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Publication Date
2006-05-20

On the Informational Role of Prices with Rational Expectations[*]

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May 10, 2006

Abstract

   Traders' expected utilities in fully revealing rational expectations equilibrium (REE) are shown to decrease as the number of informed traders is increased for an asset market model with diverse information as in Grossman (1976). It follows that no trader has any incentive to acquire information even if none of other traders does. Consequently, when information acquisition is endogenous, there exists a unique overall equilibrium with no trader acquiring information that has the fully revealing REE as an integral part, so that prices would fully reveal private information were it to be acquired by traders. This result provides a strengthening of the fundamental conflict between the efficiency with which markets spread information through the prices and the incentive to acquire information. Both the existence and the no information acquisition feature of the overall equilibrium do not depend on whether traders are endowed with the risky asset or not. (*JEL* D82, D84, G14)

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1 Introduction

When privately informed traders interact on markets, the resulting prices reflect the private information they use in making their decisions. Since prices are observable, a trader may attempt to infer additional information from prices, which is potentially valuable to the trader. The model of REE provides both a coherent description of this phenomenon and a solution for how well prices aggregate and transmit private information.

In REE each trader maximizes expected utility subject to budget constraint, markets clear, and each trader's expectations properly reflect not only his private information but also whatever information that can be inferred from the prices. The notion of REE is particularly well suited to the analysis of financial markets because asymmetry and diversity of information are important elements of these markets.

In a seminal paper, Grossman (1976) proposes an asset market model having both risk aversion and asymmetric information. In that model, there are two assets, one with a certain return and one with an exogenous unknown payoff. Trade takes place in the first period and consumption in the second period. The risk-free asset is in perfectly elastic supply while the risky asset is in perfectly inelastic supply. Traders hold a common prior about the future payoff of the risky asset. However, before entering into trade, they receive diverse private signals concerning the uncertain payoff. Under some distributional assumptions for the uncertain payoff and the signals and under constant absolute risk aversion (CARA) utility functions for the traders, Grossman shows remarkably that the model has a *fully revealing* REE: for every realization of the private signals, the REE price mapping is a *sufficient statistic* for the uncertain payoff of the risky asset given the realized private signals.[1]

Therefore, in fully revealing REE, knowing the equilibrium price vector no trader gains additional information from observing his own signal. This results in a paradoxical result that has come to be called *the Grossman paradox*:[2]

> "the paradoxical nature of "perfect markets," which the model illustrates, is robust. When prices are a perfect aggregator of information it removes private incentives to collect information ⋯ If there is no noise and information collection is costly, then a perfect competitive market will break down because no equilibrium exists where information collectors earn a return on their information, and no equilibrium exists where no one collects information." (Grossman 1976, pp. 574, 584).

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[1]DeMarzo and Skiadas (1998) show that fully revealing REE for Grossman's (1976) model is unique.

[2]See for example Blak and Tonks (1992, p. 292).

2

The incentives for private information acquisition can be weakened in two ways. First, one can infer information acquired by other traders from prices without paying any cost. Second, information acquisition can result in a less preferable equilibrium. Although much attention has been devoted to the former type of disincentives in the literature, the latter type was not left unnoticed. Hirshleifer (1971) first notes that, in an economy with risk-averse traders, better information does not necessarily lead to a better competitive equilibrium allocation. This social valuelessness of information is known as the *Hirshleifer effect*.[3] Shubik (1977) considers a competitive model of exchange with uncertainty, in which he shows that information can have negative social value. In this paper we investigate the significance of the latter type of disincentives with rational expectations and, in particular, possible impacts of these disincentives on the paradox.[4]

We show that the fully revealing REE of the Grossman's model implies that traders' expected utilities, both before (*ex ante*) and after signal realizations (*ex post*), all decrease as the number of the informed is increased, regardless of whether signals are costly or not. Intuitively, the more informed traders there are, the less preferable the fully revealing REE becomes, because competition due to more information always drives up the expected period 0 price of the risky asset.

