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Access and the Choice of Transit Technology

by

Karthikgeyan Sivakumaran

A dissertation submitted in partial satisfaction of
requirements for the degree of

Doctor of Philosophy

in

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in the

GRADUATE DIVISION

of the

UNIVERSITY OF CALIFORNIA, BERKELEY

Committee in Charge:

Professor Michael Cassidy, Co-Chair

Professor Samer Madanat, Co-Chair

Professor Elizabeth Deakin

Fall 2012

Access and the Choice of Transit Technology

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by

Karthikgeyan Sivakumaran

Abstract

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Karthikgeyan Sivakumaran

Doctor of Philosophy in Engineering - Civil and Environmental Engineering

University of California, Berkeley

Michael Cassidy and Samer Madanat, Co-Chairs

An urban transit system can be made more cost-efficient by improving the access to it. Efforts in this vein often entail the provision of greater mobility, as when high-speed feeder buses are used to carry commuters to and from trunk-line stations. Other efforts have focused on the creation of more favorable land-use patterns, as occurs when households within a Transit-Oriented Development (TOD) are tightly clustered around trunk stations. The efficacy of these mobility and land-use solutions are separately examined in the present work. To this end, continuum approximation models are used to design idealized transit systems that minimize the generalized costs to both the users and the operators of those systems.

The assessments unveil how the choice of rolling-stock for the trunk-line portion of a transit network can be influenced by its access mode. If transit is accessed solely (and slowly) on foot, then the optimal spacings between lines, and between the stations along those lines, are small. This can place capital-intensive rail systems at a competitive disadvantage with transit systems that feature buses instead. When access speeds increase, the optimal spacings between lines and stops expand as well. Hence, if accessed by fast-moving feeder buses, Metro-rail or bus-rapid transit can become preferred trunk-line options.

By comparison, the influence of altered land use patterns brought by TODs was found to be less dramatic. While clustering households around Metro-rail stations justifies larger spacings between the stations, it yields only modest reductions in generalized costs because the larger spacings penalize transit users who reside outside of the TODs.

To my parents and brother

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List of Symbols

User Parameters

Symbol	Description	Units
ρ	Travel demand density	trips/km ² -hr
λ	Total travel demand rate	trips/hr
μ	Average user wage	\$/hr
v_a	User access speed	km/hr
t_c	Critical trip time/Maximum acceptable time aboard feeder bus	hrs
d_c	Critical trip length	km

Parameters: City Geometry

Symbol	Description	Units
L	Length of rectangular urban area	km
W	Width of rectangular urban area	km
L_R	Length of peripheral urban area/satellite city	km
w	Width of peripheral urban area/satellite city	km

Parameters: Trunk System

Symbol	Description	Units
C_{I-L}	Infrastructure line cost (amortized) for trunk network	\$/km-hr
C_{I-S}	Infrastructure stop cost (amortized) for feeder network	\$/km-hr
C_V	Distance-related operating cost for trunk network	\$/vehicle-hr
C_M	Time-related operating cost for trunk network	\$/vehicle-km
C_{F-S}	Infrastructure stop cost for feeder network	\$/stop-hr
C_{F-V}	Distance-related operating cost for feeder network	\$/vehicle-hr
C_{F-M}	Time-related operating cost for feeder network	\$/vehicle-km
τ	Lost time at each trunk station for trunk vehicles	hrs
τ_f	Lost time at each feeder stop for feeder vehicles	hrs
t_r	Time cost for transfers between trunk vehicles	hrs
t_{f-t}	Time cost for transfers between feeder and trunk vehicles	hrs
v_t	Trunk vehicle cruising speed	km/hr
v_f	Feeder vehicle cruising speed	km/hr
K	Trunk vehicle capacity	passengers
K_f	Feeder vehicle capacity	passengers
Q_{max}	Maximum allowable trunk vehicle flow	vehicles/hr
Q_{f-max}	Maximum allowable feeder bus flow from trunk stations	vehicles/hr

Design Variables: Grid Trunk Network

Symbol	Description	Units
r_L	Longitudinal trunk line spacing	km
r_W	Latitudinal trunk line spacing	km
s_t	Trunk station spacing	km
p_L	# of stations between adjacent longitudinal trunk lines	stations
p_W	# of stations between adjacent latitudinal trunk lines	stations
H	Trunk headway	hrs

Design Variables: Feeder Bus Network

Symbol	Description	Units
r_f	Feeder line spacing	km
s_f	Feeder stop spacing	km
h_f	Feeder bus headway	hrs
k	Integer multiple for coordinated service, s.t. $h_f = kH$	-

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1 INTRODUCTION

An urban transit system can be made more cost-efficient by improving the access to it. Efforts in this vein often entail the provision of greater mobility, as when high-speed feeder buses are used to carry commuters to and from trunk-line stations. Other efforts have focused on the creation of more favorable land-use patterns, as occurs when households within a Transit-Oriented Development (TOD) are tightly clustered around transit stations. The efficacy of these mobility and land-use solutions are separately examined in the present work. Section 1.1 describes the motivation for exploration of this topic, Section 1.2 outlines the two specific research questions of interest, and Section 1.3 outlines the remainder of this dissertation.

1.1 Motivation

The choice of rolling stock is an important decision when planning either a new urban transit system, or extensions to an existing one. This choice for any city would be based in part on the city's characteristics; e.g. its spatial dimensions, the demand for travel within its boundaries, and the socio-economic attributes of its citizens. When accounting for these factors, previous studies find that capital-intensive rail systems tend to be less cost-effective than bus-rapid transit (BRT) or ordinary bus systems (Cox, 2002; Daganzo, 2010; Estrada et al., 2011; Tirachini et al., 2009). However, these earlier comparisons typically assume that access to a city's transit system would occur entirely by walking and that demand for travel would be distributed more-or-less uniformly over the city.

One wonders if these assumptions unfairly place rail at a competitive disadvantage. After all, a rail system is usually designed with large spacings, both between its lines and between the stations along those lines. To design a rail system otherwise would be prohibitively expensive. But when transit systems are designed with large spacings, such that walking time to stations is long and connectivity is poor, ridership can suffer, such that transit systems soon find themselves in search of subsidies. Some researchers have identified access to transit to be an even more important factor than travel time for users choosing between auto and transit (Beimborn et al., 2003). Others (Heilbrun, 1987) noted poor access, a consequence of large station spacings, as a key reason for the BART (Bay Area Rapid Transit) system's relatively low ridership levels. Despite all this, the role of access in the viability of an urban transit system has been often neglected in the existing literature.

1.2 Research Questions

Two questions therefore emerge: can the use of a faster-moving access mode (e.g. feeder buses) render capital-intensive technologies (e.g. rail) a more economically feasible option for a large, hierarchical transit system? Similarly, can clustering the demands for travel around its stations (e.g. so-called transit-oriented development or TOD) make rail systems more economically viable?

These questions are examined in the present work. To this end, continuum approximation models are used to estimate the generalized costs imparted to both the users and operators of certain idealized transit systems. These costs are compared across three technology options for trunk-line service: heavy rail, BRT and ordinary buses. The

effects of augmenting access to rail and BRT systems via feeder buses are explored, as are the effects of TOD.

The work finds the following. First, by considering the possibility of accessing trunk lines not only by walking but also by bus, many more cases – that is across many more cities of varying characteristics - arise where capital-intensive technologies like BRT and rail become cost-effective. Thus, the assessments unveil how the choice of rolling-stock for the trunk-line portion of a transit network can be influenced by its access mode. If transit is accessed solely (and slowly) on foot, then the optimal spacings between lines, and between the stations along those lines, are small. This can place capital-intensive rail systems at a competitive disadvantage with transit systems that feature buses instead. When access speeds increase via feeder buses, the optimal spacings between lines and stops expand as well. Furthermore, with a greater proportion of the transit system served by the lower-cost feeder bus network, the agency cost is reduced while trunk vehicles achieve a higher line-haul speed, to the benefit of their riders. Hence, if accessed by fast-moving feeder buses, rail or bus-rapid transit can become more viable trunk-line options.

With respect to TOD, the models further show that even the most successful policies to promote demand density around transit stations still produce only modest cost reductions for the rail option. This is found to be true even when the system is designed with TOD in mind. This is because concentrating demand around trunk stops can lead to large stop spacings, and in turn increased access distances for peripheral users. This increased access distance is particularly onerous if the only available access mode is walking. Nonetheless, rail can still be the appropriate trunk-line mode in certain instances, such as heavily traveled corridors where street space is lacking and high people-carrying capacities are needed.

1.3 Dissertation Outline

The remainder of this dissertation is broken down as follows. Section 2 provides background to the exploration of access, first describing the related literature, and then outlining the urban profile and transit network configuration analyzed in this work. Section 3 first applies this simplified transit network design model to cities of various shape, size, wage, and demand levels to determine the optimal network configuration and transit technology under the following two assumptions: 1) walking is the only access mode available to transit users and 2) travel demand is uniformly distributed. These two assumptions are then relaxed. Different transit technologies are compared for higher access speeds, when this higher access speed comes at no additional agency cost. Then, transit technologies are then compared coupled with a fixed-route feeder bus system. Section 4 relaxes the assumption of uniform demand by exploring the influence of clustered travel demand in the form of transit-oriented development (TOD) around rail stations. Section 5 offers closing remarks regarding the findings of this work and potential areas for future research.

2 BACKGROUND

Section 2.1 offers a description of the literature related to both transit system planning and land use as it relates to transit-oriented development (TOD). Section 2.2 describes the idealized modeling approach and the urban profiles under examination, and Section 2.3 outlines the three high-level “trunk” transit network configurations explored in later sections. Section 2.4 closes with a description of the three trunk transit technologies – bus, BRT, and rail – to be compared in this work.

2.1 Literature Review

There is a vast literature on the design of transit systems, which can be roughly classified into two categories: discrete models and continuum approximation (CA) models. For models of the first type, inputs tend to be numerous, detailed and case specific. For example, travel demands are specified in detail by means of (possibly time-dependent) origin-destination matrices. Even details of local topography and other factors that might constrain network layouts are common inputs to these so-called discrete models. As a result, models of this type invariably take cumbersome mathematical forms, such that solutions come via numerical sensitivity analysis. Thus when using these models to design either trunk networks (Zhao & Zheng, 2008) or feeder systems (Chien and Yang, 2000; Kuah & Perl, 1989; Kuan et al., 2006; Martins & Pato, 1998; Shrivastav and Dhingra, 2002; Shrivastava & O’Mahony, 2006; Uchimura et al., 2002; Verma and Dhingra, 2006) or to design both in joint fashion (Peng & Fan, 2007), the process is necessarily limited to the exploration of few and narrowly-defined alternatives.

In contrast, the second class of models, termed continuum approximations (CAs), use small numbers of continuous functions as their inputs. These may include the spatial and temporal distributions of: travel demand, the separations between parallel routes and the separations between stops that lie along these lines (Daganzo, 1996; Newell, 1973). By abstracting complex transit systems in this more approximate fashion, CAs take relatively simple closed forms that can directly unveil how inputs influence optimal design. Generic insights of this kind are ideal for high-level decision-making.

Hence, CAs have been used: to compare costs across distinct service strategies for feeder systems (Byrne and Vuchic, 1972; Chang & Schonfeld, 1991; Chang & Yu, 1996; Chien et al, 2002; Clarens and Hurdle, 1976; Diana et al, 2007; Hurdle, 1973; Li, 2009); to explore broader design alternatives for hierarchical trunk and feeder systems where the types of rolling stock to be used were specified a priori (Aldaihani et al, 2004; Wirasinghe, 1977; Wirasinghe, 1980); and as previously noted, to compare various rolling-stock alternatives for trunk networks under the assumption that users access these networks on foot (Cox, 2002; Daganzo, 2010; Estrada, 2011; Tirachini et al., 2009). Work has even compared the costs of various combinations of rolling-stock technologies (bus and rail) for trunk services, though not at the full network level. Instead, all trunk-line trips were assumed to occur along a single corridor (Fisher & Viton, 1975; Keeler, 1975). Furthermore, different access modes for the same trunk technology (e.g. walk-rail and feeder bus-rail) were not considered. In light of their advantages and proven track record, CAs are used in the present work.

With regards to transit-oriented development (TOD), while several studies have systematically examined the before-and-after effects of clustered demand around transit stations, the author knows of no works that have analytically examined the a priori influence of TOD on trunk network configuration or choice of trunk technology. CAs will also be applied to this analysis of TOD.

2.2 Idealized Modeling Approach

This section outlines the idealized urban profile and cost components analyzed in the sections that follow.

2.2.1 Urban Profile

The urban profile examined is that of a rectangular urban area of size $L \times W$ with a dense grid road network; see Figure 2.1.

Figure 2.1 Rectangular city.

The area's overall trip generation rate is given by λ [trips/hr]. This trip generation rate is assumed to display a many-to-many (M-M) profile in which origins and destinations are uniformly and independently distributed throughout the urban area. While uniformity is not necessarily reflected in real-world travel demand patterns, a transit system designed from these idealized O-D patterns can be adjusted to accommodate a city's real demand patterns; e.g. see Estrada (2011). Further, travel demand is assumed as exogenous to the transit system's design. Note in this regard that the system-optimal solution given in the analyses that follow is one that minimizes the sum of user and agency costs, and might be achieved by suitably pricing transit and the alternative travel modes (Daganzo, 2012).

For various combinations of city shape, city size, demand pattern, and demand level, continuum approximation models are developed in order to estimate the generalized costs imparted to both the users and the operators of certain idealized transit systems.

2.2.2 User Cost Assumptions

Users of the designed transit system are assumed to incur costs corresponding to three components of their travel time: A , the time spent accessing and egressing transit stations; Y , the time spent at transit stations, inclusive of both the time spent waiting for a transit vehicle and the time t_r spent transferring from one vehicle to another; and T , the time spent on-board transit vehicles.

The total user cost, Z_{User} , is thus given by the sum of these three components:
$$Z_{User} = A + Y + T.$$

2.2.3 Agency Cost Assumptions

A transit network will be designed for the rectangular city of Figure 2.1, with its vehicles assumed to operate at headway H [hrs] and passenger capacity K [passengers]. The transit agency is assumed to incur capital costs influenced both by the total length of the system's infrastructure, I_L , and the total number of stations in the network, I_S . The agency will further incur hourly operating costs, related to both the total distance collectively traveled by vehicles per hour, V , and the required transit vehicle fleet size, M .

Each of the four factors above has a monetary unit cost: C_{I-L} [\$/hr/km]; C_{I-S} [\$/hr/station]; C_V [\$/km/veh]; and C_M [\$/hr/veh].¹ These can be transformed into the equivalent times that an average user spends in the system by dividing each of the above factors by $\lambda\mu$, where the city's prevailing wage rate, μ [\$/hr], is assumed to be roughly representative of the monetary value of user time. The four components of agency cost, with dimensions of unit time, are thus obtained: $\pi_{I-L}I_L$, $\pi_{I-S}I_S$, π_VV , and π_MM .

The total agency cost, Z_A , is given by the sum of the aforementioned components:
$$Z_{Agency} = \pi_{I-L}I_L + \pi_{I-S}I_S + \pi_VV + \pi_MM$$

2.3 Transit Network Configuration: Rectangular Grid

Assume now that the rectangular city of Figure 2.1 is served by a transit system in the form of a rectangular grid, such that all trips within the network can be completed with a single transfer (see Figure 2.2). Its parallel lines, running in the x - and y - directions, are spaced at distances of r_W and r_L , as shown in the figure. As in Estrada et al. (2011), these line spacings are assumed to be integer multiples of the spacing between stations, s_t , such that $r_L = p_L s_t$ and $r_W = p_W s_t$.

2.3.1 User Costs

With regards to the user access time, A [hrs], half of all users will access a trunk line by traveling first in the y - direction for an average distance of $\frac{1}{4}r_W$, while the other half will travel in the x -direction for an average distance of $\frac{1}{4}r_L$. The average distance to access a

---

¹ For the present assessments, C_{I-L} and C_{I-S} will be linearly amortized over the assumed life of the system; see Appendix A.

station along a trunk line is thus $\frac{1}{4}s_t$. It is assumed that users cover similar distances when traveling from their final transit stations to their destinations off the network. Hence,

$$A = (0.5r_L + 0.5r_W + s_t) \frac{1}{2v_a} = (0.5p_L s_t + 0.5p_W s_t + s_t) \frac{1}{2v_a} \quad (2.1)$$

It is assumed that a user waits, on average, half of a vehicle headway, H , both at her origin station and again at her transfer station, and added to this waiting time is the transfer time, t_r . Hence, $Y = H + t_r$.

