Title
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Why do People fit to Benford's Law? – A Test of the Recognition Hypothesis

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Abstract

Burns & Krygier (2015) demonstrated that people could exhibit a strong bias towards the smaller first digits, in a way similar to that described by Benford's law. This paper sought to expand the scope of this phenomenon and to test a possible explanation, the Recognition Hypothesis that a Benford bias is due to life-long environmental exposure to this statistical relationship. Participants completed three numerical tasks: A Generation Task requiring answering trivia questions; a Selection Task requiring selecting between two numerical responses; and an Estimation task requiring estimating the number of jelly beans in a jar. The results found no evidence of any first digit effect in the Recognition Task, some evidence of Benford bias in the Generation Task and strong evidence in the Estimation Task. Future research should focus on alternatives to the Recognition Hypothesis and investigate the parameters of Benford bias in generation tasks.

Keywords: Benford's law, number generation, number selection, statistical learning

Introduction

Contemporary society often demands that people produce numbers when making decisions, for instance, estimating the value of an item. Decision making under uncertainty has been investigated by looking for heuristics and biases, but little research has focused on estimation other than that into anchoring (Tversky & Kahneman, 1974). Thus, understanding how people generate numerical estimates could provide new insights into many types of decisions and potential biases in such decisions. To make progress towards understanding estimation we need well-established phenomena that could serve as investigation tools, but these have been lacking.

A possible tool arises from research into the extent to which people's estimations fit to Benford's law. Benford's law is a well-established phenomenon that the first digit frequencies of numerous naturally occurring datasets follow a log distribution where digit-1 occurs 30% of the time while digit-9 has no more than a 5% of occurrence (Miller, 2015). Benford (1938) demonstrated this empirically (Figure 1) for data but there is now evidence that people spontaneously generate a first-digit bias that approximates Benford's law when estimating numbers. The results reported by Burns (2009), Burns and Krygier (2015), and Diekmann (2007) did not find a perfect fit of human data to Benford's law, but its pattern accounts for a large amount of variance in human first digit data. Thus, people appear to have a Benford bias.

Understanding why people have a Benford bias could provide insight into the process of number estimation. A potential explanation relies on the assumption that people will have been frequently exposed to this statistical relationship during their lifetime, given how ubiquitous it is for data. The potential for unconscious acquisition of this universal law leads to a Recognition Hypothesis. This paper will test this hypothesis and try to extend our understanding of Benford bias.

Background of Benford's law

Benford's law for first digits suggests that as long as a domain is numerical, spans multiple magnitudes and has no assigned boundaries, data's leading digits frequencies have the monotonic decline of a logarithm distribution (Benford, 1938). Although it applies to other digits, it is known as a first digit phenomenon because such its skewness is so large when compared to the distributions of the other digit places.

The Benford's law distribution has been discovered to apply to data from many domains. Classic examples of the first digit phenomenon are financial indicators like GDP and stock exchange data, mathematics topics like Fibonacci numbers and random matrix theory, and physical observations such as the energy level of particles (Miller, 2015). It also applies to human domains such as internet traffic records (Arshadi & Jahangir, 2014), criminal rates (Hickman & Rice, 2010), counts of friends and followers on Facebook (Golbeck, 2015), gambling behaviour (Chou et al., 2009), and brain activity (Kreuzer et al., 2014).

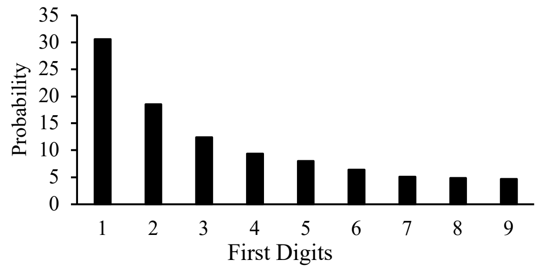

Figure 1: The percentage of times the digits 1 to 9 are used as first digits, as determined from 20229 observations (Benford, 1938).