Consequently, information collectors do not earn any return on their information. Furthermore, no trader has any incentive to acquire information even if none of other traders does. Hence, when information acquisition is endogenous, contrary to the non-existence, there exists a unique overall equilibrium with no trader acquiring information that has the fully revealing REE as an integral part, so that prices would fully reveal private information were it to be acquired by traders. These results strengthen the fundamental conflict between the efficiency with which markets spread information through the prices and the incentive to acquire information. The results also imply that with rational expectations, there will not be a socially wasteful overproduction of information nor will there be any private value from information acquisition, as argued otherwise without rational expectations by Hirshleifer (1971) and Marshall (1974).

The results are robust with respect to traders' endowments, in the sense that

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[3]We refer the reader to Brunnermeier (2001, p. 36) for a discussion of the Hirshleifer effect.

[4]In yet another seminal paper, Grossman and Stiglitz (1980) consider an asset market model in which traders are divided into two groups: the group of informed traders and the group of uninformed traders. All informed traders receive the same instead of diverse information as in Grossman (1976). In the conclusion of this paper, we briefly discuss an application of the results in this paper to Grossman and Stiglitz model.

3

they do not qualitatively depend on whether traders are endowed with the risky asset or not. However, while all traders are better off in the fully revealing REE than they were prior to trading when they are not endowed with the risky asset, strict Pareto improvement is not always possible when traders are endowed with the risky asset. Analysis of the Grossman paradox regarding the informational role of prices is in Section 2. Conclusion is in Section 3.

2 Informational Role of Prices, Grossman Paradox, and Analysis

In this section we first quickly review Grossman's (1976) model and its fully revealing REE. We then present our reexamination of the Grossman paradox. Due to the connection of the fully revealing REE allocation with the Walrasian equilibrium (WE) allocation, we begin with our analysis of WE for the model.

2.1 Grossman's (1976) Model

There are two periods, 0 and 1, and two assets, one risk-free asset and one risky asset. The price of the risk-free asset in period 0 is normalized to be 1 and the return on it is constant and equal to $r \geq 0$. The price for the risky asset in period 0 is denoted by P_0. Each unit of the risky asset has an (unknown) exogenous payoff $\tilde{P}_1$ in period 1. Both P_0 and $\tilde{P}_1$ are in units of the period 0 risk-free asset.[5] Total supply of the risky asset in period 0 is fixed at $\bar{X}$, but the supply of the risk-free asset is perfectly elastic. Consumption takes place only in period 1.

There are n traders. Each trader i has an exponential utility function $U_i(\tilde{W}_{1i}) = -e^{-a_i \tilde{W}_{1i}}$ over wealth $\tilde{W}_{1i}$ in period 1 with constant coefficient of absolute risk aversion $a_i > 0$. Trader i does not have any endowment of the risky asset. He purchases both the risk-free and the risky assets in period 0 with his endowed wealth W_{0i}. Let I_i denote information available for trader i. If the uncertain wealth $\tilde{W}_{1i}$ is normally distributed conditional on I_i, then it follows from the exponential form of trader i's utility function that trader i's expected utility conditional on I_i is given by

$$E[U_i(\tilde{W}_{1i})|I_i] = -exp\{-a_i[E[\tilde{W}_{1i}|I_i] - \frac{a_i}{2}Var[\tilde{W}_{1i}|I_i]]\},$$

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[5]We follow the notation in Grossman (1976).

4

where $E[\tilde{W}_{1i}|I_i]$ and $Var[\tilde{W}_{1i}|I_i]$ denote the conditional mean and the conditional variance of $\tilde{W}_{1i}$. Consequently, maximizing $E[U_i(\tilde{W}_{1i})|I_i]$ is equivalent to maximizing

$$E[\tilde{W}_{1i}|I_i] - \frac{a_i}{2}Var[\tilde{W}_{1i}|I_i]. \tag{1}$$

Given P_0, trader i chooses a portfolio (X_{Fi}, X_i) in period 0, with X_{Fi} units of the risk-free and X_i units of the risky assets, to maximize (1) subject to budget constraint:

$$X_{Fi} + P_0 X_i = W_{0i}. \tag{2}$$

From (2), $\tilde{W}_{1i} = (1+r)W_{0i} + [\tilde{P}_1 - (1+r)P_0]X_i$. Hence, if $\tilde{P}_1$ is normally distributed conditional on I_i, then the optimal demand for the risky assets for trader i is given by:

$$X_i^d = \frac{E[\tilde{P}_1|I_i] - (1+r)P_0}{a_i Var[\tilde{P}_1|I_i]}. \tag{3}$$

Equation (3) shows that trader i's demand is independent of his wealth. This is because constant absolute risk aversion implies zero income effect.

