}^{1} \frac{dw}{w} \tag{41}$$

$$\times \mathcal{F}_i(x_1,M^2)\mathcal{F}_j(x_2,M^2)\mathcal{D}_k(x_3,M^2)$$

$$\times \left[\frac{\pi\alpha_s^2(Q^2)C_F}{Nsv}\,\frac{1+v^2}{(1-v)^2}\,\delta(1-w) + \frac{\alpha_s^3(Q^2)}{2\pi}\mathcal{K}(s,v,w)\right]$$

where $x_1 = VW/x_3vw$, $x_2 = (1-V)/x_3(1-v)$, and $s = x_1x_2S$. Note that we have now dropped the term with the gluon distribution function. In addition, α_s had been replaced by the running coupling constant $\alpha_s(Q^2)$ evaluated at a scale Q^2 to be determined (see eq. 44 and the discussion which follows).

4. CALCULATION OF THE ORDER α_s^3 CONTRIBUTION

A. Elastic Graphs

In this section we present our results on the virtual gluon corrections to the basic quark-quark scattering cross-section (see Fig. 3). Our caluclations are performed in the Feynman gauge and as before, dimensional regularization is used to control both the infrared and ultraviolet singularities. The ultraviolet singularities appear as poles at $\varepsilon = 0$ in the momentum integrations, whereas the infrared and mass singularities appear as poles at $\varepsilon = 0$ in the Feynman parameter integrations.

The only subtlety in this procedure is in the evaluation of the wave function renormalization on external on-shell quark lines. We follow the procedure of subtracting the ultraviolet pole before taking the on shell-limit. These graphs then give a contribution containing poles in ε equal to minus the quantity subtracted. Our renormalization scheme is the so called $\overline{MS}$ [12] scheme which requires the subtraction of all the ultraviolet poles together with their attendant Euler constant and $\ln 4\pi$. We have chosen this prescription scheme because it leads to small corrections and hence presumably a well ordered perturbation series in both deep inelastic scattering[12,13] and in the e^+e^- annihilation total cross-section.[14] These processes are our major source of information about the scale of the strong coupling constant Λ.

The results of our calculation are shown in Table 1. The total contribution of the elastic graphs may be written as:

$$\frac{d\sigma}{dv} = \frac{\pi\alpha_s^2(Q^2)\mu^{2\varepsilon}C_F}{Ns\Gamma(1-\varepsilon)} \left[\frac{4\pi\mu^2}{sv(1-v)}\right]^{\varepsilon} \left[\frac{1+v^2-\varepsilon(1-v)^2}{(1-v)^2}\right] \times \left\{1 - \frac{\alpha_s(Q^2)}{2\pi}\left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{(1-2\varepsilon)} A(v)\right\} \tag{42}$$

where the function $A(v)$ is given by

$$
\begin{aligned}
A(v) = \; & C_F \Bigg[\frac{4}{\varepsilon^2} + \frac{2}{\varepsilon}(3 - 4\ \ln v - 2\ \ln(1-v) + 16 + \frac{2\pi^2}{3} \\
& + 2\ \ln^2(1-v) - 2(3 - 4\ \ln v)\ \ln(1-v) \\
& + 2\left(\frac{1-v^2}{1+v^2}\right)\left(\pi^2 + \ln^2\left(\frac{v}{1-v}\right) + \ln^2(1-v)\right. \\
& \left. + 2\ \ln(1-v) - \frac{2\ \ln v}{1+v}\right)\Bigg] \\
& + N\Bigg[\frac{2}{\varepsilon}\ \ln\ \left[v(1-v)\right] - 2\ \ln^2(1-v) - 2\ \ln v\ \ \ln(1-v) - \pi^2 - \frac{85}{9} \\
& - \frac{1}{2}\left(\frac{1-v^2}{1+v^2}\right)\left(\pi^2 + \ln^2\left(\frac{v}{1-v}\right) + 2\ \ln^2(1-v)\right. \\
& \left. + \frac{2}{1+v}\ \left[\ln\left(\frac{1-v}{v}\right) + 2v\ \ln(1-v)\right]\right)\Bigg] \\
& + \frac{10f}{9} + \left(\frac{11N - 2f}{3}\right)\ \ln\left(\frac{-t}{Q^2}\right)
\end{aligned}
\tag{43}
$$

where $t = -s(1-v)$. In eq. 42, we have introduced the running coupling constant evaluated at an arbitrary scale Q^2:

$$
\left[\frac{\alpha_s(Q^2)}{\alpha_s}\right]^2 = 1 - \frac{\alpha_s}{2\pi}\left(\frac{11N - 2f}{3}\right)\ \ln\left(\frac{Q^2}{\mu^2}\right) \tag{44}
$$

For the particular set of diagrams which we have considered, the choice $Q^2 = -t$ is the most suitable (causing the last term in eq. 43

to vanish). This is a consequence of the fact that the lowest order graphs which we are considering all have a t-channel pole. This would not be true if a more complete set of graphs were considered.

B. Inelastic Graphs

There are five inelastic graphs which contribute to $\frac{d\sigma}{dvdw}$ to order α_s^3. They are shown in figure 4. When the total amplitude is squared, one obtains 15 distinct terms. It proves convenient to choose a particular frame in which to perform the phase space integration. We chose the Gottfried-Jackson frame [15] where the gluon and the unobserved quark have no net three-momentum. The 15 terms were then added; all algebraic manipulations were performed using MACSYMA*. Next, the integral over phase space in $n \neq 4$ dimensions was computed. All the integrals encountered were either straightforward or expressible in terms of one basic integral. We discuss this integral and details of the n-dimensional three body phase space in Appendix B.

The final result for $\frac{d\sigma^{inel}}{dvdw}$ is long and therefore we do not reproduce it here. We then have to add this result to the elastic cross section. Before we can do this we must expose the mass singularities of $\frac{d\sigma^{inel}}{dvdw}$ which occur at $w = 1$. To do this, we proceed as follows. Schematically, we have found that:

$$\frac{d\sigma^{inel}}{dvdw} = A + B\,(1 - w)^{-1-\varepsilon} + C(1 - w)^{-1-2\varepsilon} \qquad (45)$$

* MACSYMA was developed by the Mathlab Group of the MIT Laboratory for Computer Science.

where A, B, and C are complicated functions of v and w but are regular at $w = 1$ and contain simple poles at $\varepsilon = 0$. We now use the identity:

$$(1 - w)^{-1-\varepsilon} = \frac{1}{(1 - w)_+} - \varepsilon\left(\frac{\ln(1 - w)}{1 - w}\right)_+ - \frac{1}{\varepsilon}\,\delta(1 - w) + O(\varepsilon^2) \quad (46)$$

Inserting eq. 46 into eq. 45, we find that we can write the cross section in the following form:

$$\frac{d\sigma^{inel}}{dvdw} = \left(\frac{A_1}{\varepsilon^2} + \frac{A_2}{\varepsilon} + A_3\right)\delta(1 - w)$$

$$+ \left(\frac{B_1}{\varepsilon} + B_2\right)\frac{1}{(1 - w)_+} + C_1\left(\frac{\ln(1 - w)}{1 - w}\right)_+ \quad (47)$$

We can now add this expression to the elastic cross section (which is proportional to $\delta(1 - w)$). One finds immediately that the terms proportional to ε^{-2} cancel as required. Therefore, the function $k(s,v,w)$ defined by eq. 38 contains terms proportional to ε^{-1} and terms which are finite as $\varepsilon \to 0$. Inserting this resulting expression into eq. 40, we have verified that the terms proportional to ε^{-1} cancel and $\mathcal{K}(s,v,w)$ is finite as $\varepsilon \to 0$.