A practical application of Benford's law is detecting fraud (e.g., Nigrini, 1996). Therefore, it has been incorporated into the auditing and accounting process, and used for detecting falsified data (Miller, 2015).

In short, Benford's law is a robust phenomenon describing the regularities of the leading digits aggregated from the unrelated datasets from nature and society.

Psychological Explorations of Benford's law

Although Benford's law is a well-established phenomenon of natural data, whether it is a psychological phenomenon has been unclear. Dehaene (1997) examined the role of the leftmost digit when trying to answer why words indicating small numerals are more frequent than larger ones in many languages. He dismissed the potential explanatory power of Benford's law for the findings from his tasks. Earlier research also seemed to support this dismissal of Benford's law as relevant to cognitive processes.

Early evidence against Benford's law

Early explorations by behaviourial science of the leading digits produced by humans were very few; to our knowledge, only four papers were published. The studies from Hsü (1948), Hill (1988), Kubovy (2009), and Scott et al. (2001) simply asked participants to generate a random number, such as a four-digit number created out of their head. Aggregation of the frequencies of the first digits created showed substantial deviation from a uniform distribution, notably, digit-1 was generated more frequently than expected by a uniform distribution. However, the patterns were inconsistent and the first-digit frequencies did not exhibit the monotonic decline suggested by Benford's law.

Psychological evidence supporting Benford's law

The negative outcome from random number generation appeared robust, so the psychological investigation of Benford's law was limited until support emerged from the studies of Diekmann (2007) and Burns and Krygier (2015, see also Burns, 2009), who employed alternative methods.

Diekmann (2007) discovered that the first digits of published unstandardized regression coefficients closely approximated Benford's law. Following this discovery, he asked students in sociology or economics to fabricate multiple "plausible values" of regression coefficients in response to a set of controversial hypotheses from neoclassical economics. This experiment obtained a reasonable fit of the first-digit to Benford's law, although there were spikes for digit 5 and 8.

Independent of Diekmann's experiments, a study from Burns and Krygier (2015) demonstrated the first digit phenomenon by asking people to produce numbers in meaningful domains. These studies contrasted with the ones that failed to find evidence of people generating numbers that fit to Benford's law by asking them to generate non-arbitrary numbers. Arbitrary numbers, like the random numbers Hsü (1948), Hill (1988), Kubovy (2009), and Scott et al. (2001) asked their participants to generate, cannot be estimated or calculated. Diekmann's participants did not know the correct regression coefficients and Burns and Krygier (2015) did not know the correct answer to questions like "How long is the Indus River?" However, they know the question is meaningful and the answer is non-arbitrary such that a reasonable, though imperfect, answer might be estimated.

Burns and Krygier (2015) gave participants a set of nine non-arbitrary questions from the meaningful domains used in Benford's original observations, like national debts and power calculation. The items were scrutinised to avoid well-known fields and were chosen so that each digit 1 through 9 was equally often the first digit of the correct answers. Hence, both true and random answers should yield a flat first-digit distribution. The pattern of the first digits showed a closer fit to Benford's law than to the correct (flat) distribution, except for a peak for digit-5. In Burns and Krygier's second study they designed an 81-item pool, structuring the questions from nine different meaningful domains (e.g., infant mortality rate) with nine different targets (e.g., Afghanistan). Consistent with the observations of their first study, a closer fit to Benford's distribution was captured with an elevation of digit-5. This replication further validated that Benford bias can be detected in behavioural data. The data was not a perfect fit to Benford's law, and it would be a shock if it captured all the variance. However, the size of the Benford bias can be estimated by calculating the effect size (eta^2) for the linear contrast weighted by the proportions predicted by Benford's law for the observed proportion of participants' first digits. In Burns and Krygier's studies this weighted linear contrast accounted for 52.1% and 42.7% of the variance in observed first digits proportions.

Explaining Benford bias

Further replication studies of Benford bias have been made (Burns, 2020; Burns, Tripodi, Chi, Krygier, & Birney, under revision), so it appears to be a robust phenomenon of non-arbitrary numerical estimates. What explains this phenomenon requires further investigation.