Before entering into trade each trader receives a signal y_i about $\tilde{P}_1$ that satisfies

$$y_i = P_1 + \epsilon_i,$$

where P_1 denotes a realization of $\tilde{P}_1$ and ϵ_i is the noise term. It is assumed that ϵ_i is normally distributed with mean 0 and variance $\sigma_\epsilon^2 = 1$, so that each trader i gets a signal of equal precision $Var(\tilde{P}_1|y_i) = \frac{\sigma^2}{1+\sigma^2}$. Furthermore, the random variables ϵ_i are jointly normally distributed with covariance $Cov(\epsilon_i, \epsilon_j) = 0$, for $i \neq j$, and $\tilde{P}_1$ is independent of ϵ_i and is normally distributed with mean $\bar{P}_1$ and variance σ^2.

To analyze traders' incentives to acquire information, we now slightly generalize Grossman's (1976) model by allowing the possibility that not all traders are informed. That is, we now divide the traders into two groups. Group N_1 consists of traders who receive signals and group N_0 consists of traders who do not receive signals. Set $N = N_0 \cup N_1$. Let n_1 denote the number of traders in N_1 and let n denote the total number of traders. It is assumed that $n < \infty$.

2.1.1 Walrasian Equilibrium

Traders in the Walrasian model do not infer information from the prices. In this case, $I_i = \{y_i\}$ if i is informed; I_i is the set of all signal values if i is not informed. Thus,

$E[\tilde{P}_1|I_i] = \bar{P}_1$ and $Var[\tilde{P}_1|I_i] = \sigma^2$ when i is uninformed; and by the distributional assumptions, it follows that

$$E[\tilde{P}_1|I_i] = \bar{P}_1 + \frac{\sigma^2}{1+\sigma^2}(y_i - \bar{P}_1) \text{ and } Var[\tilde{P}_1|I_i] = \frac{\sigma^2}{1+\sigma^2}$$

when i is informed (see Theorem 3 in DeGroot 1975, p. 269). Furthermore, the period 1 wealth $\tilde{W}_{1i}$ of trader i conditional on I_i is normally distributed. Consequently, (3) can be simplified to

$$X_i^d = \frac{\bar{P}_1 - (1+r)P_0 + \sigma^2[y_i - (1+r)P_0]}{a_i\sigma^2}, \ i \in N_1$$

and

$$X_i^d = \frac{\bar{P}_1 - (1+r)P_0}{a_i\sigma^2}, \ i \in N_0.$$

Using the above simplified demand functions together with the market clearing condition for the risky asset calling for the equality between demand and supply, it can be verified that the Walrasian equilibrium with n_1 informed traders is given by:

$$\begin{cases} P_0^*(y) = \frac{\frac{\bar{P}_1}{\sigma^2}\sum_{j \in N}\frac{1}{a_j} + \sum_{j \in N_1}\frac{y_j}{a_j} - \bar{X}}{(1+r)[\frac{1}{\sigma^2}\sum_{j \in N}\frac{1}{a_j} + \sum_{j \in N_1}\frac{1}{a_j}]}, \\ X_i^*(y) = \frac{(\sum_{j \in N_0}\frac{1}{a_j})[y_i - \bar{P}_1] + (1+\sigma^2)\sum_{j \in N_1}\frac{1}{a_j}[y_i - y_j] + (1+\sigma^2)\bar{X}}{a_i[\sum_{j \in N}\frac{1}{a_j} + \sigma^2\sum_{j \in N_1}\frac{1}{a_j}]}, \ i \in N_1, \\ X_i^*(y) = \frac{\sum_{j \in N_1}\frac{1}{a_j}[\bar{P}_1 - y_j] + \bar{X}}{a_i[\sum_{j \in N}\frac{1}{a_j} + \sigma^2\sum_{j \in N_1}\frac{1}{a_j}]}, i \in N_0, \end{cases}$$

where y denotes the collection of the signals of the informed traders.