5. THE QUARK-QUARK SCATTERING CORRECTION TERM

We now write out explicitly our result for $\mathcal{K}(s,v,w)$. At this point, we have not chosen a particular form for $f_{Gq}(x)$. We also leave unspecified the choice of the factorization scale M^2. Since $\mathcal{K}$ is regular as $\varepsilon \to 0$, we may set $\varepsilon = 0$ everywhere. The result is:

$$\mathscr{K}(s,v,w) = \frac{\pi C_F}{sNv}\Bigg\{\left(\frac{1+v^2}{(1-v)^2}\right)\left(c_1\delta(1-w) + \frac{c_2}{(1-w)_+} + c_3\left(\frac{\ell n(1-w)}{1-w}\right)_+\right)$$

$$+ \left[C_F\left(\frac{1+v^2}{(1-v)^2}\right)\left(\frac{9}{2} + 2 \ \ell n\left(\frac{v^2}{1-v}\right)\right)\delta(1-w)\right.$$

$$\left. + \frac{c_4}{(1-w)_+}\right] \ell n\left(\frac{s}{M^2}\right)$$

$$+ c_5\ \ell n\ v + c_6\ \ell n(1-vw) + c_7\ \ell n(1-v+vw)$$

$$+ c_8\ \ell n(1-v) + c_9\ \ell n\ w + c_{10}\ell n(1-w) + c_{11}$$

$$+ c_{12}\ \frac{\ell n(1-v+vw)}{1-w} + c_{13}\ \frac{\ell n\ w}{1-w} + c_{14}\ \frac{\ell n\left(\frac{1-vw}{1-v}\right)}{1-w}\Bigg\}$$

$$- \frac{\pi}{sN}\left(\frac{1+v^2w^2}{1-v}\right)\tilde{f}_{Gq}\left(\frac{1-v}{1-vw}\right)\left[\frac{C_F}{vw} + \frac{N}{(1-vw)^2}\right] \qquad (48)$$

where the coefficients c_i $(i = 1,2,\cdots 14)$ are functions of v and w and are given in Appendix C. For convenience, we have defined:

$$\tilde{f}_{Gq}(x) \equiv 2\pi f_{Gq}(x) - P_{Gq}(x)\left(-\frac{1}{\varepsilon} + \gamma_E - \ell n\ 4\pi\right) \qquad (49)$$

The complexity of the above expression prohibits us from making any quick observations as to the size of $\mathscr{K}$. We proceeded as follows: Inserting the expression for $\mathscr{K}$ (with $N = 3$, $C_F = \frac{4}{3}$ and

$f = 4$) into eq. 41,* we compared the hadronic cross section thus obtained to that predicted by using the Born parton cross section alone. We denote this ratio by $\mathscr{R}$. To simplify the calculation we work at $\theta_{cm} = 90^{\circ}$ for the one-hadron inclusive production, which corresponds to the observation of the hadron at $P_T^2 = SVW(1-V)$, where V and W satisfy $V(1 + W) = 1$. We used scale-breaking** valence quark distribution functions and scale-breaking fragmentation functions given by Feynman, Field, and Fox[2] for $\mathscr{F}(x, M^2)$ and $\mathscr{D}(x, M^2)$. We still have two uncertainties to deal with: the value of M^2 and the functional form for f_{Gq}.

We first fixed all our parameters and tried three forms for f_{Gq} given by eqs. 19, 21, and 22. We observed that eqs. 19 and 22 gave almost identical results for $\mathscr{R}$. When we used $f_{Gq}(x) = f_{qq}(1 - x)$ $(x \neq 0)$, we found that the size of the correction term decreased somewhat as shown in Fig. 5. Thus, for the remaining calculations, we settled on this latter choice for f_{Gq}. Note that our results are only mildly sensitive to this particular choice.

In order to decide on the optimal choice for M^2, we looked at four possibilities: $M^2 = s$, $M^2 = -t$, $M^2 = tu/s$ and $M^2 = 2stu/(s^2 + t^2 + u^2)$, where s, t, and u are parton variables.***

---

* Note that the limits of the w integration do not go from 0 to 1. As a result, the "plus" distributions must be modified as explained in Appendix A (see eqs. A4 and A5).

** The scale of the running coupling constant used here is $\Lambda = 400$ MeV.

*** Note that t and u are related to v and w by $t = -s(1-v)$ and $u = -svw$.

In fig. 5, we plot the results of two of these choices (at $\sqrt{S} = 27$ GeV). The smallest result for $\mathcal{R}$ occurs for $M^2 = tu/s$ (which is equal to the transverse momentum squared of the outgoing quark which fragments into the observed hadron). The largest value of $\mathcal{R}$ occurs for $M^2 = s$, with other choices of M^2 giving intermediate results.* Even with an optimal choice for M^2, we see that the QCD corrections (indicated by the deviation of $\mathcal{R}$ from 1) are large and positive. As P_T increases, $\mathcal{R}$ changes little despite the fact that $\mathcal{R}$ is the ratio of two steeply falling cross sections. Note that in the region of small p_T our calculation is not applicable because we are no longer in the perturbative domain. But even away from the kinematic boundaries where we would hope to apply QCD perturbation theory, we see that the corrections are too large to justify its use. As we increase S, the size of the running coupling constant should decrease and push down the size of the QCD correction term. A comparison of $\sqrt{S} = 27$ GeV and $\sqrt{S} = 1000$GeV is shown in Fig. 6. Due to the slow logarithmic decrease of the running coupling constant, the QCD corrections are still large. Thus, we expect that even at ISABELLE energies, QCD perturbation theory as applied to large p_T physics will remain suspect.

One may argue whether a redefinition of the coupling constant by changing the value of Λ could reduce the size of the correction

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* We find that the hadronic cross section obtained by using the Born parton cross section alone varies on the average by 20% by changing M^2 as above due to the use of scale-breaking distribution functions.

term. In fact, the term $(85N - 10f)/9$ which appears in c_1 (see eq. 48 and Appendix 3) accounts for a large portion of the QCD correction. This term could be absorbed by a change in Λ because c_1 is proportional to the Born cross section. However, such a change in Λ would destroy the well behaved perturbation expansion in deep inelastic scattering[12, 13] and $\sigma_t(e^+e^- \to \text{hadrons})$[14]. We conclude that the large QCD corrections we find at present energies connot be defined away.