One possible explanation is captured by what we have called the Recognition Hypothesis. This hypothesis assumes that if the world surrounds people with data consistent with Benford's law, then people may become sensitive to this statistical relationship. Many researchers (e.g., Bargh, & Ferguson, 2000; Gigerenzer & Todd, 1999) have emphasized the ways that decision making may be influenced by implicit knowledge of the regularities in the data they encounter in their environment. Thus, the picture this hypothesis presents of number estimation is as a process that can be strongly driven by our general experience of numbers in our environment.

If the bias towards the smaller first digits can be attributed to implicit learning through exposure, then such a preference should not be limited to the tasks involving number generation. A bias towards smaller first digits should also be present when participants are asked to try to recognize correct answers, such as when they are given numerical answers and asked to select the correct one. Thus, comparing number generation to number selection tasks provides a way to test the Recognition Hypothesis.

Previous empirical research

To test the Recognition Hypothesis, Burns and Krygier (2015) introduced a selection task in their second study. This used similar numerical questions as used for their generation task. However, instead of producing a number as a response, participants chose an answer amongst nine numerical options, each with a different first digit. The questions were randomly selected from an 81-item pool (nine meaningful domains by nine targets), and the leading digits of the correct responses to the nine questions were equally distributed. If people have Benford bias because of long-term exposure to it in the environment, the options with lower first digits should more frequently be selected than the ones with higher first digits. Contrary to this prediction, except for a small elevation for digit-1, the relative frequencies of the first digits chosen by participants were close to a flat distribution. It seemed that people showed no clear preference for any first digits when selecting answers. However, it was argued that a flattened distribution might be a result of random answering due to providing many options. Iyengar and Lepper (2000) argued that increasing the number of choices offered to an unknown question might restrain the cognitive process in decision making, thus leading to random responding.

To avoid the possibility of distorting responses by providing too many options, Tripodi (2016) in an unpublished study gave participants a single answer and asked participants if they thought it was correct. For example, "Is the total area of Greece 131940 (km^2)? Choose 'yes' or 'no'." Half of the items presented an incorrect value, and participants were told to expect about half the answers to be incorrect. Participants answered 18 questions from non-arbitrary domains (e.g., areas of countries) and 18 from arbitrary domains (e.g., specific phone numbers). Neither non-arbitrary nor arbitrary domains produced evidence of any first-digit preference let alone a monotonic decline from digit-1 to digit-9.

The findings from the selection tasks exhibited a remarkably different pattern to the ones obtained from the generation tasks and thus they challenged Recognition Hypothesis. However, intuitively the Recognition Hypothesis is very appealing because of how well it fits with other evidence that implicit learning of statistical relationships in the environment can influence decision-making. So before ruling out the utility of the Recognition Hypothesis, we wanted to test it with what we considered to be the most sensitive paradigm for a selection task by directly contrasting the first digits.

A New Paradigm

The previous selection tasks offered either nine options or a single potential answer, thus, neither of them directly contrasted the first digits. So, in the current experiment we offer pairs of numbers with a lower and a higher first digit to see if people will consistently favour the smaller first digit. For example, a forced choice was offered between 1xx and 3xx where x's are random digits.

As well as continuing to examine the explanatory power of the Recognition Hypothesis, the present experiment included other aspects designed to further expand our understanding of Benford bias by manipulating other aspects. As well as the selection task, participants completed a number generation task similar to Burns and Krygier (2015) in which we tested the effect of number type by asking participants to generate both non-arbitrary and arbitrary numbers, a distinction we also tested in the selection task.

