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Resistance of a long wire in the quantum Hall regime

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We study the two-probe transport in a long narrow channel of two-dimensional electron liquid (a quantum wire) in a strong magnetic field normal to the plane. The wire is split into alternating parallel strips of compressible and incompressible liquids. When the temperature is not too low, in every transverse cross section of the wire the electrochemical potential is different in different compressible strips but is uniform within each strip. The equilibration rate between two neighboring compressible strips falls off exponentially with the distance between them (the width of the separating incompressible strip). Assuming a parabolic confinement potential and small disorder potential, we find the magnetic field dependence of the strip widths. Using only this dependence we calculate the shape of the magnetoresistance peaks. We show that the low-field sides of these peaks are much steeper than the high-field sides, in agreement with recent experimental data.

I. INTRODUCTION

Transport in a narrow strip of two-dimensional electron liquid (a quantum wire) in a strong magnetic field has been viewed as the propagation of current through distinct quasi-one-dimensional edge channels. The edge channels are located spatially where Landau levels intersect with the level of the chemical potential. It is assumed that inter-edge-channel tunneling is suppressed^{1,2} if the separation between edge channels exceeds several magnetic lengths. Therefore, transport in a short clean wire in high magnetic field is ballistic.

The conventional picture of edge channels in a noninteracting system is that of narrow lines. They come in pairs of opposite direction of current flow and their total number is even. The functional dependence of the ballistic two-probe resistance (2PR) on magnetic field is a set of plateaus with quantized values of h/e^2k , $N = 2k$ being the number of the edge channels. The transitions between two adjacent plateaus occur abruptly as the number of edge channels increases with the lowering of the field.

The structure of the edge channels is different for interacting electrons. In this case edge channels are strips of finite width located where compressible liquid is formed.³⁻⁶ The total number of edge channels (compressible strips) may be both odd and even. It is odd ($N = 2k + 1$) if the central strip is compressible. Transport properties also differ from the previous case. In the interacting case 2PR takes quantized values only in narrow ranges of magnetic field, in which the central strip is incompressible. When the central strip is compressible, 2PR takes intermediate values constituting smooth interplateau rises.⁶ In both cases, 2PR in the ballistic regime pertinent to short clean wires, *monotonically increases* with magnetic field.

A recent experiment by Mani and von Klitzing⁷ examined the 2PR of an array consisting of several

hundred long narrow ($\sim 0.2 \times 100 \mu\text{m}^2$) wires. The wires were fabricated by shallow-mesa etching from a high-mobility ($\mu = 0.5 \times 10^6 \text{ cm}^2/\text{V s}$ at $T = 4.2 \text{ K}$) GaAs-Al_xGa_{1-x}As heterostructure. In contrast to the ballistic picture, the 2PR observed in this experiment is distinctly nonmonotonous. Instead of smooth rises 2PR exhibits huge peaks between adjacent quantized plateaus. The resistance in peaks is 10–12 times greater than the quantized values. Another peculiar feature of the 2PR peaks is their strong asymmetry at high temperatures, where the low-field sides of the peaks are much more narrow than the high-field ones. The widths of low-field sides are virtually temperature independent, while the high-field sides shrink with lowering of the temperature making the peaks appear more and more symmetrical. Qualitatively similar results were also reported by Wróbel *et al.*⁸

A nonmonotonic behavior of magnetoresistance was obtained by Kinaret and Lee⁹ by studying long narrow samples with static disorder (with no interaction) numerically. Their results show conductance dips near magnetic fields corresponding to steps in quantized resistance. They qualitatively explained this behavior in terms of an increased backscattering of the topmost edge channels as the spatial separation between them is small at these magnetic fields. They found the conductance dips to scale exponentially with the length of the wire and related this to the onset of localization. However, in the experiment the dependence of 2PR on the length of the wires appears to be no stronger than a linear one. This suggests that in the experimental range of temperatures and lengths inelastic processes break the quantum coherence and, along with the backscattering, an irreversible *equilibration* of the edge currents takes place.

If the wire is narrow and sufficiently long the edge channels equilibrate as electrons travel away from the probes. The electrons tend to redistribute in order to achieve an equal value of the electrochemical potential in

each channel, however suppressed those processes are. The rate of the equilibration between two given edge channels is characterized by their equilibration length.