6. CONCLUSIONS

We have computed the higher order QCD corrections to the quark-quark scattering process which contributes to the production of hadrons at large p_T in hadron-hadron collisions. The mass singularities are absorbed into the distribution and fragmentation functions which are defined in deep inelastic scattering and semi-inclusive e^+e^- annihilation. The calculation enables us to determine whether the perturbation expansion is well behaved and consequently whether the lowest order prediction is reliable. As in many other processes[5, 16, 17], we find that the corrections are large and we are in the position that α_s is not small enough at present energies to give a well ordered perturbation series.

The complexity of our answer does not allow us to unambiguously identify the source of the large correction. Unlike the case of Drell-Yan[5], the large corrections here do not appear to be a consequence of the soft radiation. We feel that the large corrections in our case are in part due to the fact that the eight vector gluons play a large role by giving rise to large color factors in certain graphs. Another source of the large corrections could be that the quark-quark Born cross section begins at order α_s^2; hence the number of diagrams associated with the first radiative corrections is larger than in (electro-weak) current induced processes. We feel that these features would persist in a complete calculation in which all possible contributing parton processes are taken into account. The ultimate conclusion of this paper is not encouraging - the QCD perturbation expansion is out of control in another process.

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APPENDIX A

The distributions $\frac{1}{(1-w)_+}$ and $\left(\frac{\ln(1-w)}{1-w}\right)_+$ are defined by:

$$\int_0^1 \frac{f(w)dw}{(1-w)_+} = \int_0^1 \frac{[f(w) - f(1)]}{1-w}\, dw \tag{A1}$$

$$\int_0^1 f(w)\left(\frac{\ln(1-w)}{1-w}\right)_+ dw = \int_0^1 [f(w) - f(1)]\left(\frac{\ln(1-w)}{1-w}\right) dw \tag{A2}$$

Note that the range of integration here is always assumed to be $0 \leqslant w \leqslant 1$. However, in eq. (41), we see that the w integration range is $A \leqslant w \leqslant 1$ where $A \equiv VW/x_3 v$. It is convenient to introduce a more general distribution $\frac{1}{(1-w)_A}$ such that for any $A < 1$,

$$\int_A^1 \frac{f(w)dw}{(1-w)_A} = \int_A^1 \frac{[f(w) - f(1)]}{1-w} dw \qquad (A3)$$

and a similar definition for $\left(\frac{\ell n(1-w)}{1-w}\right)_A$. We find that:

$$\frac{1}{(1-w)_+} = \frac{1}{(1-w)_A} + \ell n(1-A)\delta(1-w) \qquad (A4)$$

$$\left(\frac{\ell n(1-w)}{1-w}\right)_+ = \left(\frac{\ell n(1-w)}{1-w}\right)_A + \frac{1}{2}\ell n^2(1-A)\delta(1-w). \qquad (A5)$$

This allows us to express $\mathcal{K}(s,v,w)$ (eq. 48) in terms of these new distributions. Integrating eq. (41) from $w = A$ to $w = 1$ is then straightforward. Note that we could have accomplished the same result by replacing eq. (46) with:

$$(1-w)^{-1-\varepsilon} = \frac{1}{(1-w)_A} - \varepsilon\left(\frac{\ell n(1-w)}{1-w}\right)_A - \frac{1}{\varepsilon}(1-A)^{-\varepsilon}\delta(1-w) + O(\varepsilon^2). \qquad (A6)$$

Finally, we wish to make some technical remarks regarding eq. (40). Consider for example $H_{qq}(x)$ where $x = \frac{1-v}{1-vw}$. We recall from the definition (see eq. 34) that $H_{qq}(x)$ contains terms like $\frac{1}{(1-x)_+}$ and $\left(\frac{\ell n(1-x)}{1-x}\right)_+$. We would like to express these distributions in terms of $\delta(1 - w)$, $\frac{1}{(1 - w)_+}$, and $\left(\frac{\ell n(1 - w)}{1 - w}\right)_+$ because it is the point $w = 1$ (the elastic limit) where the soft singularities originate. Note that if desired, we may use eqs. A4 and A5 at the end to account for $A \neq 0$. Using the example just mentioned, we evaluate $\frac{1}{(1 - x)_+}$ where $x = \frac{1 - v}{1 - vw}$. We may write

$$\left(\frac{1 - vw}{v(1 - w)}\right)_+ = \frac{1 - vw}{v(1 - w)_+} + C\delta(1 - w) \tag{A7}$$

To determine C, integrate both sides of eq. A7 from $w = 0$ to $w = 1$. The left hand side can then be evaluated by changing variables to x and using eqs. A3 and A4. The result:

$$C = \left(\frac{1 - v}{v}\right) \ \ell n \left(\frac{v}{1 - v}\right) \tag{A8}$$

Similarly, we may derive other identities which are needed when the right hand side of eq. (40) is simplified. As a final example, note that

$$\left(\frac{1}{v(1 - w)}\right)_+ = \frac{1}{v(1 - w)_+} + \frac{1}{v} \ \ell n v \delta(1 - w) \tag{A9}$$

That is, one cannot simply factor out the v from the denominator.

APPENDIX B

We believe that our treatment of massless three particle phase space in n space time dimensions contains some new features and may be useful in other contexts so we give a few details of our technique in this appendix. The three particle phase space for the process

$$p_1 + p_2 \to p_3 + p_4 + k$$

may be written as

$$(PS)_3 = \int \frac{d^n p_3}{(2\pi)^{n-1}} \frac{d^n p_4}{(2\pi)^{n-1}} \frac{d^n k}{(2\pi)^{n-1}} (2\pi)^n \delta^n(p_1+p_2-p_3-p_4-k)\delta^+(p_4^2)\delta^+(k^2)\delta^+(p_3^2) \tag{B1}$$

Introducing the variable $p_{4k} = p_4 + k$ and defining $s_2 \equiv p_{4k}^2$ we may write eq. (B1) as

$$(PS)_3 = \int \frac{1}{(2\pi)^{5-4\varepsilon}} ds_2 d^n p_3 d^n p_{4k} \delta^+(p_3^2) \delta(p_{4k}^2 - s_2) \delta^n(p_1 + p_2 - p_3 - p_{4k})$$

$$\times \; d^n k \, d^n p_4 \delta^+(k^2) \, \delta^+(p_4^2) \delta^n(p_{4k} - p_4 - k) \tag{B2}$$

where $\varepsilon = \frac{4 - n}{2}$.