In addition, we examined whether the bias towards smaller digits is a phenomenon of the first digit of a number or a phenomenon of the first digit *written down.* Unlike the concept of being the "first digits" of data nature, the Benford bias observed from human activities could be alternatively interpreted as a result of the initial digit created out of the mind. Under certain circumstances, the initial digits produced are not always the leftmost digits. For instance, a person at an auction of a million-dollar house does not expect the leading digit to change too often, so the estimation may focus on the second or later digits in the number estimated. Hence, to create a situation where the first digit generated is not the first digit of an answer, we varied whether participants entered their answers normally (from the first digit of an answer) or were instructed to enter the answer backwards (from the last digit of an answer). Thus, in the backwards condition the digit that was the right-most digit of the answer was the initial digit produced. By investigating the influence of this backwards answering, it might help us to understand the robustness of the bias towards the first digit as the leftmost digit of an answer.

Finally, a new task for quantity estimation was introduced as another form of number generation. Unlike the trivia questions which asked participants to draw on their knowledge and memories, participants estimated quantities based on visual stimuli, in this case jars of jelly beans. This allowed us to test the generalizability of our effects and is a form of number generation people encounter more often in everyday life. In addition, we manipulated the amount of information presented so we could further examine how the amount of cognitive processing may affect number estimation.

Methods

Participants

173 first-year psychology students participated for partial course credit. They had an average age of 20 (SD = 3.608), ranging between 18 and 42, with 118 females (68.2%) and 55 males (31.8%). 48.6 % were English speakers, while 28.9% were Chinese speakers.

Procedure and materials

Each participant went through a Number Generation Task (18 items) followed by a Number Selection Task (50 items) and a Number Estimation Task (9 items).

Number Generation. This task presented a block of 18 number generation items. Nine of the items asked questions from non-arbitrary domains whereas the other nine were from arbitrary domains. The domains asking for non-arbitrary numbers consisted of science constants, human/livestock populations, shares trading on the NYSE, electricity consumption, square roots, GDP, national external debt, area by country, and the gross profit of films. The domains for arbitrary numbers concerned cheque numbers, postcodes, raffle ticket winning numbers, and vehicle registration numbers, for which an answer was not calculable.

The order of the 18 items was randomized, so questions from arbitrary and non-arbitrary domains were mixed. When entering their answers, participants saw a line of ten boxes on the screen. As they typed a number each digit appeared in a box. For half of the participants as digits were typed, they filled the boxes left to right. The other half of the participants were instructed to produce the number in reversed order (i.e., begin with the last digit), so as they typed their number it filled the boxes from right to left.

Items were randomly drawn from an 81-item pool. For the non-arbitrary numbers, the questions were selected so that each digit from 1 to 9 was the first digit of the correct answers equally often. Examples of items:

Non-arbitrary numbers

"What is the total area of Ireland (km^2)? ______ "

Arbitrary numbers

"Write the order number of a purchase of a fridge. ______"

Number Selection. This task had a between-subject design with 50 forced-choice items from either non-arbitrary or arbitrary domains, which were randomly drawn from a 250-item pool. The domains asking for non-arbitrary numbers consisted of national external debt, selling price of a property, the square of a number, the water area by country, and internet hosts. The domains for arbitrary numbers are associated with contact numbers, Australian Business Number (ABN), International Standard Book Number (ISBN), online post IDs, and IP addresses. The questions asked participants to choose an answer from two potential alternatives. Examples of items with possible answers were:

Non-arbitrary numbers

"How many Internet hosts were listed in Kenya by 2012?" [71018] [98280]

Arbitrary numbers

"What are the leading digits of the IP address of Reddit.com?" [336318719] [520154151]

Only first digits 1, 3, 5, 7 and 9 were provided as the leading digits in the task, which enabled us to give five examples of 10 different first-digit pairs.

Quantity Estimation. The task required the participant to estimate the number of jelly beans in pictures of nine jars presented in a random order. The nine jars were the same size and contained precisely 150, 250, 350, 450, 550, 650, 750, 850, or 950 jelly beans, so that each digit from 1 to 9 was equally often the first digit of the correct answer.

The task had three levels of visual quality: Blurry, 2-D, or 3-D (see Figure 2). In the Blurry condition a static image of the jar blurred the boundaries between the visible jelly beans. In the 2-D condition the pictures were static unblurred images of the jars. In the 3-D condition a ten-second video started with the 2-D image and then swept up in an arc to finish with a top view of the jar. This video could be replayed. In this way the total amount of information that participants might draw upon when making an estimation was varied.