Notice the above equations imply that in WE trade depends on signals. This is due to the fact that traders do not infer information from prices; hence, their updated beliefs using the private information are different which results in their trades depending on signals. Notice also when traders are all uninformed, the WE prices and demands reduce to

$$\begin{cases} P_0^* = \frac{\bar{P}_1}{1+r} - \frac{\sigma^2\bar{X}}{(1+r)\sum_{j \in N}\frac{1}{a_j}}, \\ X_i^* = \frac{\bar{X}}{a_i\sum_{j \in N}\frac{1}{a_j}}, \ i \in N. \end{cases} \quad (4)$$

The risky asset allocation in (4) coincides with the allocation according to the *optimal risk sharing rule* (see Kreps 1990, p. 173).

6

2.1.2 Fully Revealing REE and Grossman Paradox

The preceding analysis shows that prices in WE depend on signals of the informed traders. A trader can thus infer additional information from prices. Let $P_0(y)$ denote the price mapping. With information inference from prices, the conditional mean and conditional variance trader i now uses become $E[\tilde{P}_1|y_i, P_0(y)]$ and $Var[\tilde{P}_1|y_i, P_0(y)]$ when i is informed; $E[\tilde{P}_1|P_0(y)]$ and $Var[\tilde{P}_1|P_0(y)]$ when i is uninformed. The following definition of a rational expectations equilibrium is taken from Grossman (1976, p. 576, 1981, pp. 549-550).

Definition 1 *A REE is a pair (P_0^*, X^*) of mappings P_0^* and $X^* = (X_1^*, \cdots, X_n^*)$ from collections of realized signals of informed traders to prices and allocations, respectively, such that for any collection of realized signals y, (i) $X_i^*(y)$ maximizes trader i's expected utility conditional on $I_i = \{y'|P_0^*(y') = P_0^*(y), y_i' = y_i\}$ if i is informed and on $I_i = \{y'|P_0^*(y') = P_0^*(y)\}$ if i is uninformed subject to budget constraint (2) at price vector $P_0^*(y)$; (ii) $\sum_{i=1}^n X_i^*(y) = \bar{X}$.*

A REE is *fully revealing* if for every realization of the private signals y, the REE price vector $P_0^*(y)$ is a *sufficient statistic* for $\tilde{P}_1$ given y (see Grossman 1976, 1981 and Admati 1989). By Theorem 1 in Grossman (1976), there exits a fully revealing REE with [6]

$$P_0^*(y) = \alpha_0 + \alpha_1 \bar{y}, \tag{5}$$

where

$$\bar{y} = \frac{1}{n_1} \sum_{j \in N_1} y_j,$$

$$\alpha_0 = \frac{\bar{P}_1 \sum_{j \in N} \frac{1}{a_j} - \sigma^2 \bar{X}}{(1+r)(1+n_1\sigma^2) \sum_{j \in N} \frac{1}{a_j}},$$

$$\alpha_1 = \frac{n_1 \sigma^2}{(1+r)(1+n_1\sigma^2)},$$

where n_1 denotes the number of informed traders. Notice when $n_1 = 0$, the above fully revealing REE price reduces to the WE price in (4). Proposition 8 in DeMarzo and Skiadas (1998) implies that the fully revealing REE in Grossman's model is unique.

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[6]Grossman proves the result for the case where all traders are informed. His proof also applies equally well when some but not all of the traders are informed.

The REE price mapping $P_0^*(y)$ in (5) is a sufficient statistic for $\tilde{P}_1$ given y. This led Grossman to identify a paradoxical result about perfect markets. No trader will have any incentive to acquire costly information because equilibrium prices fully reveal all the information acquired by other traders and prices can be observed for free. However, if no trader acquires information, price will not be informative, in which case it would be in the interest of any one trader to acquire information. He concludes that with endogenous costly information acquisition, no equilibrium exists.