Finally, the average time spent on-board trunk vehicles, T , is the product of the user's expected on-board travel distance, $(L + W)/3$, and the vehicles' pace, $\frac{1}{v_c} = \frac{1}{v} + \frac{\tau}{s_t}$. Hence,

$$T = (L + W) \frac{1}{3v_c} = \left(\frac{L+W}{3v_t}\right) + \left(\frac{(L+W)\tau}{3}\right) s_t^{-1}. \quad (2.2)$$

The sum of A , Y , and T gives the user cost expression:

$$Z_{User} = A + Y + T = \left(\frac{(0.5p_L + 0.5p_W + 1)}{2v_a}\right) s_t + H + t_r + \left(\frac{L+W}{3v_t}\right) + \left(\frac{(L+W)\tau}{3}\right) s_t^{-1} \quad (2.3)$$

Figure 2.2 Rectangular city ($L \times W$) with a rectangular grid transit system

2.3.2 Agency Costs

The length of the transit system's line infrastructure is given by $I_L = LW \left(\frac{1}{r_W} + \frac{1}{r_L} \right)$ and the stations along this line infrastructure are spaced at s_t [km]. Thus, the number of

stations within the network is $I_S = \frac{I_L}{s_t}$. The total distance collectively traveled by vehicles per hour, V , is the product of vehicle flow, $1/H$, and $2I_L$, since service is bi-directional along the entire length of the network. Hence, $V = \frac{2I_L}{H}$ and the required fleet size of trunk vehicles, M , is simply the product of V and the expected vehicle pace, $\frac{1}{v_c}$, defined as $\frac{1}{v_c} = \frac{1}{v_t} + \frac{\tau}{s_t}$.

Thus, in summary:

$$I_L = LW \left(\frac{1}{r_w} + \frac{1}{r_L} \right) = LW \left(\frac{1}{p_w} + \frac{1}{p_L} \right) s_t^{-1} \quad (2.4)$$

$$I_S = \frac{I_L}{s_t} = LW \left(\frac{1}{p_w} + \frac{1}{p_L} \right) s_t^{-2}, \quad (2.5)$$

$$V = \frac{2I_L}{H} = 2LW \left(\frac{1}{p_w} + \frac{1}{p_L} \right) s_t^{-1} H^{-1}, \quad (2.6)$$

$$M = \frac{V}{v_c} = \frac{2LW}{v_t} \left(\frac{1}{p_w} + \frac{1}{p_L} \right) s_t^{-1} H^{-1} + 2LW\tau \left(\frac{1}{p_w} + \frac{1}{p_L} \right) s_t^{-2} H^{-1} \quad (2.7)$$

Multiplying each of these by their corresponding generalized cost conversion factors, such that monetary costs are expressed in units of time, the total agency cost is given by the sum of the aforementioned four cost components:

$$\begin{aligned} Z_{Agency} = & \pi_{I-L} LW \left(\frac{1}{p_w} + \frac{1}{p_L} \right) s_t^{-1} + \pi_{I-S} LW \left(\frac{1}{p_w} + \frac{1}{p_L} \right) s_t^{-2} + \dots \\ & \dots \left(\pi_V + \frac{\pi_M}{v_t} \right) (2LW) \left(\frac{1}{p_w} + \frac{1}{p_L} \right) s_t^{-1} H^{-1} + \pi_M 2LW\tau \left(\frac{1}{p_w} + \frac{1}{p_L} \right) s_t^{-2} H^{-1} \end{aligned} \quad (2.8)$$

2.3.3 Objective Function and Constraints

The sum of the user and agency cost expressions gives the total generalized cost, Z , as a function of the four decision variables p_L, p_w, s_t and H :

$$Z(p_L, p_w, s_t, H) = A + Y + T + \pi_{I-L} I_L + \pi_{I-S} I_S + \pi_V V + \pi_M M \quad (2.9)$$

Additionally, the transit system is constrained by its passenger-carrying capacity, K , for lines in both the N-S and E-W directions. The expected passenger load is given by the product of the trip-making density (ρ), each transit line's catchment area (either $Wp_L s$ or $Lp_w s$), the transit vehicle headway (H), and the factor $\frac{1}{4}$. The factor $\frac{1}{4}$ is derived from the bi-directionality of each line and the lines running orthogonal to the direction in question, each halving the effective line passenger demand. Thus, the objective function is constrained by both $\frac{1}{4}\rho Wp_L sH \leq K$ and $\frac{1}{4}\rho Lp_w sH \leq K$.

Additional constraints are also applied to the decision variables p_L, p_w and s_t , such that the grid contains at least two lines in either direction: $p_L s_t \leq \frac{L}{2}$ and $p_w s_t \leq \frac{W}{2}$.

Additionally, each transit line is constrained by its service capacity, Q_{max} [vehicles/hr], which differs by transit technology. This maximum allowable vehicle flow can depend on several factors, including station dwell times, the minimum spacing allowed between vehicles, the number of berths at each station, and the assumed operating margin beyond any theoretical capacity values for the line. The flow of vehicles along each line, H^{-1} , is constrained such that $H^{-1} \leq Q_{max}$.

Thus, this transit network design problem can be expressed as the following mixed-integer nonlinear mathematical program:

$$\text{Minimize} \quad Z(p_L, p_W, s_t, H) = A + Y + T + \pi_{I-L}I_L + \pi_{I-S}I_S + \pi_VV + \pi_MM$$

subject to:

$$(\frac{1}{4}\rho W p_L) s H \leq K \tag{2.10}$$

$$(\frac{1}{4}\rho L p_W) s H \leq K \tag{2.11}$$

$$p_L s_t \leq \frac{L}{2} \tag{2.12}$$

$$p_W s_t \leq \frac{W}{2} \tag{2.13}$$

$$H^{-1} \leq Q_{max} \tag{2.14}$$

For any combination of non-negative p_L and p_W , the objective function and all constraints are convex in s and H .

2.4 Transit Technology Parameters

The transit system will utilize one of three rolling-stock technologies: regular bus, BRT, or rail. For this work, the defining characteristics for each technology are as follows:

- **Bus** - minimal infrastructure costs and travel alongside regular traffic
- **BRT** - the use of articulated buses operating on grade-separated guideways
- **Rail** - some tunneling and ROW costs, with some portion of the network underground

The operating and cost parameter values for each of these systems are given in Tables 2.1 and 2.2 below, and details regarding how these values were selected are given in Appendix A. The line capacity estimates of Table 2.1 were derived from the 2nd edition of the Transit Capacity and Quality of Service Manual (2003). While this reference suggests that rail could theoretically support a train flow of 30 trains per hour, or alternatively service at 2-minute minimum headways, a more conservative service capacity constraint of 15 trains per hour is used here. BRT line capacities are assumed higher than bus line capacities due to the inclusion of transit-preferential treatment such as traffic signal priority, queue jumps, and curb extensions. Both capacities are taken to

be lower than the inverse of their respective assumed station dwell times, τ .

Table 2.1 Operating parameters for three technologies: Bus, BRT, and Rail.

	Time per Transfer t_r [secs]	Lost Time per Station τ [secs]	Cruising Speed v_t [km/hr]	Passenger Capacity K [pax]	Line Capacity Q_{max} [vehicles/hr]
Bus	10	30	25	80	20
BRT	20	30	40	120	30
Rail	60	45	60	1000	15

While there may be some argument over the precise value of cost and operating parameters, of greatest importance in the analysis that follows is the cost and speed hierarchy across the three technologies, with respect to both infrastructure (π_{I-L} , π_{I-S}) and operating (π_V , π_M) cost components: Bus is the cheapest and slowest mode, and rail the most costly but fastest. Further note that the operating and infrastructure cost parameters for the agency can vary according to the city's prevailing wage rate; and these costs will vary by technology, since a rail system will likely have a smaller percentage of costs attributable to labor in comparison to a bus system. For the assessments that follow, two levels of wage rate are examined: "Low" (\$3/hr), as roughly occurs in a city like Beijing and "High" (\$20/hr), as in a city like Los Angeles. The operating and infrastructure cost parameter values are provided for these two wage levels in Table 2.2 below.

Table 2.2 Cost parameters for three technologies: Bus, BRT, and Rail.

	Infrastructure Cost – Lines C_{I-L} [\$ /km-hr]		Infrastructure Cost – Stations C_{I-S} [\$ /station-hr]		Operating Cost – Fleet Size C_M [\$ /veh-hr]		Operating Cost – Distance C_V [\$ /veh-km]
	Low Wage	High Wage	Low Wage	High Wage	Low Wage	High Wage	
Bus	7	10	0.49	0.7	21	63	0.59
BRT	190	270	4.9	7	28	84	0.66
Rail	690	990	340	490	130	200	2.2

3 ACCESS AND THE CHOICE OF TRANSIT TECHNOLOGY: MOBILITY

This section explores how transit access improvements can enhance the cost effectiveness of a given trunk network and make capital-intensive transit technologies, such as BRT and rail, more viable options for transit service.

Section 3.1 explores the influence of access *speed* alone to a transit system on the choice of optimal transit technology. The differences in both transit technology and system performance are compared for cases when users access the system via walking

Section 3.2 explores the influence of access *mode* to a transit system on the choice of optimal transit technology. In contrast to Section 3.1, the higher access speeds provided by feeder bus service come at additional agency expense. A model of hierarchical trunk-feeder service is derived in this section, and compared to the Walk-Only models of Section 3.1.

3.1 Improved Mobility via Faster Access Speed

Using the parameters from Table 2.1, the cost function from Section 2.3.3 is now minimized in order to determine the optimal rolling-stock technology for a grid transit network serving various combinations of trip-making density and city size. Transit parameters are again taken from Tables 2.1 and 2.2. Assumed in this section is a square city with $L = W$.² The city length L is referred to as a proxy for city size. Since a square city is assumed, the transit network is also assumed to take the form of a square lattice, such that $p_L = p_W = p$ and $r_L = r_W = r$. To limit the number of integer possibilities, $p \in \{1, 2, 3, 4, 5\}$.

Since several studies (e.g. Small & Verhoef, 2007) have found that out-of-vehicle walking time to be more onerous than time spent inside vehicles or waiting at transit stations, a relatively low walking speed of 2 km/hr is used in the following assessments in order to reflect this increased time cost. These results for walking are then compared against those for a hypothetical “high-speed” (e.g. bicycle) access mode, to offer an average speed of 14 km/hr.³

With walking as the only assumed access mode, the shading in Figure 3.1 shows that for cities with a low average wage, ordinary buses would be the optimal trunk technology for wide ranges of trip-making density and city length (size). The high-wage cities in Figure 3.2 are different in that BRT becomes the most cost-effective technology for a wider range of larger and denser cities. Rail is never a preferred technology across the ranges of trip-making density and city size examined, even for cities with a high wage.

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² Inspection of Equation 2.9 reveals that, for rectangular-shaped cities of given area where $L \neq W$, generalized cost is rather insensitive to the ratio L/W ; see Appendix D. Hence, the illustrations here are limited to square cities.

³ As a reference, Moritz (1997) found the average bicycle commuter speed to be 28 km/hr; however, a much lower speed of 14 km/hr is used in this analysis since access travel typically takes place over local streets, requiring the crossing of several intersections.

Next, consider the results in Figures 3.3 and 3.4 for low- and high-wage cities when a “high-speed” access mode ($v_a=14$ km/hr) is used. Given this increase in access speed, both BRT and rail often now become more preferable.

Figure 3.1
Optimal Technology for Walk Access
Low-Wage City

Figure 3.2
Optimal Technology for Walk Access
High-Wage City

Figure 3.3
Optimal Technology for “High-Speed” Access
Low-Wage City

Figure 3.4
Optimal Technology for “High-Speed” Access
High-Wage City

Table 3.1 Design results for Low- and High-Speed Access for $\rho = 250$ trips/km²-hr and $L = 20$ km

	BRT		Rail	
	Walk	High-Speed	Walk	High-Speed
Line spacing, r [km]	0.76	2.05	1.4	3.3
Station spacing, s_t [km]	0.76	2.05	1.4	3.3
Headway, H [min]	3.8	2	4	4
Commercial speed, v_c [km/hr]	27	34	38	45
Total Generalized Cost [min]	68	40	94	45
Total Agency Cost [min]	13	5	27	10
User Cost [min]	55	35	67	35

Table 3.1 provides some insight into the reasons behind this increased preferability for BRT and Rail for higher access speeds. The table shows the difference in network design variables and parameters at optimality for a large, relatively dense city, such as Paris, with $\rho = 250$ trips/km²-hr and $L = W = 20$ km. Similar to the results of Table 3.1, increases in access speed promote larger line and station spacings, which in turn linked with reduced agency cost and higher commercial vehicle speeds.

Note from the table that both BRT and rail headways diminish when the access speed is 14 km/hr rather 2 km/hr. Yet, this increased service frequency does not increase the agency's total cost.

Idealized Transit Model

A simplified version of the Grid transit network model can be used to further illustrate the mechanism behind this increased viability: higher access speeds induce larger line and station spacings, which in turn 1) reduce the cost to the agency and 2) reduce the stopping cost to the user, promoting higher commercial speeds.

For the idealized model, in order to facilitate closed-form solutions for the optimal design variables and cost, the following assumptions are used:

- The line spacing, r , is not restricted to be an integer multiple of s_t .
- Infrastructure costs are considered “sunk” costs. Thus, $\pi_{I-L}, \pi_{I-S} = 0$.
- The trunk vehicle commercial speed does not directly account for the time lost due to acceleration/deceleration and to serving passengers at stations. Instead, a commercial speed 20% slower than the cruising speed is used: $v_c = 0.8v_t$

Modifying the user cost components of Equation 2.3 accordingly, the user access, waiting, and travel time costs are now given by:

$$Z_{User} = A + Y + T = \left(\frac{1}{2v_a}\right)r + \left(\frac{1}{2v_a}\right)s_t + H + t_r + \left(\frac{L+W}{3v_t}\right) + \left(\frac{(L+W)\tau}{3}\right)s_t^{-1}$$

Similarly modifying the agency cost components of Equation 2.8, the hourly agency cost rate is now given by

$$Z_{Agency} = 2LW \left(\pi_V + \pi_M \frac{1}{v_c} \right) r^{-1} H^{-1}$$

Therefore, the revised objective function is now:

$$Z = Z_{User} + Z_{Agency}$$

$$Z = \left(\frac{1}{2v_a} \right) r + \left(\frac{1}{2v_a} \right) s_t + H + t_r + \left(\frac{L+W}{3v_t} \right) + \left(\frac{(L+W)\tau}{3} \right) s_t^{-1} + 2LW \left(\pi_V + \frac{\pi_M}{v_c} \right) r^{-1} H^{-1} \quad (3.1)$$

Given that each of these terms is convex, and the sum of convex terms is itself convex (Bazaraa et al., 2006), the first-order conditions for optimality apply. Minimizing this function with respect to r , s_t , and H yields

$$r^* = \left(4v_a^2 (2LW) \left(\pi_V + \left(\frac{\pi_M}{v_t} \right) \right) \right)^{\frac{1}{3}} \quad (3.2)$$

$$s_t^* = \left(\frac{2}{3} v_a \tau (L + W) \right)^{\frac{1}{2}} \quad (3.3)$$

$$H^* = \left(\frac{1}{2v_a} (2LW) \left(\pi_V + \pi_M \left(\frac{1}{v_t} \right) \right) \right)^{\frac{1}{3}} \quad (3.4)$$

Given these close-form solutions for the decision variables, the agency and user cost components, in addition to total generalized cost, can be expressed solely in terms of the input parameters:

$$Z_{Agency}^* = \left(\frac{2LW}{v_a} \left(\pi_V + \frac{\pi_M}{v_c} \right) \right)^{\frac{1}{3}} \quad (3.5)$$

$$Z_{User}^* = 2 \left(\frac{2LW}{v_a} \left(\pi_V + \frac{\pi_M}{v_c} \right) \right)^{\frac{1}{3}} + \left(\frac{(L+W)}{6v_a} \tau \right)^{\frac{1}{2}} + t_r + \left(\frac{L+W}{3v_t} \right) \quad (3.6)$$

$$Z^* = 3 \left(\frac{2LW}{v_a} \left(\pi_V + \frac{\pi_M}{v_c} \right) \right)^{\frac{1}{3}} + \left(\frac{(L+W)}{6v_a} \tau \right)^{\frac{1}{2}} + t_r + \left(\frac{L+W}{3v_t} \right) \quad (3.7)$$

Using these expressions, agency and user cost curves are plotted as functions of access speed for Bus, BRT, and Rail in Figures 3.5 and 3.6, respectively. The same case study parameters are applied from Table 3.1: $\rho = 250$ trips/km²-hr and $L = W = 20$ km.

Figure 3.5 Bus, BRT, and Rail Agency Cost Curves as a Function of Access Speed

Given the noticeable reductions to both user and agency cost, generalized cost is reduced, as seen in Figure 3.7. While the agency cost curves for all three technologies appear to decrease similarly, note the differences in sensitivity to access speed with respect to generalized cost, and how higher access speed improves the viability of Rail and BRT with respect to Bus. The exact hierarchy of technologies is not of importance here since infrastructure costs were neglected in this analysis. Nonetheless, a similar sensitivity to access speed might be hypothesized for models that include infrastructure costs given similar speed-cost hierarchy across both operating and infrastructure costs.