Figure 2. The example of a Blurry, a 2-D, and a 3-D image.

Results

Analysis of Number Selection Task

If Benford bias applies to the number selection task, then we predicted that participants would be more likely to choose the number with the lower first digit. Furthermore, this preference should be stronger for pairs of lower first digits because according to Benford's law the difference in frequency of exposure is greater the lower the first digits are.

However, overall the proportion of questions participants answer by selecting the lower first digit ($M = .502$, $SD = .116$) was not significantly different to 0.500, $t(168) = 0.224$, $p = .823$. A one-way ANOVA showed that the number type (non-arbitrary vs. arbitrary) failed to substantially impact the frequencies of people's choice of a lower first digits, $F(1, 167) = 2.698$, *ns*.

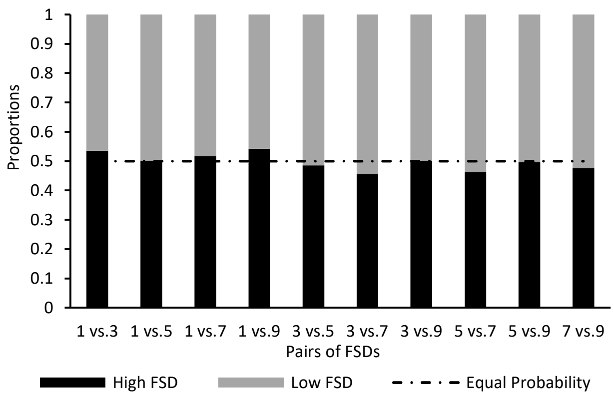

Figure 3. The proportions of a higher and a lower First Significant Digit (FSD) selected for all ten pairs of choices, compared against the equal probability in Number Selection.

To investigate if there was any relationship between the first digit and the number chosen, we examined the ten pairs of first digits in detail (see Figure 3). Only one pair, digit-3 vs. digit-7 ($\chi^2[1] = 6.277$, $p < .05$), showed a greater proportion of choosing the lower first digit over a higher one. For two pairs, digit-1 vs. digit-3 ($\chi^2[1] = 4.383$, $p < .05$) and digit-1 vs. digit-9 ($\chi^2[1] = 5.952$, $p < .05$), participants slightly preferred the choice with the higher first digit. Therefore, nine out of ten pairs failed to be consistent with Benford bias (see Figure 3).

Analysis of Number Generation Task

Figure 4 shows that the distribution of the first digit proportion for non-arbitrary numbers was flatter than found in previous studies, especially with regards to digit-1 (*M*=.169, *SD*=.148). A repeated measures ANOVA of first digit by number type (arbitrary vs non-arbitrary) found a non-significant interaction, $F(8, 1224) = .637$, *ns*., suggesting that the pattern for the non-arbitrary and arbitrary numbers was similar. A linear contrast weighted by Benford's law on proportions of responses using each first digit explained 24.8% variances for non-arbitrary numbers ($F[1,153] = 50.383$, $p < .001$, $\eta^2 = .248$) and 17.5% variances for Arbitrary numbers ($F[1,153] = 32.374$, $p < .001$, $\eta^2 = .175$). So non-arbitrary numbers failed to produce a statistically reliable greater Benford bias than the arbitrary ones.

The pattern for the smaller first digits in non-arbitrary numbers was flatter than that of Benford's law. This could have been due to forward/backwards manipulation of how numbers were entered. However, a repeated-measures ANOVA failed to produce a significant interaction between the answer order and the nine digits for the non-arbitrary numbers, $F(8, 1216) = .501$, *ns*.