Equilibration lengths in quantum wires were investigated in numerous experiments with “nonideal” probes, which inject unequal fractions of current into different edge channels (e.g., Refs. 10 and 11; see also Refs. 12 and 13). These experiments demonstrate that equilibration lengths may be as big as hundreds of micrometers, i.e., the suppression of interedge channel tunneling may be considerable.

The comprehensive theory of the equilibration among compressible liquids cannot be developed without a deep insight into the structure of the compressible liquid itself. On the other hand, the sharp exponential dependence of the inter-edge-channel equilibration on spatial separations among the strips can be taken into account rather easily. In this paper, we suggest a “high-temperature” theory of the two-probe transport in a long wire based only on the variations of this suppression factors with magnetic field. We argue that to find the 2PR of the wire to the exponential accuracy it is sufficient to know solely the dependencies of the widths of the strips on magnetic field. In the framework of our approach, we explain the origin of the peaks, their asymmetry and obtain their linear scaling with length. We are not able at present to interpret the temperature dependencies observed in the experiment. We believe they are related to the temperature-dependent tunneling rate into the compressible liquid and its transport properties.

The paper is organized as follows: the description of the model is given into the next two sections. In Sec. II we outline the model and calculate the 2PR of the wire under several simplifying assumptions. In Sec. III we justify these assumptions and demonstrate the way to treat boundary effects in the regions adjacent to the probes. In Sec. IV we present our results and compare them with the experimental data of Ref. 7. We make several concluding remarks in Sec. V.

II. THE MODEL

Consider a long narrow channel—a quantum wire—of length L and width $2b$ defined in 2D electron liquid, e.g., by mesa etching.¹⁴ We assume that the system is uniform in the longitudinal y direction, $-L/2 \leq y \leq L/2$ and that the transverse coordinate x ranges as $-b \leq x \leq b$. The electron density is depleted near mesa walls and is the largest at the center of the channel, forming a semi-circle-like profile in zero magnetic field. When a strong magnetic field in the direction normal to the x - y plane is applied, the electron liquid is split into alternating strips of compressible and incompressible liquids, with plateaus of electron density (Fig. 1) on top of the original profile near the lines of integer Landau-level occupation.⁶

Let the wire be connected to two metallic probes. Our goal is to calculate the net current I_y when a potential difference V is applied. It is well known that nonequilibrium currents are related to a nonuniform distribution of the electrochemical potential in the sample. Gener-

ally speaking, the electrochemical potential varies in each transverse cross sections of the wire as well as along the y axis.

For our purposes, it is useful to review the distribution of the electrochemical potential in the ballistic regime. Consider the case, when there is an incompressible strip at the center. In this case, the electrochemical potential is constant within each compressible strip. The distribution of the electrochemical potential $\xi(x, y)$, as a whole, is described by a set of $N = 2k$ discrete variables $\xi_i^\pm$'s. By ξ_i^+ and ξ_i^- , we designate the electrochemical potentials of the compressible strips corresponding to i th magnetic subband (Landau levels are assumed to be spin split) on the positive and negative x 's sides of the wire, respectively. Thus, the numeration starts on outermost strips and proceeds inward. In the ballistic regime these $\xi_i^\pm$'s are equal to the electrochemical potential of the respective probes:

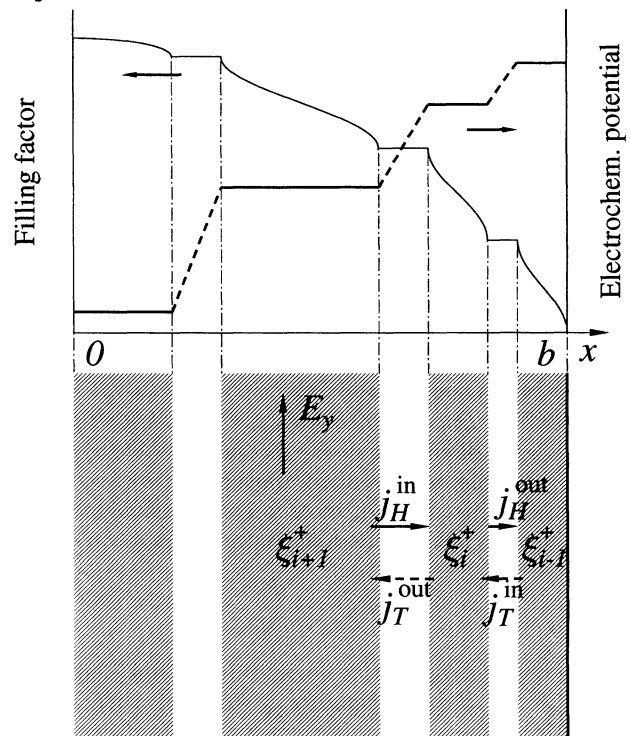


FIG. 1. Structure of the wire in a cross section. Only half of the wire on positive x 's side is shown. The top of the figure shows schematic behavior of the filling factor ν (left) and the electrochemical potential ξ (right). Compressible regions of varying, noninteger ν alternate with incompressible regions of constant, integer ν . The electrochemical potential ξ is uniform within each compressible region and changes across each incompressible one. The bottom of the figure shows the top view of the positive x 's side of a segment of the wire. The shaded regions are compressible strips; the blank regions depict incompressible strips. The boundary of the wire is shown by the solid line on the right. Each compressible strip is labeled by its electrochemical potential. The lines with arrows represent currents incoming and outgoing from the strip with electrochemical potential ξ_i^+ . Solid lines represent Hall currents on the left and on the right of this strip, the dashed lines stand for tunnelling currents between this compressible strip and the two adjacent ones.

$$\xi_i^\pm = \xi^\pm, \quad i = 1, 2, \dots, k, \quad (1)$$

where ξ^+ is the electrochemical potential of the probe at $y = -L/2$, which is bigger than that of the probe at the opposite end ξ^- by the amount eV : $\xi^+ = \xi^- + eV$; k is the filling factor of the central strip. This strip is located between two compressible strips with different electrochemical potentials. Correspondingly, it carries a nonequilibrium (Hall) current

$$I_k = \sigma_{xy} \times \frac{1}{e}(\xi_k^+ - \xi_k^-) = \frac{e}{h}k(\xi_k^+ - \xi_k^-) \equiv \frac{e}{h}k\Delta\xi_k. \quad (2)$$

We argue further that even for a long wire, where the ballistic transport no longer exists, the electrochemical potential within each compressible strip not too close to the probes is approximately x independent. However, now it varies along the wire: $\xi_i^\pm = \xi_i^\pm(y)$. Also, the drops of the electrochemical potential, $\Delta\xi_i^\pm$'s, occur across each incompressible strip, not only across the central one (this idea was earlier put forward in Ref. 15):

$$\Delta\xi_i^+ = \xi_i^+ - \xi_{i+1}^+, \quad \Delta\xi_i^- = \xi_{i+1}^- - \xi_i^-, \quad i = 1, 2, \dots, k-1, \quad (3)$$

where all $\xi_i(y)^\pm$ are taken at the same y . In this situation a generalized formula for the net Hall current applies:

$$I_y = \frac{e}{h} \sum_{i=1}^{k-1} i(\Delta\xi_i^+ + \Delta\xi_i^-) + \frac{e}{h}k\Delta\xi_k. \quad (4)$$

This formula is also valid when the total number of compressible strips, N , is $2k+1$, i.e., when the central strip is compressible. Indeed, in this case the sum of currents in the two incompressible strips closest to the center is $(e/h)k(\xi_k^+ - \xi_{k+1}^-) + (e/h)k(\xi_{k+1}^+ - \xi_k^-) = (e/h)k\Delta\xi_k$.

Besides Hall current in the incompressible strips, there is, in principle, a current of another nature in the compressible strips. Indeed, in each compressible strip there is an electric field in the longitudinal direction:

$$eE_y = -\frac{\partial\xi}{\partial y}, \quad (5)$$

which drives a dissipative current due to finite conductivity σ_{yy} . Yet if σ_{yy} is small ($\sigma_{yy} \ll e^2/h$), then as in the ballistic regime *nonequilibrium longitudinal current is a Hall current, which flows in the incompressible strips*.

Now we address the question of the electrochemical potential distribution. We find it using the concept of equilibration. The equilibration of two regions with different electrochemical potentials is achieved via exchange of electrons between them, i.e., current, in the direction from the region with the higher electrochemical potential to the region with the lower one.