Figure 3.6 Bus, BRT, and Rail User Cost Curves as a Function of Access Speed

Figure 3.7 Bus, BRT, and Rail Generalized cost Curves as a Function of Access Speed

Thus, higher access speeds can greatly improve the viability of various transit technologies, particularly expensive ones such as Rail and BRT. However, in the idealized illustrations thus far, increases in access speeds have come at no additional expense. In reality, nearly any fast-moving access mode, e.g. bicycle, automobile, feeder bus, will impart costs to transit users, the transit agency, or both. Therefore, Section 3.2 explores the design and cost impacts of high-speed access when the access mode in question is a fixed-route feeder bus system.

3.2 Improved Mobility via Faster Access Mode

This section explores how improving the access to transit systems can enhance the cost effectiveness of a trunk network and make capital-intensive transit technologies, such as BRT and rail, more viable options for transit service.

Section 3.2.1 offers an analysis of various access modes – walking and fixed-route feeder buses - to a single trunk station. The model is extended to a single trunk corridor in Section 3.2.2 and thereafter to a large rectangular urban area in Section 3.2.3. Section 3.2.4 then determines the optimal rolling-stock technology for various combinations of trip-making density and city size when feeder bus is included as an access mode, and compares these results to the walk-only results of Section 3.1.

Section 3.2.5 outlines some caveats to both the findings and the modeling approach, and offers some alternative solutions to remedy the noted.

3.2.1 Feeder Bus Access to a Single Trunk Station

Consider a single trunk station, as seen in Figure 3.8. Assume that the catchment area's width and length are given by w and s_t , and that the overall travel demand rate for the trunk station is given by $\lambda_s = \rho w s_t$ [pax/hr], where ρ is the catchment area's travel

demand density. A representative user's origin is given by the small diamond in the figure.

If all users access the trunk station via walking, the expected access distance is given by $\frac{1}{4}w + \frac{1}{4}s_t$. Dividing this access distance by the walking speed gives the expected access time cost when walking:

$$A_p = \frac{1}{4v_a}(w + s_t) \quad (3.8)$$

Figure 3.8 A Single Trunk Station, accessed via Walking

Consider the case shown in Figure 3.2 where users, rather than accessing the trunk station via walking, now do so by way of a fixed-route feeder bus system. Assume that: parallel feeder lines are spaced at r_f [km] with vehicles that operate at headway h_f [hrs]; stations for this feeder system are placed along each line at spacing s_f [km]; and upon reaching a trunk line, feeder lines alter their directions and converge to the trunk station. The feeder bus commercial speed, $v_{f,c}$ [km/hr] is assumed to be slightly lower than its cruising speed, v_f , to roughly factor in the time lost by stopping to serve passengers at each station, τ_f . This assumption will be relaxed in the sections that follow. The feeder bus cost⁴ per vehicle hour is given by C_f [\$/veh-hr]; as before, this monetary unit cost can be converted into a per-passenger time cost π_f [hrs/pax], via division by the area's overall demand rate, λ_s [pax/hr], and an assumed value-of-time, μ [\$/hr]. In this case, a representative user's travel path to the trunk station consists of: walking to the nearest feeder bus line; walking along that line to the nearest feeder bus stop, waiting at the feeder bus stop; and then traveling aboard the feeder bus until arrival at the trunk station. This representative user path is dotted in Figure 3.9.

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⁴ The infrastructure costs of feeder bus service are assumed to be negligible, and any distance-related costs (fuel, maintenance, etc.) are assumed to be accounted for in the time-related costs by means of an assumed feeder vehicle commercial speed.

In this scenario, one can develop a generalized per-passenger access cost function, A_f , to reflect both the user time costs in walking to feeder bus lines and stops, waiting for feeder vehicles, and riding feeder vehicles to the trunk station, as well as the feeder bus agency cost. The expected walking costs to feeder bus lines and stops is given by $\frac{1}{4v_a}(r_f + s_f)$, and the expected waiting time upon arriving at a feeder stop is given by $\frac{1}{2}h_f$. The expected riding time aboard feeder vehicles is given by the sum of the time spent traveling at the commercial speed, $(w + s_t)\frac{1}{4v_{f,c}}$, and the time spent dwelling at stops, $\frac{w}{4}\tau_f$.

Figure 3.9 Fixed-Route Feeder Access to a Trunk Stop

The sum of all these costs gives the expected user access cost, A_{f-user} :

$$A_{f-user} = \beta_1(r_f + s_f) + \beta_2 h_f + \beta_3 + \beta_4 \frac{1}{s_f} \quad (3.9)$$

where

$$\beta_1 = \frac{1}{4v_a}$$

$$\beta_2 = \frac{1}{2}$$

$$\beta_3 = (w + s_t) \frac{1}{4v_{f,c}}$$

$$\beta_4 = \frac{w\tau_f}{4}$$

The feeder bus agency cost is given by the product of the per-passenger hourly operating cost $\frac{C_f}{\lambda_s \mu}$ (in generalized units of user-hrs), and the required feeder bus vehicle fleet size, M_f . This fleet size is in turn given by the product of the bi-directional feeder

bus network length, $(2w + s_t) \frac{s_t}{r_f}$, the pace feeder buses travel across that network, $\frac{1}{v_{f,c}}$, and the feeder bus flow, h_f^{-1} :

$$M_f = \frac{s_t(2w+s_t)}{v_{f,c}} r_f^{-1} h_f^{-1} \quad (3.10)$$

The feeder bus agency cost is thus given by

$$\begin{aligned} A_{f-Agency} &= \frac{c_f}{\lambda_s \mu} M_f = \frac{c_f s_t}{\lambda_s \mu v_{f,c}} (2w + s_t) r_f^{-1} h_f^{-1} \\ &= \frac{c_f}{\rho w \mu v_{f,c}} (2w + s_t) r_f^{-1} h_f^{-1} = \frac{c_f}{\rho \mu v_{f,c}} \left(2 + \frac{s_t}{w}\right) r_f^{-1} h_f^{-1} = \beta_5 r_f^{-1} h_f^{-1} \end{aligned} \quad (3.11)$$

and the feeder bus generalized access cost, A_f , is

$$A_f = A_{f-User} + A_{f-Agency} \quad (3.12)$$

To formulate a closed-form solution for the decision variables, it will temporarily be assumed that $v_{f,c} = 0.8v_f$ in the feeder bus agency cost term though this assumption will be relaxed in Section 3.2.3. Coarse estimates of $v_{f,c}$ typically suffice since system designs at optimality are rather insensitive to this speed (see Kuah & Perl, 1989; Sivakumaran et al., 2012; Wirasinghe, 1983 and Footnote 5).

Minimizing A_f with respect to decision variables r_f , h_f , and s_f , the optimal spacing for the feeder bus lines, as a function of h_f , is given by:

$$r_f^*(h_f) = \beta_1^{-\frac{1}{2}} \beta_5^{\frac{1}{2}} h_f^{-1/2}. \quad (3.13)$$

Substituting (3.13) into (3.12), we minimize with respect to h and find the optimal feeder-vehicle headway,

$$h_f^* = (\beta_1 \beta_2^{-2} \beta_5)^{\frac{1}{3}}, \quad (3.14)$$

which upon substitution yields an optimal feeder line spacing of

$$r_f^* = (\beta_1^{-2} \beta_2 \beta_5)^{\frac{1}{3}} \quad (3.15)$$

Similarly, the optimal feeder stop spacing s_f^* can be written as

$$s_f^* = \left(\frac{\beta_4}{\beta_1}\right)^{1/2} \quad (3.16)$$

Substituting (3.14), (3.15), and (3.16) into (3.12), the generalized access cost A_f is given by:

$$\begin{aligned}
A_f^* &= \beta_3 + 2(\beta_1\beta_4)^{\frac{1}{2}} + 3(\beta_1\beta_2\beta_5)^{\frac{1}{3}} = \\
&= \left(\frac{1}{4v_f}\right)w + \left(\frac{1}{4v_f}\right)s_t + \frac{1}{2}\left(\frac{\tau_f}{v_a}\right)^{1/2} w^{1/2} + 3\left(\frac{c_f}{\rho\mu v_{f,c}}\left(2 + \frac{s_t}{w}\right)\right)^{1/3}
\end{aligned} \tag{3.17}$$

Thus, A_f can be expressed solely in terms of the station catchment area's dimensions, w and s_t .⁵ For illustration, suppose that the width of the station's catchment area is expressed as some multiple m of its length, such that $w = ps_t$ (i.e., assume each station catchment area's width:length ratio to be fixed). In this case, the coefficients β_3 , β_4 , and β_5 can all be expressed in terms of s_t :

$$\beta_3 = (ps_t + s_t) \frac{1}{4v_f} = \left(\frac{p+1}{4v_f}\right)s_t \tag{3.18}$$

$$\beta_4 = \frac{ps_t\tau_f}{4} = \left(\frac{p\tau_f}{4}\right)s_t \tag{3.19}$$

$$\beta_5 = \frac{c_f}{\rho\mu v_f} \left(2 + \frac{s_t}{ms_t}\right) = \frac{c_f}{\rho\mu v_{f,c}} \left(2 + \frac{1}{p}\right) \tag{3.20}$$

Then, the generalized access cost function for feeder bus service can be written as a simple function of the spacing s_t :

$$A_f^*(s_t) = \left(\frac{p+1}{4v_f}\right)s_t + \frac{1}{2}\left(\frac{p\tau_f}{v_a}\right)^{1/2} s_t^{1/2} + 1.5\left(\frac{c_f}{\rho\mu v_{f,c}}\left(2 + \frac{1}{p}\right)\right)^{1/3} \tag{3.21}$$

Similarly, the access cost function for walking can be written as

$$A_p = \frac{1}{4v_a}(w + s_t) = \frac{1}{4v_a}(ps_t + s_t) = \frac{1}{4v_a}(p + 1)s_t. \tag{3.22}$$

Then, assuming that a feeder bus system with $v_f = 20$ km/hr, $\tau_f = 20$ seconds, and $\frac{c_f}{\mu} = 4$ is serving an area with demand density $\rho = 250$ trips/km²-hr and $p = 1$, one can plot both A_p and A_f as functions of s_t , confirming that feeder bus access becomes more cost-effective as the size of catchment area increases (Figure 3.10).

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⁵ Note the insensitivity of the feeder line spacing, headway, and cost optimal solutions to the assumed feeder bus commercial speed v_f , e.g. $h_f^*, r_f^* \sim v_{f,c}^{1/3}$. This suggests that even if the true feeder bus commercial speed was half the assumed value, the optimal solutions for the feeder bus headway and line spacing would differ by little more than 20%.

Figure 3.10 Access Cost Comparison of Feeder Bus and Walking

Alternatively, one might simply note that the “average” feeder bus access cost with respect to s_t is constant in the case of walk access, and is falling for the case of feeder bus access:

$$\frac{A_p(s_t)}{s_t} = \frac{1}{4v_a}(p+1) \quad (3.23)$$

and

$$\frac{A_f(s_t)}{s_t} = \left(\frac{p+1}{4v_f}\right) + \frac{1}{2}\left(\frac{p\tau_f}{v_a}\right)^{1/2} s_t^{-1/2} + 1.5 \left(\frac{c_f}{\rho\mu v_{f,c}} \left(2 + \frac{1}{p}\right)\right)^{1/3} s_t^{-1} \quad (3.24)$$

3.2.2 Feeder Bus Access to a Single Trunk Line

The fixed-route feeder bus model of Figure 3.2 can now be extended to provide service to a single trunk line. This corridor-level model provides the framework for the hierarchical trunk-feeder models of Section 3.3, and alternatively can be seen as a model of feeder bus service for peripheral or “satellite” cities connected to some large urban core. Some real-world examples of such satellite cities can be found surrounding several Chinese cities, including Chengdu and Beijing.

Consider a long trunk line corridor serving some peripheral area of length L_c and width w . The trunk line extends from the CBD to the area’s far edge over a distance R_c , and delivers passengers to/from this singular destination/origin. For illustration, consider Figure 3.11, which shows a trunk line with three stops contained within the service area. For this model, one can visualize the mapping of a single trunk stop’s catchment area (refer back to Figure 3.9) to multiple trunk stations along the same corridor. With this mapping, the per-passenger cost function of Eqn. 3.12 remains unchanged:

$$\beta_5 = \frac{c_f L_c}{\lambda_c \mu v_{f,c}} (2w + s_t) = \frac{c_f}{\rho w \mu v_{f,c}} (2w + s_t) = \frac{c_f}{\rho \mu v_{f,c}} \left(2 + \frac{s_t}{w}\right)$$

Thus, both the feeder bus operating cost and user cost components remain the same as that of Eqn. 3.12, and Equation 3.17 holds at optimality for this scenario as well.

At this level of analysis, one might additionally consider the components of user cost related to waiting at trunk stations, given an exogenous trunk vehicle headway H . Here, access to the trunk line might be even further beneficial to both the users and operators of the transit system by setting the feeder bus headway to some integer multiple, k , of the trunk headway, i.e. $h_f = kH$, in order to achieve schedule coordination. The potential cost-saving benefits of this schedule coordination strategy are explored in Appendix E. Given that this strategy typically induces larger headways, it may be additionally necessary to consider the passenger-carrying capacity of feeder bus vehicles, K_f [hrs], in any analysis. This can be achieved by constraining the maximum feeder bus passenger load, $\frac{1}{2}\rho w r_f h_f$: $\frac{1}{2}\rho w r_f h_f \leq K_f$.

Figure 3.11 Trunk line connecting a peripheral service area to the CBD within a rectangular grid

At a higher level of analysis, one might also consider the trunk system's agency cost by incorporating the trunk vehicle headway, H , as an additional decision variable rather than an input parameter. In this case, setting $h_f = H$ can prove cost-effective. In the case where s_t is fixed, a reasonable assumption for short-term operational planning, coordinating trunk and feeder headways can reduce the agency costs to both the trunk and feeder systems by means of a larger coordinated headway at optimality; see Appendix E. However, given this larger operating headway, transit systems with smaller vehicles (e.g. bus or BRT) may be forced to operate at sub-optimal (smaller) headways if transit agencies do not wish to leave residual passenger queues at stops during periods of high

travel demand. This issue can be partially remedied by platooning vehicles or partitioning the service area into “express” service zones. A brief analysis of these two strategies is given in Appendix F.

As a further caveat, the confluence of several feeder bus lines at a single trunk station can lead to station congestion where there is insufficient space for storing feeder buses. Thus, similar to the line capacity constraints of Section 2, a constraint for feeder bus vehicle flow should be applied: $s_t r_f^{-1} h_f^{-1} \leq Q_{f-max}$, where Q_{f-max} is the maximum feeder bus service rate that can be supported by the trunk station. This issue of feeder bus storage becomes even more important when feeder and trunk schedules are coordinated. In the case of schedule coordination, where $h_f = kH$, the number of simultaneous feeder bus arrivals is given by the product of the number of feeder bus lines converging at the station, $\frac{s_t}{r_f}$, and the fraction of those lines that converge simultaneously, $\frac{H}{kH}: \frac{1}{k} s_t^{-1} r_f^{-1}$. This value is constrained by the number of available feeder bus berths. In spite of this constraining factor, a “pulse” scheduled transit system, in which several transit lines converge at the same location and time, has been successfully implemented in real-world cities, such as Zurich, Switzerland (Cervero, 1998).

3.2.3 Feeder Bus Access to a Trunk Network

The models of section 3.2.2 are now extended to an entire trunk network. In addition to the aforementioned costs of feeder bus access, the user time cost of the intermodal transfer between a feeder bus and a trunk vehicle is included by adding to the per-passenger transfer cost, t_{f-t} , a value dependent on the assumed trunk mode.⁶ Furthermore, the feeder bus line spacing is assumed to equal its stop spacing, i.e. $r_f = s_f$ (i.e. a square lattice). Beyond the benefits of simplicity, this assumption allows users to transfer between orthogonal feeder lines without walking to a different stop.

To obtain a more accurate accounting of agency costs for hierarchical transit systems, the following assumptions are made:

- The time lost due to stopping at feeder bus stations will now be more accurately reflected in the feeder bus operating cost term, such that the feeder bus vehicle pace during passenger collection is now $\frac{1}{v_{f,c}} = \frac{1}{v_f} + \frac{\tau_f}{s_f} = \left(\frac{1}{v_f}\right) + (\tau_f) r_f^{-1}$. Note that the pace along station detours is $\frac{1}{v_f}$, since stopping is not required along this section.
- The infrastructure cost of trunk stations in hierarchical networks, π_{I-S} is assumed proportional to the feeder bus storage capacity provided by trunk stations.