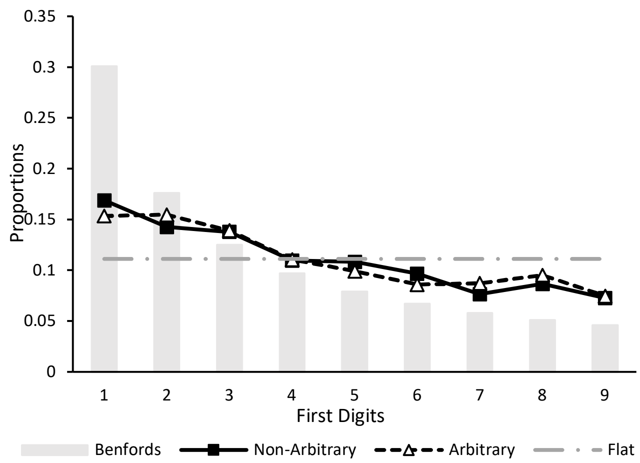

Figure 4. The proportions of first digits from non-arbitrary and arbitrary numbers, compared against Benford's law and flat (correct) distributions in Number Generation.

Analysis of Quantity estimation Task

The distributions of first digits produced for the three types of images (Blurry, 2-D, or 3-D) are presented separately in Figure 5. Overall, digit-1 (*M*= .244, *SD* = .169) was the most frequent first digit generated by the participants, while digit-9 (*M* = .034, *SD* = .061) was the least frequent first digit. A linear contrast analysis weighted by Benford's law suggested the monotonic decline of the first digits did not substantially differ across three image types, $F(2, 158) = .448$, *ns*. The variances in proportions of first digits participants used in all three conditions showed strong Benford biases, as indicated by large effect sizes for each picture type: Blurry: $F(1,51) = 59.069$, $p < .001$, $\eta^2 = .537$; 2-D: $F(1,54) = 74.580$, $p < .001$, $\eta^2 = .580$; 3-D: $F(1,53) = 70.880$, $p < .001$, $\eta^2 = .572$. So athough 3-D images contain more information than 2-D, they did not change the first-digit distributions.

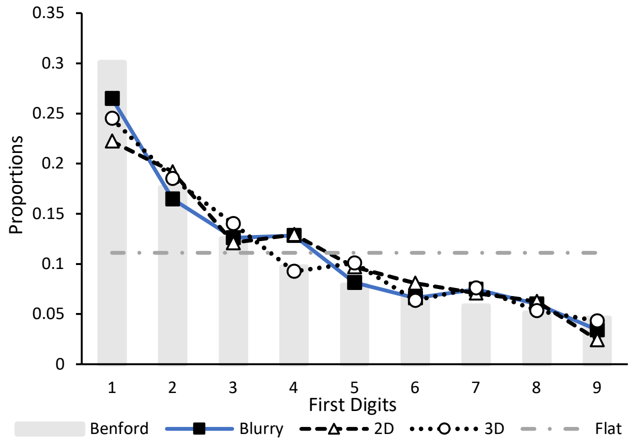

Figure 5. The proportions of the FSDs from the estimates in response to three types of pictures, compared against Benford's law and flat (correct) distributions in Number Estimation.

Discussion

We found evidence of Benford bias in the number generation and quantity estimation tasks, but not in the selection task. As Burns and Krygier (2015) and Tripodi (2016) also showed, there was no evidence of a systematic difference in preferences for any first digit when participants were given a number to select as opposed to having to generate a number, let alone differences consistent with a Benford bias. If people implicitly learned that the lower first digits occurred more frequently than the higher ones due to exposure to this pattern in the environment, then selected responses should have favoured the smaller first digits. Thus, we have no evidence that supports the Recognition Hypothesis as an explanation for Benford bias. It is possible that a different paradigm may find evidence supporting Recognition Hypothesis, but after using three different methods to test it we believe it is more worthwhile to move onto testing other hypotheses for explaining Benford bias.

One limitation of this selection task was that both possible answers had the same magnitude therefore the smaller number always had the lower first digit. So a preference for larger numbers could counteracted preference for lower first digits, but in that case there should still have been differences due to which particular digits were paired in a question.