2.2 Endogenous Information Acquisition

To analyze traders' incentives to acquire information, we look at net benefits information acquisition brings to a trader. Accordingly, we compare a trader's *ex ante* (before signal realization) expected utility due to information acquisition with his *ex ante* expected utility without information acquisition.

To do this, we consider information acquisition and trade as taking place in two stages. In stage one, each trader decides individually whether to acquire information or not. A trader who decided to acquire information becomes informed (i.e. receives a private signal) and pays a cost if necessary. In stage two, traders enter into trade given their choices in stage one.

We analyze traders' choice problems by backward induction. First, traders' individual portfolio selections are determined using the fully revealing REE for given choices in stage one.[7] As in Grossman (1976), we assume price-taking behavior and rational expectations on individual traders in this stage. Second, traders' individual choices in stage one in turn are determined by maximizing their expected utilities in the subsequent fully revealing REE.

2.2.1 The Existence of an Equilibrium

By (5) and by the distributional assumptions on the period 1 payoff of the risky asset and on signals, Theorem 3 in DeGroot (1975, p. 269) implies that in the fully revealing REE at signal realization y,

$$E[\tilde{P}_1|I_i] = \frac{\bar{P}_1 + n_1\sigma^2\bar{y}}{1 + n_1\sigma^2} \qquad (6)$$

---

[7]When a trader decides to acquire information in stage one, he will be expected by other traders to act as an informed trader in stage two. Thus to apply the fully revealing REE, we take the set of informed traders to be the set of those who decide to acquire information in stage one.

and

$$Var[\tilde{P}_1|I_i] = \frac{\sigma^2}{1 + n_1\sigma^2}. \tag{7}$$

Plugging the conditional mean in (6), the conditional variance in (7), and the price vector $P_0 = P_0^*(y)$ in (5) into (3) results in the following demand for the risky asset of trader i:

$$X_i^*(y) = \frac{\bar{X}}{a_i \sum_{j \in N} \frac{1}{a_j}}. \tag{8}$$

Notice trader i's demand for the risky asset in (8) does not depend on the price or signals. As explained in Admati (1989), with reational expectations a change in price results in an *information effect* in addition to the usual income and substitution effects, because a price change induces traders to change their expectations due to information inference. The substitution and information effects exactly offset each other in Grossman's (1976) model. Since income effect is zero, total effect due to a price change is zero.

Notice also trader i's demand coincides with his demand in WE given by (4). This can be explained as follows. First, when traders are all uninformed, the WE allocation is *ex ante* Pareto optimal. Second, with rational expectations, traders do not trade only on the basis of private information as implied by the *No-Trade Theorem* in Milgrom and Stokey (1982).

Proposition 1 *Traders' expected utilities, both ex ante (before signal realizations) and interim (after signal realizations), in the fully revealing REE decrease as the number of informed traders is increased.*

Proof. Recall that budget constraint (2) implies $\tilde{W}_{1i} = (1+r)W_{0i} + [\tilde{P}_1 - (1+r)P_0]X_i$. This together with (6), (7), and (8) implies that in the fully revealing REE given signal realization y, the difference between the conditional mean and the conditional variance in (1) can be simplified to:

$$\begin{aligned}
&E[\tilde{W}_{1i}|I_i] - \frac{a_i}{2}Var[\tilde{W}_{1i}|I_i] \\
&= (1+r)W_{0i} + \frac{\sigma^2\bar{X}^2}{a_i(\sum_{j \in N} \frac{1}{a_j})^2(1+n_1\sigma^2)} - \frac{a_i}{2}\frac{\sigma^2\bar{X}^2}{a_i^2(\sum_{j \in N} \frac{1}{a_j})^2(1+n_1\sigma^2)} \\
&= (1+r)W_{0i} + \frac{\sigma^2\bar{X}^2}{2a_i(\sum_{j \in N} \frac{1}{a_j})^2(1+n_1\sigma^2)}
\end{aligned} \tag{9}$$

for all i, where n_1 is the number of informed traders. Equation (9) shows that the difference does not depend on signal realizations y. Hence, integrating *interim*