The feeder system shown in Figure 3.9 can be mapped to each trunk line in the trunk system of Figure 2.3 by setting $w = r_L = p_L s_t$ for N-S lines and $w = r_W = p_W s_t$

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⁶ An intermodal transfer between feeder bus and rail could be expected to be more costly than one between feeder bus and bus. Thus, it is assumed that $t_{f-t} = 30$ seconds for Bus, $t_{f-t} = 60$ seconds for BRT, and $t_{f-t} = 120$ seconds for rail.

for E-W lines. Since the analyses that follow assume square cities, it is further assumed that the trunk line structure follows a square lattice, $p_L = p_W = p$. The per-passenger user costs are described in the sub-sections that follow.

Note that as many as three transfers (two intermodal trunk-feeder transfers at origin and destination and a trunk system transfer) may be needed within this grid trunk-feeder system, and this concern is addressed in Section 3.2.5. In the meantime, note that this choice of network structure (with its attendant need for multiple transfers) is not an attempt to necessarily advocate its use in real settings, but rather to facilitate more general insights into the design of hierarchical transit systems.

User Costs

The revised hierarchical model considers the possibility that users will utilize alternative travel paths. For example, rather than utilizing a feeder-trunk-feeder trip chain, feeder-only travel may be preferable for trips of shorter length. Such trips might be preferred primarily because they require fewer transfers.

To roughly account for these feeder-only trips, assume that users of a hierarchical trunk-feeder system must choose between a feeder-trunk-feeder trip chain and a feeder-feeder trip chain. Assume too that the users will estimate the feeder bus trip time to reach their destinations, and if this estimated time falls below some critical time t_c , the users will only utilize the feeder system. This proportion of “Feeder-Only” users is given by P_F . The remaining proportion who utilize the trunk system is given by $P_T = 1 - P_F$.

Users are assumed to estimate their feeder trip times given an optimistic feeder travel speed of v_f , such that a critical trip length d_c is defined by $d_c = v_f t_c$. Since origins and destinations uniformly distributed, the trip length can be represented by a random variable Z , and the proportion P_F of Feeder-Only users is given by the probability $P_F = P(Z \leq d_c)$. The expected travel distance $\bar{d}_{feeder}$ aboard the feeder bus system for these Feeder-Only users is given by the conditional expectation $\bar{d}_{feeder} = E[Z|Z \leq d_c]$. Likewise, the proportion of trunk users and their expected travel distance aboard the trunk is given by $P_T = P(Z \geq d_c)$, with $\bar{d}_{trunk} = E[Z|Z \geq d_c]$. These conditional expectations and probabilities were derived using the probability distribution for trip length derived by Fairthorne (1965); formulas are given in Appendix B.

The weighted sum of their generalized user access (A_{f-User}), out-of-vehicle station (Y), and riding (T) cost components gives the per-passenger user cost across these two groups. These components are derived below.

Both groups share the same costs in walking to feeder bus stations and waiting for feeder bus vehicles: $\left(\frac{1}{v_a}\right)r_f + h_f$. However, only Trunk users will utilize the feeder bus system as an access mode to trunk stations and will thus incur an additional cost $\frac{(p_L+1)}{2v_f}s_t + \left(\frac{p_L\tau_f}{2}\right)s_tr_f^{-1}$. The weighted sum of these cost components gives

$$A_{f-User} = \left(\frac{1}{v_a}\right)r_f + h_f + P_T \frac{(p_L+1)}{2v_f}s_t + P_T \left(\frac{p_L\tau_f}{2}\right)s_tr_f^{-1} \quad (3.25)$$

Additionally, only trunk users experience the waiting (H) and transfer costs, both intermodal ($2t_{f-t}$) and intramodal (t_r), of the trunk system, so

$$Y' = P_T(H + 2t_{f-t} + t_r) \quad (3.26)$$

With regards to the in-vehicle travel time component T' , Feeder-Only users incur travel time cost while detouring to trunk stations in addition to their expected travel time $\frac{\bar{d}_{feeder}}{v_{f,c}}$. Since a square lattice is assumed for trunk lines ($r_L = r_W = ps_t$), the number of detours of length $\frac{s_t}{2}$ experienced by Feeder-Only users is $\frac{\bar{d}_{feeder}}{ps_t}$. Denoting the travel time of Feeder-Only users as T_1 :

$$T_1 = \frac{\bar{d}_{feeder}}{v_{f,c}} + \left(\frac{\bar{d}_{feeder}}{ps_t}\right)\left(\frac{s_t}{2}\right)\frac{1}{v_f} = \bar{d}_{feeder}\left(\frac{1}{v_f} + \frac{1}{2pv_f}\right) + (\bar{d}_{feeder}\tau_f)r_f^{-1} \quad (3.27)$$

The in-vehicle travel time for trunk users is similarly

$$T_2 = \frac{\bar{d}_{trunk}}{v_t} + (\bar{d}_{trunk}\tau)s_t^{-1} \quad (3.28)$$

Their weighted sum gives the per-passenger travel time component T' , Feeder-Only users will travel aboard feeder bus vehicles a distance $\bar{d}_{feeder}$ at speed $v_{f,c}$ with some additional distance traveled at speed v_f due to feeder bus detours at trunk stations. Since a square lattice is assumed for trunk lines ($r_L = r_W = ps_t$), the number of detours of length $\frac{s_t}{2}$ experienced by Feeder-Only users is $\frac{\bar{d}_{feeder}}{ps_t}$. Denoting their travel time as T_1 , the feeder riding cost of these users is given by:

$$T' = P_F\frac{\bar{d}_{feeder}}{v_f}\left(1 + \frac{1}{2p}\right) + P_F(\bar{d}_{feeder}\tau_f)r_f^{-1} + P_T\frac{\bar{d}_{trunk}}{v_t} + P_T(\bar{d}_{trunk}\tau)s_t^{-1} \quad (3.29)$$

The revised user cost for the radial trunk-feeder system is thus $A_{f-User} + Y' + T'$.

Agency Cost

The infrastructure cost of feeder bus stops is proportional to their number, which is given by the product of the feeder line-haul network length ($2LWr_f^{-1}$), and the feeder stop density r_f^{-1} . Thus, the feeder stop infrastructure cost is $(\pi_{F,S}2LW)r_f^{-2}$. The length of the feeder bus network, which includes detours to trunk stations, is given by twice the value of the network length term in Equation 3.11, but with w equal to ps_t : $LW\left(4 + \frac{2}{p}\right)r_f^{-1}$. Multiplying this value by the flow of feeder buses, h_f^{-1} , the distance-related feeder bus cost component is $\pi_{F,V}LW\left(4 + \frac{2}{p}\right)r_f^{-1}h_f^{-1}$. The time-related costs while in

motion are proportional to the network length, the cruising vehicle pace, $\frac{1}{v_f}$, yielding a cost component of $\frac{\pi_{F,M}}{v_f}LW\left(4 + \frac{2}{p}\right)r_f^{-1}h_f^{-1}$. Finally, the time-related costs while stopping are given by the product of the assumed feeder bus dwell time τ_f , the number of bus stops along the feeder network $4LWr_f^{-2}$, and the feeder bus flow, h_f^{-1} , yielding a time-related cost component of $\pi_{F,M}LW(4\tau_f)r_f^{-2}h_f^{-1}$. Thus,

$$A_{f-Agency} = (\pi_{F,S}2LW)r_f^{-2} + LW\left(4 + \frac{2}{p}\right)\left(\pi_{F,V} + \frac{\pi_{F,M}}{v_f}\right)r_f^{-1}h_f^{-1} + \pi_{F,M}LW(4\tau_f)r_f^{-2}h_f^{-1}. \quad (3.30)$$

The trunk agency cost components are the same as those in Equations 2.4 through 2.7 and the total per-passenger agency cost is $Z_{agency} + A_{f-Agency}$.

Objective Function and Constraints

The objective function for the grid hierarchical trunk-feeder system is given by the sum of the user cost components above and the agency cost components from Section 3. Additionally, the constraints represented by Equations 2.11 through 2.14 for passenger-carrying capacity, line capacity, and line and station spacing for the trunk system are also applied. However, now the physical constraints of the feeder bus system are also considered.

The passenger-carrying capacity of the feeder bus system, K_f , must support the passenger load from both Feeder-Only and trunk users. The load contribution from Feeder-Only users is given by the product of: their trip-making density, $P_F\rho$; each transit line's catchment area, either Wr_f or Lr_f ; the feeder vehicle headway h_f ; and the factor $\frac{1}{4}$. The load contribution from trunk users is given by the product of their demand density, 0.5ρ , the feeder segment catchment area, $\frac{r_f r_L}{2} = \frac{1}{2}p s_t r_f$, and the feeder vehicle headway h_f . The feeder passenger capacity constraint is thus

$$P_F\left(\frac{\rho L}{4}\right)r_f h_f + P_T\left(\frac{\rho p}{4}\right)s_t r_f h_f \leq K_f \quad (3.31)$$

Recall that the feeder bus storage capacity at trunk stations must also be considered: $s_t r_f^{-1} h_f^{-1} \leq Q_{f-max}$. If this constraint renders the optimization infeasible (i.e. the set of constraints are mutually inconsistent), the optimization will be repeated with sufficient relaxation of Q_{f-max} to ensure feasibility and the trunk station infrastructure cost π_{I-S} will be increased by a proportional amount to account for this

increase in feeder vehicle storage capacity.⁷ The trunk system constraints from Section 2 are also applied here, but the trunk passenger load value is discounted by P_F to reflect fewer trunk passengers.

For a given p , the aforementioned objective function and constraints collectively yield the mixed-integer nonlinear mathematical program below:

Minimize

$$Z(r_f, h_f, s_t, H, p) = A_{f-User} + Y' + T' + \pi_{I-L}I_L + \pi_{I-S}I_S + \pi_VV + \pi_MM + A_{f-Agency} \quad (3.32)$$

subject to:

$$(\frac{1}{4}\rho L p P_T) sH \leq K \quad (3.33)$$

$$p s_t \leq \frac{L}{2} \quad (2.12)$$

$$H^{-1} \leq Q_{max} \quad (2.14)$$

$$P_F \left(\frac{\rho L}{4} \right) r_f h_f + P_T \left(\frac{\rho p}{4} \right) s_t r_f h_f \leq K_f \quad (3.31)$$

$$s_t r_f^{-1} h_f^{-1} \leq Q_{f-max} \quad (3.34)$$

For simplicity, $p \in \{1, 2, \dots, 5\}$. However, unlike Equation 2.9, the objective function and constraints both contain non-convex terms, such as $s_t r_f^{-1} h_f^{-1}$ and $s_t r_f^{-1}$. Nonetheless, because all terms of the objective function and inequality constraints are posynomials⁸, this mathematical program can be converted into a convex optimization problem (for given p) via geometric programming. This transformation is now described

First, the objective function and constraints are re-written in “standard form” of a geometric program for a given integer p :

Minimize

$$Z(r_f, h_f, s_t, H) = \kappa_1 + \kappa_2 r_f + h_f + \kappa_3 s_t + \kappa_4 s_t r_f^{-1} + \kappa_5 H + \kappa_6 r_f^{-1} + \kappa_7 s_t^{-1} + \kappa_8 r_f^{-2} + \kappa_9 r_f^{-1} h_f^{-1} + \kappa_{10} r_f^{-2} h_f^{-1} + \kappa_{11} s_t^{-2} + \kappa_{12} s_t^{-1} H^{-1} + \kappa_{13} s_t^{-2} H^{-1} \quad (3.32b)$$

subject to:

---

⁷ In reality, adding additional feeder bus berths to a trunk station often provides diminishing returns to capacity. An optimistic proportional increase in capacity (and cost) is nonetheless assumed here for simplicity.

⁸ A monomial can be described as a function $f(x) = C x_1^{a(1)} x_2^{a(2)} \dots x_n^{a(n)}$ where the constant C is greater than or equal to 0 and the exponential constants are any real numbers: $a^{(j)} \in \mathbf{R}$. A posynomial is simply a sum of monomials.

$$\omega_1 sH \leq 1 \quad (3.33b)$$

$$\omega_2 s_t \leq 1 \quad (2.12b)$$

$$\omega_3 s_t H^{-1} \leq 1 \quad (2.14b)$$

$$\omega_4 r_f h_f + \omega_5 s_t r_f h_f \leq 1 \quad (3.31b)$$

$$\omega_6 s_t r_f^{-1} h_f^{-1} \leq 1 \quad (3.34b)$$

where the coefficients are defined as follows:

$$\kappa_1 = P_T(2t_{f-t} + t_r) + P_F \frac{\bar{d}_{feeder}}{v_f} + P_F \frac{\bar{d}_{feeder}}{2pv_f} + P_T \frac{\bar{d}_{trunk}}{v_t}$$

$$\kappa_2 = \frac{1}{v_a}$$

$$\kappa_3 = P_T \frac{(p_L+1)}{2v_f}$$

$$\kappa_4 = P_T \left(\frac{p_L \tau_f}{2} \right)$$

$$\kappa_5 = P_T$$

$$\kappa_6 = P_F(\bar{d}_{feeder} \tau_f)$$

$$\kappa_7 = P_T(\bar{d}_{trunk} \tau) + \pi_{I-L} \left(\frac{2LW}{p} \right)$$

$$\kappa_8 = (\pi_{F,S} 2LW)$$

$$\kappa_9 = LW \left(4 + \frac{2}{p} \right) \left(\pi_{F,V} + \frac{\pi_{F,M}}{v_f} \right)$$

$$\kappa_{10} = \pi_{F,M} LW (4\tau_f)$$

$$\kappa_{11} = \pi_{I-S} \left(\frac{2LW}{p} \right)$$

$$\kappa_{12} = \left(\frac{4LW}{p} \right) \left(\pi_V + \left(\frac{\pi_M}{v_t} \right) \right)$$

$$\kappa_{13} = \pi_M \left(\frac{4LW\tau}{p} \right)$$

and

$$\omega_1 = \frac{\rho L p P_T}{4K}$$

$$\omega_2 = \frac{2p}{L}$$

$$\omega_3 = Q_{max}^{-1}$$

$$\omega_4 = \frac{P_F \rho L}{4K}$$

$$\omega_5 = \frac{P_T \rho p}{4K_f}$$

$$\omega_6 = Q_{f-\max}^{-1}.$$

Consider now the transformed variables $\tilde{r}_f, \tilde{h}_f, \tilde{s}_t$ and $\tilde{H}$, where:

$$\begin{aligned}\tilde{r}_f &= \log(r_f) \text{ or } r_f = e^{\tilde{r}_f} \\ \tilde{h}_f &= \log(h_f) \text{ or } h_f = e^{\tilde{h}_f} \\ \tilde{s}_t &= \log(s_t) \text{ or } s_t = e^{\tilde{s}_t} \\ \tilde{H} &= \log(H) \text{ or } H = e^{\tilde{H}}\end{aligned}$$

Substituting for the original variables, and noting that any coefficient c can be expressed as $e^{\log(c)}$, a revised objective function $\tilde{Z}$ and set of constraints are obtained:

Minimize

$$\begin{aligned}\tilde{Z}(\tilde{r}_f, \tilde{h}_f, \tilde{s}_t, \tilde{H}) &= e^{\log(\kappa_1)} + e^{\tilde{r}_f + \log(\kappa_2)} + e^{\tilde{h}_f} + e^{\tilde{s}_t + \log(\kappa_3)} + e^{\tilde{s}_t - \tilde{r}_f + \log(\kappa_4)} + \\ &e^{\tilde{H} + \log(\kappa_5)} + e^{-\tilde{r}_f + \log(\kappa_6)} + e^{-\tilde{s}_t + \log(\kappa_7)} + e^{-2\tilde{r}_f + \log(\kappa_8)} + e^{-\tilde{r}_f - \tilde{h}_f + \log(\kappa_9)} + \\ &e^{-2\tilde{r}_f - \tilde{h}_f + \log(\kappa_{10})} + e^{-2\tilde{s}_t + \log(\kappa_{11})} + e^{-\tilde{s}_t - \tilde{H} + \log(\kappa_{12})} + e^{-2\tilde{s}_t - \tilde{H} + \log(\kappa_{13})}\end{aligned}\quad (3.32c)$$

subject to:

$$e^{\tilde{s}_t + \tilde{H} + \log(\omega_1)} \leq 1 \quad (3.33c)$$

$$e^{\tilde{s}_t + \log(\omega_2)} \leq 1 \quad (2.12c)$$

$$e^{\tilde{s}_t - \tilde{H} + \log(\omega_3)} \leq 1 \quad (2.14c)$$

$$e^{\tilde{r}_f + \tilde{h}_f + \log(\omega_4)} + e^{\tilde{s}_t + \tilde{r}_f + \tilde{h}_f + \log(\omega_5)} \leq 1 \quad (3.31c)$$

$$e^{\tilde{s}_t - \tilde{r}_f - \tilde{h}_f + \log(\omega_6)} \leq 1 \quad (3.34c)$$

The transformed objective function and constraints are obtained by taking the logarithm of each, yielding an equivalent mathematical program:

$$\text{Minimize } \log(\tilde{Z}(\tilde{r}_f, \tilde{h}_f, \tilde{s}_t, \tilde{H})) \quad (3.35)$$

subject to:

$$\log(e^{\tilde{s}_t + \tilde{H} + \log(\omega_1)}) \leq 0 \quad (3.36)$$

$$\log(e^{\tilde{s}_t + \log(\omega_2)}) \leq 0 \quad (3.37)$$

$$\log(e^{\tilde{s}_t - \tilde{H} + \log(\omega_3)}) \leq 0 \quad (3.38)$$

$$\log(e^{\tilde{r}_f + \tilde{h}_f + \log(\omega_4)} + e^{\tilde{s}_t + \tilde{r}_f + \tilde{h}_f + \log(\omega_5)}) \leq 0 \quad (3.39)$$

$$\log(e^{\tilde{s}_t - \tilde{r}_f - \tilde{h}_f + \log(\omega_6)}) \leq 0 \quad (3.40)$$

For a given p , the objective function is now represented by the logarithm of a sum of exponential functions, $\log(\sum e^{g_i(\tilde{x})})$, which is convex for convex $g_i(\tilde{x})$. Since all exponentiated functions are linear in $\tilde{r}_f, \tilde{h}_f, \tilde{s}_t$ and $\tilde{H}$, the transformed objective function is convex. Further, each constraint is now represented by either a linear or convex log-sum function. Thus, for a given p , the mathematical program above takes on a convex form and yields globally optimal solutions to the original problem.