Consider some transverse cross section of the wire. The reason why the electrochemical potential vary in this cross section is that there is a Hall current in the x direction due to nonzero E_y [Eq. (5)]:

$$j_H = \frac{e^2}{h}\nu(x)E_y, \quad (6)$$

where ν is the filling factor. This current tends to build up a Hall voltage, and, consequently, the electrochemical

potential difference between the positive and negative x 's sides of the wire. The equilibrating currents counteract this effect.

Apparently, the faster the equilibration within some region of the wire takes place the smaller the variations of the electrochemical potential in this region are. We consider the case of relatively high temperatures, where the intra-edge-channel equilibration is still much faster than inter-edge-channel one. The drops of the electrochemical potential are considerable between well separated compressible strips as they are connected through big tunneling resistances; on the other hand, variations of the electrochemical potential across compressible strips are slight since they have small resistances and equilibrate easily. This allows us to describe each of the compressible strip by a single value of the electrochemical potential $\xi_i^\pm(y)$, as we mentioned before.

The equilibration currents tend to bring the electrochemical potentials of different compressible strips closer together. In a long wire this results, in its turn, into the tendency to equate the derivatives of these electrochemical potentials with respect to y . These derivatives are proportional to the longitudinal electric field E_y in the strips [Eq. (5)]. Therefore, the longitudinal electric field in the wire becomes more and more uniform as we move further away from the probes. At sufficiently large distances away from the probes, the electrochemical potential vary linearly with y with the same slope at all x . The differences between the electrochemical potentials of neighboring compressible strips in the same cross section (which we need for calculating the net current) reach their limiting values and do not change.

To find, for instance, $\Delta\xi_i^+$, consider the balance of the currents across the incompressible strip with $\nu = i$ ($1 \leq i < k$) in some cross section of the wire where electric field is uniform (Fig. 1). The transverse currents must compensate each other:

$$j_T^{\text{out}} = j_H^{\text{in}} = \frac{e^2}{h}iE_y. \quad (7)$$

The tunneling current density is defined by the ratio of the electrochemical potential drop across the incompressible strip and its tunneling resistances per unit length ρ_i^+ :

$$j_T^{\text{in}} = \frac{\Delta\xi_i^+}{\rho_i^+}. \quad (8)$$

The tunneling resistance per unit length may be expressed via the length of a segment, whose tunneling resistance is h/e^2 :

$$\rho_i^+ = \frac{h}{e^2}l_i^+. \quad (9)$$

We shall call this length *the equilibration length* of the pair of compressible strips. To account for the suppression of the tunneling we choose the following form for the equilibration length:

$$l_i^+ = l \exp \left\{ \frac{1}{2} \left(\frac{w_i^+}{\lambda} \right)^2 \right\}, \quad (10)$$

where λ is the magnetic length and w_i^+ is the width of the incompressible strip. This particular form is appropriate when the single-act scattering on a short-range disorder dominates (for multiple-act scattering see Refs. 2 and 16). The parameter l of dimensionality of length is the only fit parameter in the model. Combining Eqs. (7)–(9), we find

$$\Delta\xi_i^+ = eE_y i l_i^+. \quad (11)$$

In the same manner we obtain

$$\Delta\xi_i^- = eE_y i l_i^-, \quad (12)$$

where $i = 1, 2, \dots, k-1$. As for $\Delta\xi_k$, we write it in the form

$$\Delta\xi_k = eE_y k l_k. \quad (13)$$

If the central strip is incompressible, then

$$l_k \equiv l_k^+ = l_k^-; \quad (14)$$

otherwise, (the central strip is compressible and $N = 2k + 1$)

$$\Delta\xi_k = \xi_k^+ - \xi_{k+1} + \xi_{k+1} - \xi_k^- = eE_y k l_k^+ + eE_y k l_k^-, \quad (15)$$

and, consequently,

$$l_k \equiv l_k^+ + l_k^-. \quad (16)$$

Note that the drops of the electrochemical potential between two compressible strip are the bigger the larger their equilibration lengths (growing with their separation) in agreement with our earlier statement. In this derivation we assumed that the region of uniform electric field exists. In the next section, we show that this requires $l_i^\pm \ll L$. The magnitude of this electric field may be approximately found as $E_y \approx V/L$. Substituting Eqs. (11) and (12) into the formula for the current (Eq. [4]) and dividing the potential difference V by the found value, we, finally, obtain the 2PR of the wire as

$$R = \frac{h}{e^2} L \left\{ \sum_{i=1}^{k-1} i^2 (l_i^+ + l_i^-) + k^2 l_k \right\}^{-1}. \quad (17)$$