In the Number Generation task, the first-digit distribution found some evidence of a Benford bias, but the effect size was much smaller than in Burns and Krygier (2015) or Burns et al. (under revision). Furthermore, we did not find a difference between the generation of arbitrary and non-arbitrary numbers, which was surprising given that Burns et al. (under revision) found such a difference. This might be due to two changes in the methodology which were introduced to investigate other questions about Benford bias. First, by not separating the generation of non-arbitrary and arbitrary numbers we may have led participants to be more likely to treat them the same, thus producing a convergence of the processes used to generate the numbers. In particular, all numbers appear to have been more likely to be treated as arbitrary, which would explain why the pattern for non-arbitrary numbers diverged substantially from what has previously been found. Second, by having participants type a number one digit at a time into boxes we may have disrupted people's internal process of producing a number as a whole. If the Benford bias is due to the process of generating numbers, then changes to that process may alter first-digit distributions. The possible impact of these methodological issues will be addressed in other studies and could add to our understanding of the parameters of Benford bias.

The responses generated from the quantity estimation task were consistent with the observations from Burns and Krygier (2015) in finding evidence for a strong Benford bias. The first digits of the estimates approximated Benford's law regardless of the amount of information presented in different types of pictures. Thus, it appears that the tasks with which Benford bias can be demonstrated may be extended to quantity estimation with visual stimuli.

A limitation of the quantity estimation task was that all the true jelly bean counts had the same magnitude, thus the pattern we found in the first digits could be partly due to a systematic underestimation of jelly bean counts. Of course, such an underestimation is predicted for these stimuli by Benford bias, but it is hard to determine if Benford bias is an outcome or a cause of such underestimation. This was a limitation of using similar visual stimuli for which it is hard to greatly vary magnitude. This problem is avoided in the number generation and selection tasks by using a variety of different questions that have a variety of magnitudes, so for future estimation tasks we may need to use a greater variety of visual stimuli.

The Number Recognition task in the present research was the third attempt to demonstrate that the process of selecting rather than generating numerical responses yielded different effects on the first digit of answers. The failure of the Recognition Hypothesis has somewhat surprised us given the findings that people are sensitive to learned statistical relationships under laboratory conditions (e.g., Fiser & Aslin, 2002) and how ubiquitous is Benford's law. This might suggest that the mechanism of automatic acquisition of statistical relationships requires more research to understand the full extent of the constraints on implicit statistical learning (Fiser & Aslin, 2002). It is also indicative of a substantial difference between the process of number generation and the process of number selection.

Therefore, our results are leading us to explore alternative explanations for Benford bias that focus on potential mechanisms for number generation rather than awareness or sensitivity to the first-digit pattern itself. For example, Burns et al. (under revision) proposed an Integration Hypothesis suggesting that Benford bias is a product of how people combine information when generating numbers. This is partly inspired by the mathematical analysis of the conditions under which Benford's law arises (see Berger & Hill, 2015).

The Benford bias we are detecting in people's number estimations appears to be a consequence of how people generate numbers. Therefore, understanding it should provide insight into the cognitive processes for producing numerical answers. The results for the quantity estimation task showed that Benford bias is generalizable beyond just knowledge questions. Our results are also revealing other new phenomena of number generation, such as the elevation of digit-5, which we suspect is unrelated to Benford bias. The studies of the effects of anchoring in decision making arising from Tversky and Kahneman (1974) point to the importance of understanding the number estimation process. However, apart from anchoring, there has been relatively little examination of number estimation processes. This could be due to the lack of good tools for such examination, but our results suggest Benford bias could be such a tool.

In summary, the critical findings of this experiment were the continued failure of the Recognition Hypothesis showing the need to examine alternatives; the extension of Benford bias to the estimation of unknown quantities with the visual presentation; and the disruption of first-digit distribution in the generation task likely due to methodological changes. Therefore, future research is necessary to investigate the processes leading to Benford bias, to extend our methods to examine other forms of estimations, and to further investigate the utility of plausible alternative hypotheses for explaining Benford bias and number generation in general.

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