3.2.4 Optimal Trunk Technology with Feeder Bus Access

The trunk-feeder system of the previous section can render rail and BRT systems more cost-effective, as will be shown in the sections that follow.

Figure 3.12 shows the optimal combinations of trunk and feeder technologies for low-wage (\$3/hr) square-shaped cities of varying physical length, L , and trip-making density, ρ . Figure 3.13 shows the same for high-wage (\$20/hr) cities. In both cases, the feeder bus cruising speed, v_f , is assumed 20 km/hr with $\tau_f = 20$ seconds of dwell time lost at each stop. The feeder bus stop infrastructure-, distance-, and time-related cost parameters are all assumed to be 80% those of the corresponding values for bus. The critical time is assumed to be $t_c = 0.4$ hrs (24 minutes). Thus, for trips longer than $t_c v_f = 0.4(20) = 8$ km, users will utilize the trunk system accessed by feeder buses. Users traveling shorter distances will only use the feeder bus system as their sole mode of transport. The passenger-carrying capacity of feeder buses is assumed to be $K_f = 80$, lower than the capacity for bus due to the assumed smaller size of the former.

Figure 3.12
Optimal Trunk Technology for Feeder Access
(Low-Wage Cities)

Figure 3.13
Optimal Trunk Technology for Feeder Access
(High-Wage Cities)

By visually comparing Figures 3.12 and 3.13 with their counterparts, Figures 3.1 and 3.2, one can see that the landscape is dramatically changed by having included feeder buses as an option for access. Note how rail now becomes an optimal choice for larger, denser cities, especially where wages are high. The reason for rail’s “new-found” preferability is revealed in the details of case-specific analyses. These analyses come next.

Case Studies

First considered is a large, high-wage city like Paris with: trip-making density $\rho = 250$ trips/km²-hr; city area $L = W = 20$ km; and wage rate $\mu = \$20/\text{hr}$. Table 3.2 furnishes the city’s optimal configuration and resulting costs for rail, when access occurs on foot and when it occurs via feeder buses. Note from the table the increased commercial speed afforded by feeder bus access, as well as reductions in user and agency cost.

The reason for these favorable outcomes is unveiled by comparing the optimal network configurations in Table 3.2. Note how rail’s line and station spacings both increase when the network is accessed by feeder buses. With higher-speed access, greater portions of the costly rail network are replaced with a less expensive network of feeder buses. Rail can thus become a more attractive option for trunk-line service. Continuing in this vein, next compared are generalized costs (in units of minutes/passenger) for three trunk-access combinations: (i) a bus network accessed by walking; (ii) a BRT network accessed by feeder buses; and (iii) a rail network accessed again by feeder buses. These comparisons are performed for representative cities of the world, each with its own distinct physical size and trip-making density.

Table 3.2
Trunk Network Configuration and Cost for Walk and Feeder Bus Access: Rail

	Walk Access	Feeder Bus Access
Rail line spacing, r [km]	1.4	6
Rail station spacing, s_t [km]	1.4	3
Rail headway, H [min]	4	4
Rail commercial speed, v_c [km/hr]	39	48
Total Generalized Cost [min]	94	70
Total Agency Cost [min]	27	10
Feeder-Bus Agency Cost [min]	-	5
User Cost [min]	67	60

The generalized costs for these trunk-access mode combinations in various low- and high-wage cities are presented in Tables 3.3 and 3.4, respectively. The optimal choice of combination for each city is highlighted in the tables with shading. Note from Table 3.3 that rail is never the optimum choice for low-wage cities, though it comes close in larger, denser cities like Delhi. Similarly, Table 3.10 shows that rail can be very close to optimal in larger denser cities with high wages. Given the level of uncertainty in the assumed cost, demand, and trip length distribution parameters, rail may actually be the optimal choice where cost estimates for rail are close to those for BRT, e.g. cities such as

Yokohama or Paris. Moreover, rail systems may favourably transform cities over the long run in ways that the preceding analysis cannot capture.

Table 3.3

Generalized Costs [mins/passenger]

for Low-Wage, Square Cities of Varying Trip-Making Density and Length

		TRIP-MAKING DENSITY					
		Low ~50 trips/km ² -hr		Medium ~150 trips/km ² -hr		High ~250 trips/km ² -hr	
CITY SIZE	Small ~ 10 km	<i>Mombasa, Kenya</i>		<i>Quito, Ecuador</i>		<i>Port Au Prince, Haiti</i>	
		Bus-Walk	80	Bus-Walk	62	Bus-Walk	57
		BRT-Feeder	115	BRT-Feeder	80	BRT-Feeder	68
		Rail-Feeder	209	Rail-Feeder	111	Rail-Feeder	92
	Medium ~20 km	<i>Dakar, Senegal</i>		<i>Manila, Philippines</i>		<i>Ho Chi Minh City, Vietnam</i>	
		Bus-Walk	99	Bus-Walk	83	Bus-Walk	78
		BRT-Feeder	124	BRT-Feeder	92	BRT-Feeder	81
		Rail-Feeder	168	Rail-Feeder	117	Rail-Feeder	100
	Large ~30 km	<i>Havana, Cuba</i>		<i>Rio De Janeiro, Brazil</i>		<i>Delhi, India</i>	
		Bus-Walk	119	Bus-Walk	104	Bus-Walk	99
		BRT-Feeder	136	BRT-Feeder	103	BRT-Feeder	93
		Rail-Feeder	175	Rail-Feeder	125	Rail-Feeder	110

Table 3.4

Generalized Costs [mins/passenger]

for High-Wage, Square Cities of Varying Trip-Making Density and Length

		TRIP-MAKING DENSITY					
		Low ~50 trips/km ² -hr		Medium ~150 trips/km ² -hr		High ~250 trips/km ² -hr	
CITY SIZE	Small ~ 10 km	<i>St. Louis, USA</i>		<i>Tel-Aviv, Israel</i>		<i>Barcelona, Spain</i>	
		Bus-Walk	62	Bus-Walk	52	Bus-Walk	48
		BRT-Feeder	76	BRT-Feeder	56	BRT-Feeder	50
		Rail-Feeder	96	Rail-Feeder	70	Rail-Feeder	59
	Medium ~20 km	<i>Dublin, Ireland</i>		<i>New York, USA</i>		<i>Paris, France</i>	
		Bus-Walk	82	Bus-Walk	73	Bus-Walk	71
		BRT-Feeder	88	BRT-Feeder	69	BRT-Feeder	63
		Rail-Feeder	106	Rail-Feeder	79	Rail-Feeder	70
	Large ~30 km	<i>Warsaw, Poland</i>		<i>Yokohama, Japan</i>		<i>Seoul, South Korea</i>	
		Bus-Walk	102	Bus-Walk	95	Bus-Walk	92
		BRT-Feeder	99	BRT-Feeder	81	BRT-Feeder	76
		Rail-Feeder	114	Rail-Feeder	88	Rail-Feeder	80

3.2.5 Caveat: Transfer Penalty

One drawback to the implementation of a hierarchical trunk-feeder network is the potentially significant transfer penalty imposed on its users. However, this penalty may be lessened by: (i) reducing the disutility of transfers and (ii) reducing the number of transfers. Reducing the disutility of transfers may be achieved via thoughtful design of stations, e.g. promoting same-platform transfers, in conjunction with other forms of system connections. A reduction in the number of transfers required may be achieved by means of alternative network configurations, such as the ones described below.

Trunk-Feeder System with Uni-Directional Trunk Lines

Consider the case in which trunk lines only run in one direction, with feeder bus lines running both orthogonally and parallel to trunk lines. Trips in this network require at most two transfers, as exemplified by the dotted arrows in Figure 3.14:

Figure 3.14 A Trunk-Feeder System with Uni-directional Trunk Lines

Dual BRT-Bus System

While not applicable to rail systems, BRT systems can make use of a bus' ability to travel both on local streets and on BRT corridors. Thus, a bus within a BRT system may furnish "feeder" service while traveling along local streets, but then transition to a long-haul "trunk" service upon turning onto a BRT corridor.

Hybrid Grid-Radial Trunk-Feeder System

An intriguing potential exists for feeder bus access within the hybrid grid-radial models introduced by Daganzo (2011) and Estrada (2011). These hybrid models, which consist of a grid network structure within the interior of a city and a radial network structure in its periphery, might greatly benefit from the introduction of feeder bus rings connecting all the radial line segments. Such a network would allow for travel from any point to any point using no more than two transfers, while (potentially) allowing larger station and line spacings in the region's periphery, where people are more likely to use feeder buses, and smaller station and line spacings in the region's center, where users are more likely to walk.

4 ACCESS ON THE CHOICE OF TRANSIT TECHNOLOGY: LAND USE

Now explored is the influence of land use, specifically the consolidation of trip origins and destinations, on the choice of transit technology.

4.1 The Cost of Access when Demand is Non-Uniform

First examined is the impact of zones of higher trip-making density that are clustered closely around rail stations, i.e. Transit-Oriented Development. To this end, suppose that the shaded, square-shaped zone in Figure 4.1 delineates a TOD. Assume that all users access the rail station by walking along a dense rectangular grid of N - S and E - W streets or sidewalks. Hence all points along the TOD's perimeter lie a distance, d_{max} , from the trunk station, as measured in $|x| + |y|$ space. For simplicity, assume that the rail network is a square grid, with $p_L = p_W = 1$, such that the access cost can be written as a simple function of s_t .

Figure 4.1 Trunk Station Catchment Area with TOD Zone

The literature reports that the deployment of a rail system does not increase aggregate demand for travel as much as it redistributes that demand to locations that lie closer to the system (Bollinger & Ihlanfeldt, 1997; Cambridge Systematics et al., 1998; Cervero et al, 2004; Knight & Trygg, 1977). In light of this, consider what happens when a fixed baseline demand for the catchment area, of area $s_t \times s_t$ in Figure 4.1, is redistributed such that disproportionately high fractions of that baseline are thereafter contained in the shaded TOD zone.

The trip-making density within the TOD zone, ρ_{TOD} , is taken to be some multiple of the density outside it, ρ_0 ; i.e., $\rho_{TOD} = f\rho_0$, where $f \geq 1$. The total demand for the rail station, $D_{station} = \rho s_t^2 = \frac{\lambda}{LW} s_t^2$, where ρ is the average (or baseline) density in the $s_t \times s_t$

catchment area. The $D_{station}$ is the sum of the demands within the catchment area that reside both inside and outside of the TOD, D_{TOD} and D_0 , respectively. Hence, we have:

$$D_{TOD} + D_0 = D_{station}$$

$$\rho_{TOD} 2d_{max}^2 + \rho_0(s_t^2 - 2d_{max}^2) = \rho s_t^2$$

giving a density outside the TOD of

$$\rho_0 = \rho \frac{s_t^2}{s_t^2 + (f-1)2d_{max}^2}. \quad (4.1)$$

This density, ρ_0 , can be viewed as a background density in the catchment area, and we concern ourselves with the TOD's density over and above this background, $\rho_{TOD} - \rho_0$. The average distance from anywhere within the catchment area to the trunk station is $\frac{1}{2}s_t$; and from within the TOD zone, the average distance is $\frac{2}{3}d_{max}$, as determined by geometric probability. Thus, the expected access distance, $\bar{a}$, can be expressed as

$$\bar{a} = \frac{\rho_0 s_t (\frac{1}{2}s_t) + (\rho_{TOD} - \rho_0) 2d_{max} (\frac{2}{3}d_{max})}{\rho s_t^2}, \text{ or equivalently,}$$

$$\bar{a} = \frac{\frac{4}{3}d_{max}^3(f-1) + \frac{1}{2}s_t^3}{2d_{max}^2(f-1) + s_t^2} \quad (4.2)$$

to underscore the role of f . The access and egress time component, A_{TOD} , is thus $2\bar{a}/v_a$.

4.2 Generalized Costs

Explored now are the cost impacts of demand redistribution around transit stations, i.e. TOD. These impacts are evaluated by replacing the access cost, A , in Equation 2.9 with A_{TOD} and then minimizing the resulting expression numerically. Illustrations are furnished in Figure 4.2, which presents costs for rail as functions of f . Parameters are the same as in the previous case study and are specified in the caption of Figures 4.2 and 4.3.⁹

Note from the figure that the generalized cost diminishes as f increases, indicating that TOD can improve rail's economic footing. Yet, the marginal reductions are small, and these diminish with f . Even for $f = 10$, the reduction in total generalized cost is only about 6%, as compared against a uniform trip-making density with $f = 1$.

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⁹ The maximum acceptable walking distance has typically been defined in the literature as ranging from 0.4 to 0.8 kms, with users often more willing to walk further distances for rapid transit systems. Distances within this range have also been used to define TOD areas (Cervero et al., 2004). An intermediate value of 0.5 km for d_{max} is chosen for this work.

The percent reductions to the transit agency are smaller still. As a reference, we note that TOD ridership has been found to be three- to five-times higher than regional averages in various case studies for rail systems (Cervero et al., 2004); and that the case of $f = 10$ corresponds to a TOD demand roughly five times higher than the baseline (or average) trip-making density in the idealized city used in this work.

Figure 4.2 Relationship of Cost Components to Trip-Making Density Distribution

Figure 4.3 Expected Access Distance for non-TOD Users

The marginal benefits are small because consolidating demand closer to rail stations leads to larger optimal spacings between those stations. Thus, with increasing f , the users who reside within the TOD zone are subject to an expected access distance of $\frac{2}{3}d_{max}$. Yet, those users residing within the catchment periphery (see again Figure 4.2) incur greater access distances due to the larger station spacings. Figure 4.3 shows how the expected access distance for users within the periphery increases with f . This effect

negates some of the benefits of consolidating demand via TOD when planning a transit system.

4.3 Optimal Transit Technology with TOD

The influence of TOD on the choice of rolling stock is now explored. This is done for high-wage, square-shaped cities of length, L ; varying trip-making densities, ρ ; and where access to transit occurs on foot.

The shadings in Figure 4.4 show the ranges of L and ρ for which bus, BRT, and rail are the optimal technologies in the absence of TOD, such that ρ is uniform (i.e. $f = 1$). Figure 4.5 shows the optimal choices when rail (only) is accompanied by TOD and where we optimistically assume that $f = 10$.

Comparison of the two figures shows that with TOD, rail becomes the optimal choice for small of L and ρ . The difference is rather modest, despite the optimistically assumption that TOD would redistribute demand in very favorable ways.

Figure 4.4
Optimal Trunk Technology without Rail TOD
(High-Wage Cities)

Figure 4.5
Optimal Trunk Technology with Rail TOD
(High-Wage Cities)

Nonetheless, this is not to say that TOD does not yield benefits, but only that accounting for the impacts of TOD in designing transit systems yields marginal benefits. TOD planning around existing rail stations may indeed be beneficial in reducing system-wide access costs. Furthermore, the finding suggests the importance of providing alternative access, such as feeder bus service, to those users located further away from trunk stations, even in cases where there is high demand clustered around those stations. Recall this is because the high access cost for these peripheral users can counterbalance the benefits of clustered demand.

5 CONCLUDING REMARKS

This dissertation has explored how the economic footing of capital-intensive transit systems, like rail or BRT, might be improved by enhancing access to them. Continuum approximation models were applied to idealized settings, since this approach is known to furnish useful insights that are still relevant to more complicated real-world systems. This approach was used to examine access in terms of both mobility (via the introduction of faster-moving feeder buses) and land use (via TOD).