9

expected utility $E_{\tilde{P}_1|\bar{y}}U_i[\tilde{W}_{1i}|I_i]$ over $\bar{y}$ yields trader i's *ex ante* expected utility equal to his *interim* expected utility: [8]

$$
\begin{aligned}
E_{\bar{y}}E_{\tilde{P}_1|\bar{y}}U_i[\tilde{W}_{1i}|I_i] &= E_{\tilde{P}_1|\bar{y}}U_i[\tilde{W}_{1i}|I_i] \\
&= -exp\left\{ -a_i\left[(1+r)W_{0i} + \frac{\sigma^2 \bar{X}^2}{2a_i(\sum_{j\in N}\frac{1}{a_j})^2(1+n_1\sigma^2)}\right]\right\}.
\end{aligned}
\tag{10}
$$

Equation (10) implies that trader i's expected utility before or after signal realizations decreases with the number of informed traders. $\blacksquare$

The total expected excess return from risky asset holding for trader i is $E[\tilde{P}_1 - (1+r)P_0^*(y)|I_i]X_i^*(y)$. By normality of the distributions and (5), the per-unit expected excess return is

$$
E[\tilde{P}_1 - (1+r)P_0^*(y)|I_i] = \frac{\sigma^2 \bar{X}}{(1+n_1\sigma^2)\sum_{j\in N}\frac{1}{a_j}}.
\tag{11}
$$

Together, (8) and (11) imply that the second term in the first equality in (9) is the total expected excess return from risky asset holding for trader i. By (11), the per-unit expected excess return decreases as the number of informed traders is increased. What drives this decrease is the fact that more informed implies that more information regarding the future payoff of the risky asset becomes available for every trader; hence, competition among traders bids up the expected price over signals, $E_{\bar{y}}P_0^*(y)$, which causes the expected excess return to decrease.[9] Since demand for the risky asset $X_i^*(y)$ remains unchanged as the number of informed traders changes, the total expected excess return also decreases with the number of informed traders. The loss in expected utility due to the decrease in total expected excess return outweighs the gain from improved information precision.

Trader i's expected utility in (10) has two parts. The first part corresponds to what trader i can get by putting all his endowed period 0 wealth in risk-free asset. That is, his wealth level would simply have been $(1+r)W_{0i}$ were he not to share the risk by entering into trade for the risky asset. Risk sharing brings an increase

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[8]We now use the subscript in the expectation operator to denote the random variable(s) over which the expectation is taken.

[9]By (5), it can be checked that the expected period 0 price $E_{\bar{y}}P_0^*(y)$ of the risky asset increases as the number of informed traders is increased.

in his expected utility which corresponds to the second part

$$\frac{\sigma^2 \bar{X}^2}{2a_i(\sum_{j \in N} \frac{1}{a_j})^2(1 + n_1\sigma^2)}$$

in the exponent in (10). It follows that on the basis of either *ex ante* or *interim* expected utility, trader i is always better off than he would be without entering into trade for the risky asset.

Since traders' expected utilities decrease as the number of informed traders increases, information collectors do not earn any return on their information, and more importantly, no trader will choose to acquire information even if it is free. Hence, there is a unique overall equilibrium that has the fully revealing REE as an integral part, in which no trader chooses to acquire information.

2.2.2 The Existence of an Equilibrium with Risky Asset Endowments

We now extend trader i's endowment to include $\bar{X}_i$ units of the risky asset in period 0, in addition to his wealth endowment W_{0i}. No additional source of risky asset supply besides the endowments of the traders exists. In this case, given trading price P_0 in period 0, trader i's total available wealth changes from W_{0i} to $W_{0i} + P_0\bar{X}_i$. Consequently, trader i's budget equation changes from (2) to

$$X_{Fi} + P_0 X_i = W_{0i} + P_0 \bar{X}_i. \tag{12}$$

Budget equation (12) implies $\tilde{W}_{1i} = (1+r)W_{0i} + (1+r)P_0\bar{X}_i + [\tilde{P}_1 - (1+r)P_0]X_i$.