Since we assumed that the equilibration lengths $l_i^\pm$'s are much smaller than L , this formula is valid only if R is much larger than the resistance quantum h/e^2 . This is the regime of highly dissipative transport. Note the linear dependence of the 2PR on the length of the wire.

In the opposite limit, where all the $l_i^\pm$'s are much bigger than L , the equilibration is ineffective, and the transport is quasiballistic (a “long” wire behaves like a “short” one). Naturally, the ballistic regime formula applies:

$$R = \frac{h}{e^2} \frac{1}{k}. \quad (18)$$

In our numerical simulation (see Sec. IV) only l_k used to be larger than L . Nevertheless, Eq. (18) holds in this

case as well. Indeed, let only the central incompressible strip inhibit the equilibration, then the edge states on one side of the wire are “well isolated” from those on the other. Therefore, edge channels on opposite sides have different (but common for each side) values of the electrochemical potential, which is exactly the picture of the ballistic transport.

A more rigorous treatment of the problem may be found in the next section, but for illustrating how the 2PR peaks evolve in our model, it suffices to use an interpolation formula:

$$R = \frac{h}{e^2} \left(\frac{1}{k} + L \left\{ \sum_{i=1}^{k-1} i^2 (l_i^+ + l_i^-) + k^2 l_k \right\}^{-1} \right). \quad (19)$$

It is noteworthy that the variation of the 2PR with magnetic field is determined purely by the variations of several geometrical parameters—the widths of the incompressible strips. Our justification of this model is a very sharp dependence of the equilibration rate on those widths.

Section III outlines how to calculate the electrochemical potential distribution in a more accurate way with the proper treatment of the regions close to the probes. It is not, however, essential for the understanding of our results, which are derived from Eq. (19) and presented in Sec. IV.

III. EQUILIBRATION IN THE NONUNIFORM ELECTRIC-FIELD REGIONS

If the electric field is uniform within each compressible strip, but nonuniform throughout the entire width of the wire, then we still can describe the electrochemical potential distribution by N functions of longitudinal coordinate $\xi_i^\pm(y)$'s. However, at each y these functions are not only different in magnitude, but also have different y derivatives. The Hall current given by Eq. (6) is easily expressed in terms of the $\xi_i^\pm(y)$'s only in the immediate vicinity of the compressible strips. For the strip with potential ξ_i^+ ($1 < i < k$) the Hall current entering from the side closer to the center is (Fig. 1)

$$j_H^{\text{in}} = -\frac{e}{h} i \frac{d\xi_i^+}{dy}, \quad (20)$$

and the Hall current leaving from the other side is

$$j_H^{\text{out}} = -\frac{e}{h} (i-1) \frac{d\xi_i^+}{dy}. \quad (21)$$

The other type of currents are tunneling currents:

$$j_T^{\text{in}} = \frac{\Delta\xi_{i-1}^+}{l_{i-1}^+}, \quad (22)$$

$$j_T^{\text{out}} = \frac{\Delta\xi_i^+}{l_i^+}. \quad (23)$$

Since we treat longitudinal currents in the compressible strips as negligible and because of current conservation, the transverse currents must compensate each other within this accuracy in every cross section. We write

$$j_H^{\text{in}} - j_H^{\text{out}} + j_T^{\text{in}} - j_T^{\text{out}} = 0. \quad (24)$$

This leads to

$$\frac{d\xi_i^+}{dy} = \frac{\xi_{i-1}^+ - \xi_i^+}{l_{i-1}^+} - \frac{\xi_i^+ - \xi_{i+1}^+}{l_i^+}. \quad (25)$$