The 3 particle phase space has factored into two Lorentz invariant parts which we may evaluate in any frame. Working in the rest system of $p_4 + k$ we orient the vectors p_1, p_2 and p_3 so that they lie in the plane of the n^{th} and $(n-1)^{th}$ components of the momentum. Thus we have

$$p_4 = \frac{\sqrt{s_2}}{2} (1, \cdots, \cos\theta_2 \sin\theta_1, \cos\theta_1) \tag{B3}$$

$$k = \frac{\sqrt{s_2}}{2}(1,\cdots, -\cos\theta_2\sin\theta_1, -\cos\theta_1) \quad (B4)$$

where the dots indicate $n - 3$ unspecified momentum which with the above orientation of p_1, p_2, and p_3 can be trivially integrated over. Defining $t = (p_1 - p_3)^2$ and $u = s_2 - s - t$, we introduce rescaled variables:

$$v = 1 + \frac{t}{s} \quad (B5)$$

$$w = \frac{-u}{s+t} \quad (B6)$$

Using eqs. B3 and B4, we obtain from eq. B2:

$$(PS)_3 = \frac{s}{2^8\pi^4\Gamma(1-2\varepsilon)}\left(\frac{4\pi}{s}\right)^{2\varepsilon}\int_0^1\int_0^1 v\,dv\,dw(1-v)^{-\varepsilon}(1-w)^{-\varepsilon}v^{-2\varepsilon}w^{-\varepsilon}$$
$$\int_0^\pi d\theta_2\sin^{-2\varepsilon}\theta_2\int_0^\pi d\theta_1\sin^{1-2\varepsilon}\theta_1 \quad (B7)$$

For definiteness, let us consider a term in the matrix element squared of the bremsstrahlung graphs of the form $\frac{1}{(p_1-k)^2(p_3+k)^2}$. We choose:

$$p_1 = \left(\frac{s(1-vw)}{2\sqrt{s_2}}\right)(1, 0,\cdots,0,0, 1) \quad (B8)$$

$$p_3 = \left(\frac{s(1 - v + vw)}{2\sqrt{s_2}}\right)(1, 0, \cdots, 0, \sin\psi, \cos\psi) \tag{B9}$$

where $s_2 = sv(1 - w)$ and the angle ψ is given by

$$\cos\frac{\psi}{2} = \frac{1 - v}{(1 - vw)(1 - v + vw)} \tag{B10}$$

Using eq. B7, we find that we have to evaluate an integral of the form

$$J(\psi) \equiv \int_0^\pi d\theta_2 \int_0^\pi d\theta_1 \frac{\sin^{-2\varepsilon}\theta_2 \sin^{1-2\varepsilon}\theta_1}{(1 + \cos\theta_1)(1 + \cos\theta_1\cos\psi + \sin\theta_1\sin\psi\cos\theta_2)} \tag{B11}$$

Feynman parameterizing the denominators, we find after some manipulation,

$$J(\psi) = \frac{-\pi}{\varepsilon}\left(\sin^2\frac{\psi}{2}\right)^{-1-\varepsilon} F(-\varepsilon, -\varepsilon; 1 - \varepsilon; \cos^2\frac{\psi}{2}) \tag{B12}$$

where F is the usual hypergeometric function[18]. Expanding about $\varepsilon = 0$ we find:

$$F(-\varepsilon, -\varepsilon; 1 - \varepsilon; x) = 1 + \varepsilon^2 Li_2(x) + O(\varepsilon^3) \tag{B13}$$

where

$$Li_2(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^2} \tag{B14}$$

is the dilogarithm.

By suitable rotation of axes and partial fractioning, all the bremsstrahlung integrals are either straightforward or can be reduced into the form of eq. B11.

APPENDIX C

The coefficients c_i, i = 1,2,..., 14 are defined in Eq. 48.
The are given by:

$$c_1 = -C_F \Bigg[\frac{\pi^2}{3} - 1 - 16 \ln^2 v + 2 \ln^2 (1-v) + 12 \ln v \ln (1-v) - \frac{3}{2} \ln v - \frac{9}{2} \ln (1-v) + 2\left(\frac{1-v^2}{1+v^2}\right)\left(\pi^2 + \ln^2 \left(\frac{v}{1-v}\right) + \ln^2 (1-v) + 2 \ln (1-v) - \frac{2\ln v}{1+v}\right)\Bigg]$$

$$- N \Bigg[4 \ln^2 v - 2 \ln^2 (1-v) - 2 \ln (1-v) \ln v - \pi^2 - \frac{1}{2}\left(\frac{1-v^2}{1+v^2}\right)\left(\pi^2 + \ln^2\left(\frac{v}{1-v}\right) + 2 \ln^2 (1-v) + \frac{2}{1+v}\left[\ln\left(\frac{1-v}{v}\right) + 2 v \ln (1-v)\right]\right)\Bigg]$$

$$+ \left(\frac{85N - 10f}{9}\right)$$

$$c_2 = 3 C_F \left(1 + 6 \ln v + 2 \ln (1-v)\right) - 4 N \ln [v(1-v)]$$

$$c_3 = 4 C_F$$

$$c_4 = C_F \left[\left(\frac{1+v^2w^2}{(1-vw)^2}\right) \left(\frac{1-vw}{1-v} + \frac{1-v}{1-vw}\right) + \frac{(1+w^2)(1+v^2)}{w(1-v)^2} \right.$$

$$+ \frac{[1+(1-v+vw)^2][v^2w^2+(1-v+vw)^2]}{(1-v)^2(1-v+vw)}$$

$$\left. + \frac{(1-w)(1+v^2w^2)[(1-vw)^2+v^2(1-w)^2]}{w(1-vw)(1-v)^2} \right]$$

$$+ \frac{Nv(1-w)(1+v^2w^2)[(1-vw)^2+v^2(1-w)^2]}{(1-v)^2(1-vw)^3}$$

$$c_5 = \frac{C_F}{w(1-vw)(1-v)^2} [2v^4w^4 - 2v^4w^3 - 2v^3w^3 + v^4w^2 + 25v^3w^2 - 21v^2w^2$$

$$+ vw^2 - 9v^3w + 5v^2w - 35vw - w + 2v^2 + 2]$$

$$\frac{+ Nv}{(1-v)^2(1-vw)^3} [2v^4w^4 - 8v^4w^3 + 4v^3w^3 + 3v^4w^2 + 10v^3w^2 - v^2w^2$$

$$- 4v^3w - 4v^2w - 12vw + 3v^2 - 2v + 9]$$

$$c_6 = -\frac{2C_F(1-vw)(2v^2w^2-2v^2w+2vw+v^2+1)}{w(1-v)^2} + \frac{Nv(4vw-v+5)}{(1-v)^2}$$

$$c_7 = \frac{2C_F v(2v^2w^2-2v^2w+4vw+v^2+3)}{(1-v)^2} + \frac{Nv}{1-v}$$

$$c_8 = \frac{C_F}{w(1-v)^2(1-vw)^3} [4v^6w^6 - 4v^6w^5 - 8v^5w^5 + 2v^6w^4 + 9v^5w^4$$