As noticed before, trader i's demand for the risky asset does not depend on his wealth level. This implies that there is a unique fully revealing REE with a price mapping as in (5) (see Grossman 1976, p. 584). However, unlike before, by (5)-(8) and (12), in the fully revealing REE the difference $E[\tilde{W}_{1i}|I_i] - \frac{a_i}{2}Var[\tilde{W}_{1i}|I_i]$ now depends on $\bar{y}$:

$$\begin{aligned}
&E[\tilde{W}_{1i}|I_i] - \frac{a_i}{2}Var[\tilde{W}_{1i}|I_i] \\
&= (1+r)W_{0i} + \frac{\sigma^2\bar{X}^2}{2a_i(\sum_{j \in N}\frac{1}{a_j})^2(1+n_1\sigma^2)} + (1+r)P_0^*(y)\bar{X}_i \\
&= (1+r)W_{0i} + \frac{(\bar{P}_1 + n_1\sigma^2\bar{y})\bar{X}_i}{1+n_1\sigma^2} + \sigma^2\bar{X}\frac{\bar{X} - 2a_i\bar{X}_i\sum_{j \in N}\frac{1}{a_j}}{2a_i(\sum_{j \in N}\frac{1}{a_j})^2(1+n_1\sigma^2)},
\end{aligned} \tag{13}$$

where $\bar{X} = \sum_{j \in N}\bar{X}_j$. Compared with (9), the third term on the right-hand-side of the first equality in (13) is the additional term due to endowment of the risky asset.

It is remarkable that in terms of traders' *ex ante* expected utilities the conclusion of Proposition 1 also applies to the case with traders endowed with the risky asset:

11

Proposition 2 *Traders' ex ante expected utilities in the fully revealing REE with risky asset endowments decreases as the number of informed traders is increased.*

Proof. By (13), the difference $E[\tilde{W}_{1i}|I_i] - \frac{a_i}{2}Var[\tilde{W}_{1i}|I_i]$ depends on the signals y only through the sample mean $\bar{y}$. Under the distributional assumptions, the sample mean has a normal distribution with mean $\bar{P}_1$ and variance $\sigma^2 + \frac{1}{n_1}$. It follows that integrating *interim* expected utility $E_{\tilde{P}_1|\bar{y}}U_i[\tilde{W}_{1i}|I_i]$ over the sample mean $\bar{y}$ and simplifying yield trader i's *ex ante* expected utility:

$$
\begin{aligned}
&E_{\bar{y}}E_{\tilde{P}_1|\bar{y}}U_i[\tilde{W}_{1i}|I_i]\\
&= -exp\bigg\{-a_i\bigg[(1+r)W_{0i} + \frac{\bar{P}_1\bar{X}_i}{1+n_1\sigma^2} + \frac{\bar{X}-2a_i\bar{X}_i\sum_{j\in N}\frac{1}{a_j}}{2a_i(\sum_{j\in N}\frac{1}{a_j})^2(1+n_1\sigma^2)}\sigma^2\bar{X}\\
&\quad + \frac{n_1\sigma^2\bar{P}_1\bar{X}_i}{1+n_1\sigma^2} - \frac{a_i}{2}\big(\frac{n_1\sigma^2\bar{X}_i}{1+n_1\sigma^2}\big)^2(\sigma^2+\frac{1}{n_1})\bigg]\bigg\} \qquad (14)\\
&= -exp\bigg\{-a_i\bigg[(1+r)W_{0i} + \bar{P}_1\bar{X}_i - \frac{a_i}{2}\sigma^2\bar{X}_i^2 + \frac{a_i}{2}\sigma^2\frac{\big(\frac{\bar{X}}{a_i\sum_{j\in N}\frac{1}{a_j}}-\bar{X}_i\big)^2}{1+n_1\sigma^2}\bigg]\bigg\}.
\end{aligned}
$$

It is clear from (14) that trader i's *ex ante* expected utility decreases with the number of the informed. ∎