Repeating the same argument for the strips with electrochemical potentials ξ_i^- 's, we arrive at the system of equations:

$$\pm \frac{d\xi_i^\pm}{dy} = \frac{(\xi_{i-1} - \xi_i)^\pm}{l_{i-1}^\pm} - \frac{(\xi_i - \xi_{i+1})^\pm}{l_i^\pm}, \quad i = 1, 2, \dots, k-1, \quad (26)$$

where the notations $\xi_0^\pm \equiv \xi_1^\pm$ are introduced for convenience. One more pair of equations for $\xi_k^\pm$ reads

$$\frac{d\xi_k^\pm}{dy} = \frac{\xi_k^\mp - \xi_k^\pm}{l_k} \pm \frac{(\xi_{k-1} - \xi_k)^\pm}{l_{k-1}^\pm}, \quad (27)$$

with l_k defined by Eqs. (14) and (16). The boundary conditions for this system are the same as in the ballistic regime [Eq. (1)]:

$$\begin{aligned} \xi_i^\pm(\mp L/2) &= \xi^\pm, \\ \xi^+ - \xi^- &= eV. \end{aligned} \quad (28)$$

Equations (26) and (27) may be rewritten in the standard matrix form

$$\frac{d}{dy} \boldsymbol{\xi} = \mathbf{A} \cdot \boldsymbol{\xi}, \quad (29)$$

where $\boldsymbol{\xi} = (\xi_1^-, \xi_2^-, \dots, \xi_k^-, \xi_k^+, \xi_{k-1}^+, \dots, \xi_1^+)^T$ is a column vector with $2k$ components, and $\mathbf{A}$ is the constant $2k \times 2k$ matrix of coefficients. The solution of the boundary-value problem Eqs. (26)–(28) is expressed in terms of the eigensystem of the matrix $\mathbf{A}$:

$$\boldsymbol{\xi}(y) = (\alpha_0 - eE_y y) \mathbf{z}_0 - eE_y \mathbf{z}_{\text{uni}} + \sum_{i=1}^{2k-2} \alpha_i \hat{\mathbf{z}}_i e^{\mu_i y}. \quad (30)$$

Here, $\mathbf{z}_0$ is the vector with all components equal to unity; $\mathbf{z}_{\text{uni}}$ renders the uniform field solution described in the proceeding chapter [Eqs. (11), (12)] and has components

$$\mathbf{z}_{\text{uni}}^n = -\left(\frac{k}{2} l_k + \sum_{i=n}^{k-1} i l_i^-\right), \quad n = 1, 2, \dots, k; \quad (31)$$

$$\mathbf{z}_{\text{uni}}^n = \frac{k}{2} l_k + \sum_{i=2k-n+1}^{k-1} i l_i^+, \quad n = k+1, k+2, \dots, 2k; \quad (32)$$

$\hat{\mathbf{z}}_i$'s are unit eigenvectors corresponding to nonzero eigenvalues μ_i . The nonuniformity of the electric field is attributable to the last of these. The numerical coefficients α_i 's are determined from the boundary conditions Eq. (28). Generally, the smallest absolute value of the μ_i 's is $O(1/l_{\text{max}})$, where l_{max} is the largest equilibration length, unless the values $l_i^\pm$ are specially chosen.

We are going to show that the nonuniformity of the electric field is small in the region far enough from the probes, $L/2 - |y| \gtrsim l_{\text{max}}$, if $l_{\text{max}} \ll L$. Indeed, let $-\mu < 0$ be an eigenvalue of $\mathbf{A}$ and $\hat{\mathbf{z}}$ be the corresponding unit eigenvector. The coefficient α_i , with which it enters $\boldsymbol{\xi}$ must be of the order of $eV \exp(-\mu L/2)$ to compensate for the large amplitude of this harmonic at $y = -L/2$ probe. As we move away from that probe this harmonic decays exponentially on the lengthscale of $1/\mu$. On the other hand, positive eigenvalue harmonics decay on the same lengthscale as we move away from the $y = L/2$ probe. Therefore, the middle of the wire is with a good approximation a region of a uniform field.

We remark also that the interpolation formula Eq. (19) can indeed be retrieved from Eqs. (26)–(28) if $k = 1$.

