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360 OPC on “Enactive Metaphorizing in the Mathematical Experience” by Daniela Díaz-Rojas, Jorge Soto-Andrade, and Ronnie Videla-Reyes

Enactivist How? Rethinking Metaphorizing as Imaginary Constraints Projected on Sensorimotor Interaction Dynamics

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Abstract: Welcoming their scholarly focus on metaphorizing, I critique Díaz-Rojas, Soto-Andrade and Videla-Reyes’s selection of the hypothetical constructs “conceptual metaphor” and “enactive metaphor” as guiding the epistemological positioning, educational design, and analytic interpretation of interactive mathematics education purporting to operationalize enactivist theory of cognition – both these constructs, I argue, are incompatible with enactivism. Instead, I draw on ecological dynamics to promote a view of metaphors as projected constraints on action, and I explain how mathematical concepts can be grounded in perceptual reorganization of motor coordination. I end with a note on how metaphors may take us astray and why that, too, is worthwhile.

1. In their target article, Daniela Díaz-Rojas, Jorge Soto-Andrade, and Ronnie Videla-Reyes discuss the theory, practice, and phenomenology of metaphorizing in mathematics education. Whereas I salute the authors for rekindling an important conversation bearing potentially momentous implications for education, and while I delighted in engaging with their sample mathematical problems, I interrogate what I view as epistemological incompatibility between their philosophical affinities (enactivism) and their theoretical allegiances (conceptual metaphor). I propose an alternative view of metaphor, arguing it is consistent with foundational enactivist tenets. Notwithstanding, I join the authors in viewing idiosyncratic metaphorizing as an epistemic practice that should be fostered in mathematics education, even though – and, perhaps, precisely *because*, I offer – some metaphors lead us down a garden path, so to speak, giving us pause to think about thinking.

Epistemology: What should be an enactivist account of metaphorizing?

2. Similar to the authors, I am a cognitive scientist who espouses enactivist epistemology to inform educational design and research. As such, I was left wondering how the authors make sense of learning from the enactivist philosophical and theoretical standpoint. That is, I was hoping for a more technical explanatory model detailing how embodied activity may bring forth conceptual understanding – a model that is either sufficiently specified or potentially specifiable as lending researchers analytic purchase on empirical data documenting the micro-genetics of educational interactions with artifacts, peers, and instructors. That is, I was hoping for a theoretically grounded and

pedagogically actionable epistemological position on how enactivist epistemology unfolds in human phenomenology and conscious cognition. In our own design-based research, we have been inspired by the following succinct mantra of Francisco Varela, Evan Thompson, and Eleanor Rosch (1991: 173):

> "[T]he enactive approach consists of two points: (1) perception consists in perceptually guided action and (2) cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided."

3. Thus, whereas in the target article the authors (§22) espouse central tenets of the cognitive semantics theory of conceptual metaphor as it obtains in mathematics (Lakoff & Núñez 2000), I view those tenets as epistemologically incompatible with enactivism. My position builds on analyzing a decade of empirical data evidencing the persistent irrelevance of that theory as a viable account for the emergence of new perceptuomotor competence underlying mathematical concepts (for a review, see Abrahamson 2019). In our activity architecture – the Mathematics Imagery Trainer – students spontaneously develop new perceptual orientations toward an embodied-interaction display as their means of coordinating the enactment of bimanual movements that solve motor-control tasks. Our mixed-methods methodologies triangulate students' digital manipulation, eye-gaze, and verbal–gestural utterance (Tancredi et al. 2021). We thus attempt to understand how new ways of moving bring forth new ways of thinking. Our research program, *embodied design*, is a quest to align the theory and practice of mathematics education around a coherent set of principles that draw on the embodied turn in the cognitive sciences.

4. So, how does metaphorizing work? In Abrahamson, Sánchez-García & Smyth (2016) we drew on ecological dynamics (Araújo, Davids & Renshaw 2020) to model metaphor across diverse domains of practice, including sports, somatic therapy, and mathematics. We proposed to theorize metaphor as constituting a constraint that individuals project imaginatively onto their perceptuomotor attempts to engage the environment so as to enact goal movements that achieve task performance. Under these self-imposed constraints, individuals reconfigure their perception of the environment so that it re-affords the coordination of actions. I later demonstrated this ecological-dynamics principle in an introspective phenomenological analysis of metaphor-based skill development. My study, situated in the discipline of cello playing, highlighted the pivotal role of pursuing sensorial goals, which are implicit to the action-based instructional metaphors, as orienting the emergence of new perceptually guided action (Abrahamson 2020). I further conjectured that mathematical cognition coopts an evolutionarily selected capacity to improve our perceptuomotor grasp on the environment (Abrahamson 2021). As such, our research program has sought to emulate enactivist epistemology bottom-up from sensorimotor phenomenology, even as we consider the top-down cultural framings of these explorations. Let me now consider the pedagogical implications of this epistemological position for the design of digital educational resources.