$$+ 9v^4w^4 - 9v^5w^3 - 19v^4w^3 - 8v^3w^3 + 12v^4w^2 + 29v^3w^2$$

$$+ 3v^2w^2 - 9v^3w - 11v^2w - 4vw + 2v^2 + 2]$$

$$+ \frac{Nv(2v^2w^2+v^2+4v-3)}{(1-v)^2(1-vw)}$$

$$c_9 = \frac{-C_F}{w(1-v)^2(1-vw)} [4v^3w^3 - 21v^3w^2 + 16v^2w^2 - vw^2 + 11v^3w + 5v^2w$$

$$+ 23vw + w - 3v^2 - 3]$$

$$+ \frac{Nv(2v^2w^2-5v^2w+5vw+3v^2+v+6)}{(1-vw)(1-v)^2}$$

$$c_{10} = \frac{C_F}{w(1-v)^2(1-vw)} [2v^4w^4 - 2v^4w^3 + 2v^3w^3 + v^4w^2 + 8v^3w^2$$

$$- 5v^2w^2 + 6v^2w - 14vw + v^2 + 1]$$

$$+ \frac{Nv[3v^3w^3+v^2w^2-2v^2w-5vw+3]}{(1-v)(1-vw)^3}$$

$$c_{11} = \frac{C_F}{2w(1-v)^2(1-vw)^2(1-v+vw)} [6v^6w^6 - 12v^6w^5 - 6v^5w^5 + 7v^6w^4$$

$$+ 18v^5w^4 - 23v^4w^4 + 2v^3w^4 - v^6w^3 - 16v^5w^3$$

$$+ 17v^4w^3 + 18v^3w^3 - 2v^2w^3 + 15v^5w^2 - 66v^4w^2 + 81v^3w^2$$

$$- 30v^2w^2 - 2vw^2 + 17v^4w - 56v^3w + 59v^2w$$

$$- 28vw + 2w + v^3 + 3v^2 - 3v - 1]$$

$$- \frac{N}{(1-v)^2(1-vw)^3(1-v+vw)} [2v^6w^5 - 6v^6w^4 + 3v^5w^4 - v^4w^4$$

$$+ 6v^6w^3 - 3v^5w^3 - v^4w^3 + 2v^3w^3 - 4v^6w^2 + 16v^5w^2$$

$$- 27v^4w^2 + 11v^3w^2 - 4v^5w + 10v^4w - 3v^3w + v^2w$$

$$- 2vw + 2v^4 - 8v^3 + 9v^2 - 4v + 1]$$

$$c_{12} = \frac{-8C_Fv^2}{(1-v)^2} - \frac{2N(1+v)}{1-v}$$

$$c_{13} = \frac{2C_F(11v^2+7)}{(1-v)^2} - \frac{N(5v^2+3)}{(1-v)^2}$$

$$c_{14} = \frac{8C_F(1+v)}{1-v} - \frac{N(v^2+7)}{(1-v)^2}$$

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TABLE CAPTIONS

Table 1: Computation of virtual graphs to $O(\alpha_s^3)$

We give the expressions which correspond to Fig. 3 (a) - (d). S_o is equal to the Born quark-quark cross section given in eq. 29. The sum of all the terms in this table is given in eq. 42. Terms of $O(\varepsilon)$ are neglected. Note that $t = -s(1-v)$.

TABLE 1

$$\text{(a)}\ S_o\ \frac{\alpha_s}{2\pi}\left(-\frac{4\pi\mu^2}{t}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{(1-2\varepsilon)}\left[C_F\left(-\frac{4}{\varepsilon^2}-\frac{6}{\varepsilon}-16-\frac{2\pi^2}{3}\right.\right.$$

$$\left.\left.+N\left(\frac{2}{\varepsilon^2}+6+\frac{\pi^2}{3}-2\,\ln\left(\frac{-t}{\mu^2}\right)\right)\right]$$

$$\text{(b)}\ S_o\ \frac{\alpha_s}{2\pi}\left(-\frac{4\pi\mu^2}{t}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\left(\frac{2}{3}f-\frac{5}{3}N\right)\ln\left(\frac{-t}{\mu^2}\right)+\frac{31N-10f}{9}\right]$$

$$\text{(c)}\ S_o\ \frac{\alpha_s}{2\pi}\left(-\frac{4\pi\mu^2}{t}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}(2C_F-N)\left[\frac{4}{\varepsilon^2}+\frac{4}{\varepsilon}\ln(1-v)-\frac{4\pi^2}{3}\right.$$

$$\left.-\frac{2v(1-v)}{1+v^2}\ln(1-v)-\left(\frac{1-v^2}{1+v^2}\right)\ln^2(1-v)\right]$$

$$\text{(d)}\ S_o\ \frac{\alpha_s}{2\pi}\left(-\frac{4\pi\mu^2}{t}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left(2C_F-\frac{N}{2}\right)\left[-\frac{4}{\varepsilon^2}-\frac{4}{\varepsilon}\ln\left(\frac{1-v}{v}\right)+\frac{4\pi^2}{3}\right.$$

$$\left.-\frac{2(1-v)}{1+v^2}\ln\left(\frac{1-v}{v}\right)-\left(\frac{1-v^2}{1+v^2}\right)\left(\pi^2+\ln^2\left(\frac{1-v}{v}\right)\right)\right]$$

$$S_o\equiv\frac{d\sigma^{qq}_{\text{Born}}}{dv}$$

FIGURE CAPTIONS

Fig. 1. One-hadron inclusive hadron-hadron scattering in the parton model.

Fig. 2. Born diagrams for the hard scattering of quarks and gluons.

Fig. 3. Elastic Graphs at $O(\alpha_s^3)$.

The interference between the quark-quark scattering Born term and one-loop corrections is displayed. The abbreviation w.f. stands for the corrections due to the wave function renormalization on the external legs.

Fig. 4. Gluon bremsstrahlung graphs.

Fig. 5. QCD Correction to one-hadron inclusive hadron-hadron scattering.

$\mathscr{R}$ is equal to the hadronic cross section including the QCD correction term $\mathscr{K}$ (see eqs. 41 and 48) divided by the same cross section with $\mathscr{K} = 0$. Scale-breaking distribution and fragmentation functions are used in both cross sections. We take $\theta_{cm} = 90^{\circ}$ and $\sqrt{S} = 27$ GeV. The solid lines correspond to the choice $f_{Gq}(x) = f_{qq}(1 - x)$; the dashed line corresponds to $f_{Gq}(x)$ given by eq. 22.

Fig. 6. Energy dependence of the QCD correction.

See caption to Fig. 5. We take $\theta_{cm} = 90^{\circ}$ and $f_{Gq}(x) = f_{qq}(1 - x)$.