A similar intuitive explanation as for the previous case about why expected utilities of the traders decrease as the number of informed traders is increased applies here. Furthermore, trader i's expected utility in the last equality in (14) also contains two parts. One part is what trader i can get by putting all his wealth W_{0i} into the risk-free asset and holding on to the risky asset endowment. This part corresponds to $(1+r)W_{0i}+\bar{P}_1\bar{X}_i-\frac{a_i}{2}\sigma^2\bar{X}_i^2$ in the exponent. Entering into trade for the risky asset so as to participate in the optimal risk sharing, results in an increase in his expected utility. This increase in trader i's expected utility corresponds to the second part

$$
\frac{a_i}{2}\sigma^2\frac{\big(\frac{\bar{X}}{a_i\sum_{j\in N}\frac{1}{a_j}}-\bar{X}_i\big)^2}{1+n_1\sigma^2} \qquad (15)
$$

in the exponent in (14).

When trader i's endowed amount of the risky asset $\bar{X}_i$ happens to be identical with his optimal share $\bar{X}/(a_i\sum_{j\in N}\frac{1}{a_j})$, (15) implies that gains from entering into trade for the risky asset are zero. In this case, trader i is indifferent between entering

12

into trade for the risky asset or holding on to his endowment of the risky asset. Trader i's nonparticipation in trade for the risky asset does not affect other traders' optimal sharing of the risky asset, but it reduces their potential losses due to the signal he may bring to the market with his participation.

Proposition 2 implies that information collectors still do not earn any return on their information, and more importantly, it is still not optimal for any trader to acquire information even if it is free. Again, there exists a unique overall equilibrium without information acquisition that has the fully revealing REE as an integral part, so that prices would fully reveal private information if it were to be acquired by traders.

3 Conclusion

A large amount of research in the area of financial econometrics and market efficiency theory has addressed the question of whether it is possible to use trading rules to generate abnormal financial gains from trading capital assets. Fama (1970, p. 383) defines an efficient market as one in which prices always fully reflect all available information. It follows that in an efficient market, no trader will be presented with an opportunity for making a return on an asset that is greater than a fair return for the riskiness associated with the asset.

Grossman (1976) shows that a problem associated with efficient markets is that no trader will have any incentive to acquire costly information because equilibrium prices fully reveal all information acquired by other traders and prices can be observed for free. However, if no trader acquires information, price will not be informative, in which case it would be in the interest of any one trader to acquire information. He concludes that with endogenous costly information acquisition, no equilibrium exists.

In this paper we show that in addition to the disincentives for private information acquisition as identified by Grossman, another type of disincentives results from the fact that the more informed traders there are, the less preferable the fully revealing REE becomes, because competition due to more information always drives up the expected period 0 price of the risky asset. It follows that no one has any incentive to acquire information even if none of the other traders does. As a result, there actually exists a unique overall equilibrium without information acquisition that has the fully revealing REE as an integral part, so that prices would fully reveal private information were it to be acquired by traders.

Grossman and Stiglitz (1980) consider an asset market model in which traders are divided into two groups: the group of informed traders and the group of uninformed traders. All the informed traders receive the same instead of diverse information. Grossman and Stiglitz consider a notion of REE that has the fraction of informed traders as an integral part. That is, the fraction of informed traders under their notion of REE is endogenously determined. The endogeneity comes from the equilibrium requirement that expected utilities of both informed and uninformed be the same. Traders' decisions on whether to acquire information or not are, however, not specified.

The model yields another well-known paradox that has come to be called the Grossman-Stiglitz Paradox: "In the limit, when there is no noise, prices convey all information, and there is no incentive to purchase information. Hence, the only possible equilibrium is one with no information. But if everyone is uninformed, it clearly pays some individual to become informed. Thus, there does not exist a competitive equilibrium." (See Grossman and Stiglitz 1980, p. 395). Assume the number of traders is finite and there is no noise other than that in the signal. Then, since signals are homogenous, the fully revealing REE depends on whether the signal is received or not but not on the number of traders who receive the signal. Consequently, the only possible revealing equilibrium is for one trader to acquire the costly information. However, our results imply that information acquisition reduces every information collector's expected utility in fully revealing REE. Hence, no one has any incentive to acquire information even when everyone is uninformed. It follows that when there is no noise and information is costly, there exists a unique equilibrium with no one acquiring information.

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