5.1 Summary of Research Findings

Access: Mobility

As regards mobility, this work has found that traveling to and from trunk-line stations by feeder bus, rather than by walking, can improve the cost effectiveness of transit systems. The higher access speeds dictate that the optimal spacings between trunk lines and stations both increase, such that a greater proportion of the transit system is served by the lower-cost feeder buses. Cost reductions occur even when short trips utilize only the subordinate feeder-bus system. Hence, by integrating feeder systems early in the design process, capital-intensive transit systems can become optimal or near-optimal alternatives.

Access: Land Use

As regards land use, the work has found that when walking is the only access mode, modest cost reductions come via TODs that clump travel demand close to rail stations. But the optimal spacings between these stations grow as a result, and this penalizes users who live (or work) outside of a TOD. Had models accounted for transit user sensitivity to access distance, a reduction in the rail system ridership might be expected. Hence the benefits of planning rail systems around TODs seem to be limited.

5.2 Research Contribution

The results show that 1) access can play a key role in determining the optimal choice of transit technology and 2) hierarchical systems can lend cost-saving benefits to capital-intensive transit technologies. That rapid access can improve the viability of capital-intensive transit systems is good news given the transformative effects that rail or BRT can have on a city. Thus, these modified designs may be of particular interest to cities in the developing world, many of which have yet to invest in major transit systems.

A secondary contribution lies in the application of continuum approximation modeling to design of hierarchical transit networks. Despite the idealized assumptions that underpin macroscopic models of this type, mesoscopic- (e.g. line capacity) and microscopic-level (e.g. feeder bus station capacity) constraints were readily incorporated into network design in conjunction with passenger behavior (e.g. mode segregation). Additionally, the modeling approach yielded a convex optimization problem. Since convex optimization yields global optimal solutions, this ensures that comparisons between transit technologies are between the “best-case” scenario for each. Such a guarantee cannot necessarily be made by discrete modeling approaches that utilize heuristic solution methods.

Furthermore, the simple, convex form of the model may lend itself well to the high-level planning of real-world systems. Although travel demand patterns are often non-uniform in practice, the uniformity assumptions made in this work may in fact be conservative, since planned lines can often be relocated in order to more favorably serve focused demand patterns. Adjusting the locations of lines and stops to real-world travel patterns does not necessarily result in costs far from the idealized optimal, since agency costs are far more dependent on the total number and network length of all lines, along with the service frequency on each line. Computational benefits are also offered. In analyzing a city, a discrete modeling approach for each transit technology and each hierarchical transit technology combination can suffer from combinatorial explosion.

While there are caveats to the theoretical network designs that produced the aforementioned results, alternative designs have also been proposed which address some of these caveats in certain cases. These include a hierarchical trunk-feeder system with trunk lines running in one direction and feeder lines running in the other.

5.3 Future Work

There are several potential directions for extending the analysis of transit access

- *Alternative access modes:* Other transit access modes might also be modeled, including flex-route feeder buses, taxis, and/or other forms of informal transit service, which are quite common in the developing world. Additionally, bicycle-share systems might be explored as a feeder system to a city-wide transit system, with bicycle depots placed at all transit stations in conjunction with depots optimally located throughout a city.
- *Heterogeneity in user access modes:* A more advanced model of transit service might segregate users by access distance, with those near transit stations walking to/from transit stations, and those further away riding feeder buses, bicycles, or other modes in order to access/egress transit stations.
- *Greenhouse Gas emissions:* The environmental impacts, both in regards to tailpipe emissions as well as the potential savings from diverting individual automobile trips, may also be worth further evaluation. Of particular interest may be an analysis of greenhouse gas (GHG) emissions of hierarchical-trunk feeder systems, particularly rail-feeder systems. Although a Rail-Feeder system may be cost-competitive with a BRT-Walk system, the former might potentially produce more harmful environmental impacts. Thus, an evaluation of both the environmental impacts of hierarchical systems 1) a posteriori, at cost optimality, and 2) a priori, as a cost component in the design of a transit system, may be worthwhile pursuits in comparing the pros and cons of various transit technologies.

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Appendix A. Agency Cost Parameters

The following are cost estimates for the “High” wage city, defined to be a city with an average wage of ~\$20/hr. For each mode, costs are decomposed into infrastructure and operating cost components (both distance- and time-related). First presented is Rail.

Table A1. Agency Cost Parameter Breakdown for Rail (High-Wage City)

	Parameter	Value	Comments
Infrastructure Costs	Lifespan [years]	30	Estimated 30-year life, assuming regular track maintenance
	Lifespan [hrs]	189,000	Operation 350 days/yr and 18 hrs/day
	Infrastructure Line Cost [\$ /km]	\$140,000,000	In 2001 \$, from Flyvberg et al., (2008)
	Infrastructure Line Cost [\$ /km]	\$187,000,000	Infrastructure cost in 2012 \$
	Amortized Infrastructure Line Cost, C_{I-L} , [\$ /km-hr]	\$990	Straight-line amortization
	Infrastructure Station Cost [\$ /station]	\$40,000,000	Infrastructure cost in 1983 \$, from Flyvberg et al. (2008)
	Infrastructure Station Cost [\$ /station]	\$92,000,000	Infrastructure cost in 2012 \$
	Amortized Infrastructure Station Cost, C_{I-S} , [\$ /station-hr]	\$490	Straight-line amortization
Operating Costs (Distance)	Energy Consumption per Car [kWh/car-mi]	3.6	From Sfeir and Chow (1992) regarding BART system’s energy consumption
	Energy Consumption per Train [kWh/train-mi]	36	Assumed 10-car trains
	Energy Cost per kW-hr	\$0.10	Average from Electric Power Monthly (2011)
	Energy Cost per Train-Mile [\$ /veh-mi]	\$3.60	
	C_V , Cost per veh-km [\$ /veh-km]	\$2.20	
Operating Costs (Time)	# Employees per Vehicle	5	From Wilson (2010) and Pushkarev & Zupan (1972)
	Average Wage [\$ /hr]	\$20	Based on high-wage city from UBS (2010)
	Labor Cost per hr [\$ /hr]	\$100	
	Purchase Price of Vehicle [\$]	\$20,000,000	Estimate of \$2M per train car from Wilson (2010), assuming 10-car trains.
	Vehicle Lifespan [yrs]	30	
	Depreciation per hr [\$ /hr]	\$101	Assumed straight-line depreciation
	C_M , Cost per vehicle-hr [\$ /veh-hr]	\$200	

Table A2. Agency Cost Parameter Breakdown for BRT (High-Wage City)

	Parameter	Value	Comments
Infrastructure Costs	Lifespan [years]	20	Assuming 20-year life, assuming regular track maintenance
	Lifespan [hrs]	131400	
	Infrastructure Line Cost [\$ /mile]	\$32,000,000	Infrastructure cost in 1989 \$, from Kain et al. (1992)
	Infrastructure Line Cost [\$ /km]	\$36,108,800	Infrastructure cost in 2012 \$
	Amortized Infrastructure Line Cost, C_{I-L} , [\$ /km-hr]	\$270	Straight-line amortization
	Infrastructure Station Cost [\$ /station]	\$500,000	Infrastructure cost in 2001 \$
	Infrastructure Station Cost [\$ /station]	\$670,000	Infrastructure cost in 2012 \$
	Amortized Infrastructure Station Cost, C_{I-S} , [\$ /station-hr]	\$7	Straight-line amortization
Operating Costs (Distance)			
	Maintenance Cost per Vehicle-Mile [\$ /veh-mi]	\$0.30	In 2007 \$, from Clark et al. (2007) for 40-ft buses; assumed that use of articulated buses increases maintenance cost from \$0.20/mile to \$0.30/mile
	Fuel Price [\$ /gal]	\$4.00	
	Fuel Efficiency [mpg]	6	From Clark et al. (2007)
	Fuel Cost per Mile [\$ /mile]	\$0.67	
	Cost per Veh-Mile [\$ /veh-mi]	\$0.97	
Operating Costs (Time)	C_v , Cost per veh-km [\$ /veh-km]	\$0.66	Adjusted for inflation, converted to 2011 \$
	# Employees per Vehicle	4	From Wilson (2010) and Pushkarev & Zupan (1972)
	Average Wage [\$ /hr]	\$20	Based on high-wage city from UBS (2010)
	Labor Cost per Hr [\$ /hr]	80	
	Purchase Price of Vehicle [\$]	\$500,000	From Wilson (2010) for articulated 60 ft. bus
	Vehicle Lifespan [yrs]	20	
Operating Costs (Time)	Depreciation per Hr [\$ /hr]	\$3.81	Assumed straight-line depreciation
	C_M , Cost per veh-hr [\$ /veh-hr]	\$84	

Table A3. Agency Cost Parameter Breakdown for Bus (High-Wage City)

	Parameter	Value	Comments
Infrastructure Costs	Lifespan [years]	20	Assuming 20-year life, assuming regular track maintenance
	Lifespan [hrs]	131400	
	Infrastructure Line Cost [\$/mile]	\$1,600,000	Infrastructure cost in 1989 \$, from Kain et al. (1992)
	Infrastructure Line Cost [\$/km]	\$1,805,440	Infrastructure cost in 2011 \$
	Amortized Infrastructure Line Cost, C_{I-L} , [\$/km-hr]	\$10	Straight-line amortization
	Amortized Infrastructure Station Cost, C_{I-S} , [\$/station-hr]	\$0.70	For simplicity, Bus station costs are assumed to be one-tenth the costs of BRT station costs
Operating Costs (Distance)	Maintenance Cost per Vehicle-Mile [\$/veh-mi]	\$0.20	In 2007 \$, from Clark et al. (2007) for 40-ft buses; assumed that use of articulated buses increases maintenance cost from \$0.20/mile to \$0.30/mile
	Fuel Price [\$/gal]	\$4.00	
	Fuel Efficiency [mpg]	6	From Clark et al. (2007)
	Fuel Cost per Mile [\$/mile]	\$0.67	
	Cost per Veh-Mile [\$/veh-mi]	\$0.87	
	C_V , Cost per veh-km [\$/veh-km]	\$0.59	Adjusted for inflation, converted to 2011 \$
Operating Costs (Time)	# Employees per Vehicle	3	From Wilson (2010) and Pushkarev & Zupan (1972)
	Average Wage [\$/hr]	\$20	Based on high-wage city from UBS (2010)
	Labor Cost per Hr [\$/hr]	60	
	Purchase Price of Vehicle [\$]	\$350,000	From Wilson (2010) for articulated 60 ft. bus
	Vehicle Lifespan [yrs]	20	
	Depreciation per Hr [\$/hr]	\$2.66	Assumed straight-line depreciation
	C_M , Cost per veh-hr [\$/veh-hr]	\$63	

We assume the cost parameters for low-wage cities to differ as follows. Regarding operating cost components, the hourly labor cost in low-wage cities is assumed to be \$6/hr, due to an assumed premium for semi-skilled labor. Other components (fuel, maintenance, etc.) are assumed to be the same.

For simplicity, infrastructure costs across cities of different wage are assumed to differ by means of a 30% cost reduction. This is roughly based on the cost differences seen between heavy rail projects in Latin America and those in Europe and the U.S. from Flyvberg et al. (2008).

Appendix B. Trip Length Distribution in a Rectangular City

Assume for a rectangular city of width W and length L that user origins and destinations are respectively represented by $X - Y$ coordinate pairs (x_1, y_1) and (x_2, y_2) . Random variables x_1 and x_2 are assumed to be uniformly distributed between 0 and L and random variables y_1 and y_2 are assumed uniformly distributed between 0 and W .

Take Z as a random variable representing the trip distance: $Z = |x_1 - x_2| + |y_1 - y_2|$. Fairthorne (1965) gives the probability distribution of Z as

$$\begin{aligned} f_Z(z) &= \frac{2z}{3L^2W^2} (6LW - 3z(L + W) + z^2) \quad \text{for } 0 \leq z \leq W \\ f_Z(z) &= \frac{2}{3L^2} (3L + W - 3z) \quad \text{for } W \leq z \leq L \\ f_Z(z) &= \frac{2}{3L^2W^2} (L + W - z)^3 \quad \text{for } L \leq z \leq L + W \end{aligned}$$

Consider now the users of a hierarchical grid trunk-feeder transit system. Optimistically assuming feeder buses travel at cruising (rather than commercial) speed v_f , users will choose the feeder bus system as their sole transport mode as long as their estimated feeder bus travel time falls below a critical trip time t_c . A critical trip length can then be defined as $d_c = v_f t_c$, and thus the proportion of “Feeder-Only” users is given by the probability P_F , such that $P_F = P(Z \leq d_c) = \int_0^{d_c} f(t) dt$.

The expected trip length for these Feeder-Only trips, $\bar{d}_{feeder}$, is given by the following conditional expectation:

$$E[Z|Z \leq d_c] = \frac{1}{P(Z \leq d_c)} \int_0^{d_c} t f_Z(t) dt = \frac{1}{F_Z(d_c)} \int_0^{d_c} t f_Z(t) dt$$

where $F_Z(z)$ is the cumulative distribution function of z .

Evaluation of this integral yields the following expressions for $\bar{d}_{feeder}$, depending on the value of d_c

$$\bar{d}_{feeder} = \begin{cases} E[Z|Z \leq d_c] = \frac{d_c(40LW - 15Ld_c - 15Wd_c + 4d_c^2)}{5(12LW - 4Ld_c - 4Wd_c + d_c^2)} & \text{for } 0 \leq d_c \leq W \\ E[Z|Z \leq d_c] = \frac{2Wd_c^2 + L(6d_c^2 - W^2) - \frac{W^3}{5} - 4d_c^3}{L(12W - 12d_c) - 4Wd_c + 6L^2 - 3W^2 + 6d_c^2} & \text{for } W \leq d_c \leq L \\ E[Z|Z \leq d_c] = \frac{4d_c^5 + L^5 + 5L^4W + 20(L+W)^2d_c^3 + 5LW^4 + W^5 - 15(L+W)d_c^4 - 10(L+W)^3d_c^2}{5d_c^4 + 30(L+W)^2d_c^2 + 5L^4 + 20L^3W + 20LW^3 + 5W^4 - 20(L+W)d_c^3 - 20(L+W)^3d_c} & \text{for } L \leq d_c \leq L + W \end{cases}$$

Now consider the proportion of users who opt to utilize the trunk system for their travel, P_T . This proportion of users is simply given by $P_T = 1 - P_F$. The expected trip length for these trunk users, $\bar{d}_{trunk}$, is given by the following conditional expectation:

$$E[Z|Z > d_c] = \frac{1}{P(Z > d_c)} \int_{d_c}^{L+W} t f_Z(t) dt = \frac{1}{1 - F_Z(d_c)} \int_{d_c}^{L+W} t f_Z(t) dt$$

which yields the following expressions for $\bar{d}_{trunk}$ depending on the value of d_c

$$\bar{d}_{trunk} = \begin{cases} E[Z|z > d_c] = \frac{2L^3W^2+2L^2W^3-8LWd_c^3+3Ld_c^4+3Wd_c^4-\frac{4}{5}d_c^5}{6L^2W^2-12LWd_c^2+4Ld_c^3+4Wd_c^3-d_c^4} & \text{for } 0 \leq d_c \leq W \\ E[Z|z > d_c] = \frac{10L^3+10L^2W+5LW^2-30Ld_c^2+W^3-10Wd_c^2+20d_c^3}{15W^2+20Wd_c-60LW-30d_c^2+60Ld_c} & \text{for } W \leq d_c \leq L \\ E[Z|z > d_c] = \frac{1}{5}(L+W+4d_c) & \text{for } L \leq d_c \leq L+W \end{cases}$$

Appendix C. Expected Access Distance for a TOD Zone

Consider Figure C1 below, which displays one quadrant of the TOD zone contained within a trunk station's catchment area.

Figure C1 One Quadrant of a TOD Zone

The expected access distance in x , $\bar{x}$, can be calculated by integrating the weighted access distance from 0 to d_{TOD} and dividing that value by the area's demand density:

$$\begin{aligned}\bar{x} &= \frac{\int_0^{d_{TOD}} \rho_{TOD} (d_{TOD} - x) x dx}{\rho_{TOD} \left(\frac{1}{2} d_{TOD}^2 \right)} \\ \bar{x} &= \frac{\rho_{TOD} \int_0^{d_{TOD}} d_{TOD} x - x^2 dx}{\rho_{TOD} \left(\frac{1}{2} d_{TOD}^2 \right)} \\ \bar{x} &= \frac{\left[d_{TOD} \left(\frac{1}{2} d_{TOD}^2 \right) - \frac{1}{3} d_{TOD}^3 \right]}{\left(\frac{1}{2} d_{TOD}^2 \right)} \\ \bar{x} &= d_{TOD} - \frac{2}{3} d_{TOD} \\ \bar{x} &= \frac{1}{3} d_{TOD}\end{aligned}$$

By symmetry, the expected distance in y , $\bar{y}$, can be calculated similarly, and $\bar{y} = \frac{1}{3} d_{TOD}$. The sum of $\bar{x}$ and $\bar{y}$ gives the total expected access distance for TOD users, $\bar{x} + \bar{y} = \frac{2}{3} d_{TOD}$.