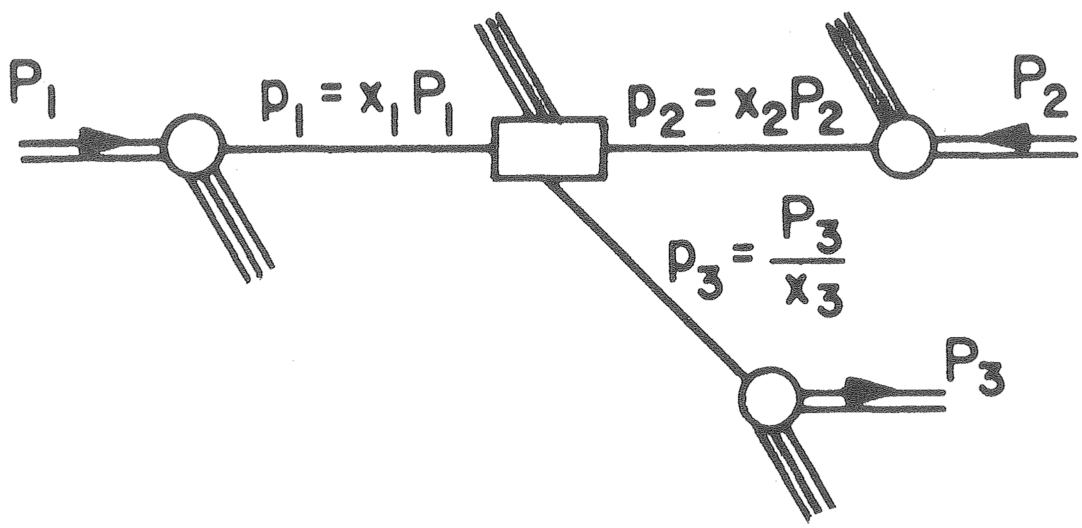

Fig. 1

XBL 802-8153

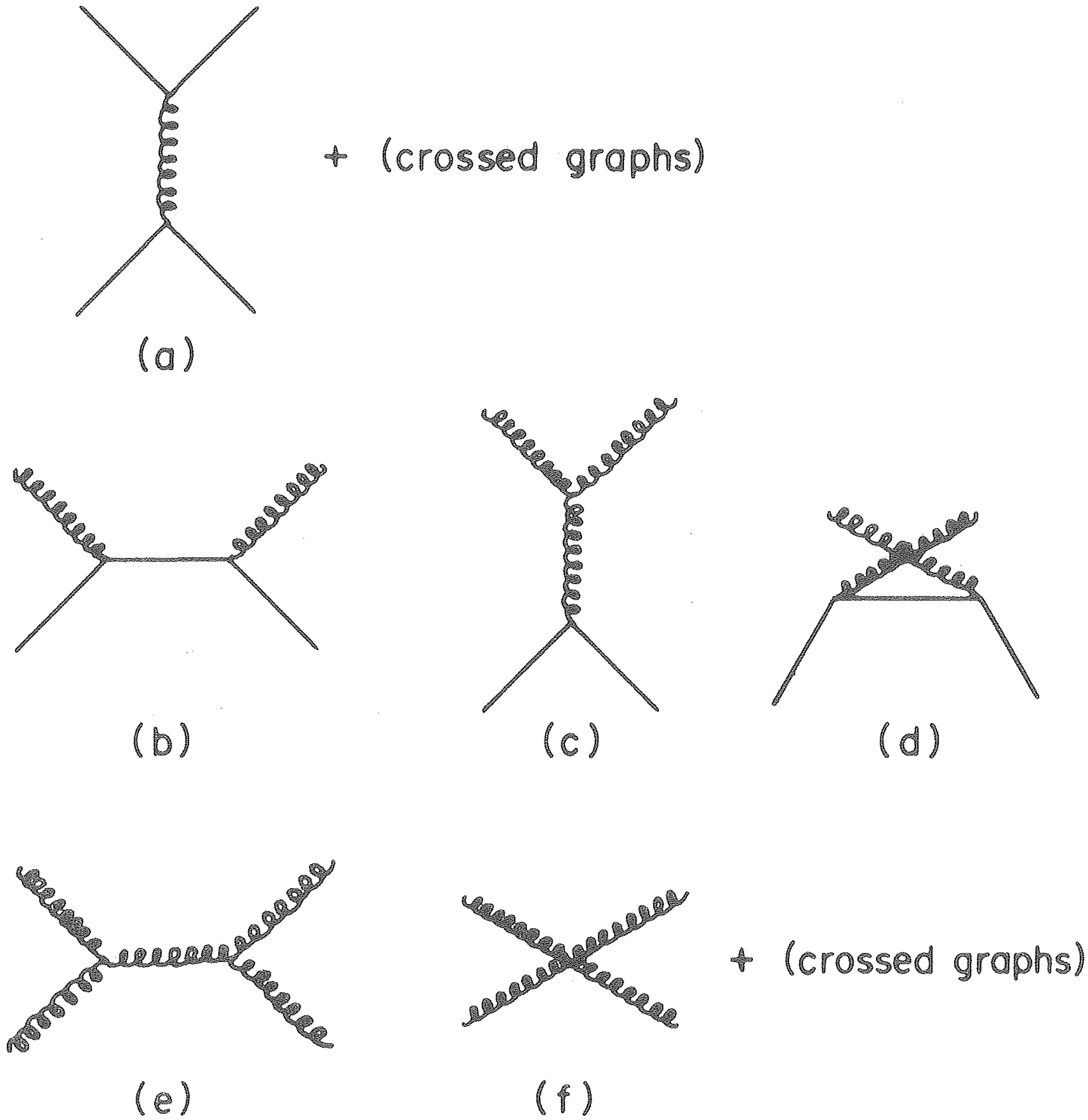

Fig. 2

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