Title

Spiral Phase Plate Modulation Method for Producing LG_{01} Mode

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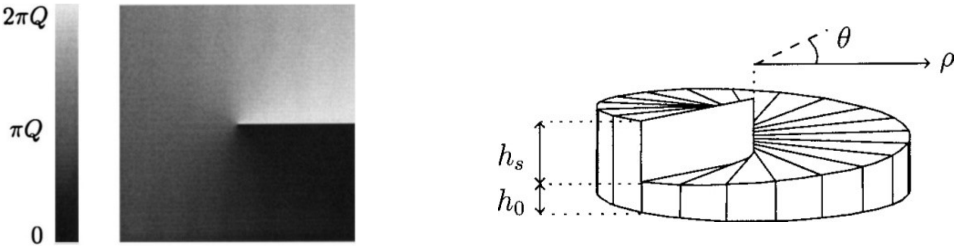
Abstract: Detailed analysis of spiral phase plate (SPP) modulation method for producing LG$_{01}$ laser mode from a Gaussian laser mode.

INTRODUCTION

An electron beam (e-beam) is used in free-electron lasers (FELs) to produce high-intensity laser beams. With a given laser beam as an input, the laser heater inside the FEL can amplify the intensity of the laser beam by inducing transverse oscillation of electrons and producing intense and coherent radiation by stimulated emission. However, the interaction between the e-beam and the laser beam would also produce microbunching instability (MBI), which is known to be detrimental to FELs. To reduce MBI, an effective method is to use an LG$_{01}$ (donut-shape) beam instead of a Gaussian beam, such that the e-beam can be propagated along the optical axis with no field intensity. There are multiple methods to produce an LG$_{01}$ mode laser beam from a Gaussian laser beam. SPP modulation method is discussed in this paper.

METHODS

An effective method to produce LG$_{01}$ is by passing a Gaussian beam through a spiral phase plate (SPP) [1]. The spiral plate modulates the phase of the beam on the cross-section to change the spatial amplitude modulation of the beam. For a uniform TEM mode passing through the spiral plate, the wavefronts of the response are distributed radially around the axis (Fig. 1).



(a) (b)

Fig. 1: (a) Spiral plate phase response (Ref. [2], Fig. 2). Q is an integer. (b) Physical design of spiral plate phase for Q = 1 (Ref. [2], Fig. 1).

A wave with wavelength λ propagating in an optical medium with thickness d and index of refraction n would experience a phase delay of $2\pi nd/\lambda$. With different thicknesses in different azimuth angles, a spiral plate can be designed for a single frequency such that the wave propagated through the SPP can experience different phase delays within the range from 0 to an integer time of 2π (Fig. 1.a). The thickness difference that would create a 2π phase difference is $h_s = \lambda/n$, which is the same as a full wavelength in the medium. The top side of the SPP is not flat, which would affect the quality of the output beam. By choosing a high index of refraction, h_s can be small compared with λ such that the top of Fig. 1.b can be approximated as a flat surface.

Omitting the constant term and assuming the top surface is flat, the phase delay $\Delta\phi$ added to the input beam can be expressed in terms of the azimuth angle θ (in rad/s) is $\Delta\phi = Q\theta$.

A coherent, fixed-polarization, normalized TEM$_{00}$ Gaussian beam can be described by [3, Ch.3.3]:

$$\hat{E}(r,\theta,z,t) = \underbrace{C_{00}[w(z)]^{-1} e^{-\left(\frac{r^2}{w^2(z)}\right)}}_{\text{spatial magnitude profile}} \underbrace{e^{j\frac{kr^2}{2R(z)}} e^{-j\tan^{-1}\left(\frac{2z}{kw_0^2}\right)}}_{\text{spatial phase profile}} \underbrace{e^{j\omega t}}_{\text{temporal phase profile}}$$

$$R(z) = z\left[1 + \left(\frac{kw_0^2}{2z}\right)^2\right], \qquad w(z) = w_0\sqrt{1 + \left(\frac{2z}{kw_0}\right)^2}$$

Where C_{00} is the normalization constant for TEM$_{00}$, k is the wavenumber, and w_0 is the minimum Gaussian beam spot size. The term for polarization is not included.

Assuming $z \ll kw_0^2/2$, $R(z) \to \infty$, $w(z) = w_0$, the Gaussian beam can be described by:

$$\hat{E}(r,\theta,z,t) = e^{-\left(\frac{r^2}{w_0^2}\right)} e^{j\omega t}$$

A phase delay $\Delta\phi = Q\theta$ would be applied to the beam by the SPP. The beam right after the SPP can be described by:

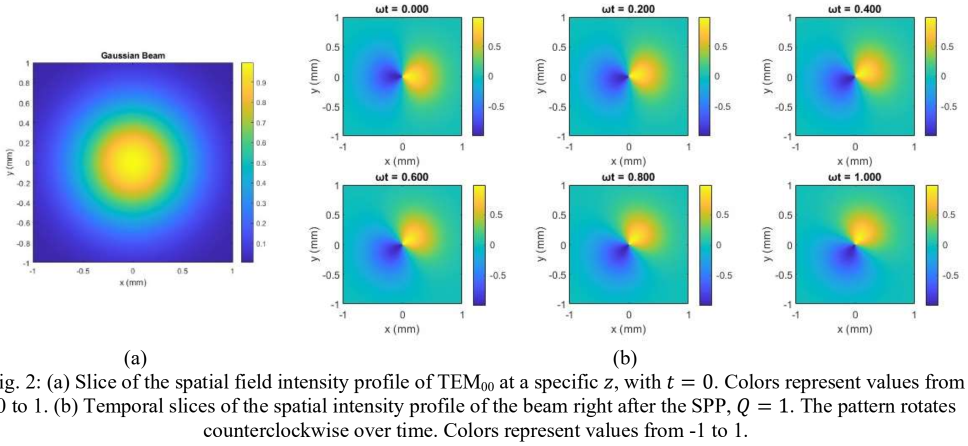
$$\hat{E}_{SP}(r,\theta,z) = e^{-\left(\frac{r^2}{w_0^2}\right)} e^{j\omega t} e^{-jQ\theta}$$

RESULTS AND INTERPRETATION

MATLAB was used for simulating the electric field intensity profiles. The parameters used in the simulations are shown in Table 1.

Table 1: Parameters used in simulations.

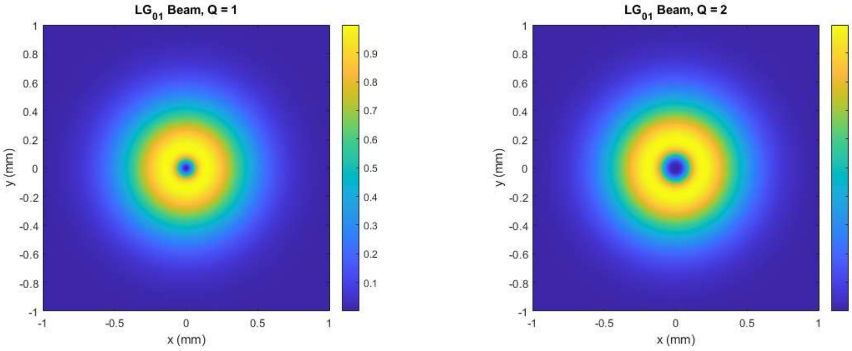
Parameter	Value
Wavelength (λ)	600 nm
Minimum Gaussian beam spot size (w_0)	600 μm
Distance from the waist (z)	1 μm ($z \ll kw_0^2/2 = 1.885\ m$)



(a) (b)

Fig. 2: (a) Slice of the spatial field intensity profile of TEM$_{00}$ at a specific z, with $t = 0$. Colors represent values from 0 to 1. (b) Temporal slices of the spatial intensity profile of the beam right after the SPP, $Q = 1$. The pattern rotates counterclockwise over time. Colors represent values from -1 to 1.

For a distance away from the SPP, for each location (r, θ) on the cross-section, optical waves from the SPP superpose to form patterns. The fields around the axis $(r \to 0)$ have a high density over phases from 0 to 2π, therefore they are destructively superposed such that the field intensity around the axis is nearly zero. The fields far away from the axis experience less destructive interference and thus maintain some intensity. Therefore, the time-averaged power profile of the beam is in donut-shape, also known as the LG_{01} mode.



(a) (b)

Fig. 3: LG_{01} beam spatial time-average power profile with (a) $Q = 1$, (b) $Q = 2$.

For a larger Q, the fields have higher density over phases from 0 to 2π around the axis, and thus destructive interferences would occur in a larger region around the axis, creating an LG_{01} pattern with a larger inner radius.

Depending on the setup of the lens system, the Gaussian beam's center might be misaligned with the designed optical axis of the SPP. To reduce the system's sensitivity to this misalignment, the input beam pattern can be enlarged by a set of concave-convex lenses [1].

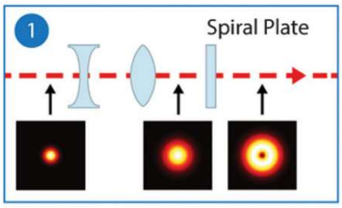


Fig. 4: Spiral plate modulation method (Ref. [1], Fig. 1, inset 1).

CONCLUSIONS

By converting the Gaussian beam to an LG_{01} beam, the e-beam in free-electron lasers can be propagated along the optical axis without being interfered with by the optical fields, and thus the MBI can be suppressed. The LG_{01} laser mode needed for suppressing MBI in free-electron lasers can be created effectively using a spiral phase plate. By choosing a proper index of refraction and the thickness distribution of the spiral plate, a spiral plate can be used to convert a Gaussian beam to an LG_{01} beam for a specific frequency.

The spiral phase plate method can be simply applied to any complicated project by adding one more optical device minimum, which is the SPP, into the system. However, it is expensive to use such an SPP, because an SPP is only designed for a specific frequency, and therefore the SPP used in any project must be customized.

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