Appendix D. Sensitivity of Cost to City Length:Width Ratio for a Grid Network

While city shape of rectangular cities (the ratio of L to W) was initially considered, the differences in generalized cost across various ratios were found to be trivial. This makes sense upon re-inspection of the objective function. Holding the total city size ($L \times W$) constant, but varying L and W accordingly, the sole cost component that changes is the expected travel time. If $b = L/W$, one finds the riding cost T proportional to the factor $\left(b^{\frac{1}{2}} + b^{-\frac{1}{2}}\right)$, which increases slowly with b (the Uniform Sprawl case is taken as an example here):

$$T = (L + W) \frac{1}{3v_c}$$

$$T = \left(A^{\frac{1}{2}}b^{\frac{1}{2}} + A^{\frac{1}{2}}b^{-\frac{1}{2}}\right) \frac{1}{3v_c}$$

$$T = A^{\frac{1}{2}} \frac{1}{3v_c} \left(b^{\frac{1}{2}} + b^{-\frac{1}{2}}\right)$$

$$T \sim \left(b^{\frac{1}{2}} + b^{-\frac{1}{2}}\right)$$

This indicates that even when the length of a city is five times its width ($b = 5$) the travel time coefficient is only roughly 34% larger than that for a square city of the same area. Given the robustness of the objective function, the optimal values of the decision variables are not drastically affected by the city shape. Figure E1 below plots the percentage change in travel time coefficient with respect to changes in the L/W ratio.

Figure D1 Percent Increase in Riding Travel Time with Elongated City Shape for Grid System

Appendix E. Cost-Saving Properties of Schedule Coordination in a Simple Trunk-and-Feeder Transit System

Karthikgeyan Sivakumaran^{*}, Yuwei Li, Michael J. Cassidy, Samer Madanat

Abstract¹⁰

The paper explores how the coordination of vehicle schedules in a public transit system affects generalized costs. We consider an idealized system that delivers its users to a common destination by requiring each to transfer from a feeder- to a trunk-line vehicle. Continuum models are used first to analyze cases in which the trunk-line vehicle schedule is given exogenously. We find that when feeder vehicles are dispatched in coordination with this exogenous trunk-line schedule, the reduction in user cost often outweighs the added cost to the feeder operation. In cases when the frequencies of trunk and feeder services can be established jointly, the models show that coordination can be Pareto improving, meaning that operator and user costs both diminish. Conditions that give rise to these cost savings are specified. Practical implications are discussed.

E.1. Introduction

By reducing the times spent in transferring between vehicles, schedule coordination can diminish the costs that a transit system imparts to its users. In the long run, this can bring broader benefits to both the transit agency and society at large by inducing greater transit ridership. These matters have been studied extensively in the literature (Chien and Schonfeld, 1998; Chowdhury and Chien, 2001; Li et al., 2009).

What seem to have garnered less attention are questions on how coordination can affect aspects of transit cost beyond just the user cost. Yet, a decision on whether to deploy some proposed coordination scheme will often depend upon the subsequent impact on transit operating costs, which include the costs of fuel, labor and vehicle maintenance. Thus, a scheme that saves user transfer time is more likely to be adopted if it also reduces, or at least does not significantly increase, these operating costs (Hickey, 1992; Schumann, 1997).

The present paper therefore explores how schedule coordination affects key costs that are imparted to transit operators as well as its users. This is done by applying continuum approximations of generalized cost to a simple transit network of many parallel feeder lines that connect to a single trunk.

Background is furnished in the following section. A continuum model is used in section E.3 to explore impacts of coordination when the schedule of trunk-line service is given exogenously. We find that by dispatching feeder vehicles in coordination with the

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¹⁰ This appendix section contains a modified version of the journal article “Cost-Saving Properties of Schedule Coordination in a Simple Trunk-and-Feeder Transit System” published in *Transportation Research Part A* 46 (2012) 131–139.

given trunk schedule, total user cost can significantly diminish while little or no extra cost is imparted to the operator of the feeder service. The continuum model is expanded and used in section E.4 to explore cases in which the trunk and feeder service frequencies can be optimized jointly. Here we find that coordination can be Pareto improving, such that costs diminish for all parties. Practical implications are discussed in section E.5.

E.2. Background

This section: reviews relevant literature (sec. E.2.1); describes our general approach to the present analyses (sec E.2.2); and presents the trunk-feeder network to be used in these analyses (sec. E.2.3).

E.2.1 Literature Review

Numerous models have been developed for designing transit systems. Most furnish values for decision variables (e.g. the distances between stops and between routes, vehicle headways, etc.) that minimize some generalized cost. For the reasons described in Section E.2, a continuum approximation (CA) approach is used for the analyses that follow in favor of simplicity. Given their advantages, CA models have been widely used for many-to-one transit-system design (Byrne and Vuchic, 1972; Chien et al., 2002; Clarens and Hurdle, 1976; Hurdle, 1973; Kuah and Perl, 1988; Wirasinghe et al., 1977; Wirasinghe, 1980). However, earlier many-to-one models related to the work at hand either ignored trunk-line costs, or have assumed nonetheless that trunk-vehicle headways were sufficiently small as to render user transfer costs negligible. These assumptions have precluded these works from exploring how vehicle coordination can influence design choices.

Other research that has explicitly examined transit vehicle coordination did so independent of design issues; i.e., the effects of coordination were studied for given network configurations. Some of these works explored vehicle control and transfer coordination strategies (Ceder et al., 2001; Chowdhury and Chien, 2001; Dessouky et al., 2003; Shafahi and Khani, 2010; Ting and Schonfeld, 2005). Another examined the social welfare and operator profits generated by off-line schedule coordination, given a fixed system layout and varying market regimes that included monopoly, oligopoly, etc. (Li et al, 2009).

E.2.2 Study Scope

The present work will formulate and use CA models for determining the optimal design (i.e. route and stop spacings, as well as service frequencies) for a simple trunk-feeder transit network. The work will do so with due consideration to the influence of schedule coordination. Initial insights will come by including a term in our CA model to describe a user's cost in transferring from a feeder- to a trunk-line vehicle. We will thereafter add another term to describe trunk-line operating cost.

We will thus consider a user's costs incurred as she: accesses a feeder line by travelling toward it; waits for a feeder vehicle; and eventually transfers to a trunk vehicle. These will be estimated for the so-called average-case user during her one-way trip. The costs of accessing a feeder-line stop while travelling parallel to the line, and of travelling aboard vehicles are ignored, since these are invariant to schedule coordination.

The costs for the system operator(s) will depend upon the vehicle-hours of service that are to be provided. These will depend on factors that include: service frequencies, the density of feeder lines, the physical lengths of those lines and vehicle travel speeds. We will assume that any cost of controlling vehicles to maintain a schedule is the same, whether or not the schedule is aimed at coordinating trunk and feeder services¹¹; and that all transit vehicles have sufficient capacities to accommodate boarding demands.

E.2.3 Case Study

The idealized network on which we will base our study is shown in Figure E1. It consists of a trunk line, operating at a headway H , that runs in the y -direction to a Central Business District (CBD) at location $(0, R)$; and parallel feeder lines. The latter are each of length L , run perpendicular and connect to the trunk, and collectively span distance L_r along the trunk. The service region is thus a rectangle of dimension $L_r \times L$, as shown in the figure.

We will assume that L_r is large enough that the service region holds many feeder lines; and that these lines can be located anywhere throughout the region, as might occur when the feeder service is provided by buses on a dense network of streets. Trunk-line stops will not necessarily be placed at every junction with the feeder lines, meaning that a feeder vehicle may be required to travel in the y -direction to reach a trunk-line stop. However, this travel distance will be assumed negligible as compared with L , a feeder vehicle's one-way travel distance in the x -direction.

Travel demand in the service region is expressed as a continuous, time-independent density function. We will assume that this density varies gradually along x and is independent of y , as might occur, for example, if development arose along the trunk line and gradually diminished at greater distances from it. We therefore denote this density function $\delta(x)$. By assuming that demand is independent of y , both the spacing between neighboring feeder lines, r , and the feeder-vehicle headway, h , will be fixed throughout the service region. This will simplify our analysis for the case when trunk and feeder schedules are established jointly (in Section E.4). This assumption of uniform demand in the y -direction also means that all feeder lines will have the same number of stops, since we will take stop locations to be unconstrained by topography and would be selected instead to minimize costs (see Kuah and Perl, 1988). We can therefore assume that feeder-vehicle speeds are the same on all lines. We select a feeder-vehicle commercial speed, v_f , that is slower than the cruise speed to roughly account for the time lost in serving passengers. (Crude estimates for commercial speed suffice, as we shall see in section E.3.)

Finally, we will assume that the distribution of trips is many-to-one, with all users bound for the CBD. A case like this might arise (approximately) during the morning rush in a mono-centric city. The system is meant to be representative of an urban peripheral

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¹¹ Since the cost of control is assumed to be invariant, it is ignored in our analyses. Moreover, new control methods for maintaining a transit schedule can be deployed quite inexpensively; see Daganzo and Pilachowski (2009) and Pilachowski (2009). Further discussion on the costs of control, and on issues regarding vehicle capacity, is offered in section E.5.

region connected to the central business district by means of a major regional trunk line, such as the BART system in the San Francisco Bay Area or the Metro in the Washington D.C. metropolitan area. Though our use of perfectly parallel feeder lines and a non-circuitous trunk line are idealized, the operating costs associated with a transit system depend on its number of lines and length of travel far more than on the circuitry of its routes. Furthermore, the assumption of a perfectly square area could easily be relaxed by allowing L to vary with y , leading to feeder routes with varying travel lengths. Thus, any additional circuitry in a real-world network might be incorporated into our idealized representation by increasing the length of the feeder bus routes at the appropriate locations.

Figure E1 Hypothetical trunk-and-feeder operating environment.

E.3. Exogenous Trunk-Line Schedules

We first consider the case in which trunk-line vehicles operate with a headway that is given and that cannot be altered to accommodate feeder operation. This case can arise when local feeder bus service is not operated by the same authorities that govern regional trunk lines. Examples include New Jersey Transit, which runs feeder lines serving commuters travelling aboard the Port Authority Transit Corporation Speedline to/from Philadelphia, and many of the San Francisco Bay Area's local transit agencies (AC Transit, WestCAT, etc.) that serve the BART system. While in recent years, "umbrella" Metropolitan Planning Organizations have aided in the consolidation of transit agencies across urban regions, service integration still remains an issue. Even in cases where the same agency oversees both regional trunk and local transit systems, "operation is normally managed by separate modes or lines" (Guo and Wilson, 2011).

We next: present CAs for estimating generalized costs when trunk and feeder services are uncoordinated (sec. E.3.1); reformulate the models to estimate costs when feeder service is operated in coordination with the exogenously-specified trunk-line schedule (sec. E.3.2); and make comparisons (sec E.3.3).

E.3.1 Cost Models for Uncoordinated Service

The CAs presented below are comparable to those derived in earlier work (Hurdle, 1973; Wirasinghe, 1980; Kuah and Perl, 1988), but have an added term to estimate the user cost of transfers. All parameters and decision variables to be used in our models are defined in Table E1. For good measure, additional terms are presented in the lower, shaded portion of that same table.

We define Z to be the generalized cost per unit time for the service region. It is the sum of four cost components: the users' access to feeder lines (along the y direction), their wait times at feeder stops, their wait (i.e. transfer delay) at the trunk stop, and the feeder operating cost. Thus,

$$Z = \alpha_1 r + \alpha_2 h + \alpha_2 H + \alpha_3 \frac{1}{h} \frac{1}{r} \quad (1)$$

where

$$\alpha_1 = \frac{c_a}{4v_a} L_r \int_0^L \delta(x) dx$$

$$\alpha_2 = \frac{c_w}{2} L_r \int_0^L \delta(x) dx$$

$$\alpha_3 = C_f \frac{2L}{v_f} L_r .$$

We minimize Z , taking r and h as decision variables. The optimal spacing for the feeder lines, as a function of h , is therefore

$$r^*(h) = \alpha_1^{-\frac{1}{2}} \alpha_3^{\frac{1}{2}} h^{-1/2} . \quad (2)$$

Substituting (2) into (1), we minimize with respect to h and find the optimal feeder-vehicle headway to be

$$h^* = (\alpha_1 \alpha_2^{-2} \alpha_3)^{\frac{1}{3}} = \left(\frac{2c_a c_f L}{v_a v_f c_w^2 \int_0^L \delta(x) dx} \right)^{\frac{1}{3}} , \quad (3)$$

which leads to an optimal feeder line spacing of

$$r^* = (\alpha_1^{-2} \alpha_2 \alpha_3)^{\frac{1}{3}} = \left(\frac{16v_a^2 c_w c_f L}{v_f c_a^2 \int_0^L \delta(x) dx} \right)^{1/3} \quad (4)$$

Substituting (3) and (4) into (1), we find that the minimum generalized cost for the service region is

$$Z^* = 3(\alpha_1 \alpha_2 \alpha_3)^{\frac{1}{3}} + \alpha_2 H = 3 \left(\frac{c_a c_w c_f}{4v_a v_f} L \left(\int_0^L \delta(x) dx \right)^2 \right)^{\frac{1}{3}} + \frac{c_w}{2} L_r \left(\int_0^L \delta(x) dx \right) H \quad (5)$$

Note from (5) how this minimum cost (excluding the waiting cost at the trunk stop) is robust to variations in the input parameters, meaning that coarse estimates for their values will yield near-optimal designs for the feeder system.¹²

Note too that (5) furnishes a cost in the absence of coordination between the trunk and feeder schedules; this is evident from the third term in (1) which takes the average transfer time to be half the trunk-vehicle headway. Schedule coordination is examined next.

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¹² Methods for transforming idealized CA design variables to real-world environments can be found in Wirasinghe (1980), Kuah and Perl (1988), and Estrada et al. (2011).

Table E1

Description of input parameters and decision variables

Term	Description	Units
$\delta(x)$	Demand density	pax/km ² -hr
v_a	User speed in accessing the transit system	km/hr
v_f	Feeder vehicle speed	km/hr
v_t	Trunk vehicle speed	km/hr
C_a	User value of access time	\$/hr
C_w	User value of waiting time	\$/hr
C_f	Feeder vehicle operating cost rate	\$/hr
C_t	Trunk vehicle operating cost rate	\$/hr
L	Width of the service region	km
L_r	Length of the service region	km
R	Distance to CBD from $y = 0$	km
r	Feeder line spacing	km
h	Feeder headway	hrs
H	Trunk headway	hrs
h_c	Feeder headway, coordinated	hrs
r_c	Feeder route spacing, coordinated	km
m	Ratio of coordinated feeder headway, h_c , to optimal uncoordinated feeder headway, h^*	-
k	Integer multiple, such that $h_c = kH$	-
Z	Generalized cost per unit time for service area	\$/hr
Z_c	Generalized cost per unit time for service area, coordinated	\$/hr

E.3.2 Coordinating Feeder Service with the Trunk

Consider a simple scheme in which we coordinate schedules within the service region by choosing a common feeder headway, h_c , that is some integer multiple of the trunk line's exogenous headway; i.e.

$$h = h_c = kH,$$

where k is any positive integer.

One could select the value of k that makes h_c closest to the optimal feeder line headway, h^* , obtained from (3). Suitable alteration to (2) yields:

$$r_c^* = \alpha_1^{-\frac{1}{2}} \alpha_3^{\frac{1}{2}} \quad (6)$$

and

$$Z_c^* = 2\alpha_1^{\frac{1}{2}} \alpha_3^{\frac{1}{2}} h_c^{-\frac{1}{2}} + \alpha_2 h_c \quad (7)$$

where the subscript c denotes coordinated operation.

E.3.3 Cost Comparisons

Selecting a line spacing as in (6) can, to a large degree, compensate for the added cost of choosing an $h_c \neq h^*$, such that the user and operating cost incurred within the feeder subsystem changes very little. To illustrate this, we note first that $h_c = kH$ can be rewritten as $h_c = mh^*$, where m is some non-negative constant that should be close to 1 (by virtue of having chosen suitable k). In the uncoordinated case, the costs within the service region for user access, for the wait for feeder vehicles and for feeder operation are each $(\alpha_1\alpha_2\alpha_3)^{\frac{1}{3}}$. In the coordinated case, the access and feeder operating costs are each $m^{-\frac{1}{2}}(\alpha_1\alpha_2\alpha_3)^{\frac{1}{3}}$, and the user wait cost for feeder vehicles is $m(\alpha_1\alpha_2\alpha_3)^{\frac{1}{3}}$. The percent difference in the sum of access, waiting, and feeder operating cost between coordinated and uncoordinated operation is therefore $\frac{2m^{-\frac{1}{2}}+m-3}{3}$. This difference is small for m close to 1, as shown in Figure E2.

Figure E2 Percent change in the sum of access, waiting, and feeder operating costs through coordination.