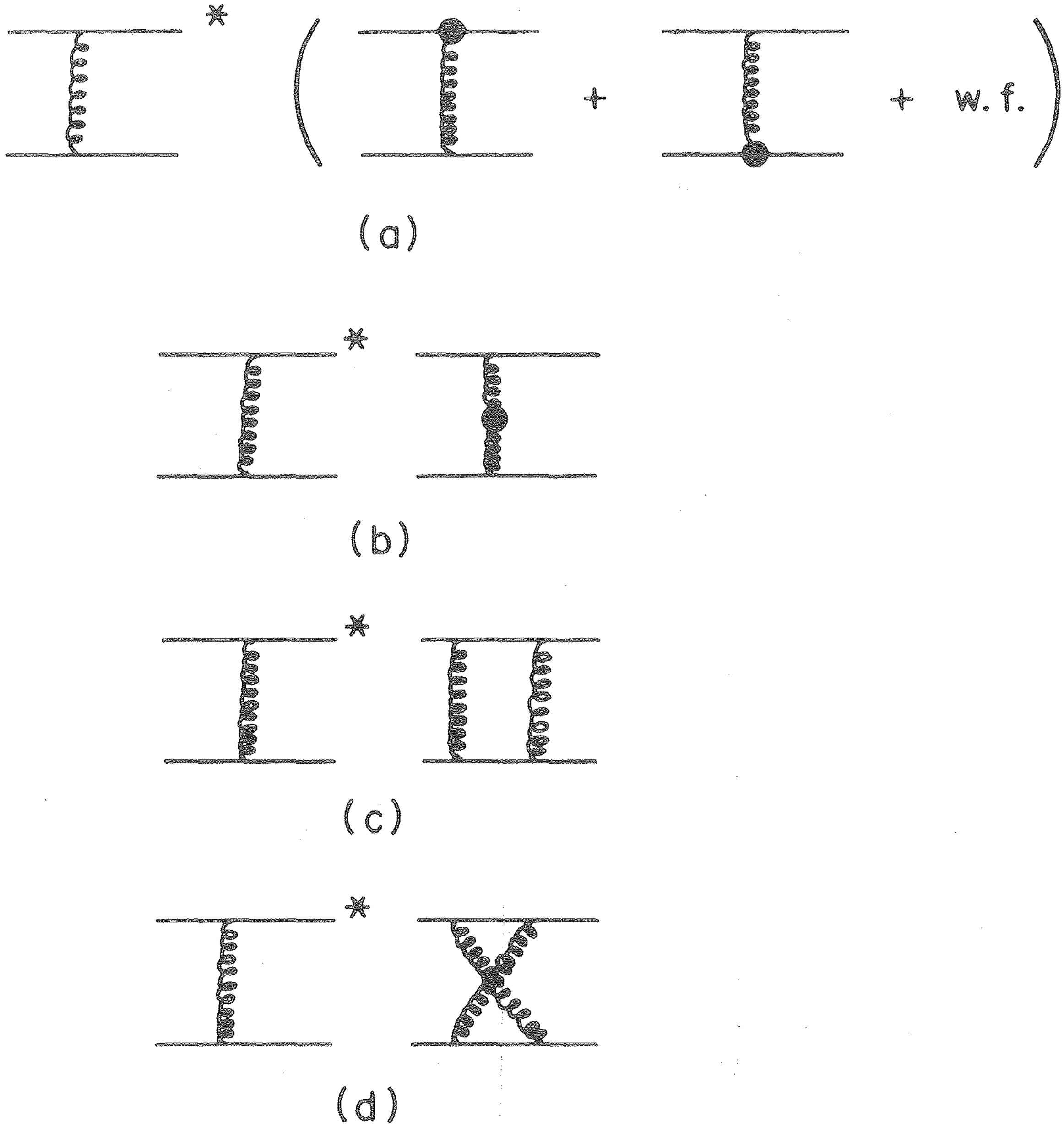

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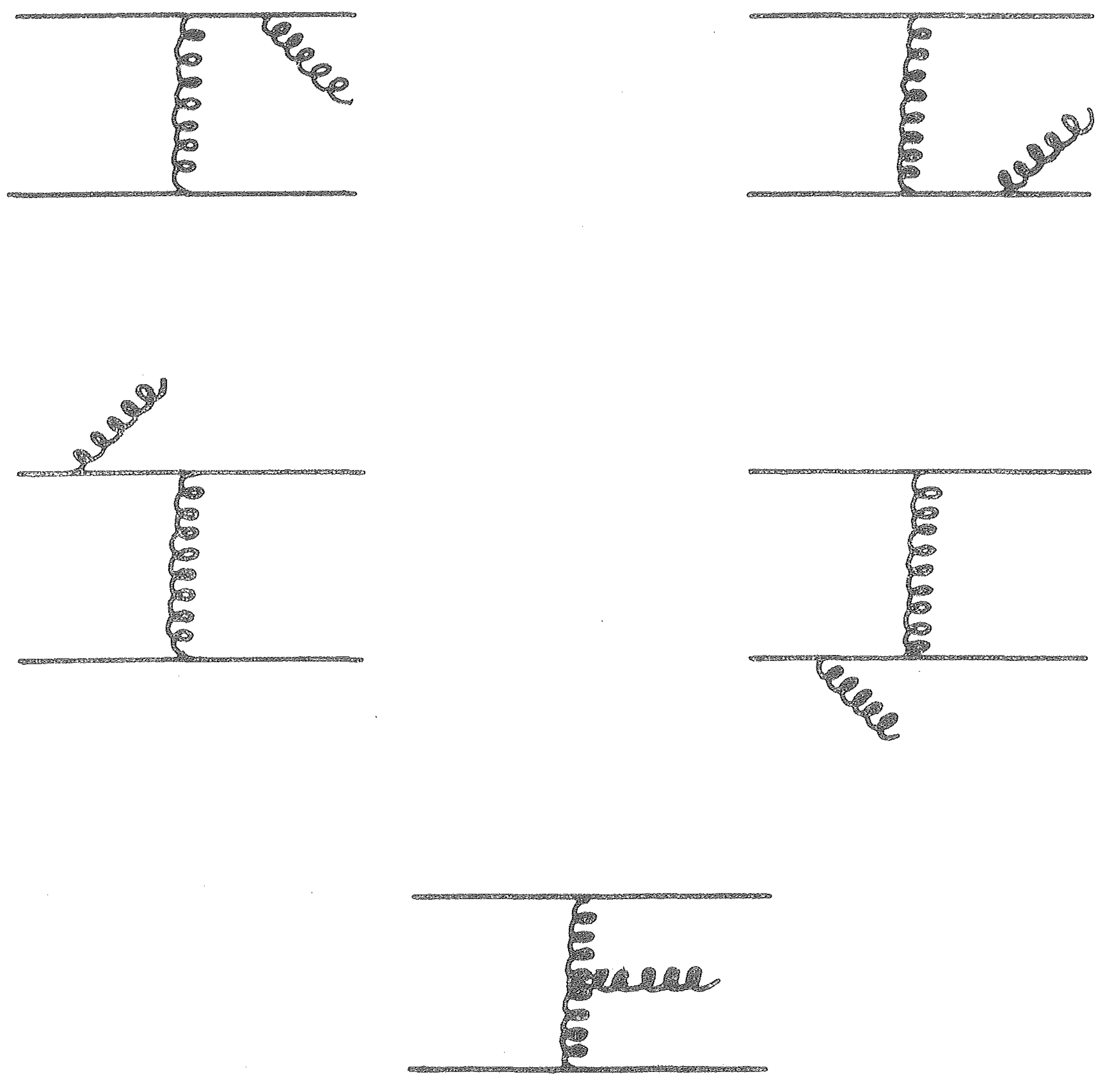