Design: Students should figure out themselves how to move in new ways

5. In §6, Díaz-Rojas, Soto-Andrade, and Videla-Reyes cite the construct of *enactive metaphor* (Gallagher & Lindgren 2015) as putatively constituting a powerful heuristic principle for designing pedagogical architectures that would operationalize enactivist philosophy in the form of interactive experiences for learning mathematical content. My own reading of this same empirical work motivating the "enactive metaphor" construct, however, suggests that the interaction design does not emulate pedagogical implications of enactivist philosophy. I agree that students engaged in those activities do enact movements, but these movements are dictated rather than discovered. Namely, the movements that students perform as they engage in the cited instructional activities are explicitly prescribed remote-control manual mini-choreographies for operating an interface, such as twisting or rotating the hand in a meticulously specified form to activate virtual objects. The activity does not attempt to solicit individuals' enactive *know-how*, instead stipulating experts' *how-to* dynamic gestures. As such, this educational interpretation of embodiment – digital bells, whistles, and all – is ironically regressive relative to the traditional mechanical manipulatives it attempts to emulate, supplant, and enhance. Imposed motor actions, I submit, eviscerate enactive discovery dynamics from the child–environment interaction design, instead grafting cookie-cutter engagement as opaque enactive prosthetics (Abrahamson 2018). As such, enactive metaphor commits Shaun Gallagher's own sin of "body snatching" (Gallagher 2015; see also Sheets-Johnstone 2015). The authors are not alone in twisting constructivist epistemology as condoning movement dictation (e.g., see Abrahamson et al. 2020; Zhang et al. 2021). This scholarly debate over how students should move to learn could be resolved empirically (Abrahamson & Abdu 2020). Such research would be timely, given the increasing proliferation of novel interactive commercial technological applications that could support these educational processes.

Pedagogical tradeoffs: Metaphorization as a double-edged sword

6. Cognitive scientists have long wondered whether metaphorizing mathematical problems bears any tradeoff for learning and problem solving. Most vociferous in this regard are David Uttal, Kathyrn Scudder and Judy DeLoache (1997), and Vladimir Sloutsky, Jennifer Kaminski, and Andrew Heckler (2005), who claimed that concretizing mathematical situations is liable to introduce distracting extraneous information into the problem space, ultimately delimiting generalization (but see critique in Trninic, Kapur, & Sinha 2020).

7. Instead of this cognitivist caveat (e.g., Gray & Holyoak 2021), we have offered an enactivist account. In Rosen, Palatnik & Abrahamson (2018) we reported on an experiment that investigated the effect of iconicity on students' perceptual solutions to a bimanual motor-control problem designed to foster the enactment of a proportional movement form. The experimental group manipulated virtual icons of hot-air balloons, while the comparison group manipulated blank circles of identical size. The iconic group were reluctant to manipulate the hot-air balloons in "hazardous" ways, such as rotating the icons; they tended to view the icons as orienting from the bottom of the

screen up along straight vertical trajectories; they construed the icons as located at specifiable heights above the bottom of the screen, which thus constituted an *ad hoc* spatial frame of reference; and they confabulated brief schematic narratives describing the exploits of the two icons in relation to each other, such as engaged in a competition to get to the top of the screen. By way of comparison, the blank-circle group appeared unconstrained by any practical considerations that draw on familiar situations, and they manipulated the virtual objects in myriad ways. We argued that the iconic group thus missed out on a repertory of sensorimotor engagements that would have grounded and enriched their mathematical insights, learning, and coordinations (Abrahamson et al. 2014). For resonant views on the brittleness of mathematical metaphorizing, see Abrahamson (2009: 179f).

8. In Abrahamson, Berland, et al. (2006) we recounted how a group of graduate students made sense of a bar-chart output from a computer-based simulation of a waiting-time probability experiment. One of the students metaphorizes the total set of waiting times data outputs as a concatenated spatial continuum of "sticks"; yet, in so doing, he inadvertently brought to bear mathematically unsubstantiated assumptions that led him to tackle the situation as though it were a partition problem. In Abrahamson, Gutiérrez & Baddorf (2012) we described a set of idiosyncratic metaphors invented spontaneously by student–tutor dyads engaged in task-based semi-structured clinical interviews concerning the expected outcomes of concrete and digital probability experiments. One student metaphorized the sample space as generated by a slithering snake, yet the snake succumbed to combinations inauspicious to slithering motility. Finally, in Abrahamson, Janusz & Wilensky (2006: Section 2.4.4) we showcased a group of middle-school students whose figural approach to a probability counting problem led them to a solution impasse. Remarkably, however, this was the only group in two classrooms who so clearly articulated the objectives of all combinatorial analysis methodologies: to come up with a rigorous system that counts all permutations without duplicating any count. In summary, our findings suggest that mathematical metaphors are not born equal. Sometimes they lead you astray, sometimes they lead you to dead ends, and yet sometimes those meanderings and dead ends prove instrumental for learning (Leron & Ejersbo 2021).

Conclusion

9. Metaphorizing is the quintessential epistemic mode of mathematical thinking. Accordingly, in their manifesto for enactivist mathematics pedagogy, Abrahamson, Dutton & Bakker (in press) encourage instructional practices that legitimize idiosyncratic metaphorizing in classroom discourse. As such, Díaz-Rojas, Soto-Andrade, and Videla-Reyes's target article is an important contribution to the field, by way of stimulating further discussion among cognitive developmental psychologists and educational researchers on enactivist theorization of mathematical metaphorizing. This further discussion might focus on which theories of metaphorical reasoning from the cognitive sciences would best appertain to and leverage enactivist literature. The particular theories of metaphorical reasoning selected by the authors as their means of reasoning through their empirical cases, I submit, do not do justice to the profundity of

enactivist insight and do not lend sufficient methodological traction on the data to meaningfully and rigorously advance the elaboration of enactivist perspectives. Instead, embodied-design technologies for mathematics education, multimodal learning analytic instruments and algorithms, and micro-phenomenological interview approaches now enable us to evaluate and advance enactivist perspectives by scrutinizing how cognitive structures arise from repeated patterns in students' explorative sensorimotor activity that ultimately enables action to be perceptually guided. Considering these resources could promote the authors' research program.

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