Since schedule coordination eliminates the user transfer cost at the trunk stop, the difference between Z_c^* and Z^* is $\left(2m^{-\frac{1}{2}} + m - 3 - \frac{m}{k}\right)(\alpha_1\alpha_2\alpha_3)^{\frac{1}{3}}$. And since $h_c = kH$, coordination yields a savings in total cost only when k is small. This makes sense: when H is small compared to h^* , there is no need for schedule coordination.

E.4. Endogenous Trunk-Line Schedules

Suppose now that the trunk-line component of the system serves only the feeder lines that reside within the service region of Figure E1. For this case, we will compare the costs of uncoordinated services against those that occur when the trunk and feeder lines operate with the same headway and in coordinated fashion. We will also demonstrate how this simple coordination scheme can be Pareto improving. CAs are presented for the uncoordinated and coordinated cases (secs. E.4.1 and E.4.2, respectively). Estimated costs are compared to identify the conditions needed to achieve Pareto improvements

(sec. E.4.3); and the cost savings are quantified for a range of operating environments on our hypothetical network (sec. E.4.4).

E.4.1 Uncoordinated Trunk-Feeder Service

The trunk and feeder headways, H and h respectively, are treated for now as separate decision variables, such that Z , the sum of relevant generalized costs for the entire service region is given by

$$Z = \beta_1 H^{-1} + \alpha_2 H + 3(\alpha_1 \alpha_2 \alpha_3)^{1/3}, \quad (8)$$

where

$$\beta_1 = C_t \frac{2R}{v_t}.$$

The first term in (8) is the trunk operating cost, obtained by multiplying the operating cost rate, C_t , with the needed number of trunk-line vehicles $\left(\frac{2R}{v_t} H^{-1}\right)$. The second term is the average transfer cost, where in the absence of coordination, the average wait time is again assumed to be $H/2$. The third term describes the remaining two user-cost components and the feeder operating cost, given in (5).

We find that

$$H^* = \sqrt{\frac{4C_t R}{C_w v_t D}}$$

and that

$$Z^* = 2\sqrt{\frac{C_w C_t R D}{v_t}} + 3\left(\frac{C_a C_w C_f L}{4v_a v_f}\right)^{\frac{1}{3}} \left(\int_0^L \delta(x) dx\right)^{\frac{2}{3}} L_r \quad (9)$$

where $D = L_r \int_0^L \delta(x) dx$ is the service region's total hourly travel demand.

Note that the minimum cost, Z^* , is expressed purely as a function of its input parameters.

E.4.2 Coordinated Trunk-Feeder Service

When trunk and feeder services are coordinated and operate at a common headway, H_c , the sum of relevant generalized costs for the service region, Z_c , is given by

$$Z_c = \beta_1 H_c^{-1} + \alpha_2 H_c + \beta_2 H_c^{-\frac{1}{2}} \quad (10)$$

where

$$\beta_2 = 2(\alpha_1 \alpha_3)^{\frac{1}{2}}.$$

The second term in (10) is the user waiting cost for a feeder vehicle (only), since we assume zero user cost in transferring to the trunk. The third term, consisting of the access to feeder lines and the feeder operating cost, comes from (7). The convexity of (10) can be confirmed by assessment of the Hessian, which is positive definite for positive r_c and H_c . Thus, the minimum cost, Z_c^* , can be obtained numerically from (10).

E.4.3 Conditions for Pareto Improvement

We now compare the cost models of secs. 4.1 and 4.2 to determine the bounds for which schedule coordination is Pareto improving. The user costs will be examined first. These diminish with coordination if the sum of the user-cost components of (10) is less than that of (9); i.e., if

$$C_w D \frac{1}{2} H_c^* + H_c^{*-1/2} (\alpha_1 \alpha_3)^{1/2} < \sqrt{\frac{C_w C_t D R}{v_t}} + 2(\alpha_1 \alpha_2 \alpha_3)^{1/3}. \quad (11)$$

Next we determine the conditions in which coordination reduces the operating cost for trunk and feeder services combined. To this end, we compare the operating-cost components of (10) and (9). If

$$C_t \frac{2R}{v_t} H_c^{*-1} + (\alpha_1 \alpha_3)^{1/2} H_c^{*-1/2} < \sqrt{\frac{C_w C_t D R}{v_t}} + (\alpha_1 \alpha_2 \alpha_3)^{1/3}, \quad (12)$$

the trunk and feeder operating cost is reduced.

If both the trunk and feeder services are furnished by a single entity, coordination will be Pareto improving whenever (11) and (12) both hold. Of course, trunk and feeder services are often provided separately by distinct agencies. In this latter case, coordination can still reduce the costs for every party involved.

To see why this is true, note first from (12) that the operating cost for feeder service (alone) is reduced if $H_c^{*-1/2} (\alpha_1 \alpha_3)^{1/2} < (\alpha_1 \alpha_2 \alpha_3)^{1/3}$, which can also be expressed as

$$H_c^* > \left(\frac{(\alpha_1 \alpha_3)^{1/2}}{(\alpha_1 \alpha_2 \alpha_3)^{1/3}} \right)^2. \quad (13)$$

Referring to (3), we can see that (13) reduces to the inequality $H_c^* > h^*$.

As regards the operating cost of trunk service (alone), inspection of (8) and (10) reveals that the optimal trunk-vehicle headway is guaranteed to be larger with coordination. This is because i) the coordinated model, given by (10), shifts the waiting cost ($\alpha_2 H$) at trunk stops to feeder stops, and ii) this waiting cost is now weighed against both the trunk and feeder operating costs (rather than solely against the trunk operating cost). These two factors combined assure that coordination results in a larger trunk headway. The larger headway leads to a lower trunk operating cost.

E.4.4 Illustrations of Pareto Improvement

To illustrate the benefits of coordination, we present two scenarios. The first corresponds to a region where the average wage rate is low, as typically occurs in a developing country; and the second to a region with a high average wage rate, as in a more industrialized country. We assume in both scenarios that the user value of time, for both access and waiting, is equivalent to the user wage rate. Additionally, while some components of the hourly cost rates for both the feeder and trunk systems will remain roughly the same across the two scenarios (fuel, depreciation, etc.), the overall hourly operating cost will differ due to differences in labor costs. Accordingly, the input cost parameters for the developing country (“low”) and the industrialized country (“high”) are shown in Table E2.

We assume that both scenarios are governed by the same system characteristics: $v_a = 5$ km/hr, $L_r = 10$ km, $L = 5$ km, and $R = 20$ km (refer again to Table E.1 for the definitions of these parameters). Feeder- and trunk-vehicle speeds are chosen to reflect typical urban bus and light rail speeds: 15 and 30 km/hr, respectively.

Both scenarios are analyzed under a range of demand densities for the trip origins. For simplicity, uniform densities are used¹³. These uniform densities ranged from 10- to 200 pax/ km²-hr, in increments of 10 pax/ km²-hr, such that the total demand in the service region ranged from 500- to 10,000 pax/hr.

Table E2 Cost parameter values for two scenarios

Term	Low Value [\$ /hr]	High Value [\$ /hr]
C_a	3	15
C_w	3	15
C_f	40	100
C_t	180	300

Costs saved through coordination are shown as functions of these demands in Figures E3 and E4. Figure E3 displays savings for the low-cost scenario, and Figure E4 for the high-cost one. Both figures present curves for the savings in total user cost, total (trunk and feeder) operating cost and feeder operating cost alone. Note that costs are saved in all cases.

Figure E3 Cost Savings from Coordination, for “Low” Cost Parameter Values

¹³ For non-uniform demand densities that varied with x , similar results were obtained. This is because the user costs in the model are proportional to the total demand, and not to the distribution of this demand.

Figure E4 Cost Savings from Coordination, for “High” Cost Parameter Values

E.5. Conclusion

By applying continuum approximations to a simple trunk-feeder transit network, we find that schedule coordination can save more than just user costs. In cases when the vehicle headway on the trunk is given exogenously, service can be coordinated by operating the feeder vehicles at headways that are integer multiples of those on the trunk. By suitably adjusting the feeder-line spacing, this simple coordination scheme can eliminate user waiting cost at the trunk station, while often adding little or no cost elsewhere in the system. Better still, if the headways for trunk and feeder vehicles are both decision variables, schedule coordination can often be Pareto improving and benefit all parties.

E.5.1 Limitations

We acknowledge that these findings came by analyzing an idealized network, and by adopting a number of (often simplifying) assumptions. We further concede that there are limits on the extent to which these findings can be exploited. For example, when a trunk line’s schedule is exogenous, an operator does not always have free reign to locate the feeder lines in response to that schedule. When this freedom does exist, moreover, feeder-line spacing would usually be optimized for a limited portion of the day (e.g. the morning rush), since the headways scheduled for a trunk tend to change over the day as demand changes. When demand does not follow a many-to-one profile, the benefits of coordination should be weighed against the increase in waiting costs incurred by local feeder bus riders not destined for trunk stops.

Furthermore, we did not account for any real-world “control” costs of coordination, which might include global-positioning systems and wireless communication technology. In some cases, the cost of control may exceed and negate the benefits of coordination, particularly when the optimal coordinated headway takes a low value. Finally, the passenger-carrying capacity of trunk-line vehicles can also limit possibilities, since the joint selection of a feeder- and trunk-vehicle headway tends to

expand the latter. This limitation can be remedied at relatively low cost when trunk vehicles can be enlarged; e.g. by using articulated buses or by adding cars to trains. Or, an operator might increase capacity by dispatching trunk vehicles in small platoons, though the resulting increase in trunk operating cost might then become large.

E.5.2 Practical Advice and Policy Implications

All of the above notwithstanding, the present findings can inform transit system design. In those instances when trunk and feeder services are provided by distinct agencies, the findings speak to the benefits that might come via institutional cooperation. Metropolitan Planning Organizations might encourage the cooperation needed for schedule and service coordination through financial incentives; for example, by allocating the savings to both the feeder and trunk agencies.

Given the potential benefits for both users and operators, special attention should be paid to the design of transfer facilities, since a poor design may render transferring so onerous to users that any coordination benefits are minimal in comparison. Thus, for future investments, transfer facilities should be designed to allow efficient transfers not just between the same type of vehicles (e.g. cross-platform transfers within a rail system), but also between different types of vehicles (e.g. small feeder buses and commuter rail cars).

The benefits achieved by coordination may also motivate transit agencies to explore alternative schemes for delivering service. Consider, for example, a case in which many-to-one service is provided on a network with a long trunk line that spans a long service region, as suggested in Ghoneim and Wirasinghe (1987). Pareto improvements might come by partitioning the network into narrower sub-regions and assigning trunk-vehicles to serve sub-regions in dedicated fashion. A better understanding of the cost-saving potential of schedule control might ultimately give rise to any number of innovations in transit service. The present paper represents a step forward in this regard.

Appendix F. Strategies to Accommodate Demand along a Single Transit Line

Explored now are strategies that ensure that trunk vehicles do not leave residual passenger queues at stations; the notation and findings from Appendix E are applied here. A trunk line serving a many-to-one demand pattern of rate λ [passengers/hr] must carry the full passenger load from the service area, increasing the likelihood that the vehicle capacity becomes a binding constraint. If K is a trunk vehicle's capacity [passengers], the maximum allowable trunk headway is given by $\frac{K}{D}$ hrs; if $\frac{K}{D}$ is less than the optimal trunk headway, the trunk operator may be forced to provide suboptimal service. Two possible strategies to ensure sufficient passenger-carrying capacity for trunk lines include platooning (Section F.1), and partitioning the service area (Section F.2). Both alternatives are examined below.

F.1 Platooning

One alternative is to platoon trunk vehicles, such that multiple vehicles are sent along the trunk together. This allows the trunk operator to meet capacity constraints while operating at larger headways, since a platoon's maximum allowable headway would be $H_{max,m} = \frac{mK}{D}$, where m is equal to the trunk platoon size (in vehicles).

Under this strategy, two possible scenarios arise within both the uncoordinated and coordinated service models: either 1) the optimal headway is less than $H_{max,m}$, or 2) $H_{max,m}$ is a binding constraint.

For uncoordinated service, platooning is unnecessary when the maximum allowable headway is a binding constraint. A platoon of m vehicles operating at a headway $H = \frac{mK}{D}$ hrs would generate the same operating cost as a single vehicle operating at a headway $H = \frac{K}{D}$ hrs, but would be inferior since the user transfer time is always higher. Thus, for the uncoordinated case, if $H^* < H_{max,1}$, then

$$Z^* = 2\sqrt{\frac{C_w C_t D R}{v_t}} + 3 \int_0^{L_r} (\alpha_1 \alpha_2 \alpha_3)^{1/3} dy$$

Otherwise,

$$Z^* = C_t \frac{2R D}{v_t K} + C_w \frac{K}{2} + 3 \int_0^{L_r} (\alpha_1 \alpha_2 \alpha_3)^{1/3} dy$$

For coordinated service, platooning should be examined for all feasible platoon sizes (m). Thus, for each value of m , if $H_{c,m}^* < H_{max,m}$, then

$$Z_c^*(m) = m\beta_1 H_{c,m}^{*-1} + \beta_2 H_{c,m}^* + \beta_3 H_{c,m}^{*\frac{-1}{2}}$$

Otherwise,

$$Z_c^*(m) = C_t \frac{2R D}{v_t K} + C_w \frac{mK}{2} + (2 \int_0^{L_r} (\alpha_1 \alpha_3)^{\frac{1}{2}} dy) \left(\frac{D}{mK} \right)^{-\frac{1}{2}}$$

After determining $Z_c^*(m)$ for each feasible m , we then take the minimum¹⁴ coordinated cost Z_c^* across all m . The optimal operating strategy is finally given by the smaller of Z^* and Z_c^* . Assuming a demand density $\delta(x, y) = 10y$, such that $D = \int_0^{L_r} \int_0^{L(y)} 10y dx dy$, and using the input parameters provided in Table F1, Figure F1 shows the uncoordinated cost curve together with the coordinated generalized cost curves for $m = 1, 2, 3$ and 4 . Visual inspection of the figure shows that the optimal operating strategy is a coordinated trunk-feeder transit system with platoon size $m = 2$. This is not always the case; given a different set of parameter values, uncoordinated service can be the optimal strategy.

Table F1 Input parameters and corresponding values

Parameter	Description	Units	Value
v_a	User speed in accessing the transit system	km/hr	5
v_f	Feeder vehicle speed	km/hr	30
v_t	Trunk vehicle speed	km/hr	50
C_a	User value of access time	\$/hr	9
C_w	User value of waiting time	\$/hr	9
C_f	Feeder vehicle operating cost rate	\$/hr	100
C_t	Trunk vehicle operating cost rate	\$/hr	200
$L(y)$	Lateral distance from the trunk line to the edge of the service area	km	5
L_r	Longitudinal length of the service area	km	10
R	Distance to CBD from $y = 0$	km	20

The above analysis can be applied to bus or bus rapid transit (BRT) trunk lines, but is unsuitable for a rail trunk line. Vehicles on the latter already travel in platoons, and each “vehicle” does not require a separate driver. Consequently, the operating cost term cannot be expressed as a linear function of m , but should instead reflect economies of scale.

¹⁴ Note that the coordinated platoon cost function is convex with respect to m . Furthermore, m will be bounded by the available vehicle storage at each trunk station, which we assume to be four trunk vehicles.

Figure F1 Generalized Cost Curves for Uncoordinated and Coordinated Platooned Service

F.2 Partitioning

Another alternative to ensure sufficient trunk-vehicle capacity is to partition the service area during peak-periods, such that different trunk vehicles are assigned to different regions of the given service area. This strategy can reduce overall trunk vehicle-miles-traveled while also reducing riding cost for passengers originating from service areas farthest from the CBD, as stops within subsequent service areas are bypassed.

As an example, Figure F2 shows one way to partition a service area given the parameters used in the platooning example above. The service area has been partitioned such that each partitioned area contains equal hourly demand. If the headway $H_{c,1}^*$ is adopted for Service Areas 1 and 2, each area can be sufficiently served by platoon size $m = 1$, and overall generalized cost is reduced in comparison to platooned service (see Table F2), even without considering the savings in riding cost.

Because a precise trunk operating cost term was not used for the above metro system example, a comparison between BRT and metro systems may be unfair. Nonetheless, there are other cost considerations which can be included in future work which compares different trunk modes. The additional social cost of greenhouse gas emissions could also be considered in the generalized cost, as well as the initial capital investment. The capital cost can be amortized over the lifetime of the project and included as a fixed cost component within the generalized cost expression.

Figure F2 A Partitioned Service Area

Table F2 Cost Comparison between Platooned and Partitioned Service

	Platooning	Partitioning
User Cost Rate [\$/hr]	\$3,050	\$3,050
Operator Cost Rate [\$/hr]	\$5,160	\$4,200
Total Cost Rate [\$/hr]	\$8,210	\$7,250

Figure F3 Metro Cost Curves for Uncoordinated and Coordinated Service