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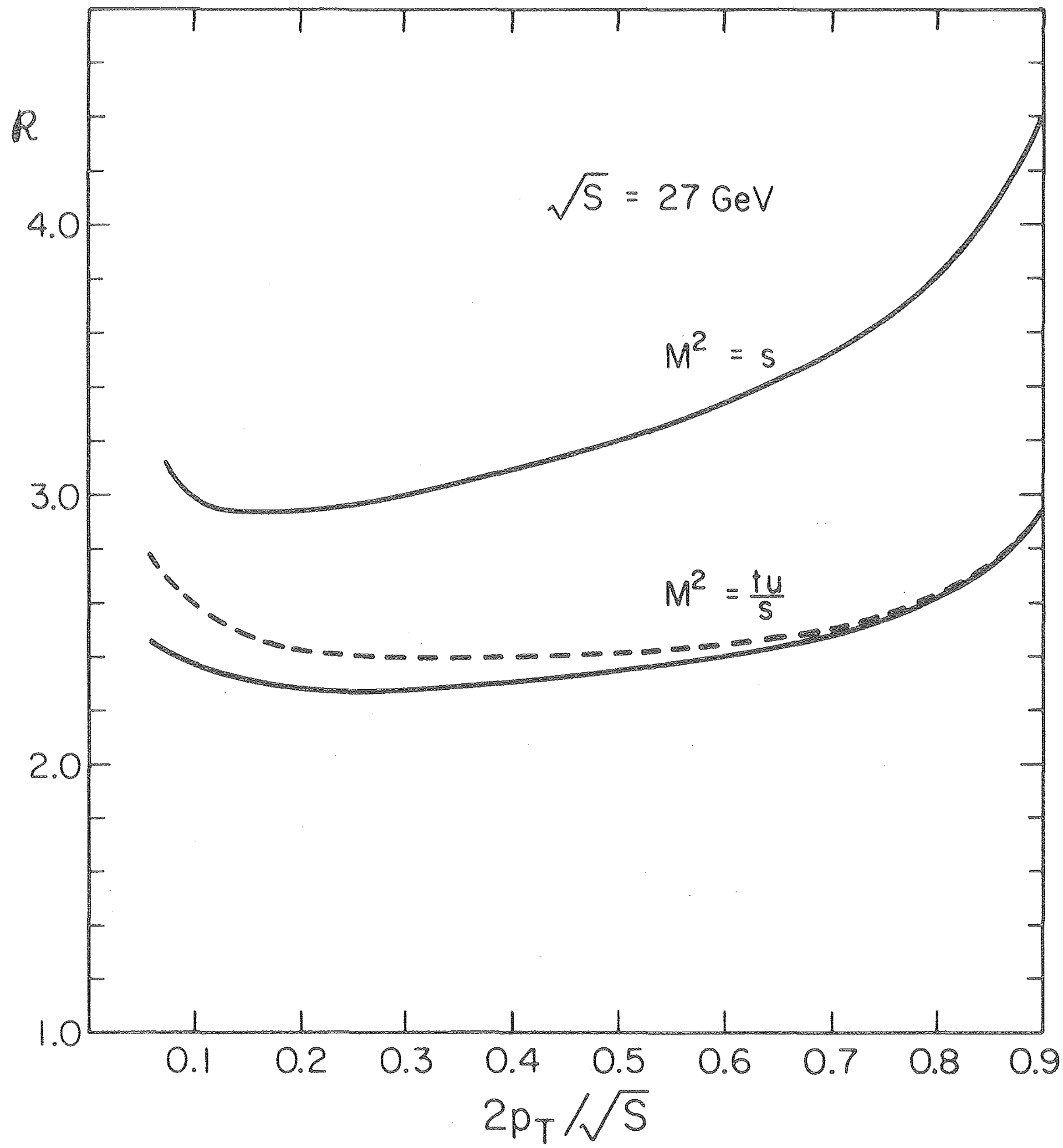

Fig. 5

XBL 802-8157

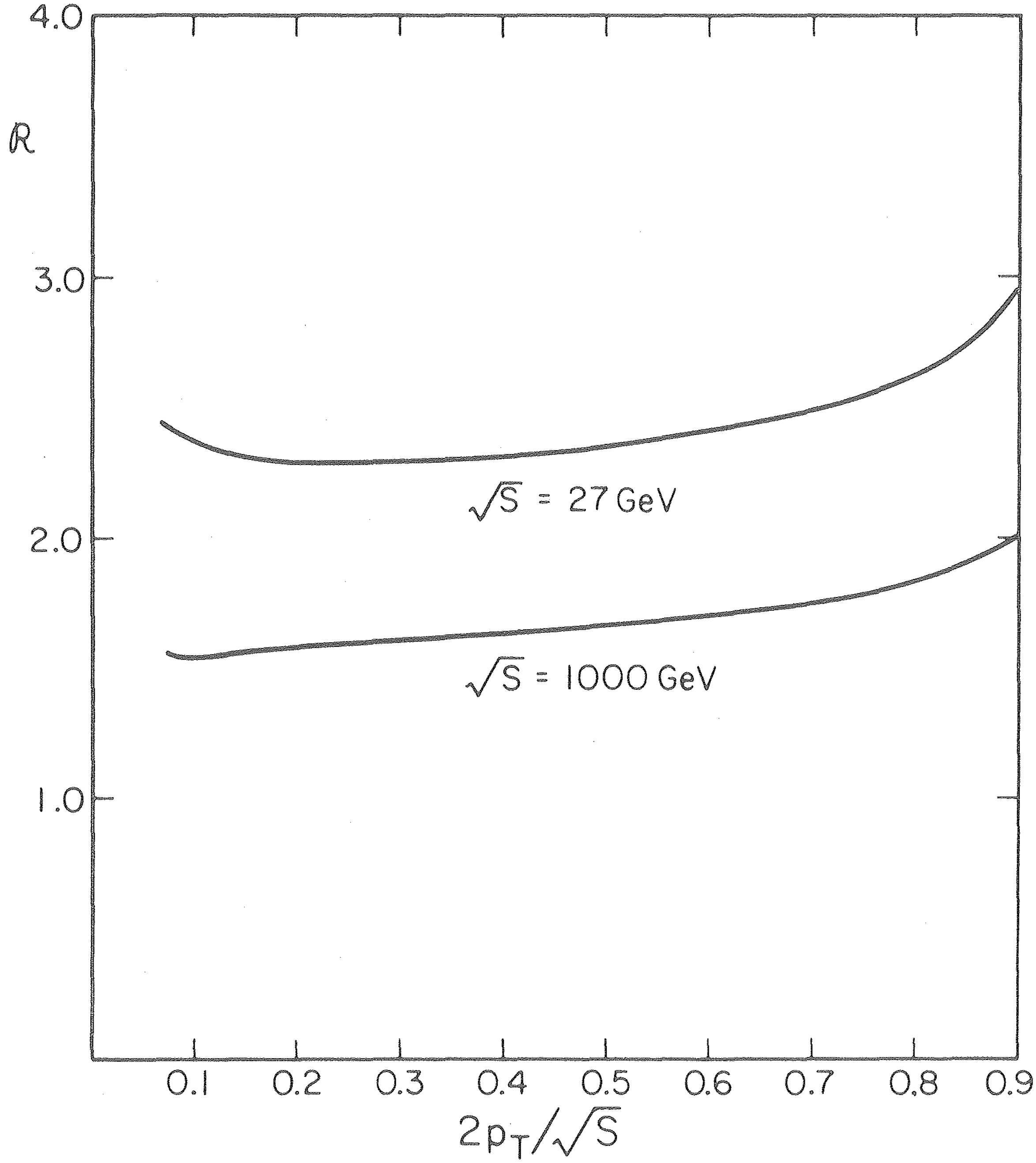

Fig. 6

XBL 802-8158