IV. DISCUSSION OF THE RESULTS

For the numerical simulation we used a program developed earlier.¹⁷ This program calculates the one-dimensional charge distribution in a quantum wire in a strong magnetic field. For the purpose of the simulation the x axis is divided into a number of small intervals with different values of electron density. Electrons belonging to the intervals interact by two-dimensional Coulomb law. For a given density distribution the energy of the system is calculated as the sum of the energy in the external confining potential, the energy of Coulomb interaction and the magnetic energy corresponding to the number of occupied Landau levels in each interval. Starting from some initial distribution, the density is iteratively redistributed among the intervals in order to lower the total energy of the system until a desired accuracy is achieved. The confining potential was chosen to be parabolic, and disorder potential was disregarded, since in a clean wire it may significantly affect only the equilibration, but not the distribution of the electron density. The only information we needed from this program was the widths of the incompressible strips in the range of interest of the magnetic field.

The results of the calculations using Eq. (19) for spin-split Landau levels are presented in Fig. 2. The parameters of the simulation matched the experimental ones (see the caption text). The value of an exchange-enhanced g factor¹⁸ was taken to be 8.0. The value of the single fit parameter in our model $l = 2.0 \mu\text{m}$ was chosen to match the experimentally observed height of the $0 \downarrow$ peak.

The peaks are strongly asymmetrical in shape with very sharp low-field slopes and smooth high-field ones. However, these results should be treated as illustrative only. Near the peaks the exponential factors in the $l_i^\pm$'s turned out to be small ($\lesssim 2$), and the very tips of the

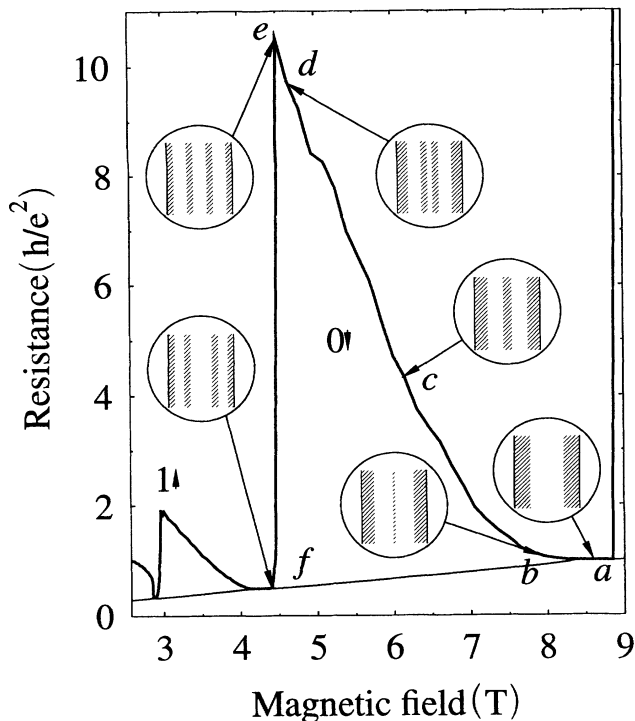


FIG. 2. The two-probe resistance R as a function of magnetic field as given by our numerical simulation. Parameters of the simulation: the half width of the wire $b = 1000 \text{ \AA}$, length $L = 57 \text{ \mu m}$, electron density at the center $n = 1.5 \times 10^{11} \text{ cm}^{-2}$, exchange enhanced g factor 8.0. The fit parameter $l = 2.0 \text{ \mu m}$ was chosen to match the experimentally observed height of the $0 \downarrow$ peak. 2PR in the ballistic regime as obtained following Ref. 6 is shown with a thin line for comparison. Due to narrowness of the quantized plateaus it looks almost like a straight line. Diagrams in circles depict the top view of short segments of the wire in magnetic fields corresponding to the positions pointed to by arrows. The meaning of shaded and blank areas is the same as in Fig. 1, but as opposed to that figure, now both sides of the wire are shown. Positions (a)–(f) represent several stages in the evolution of the resistance peak as discussed in Sec. IV.

peaks are not reliable. For this matter, although the exponential factors are smaller at the peak $1 \uparrow$ than at peak $0 \downarrow$ (due to a bigger magnetic length λ), the former peak is still lower due to bigger i^2 prefactors in Eq. (19). Surprisingly, the height of this peak is close to the experimental value.

In order to understand the detail of the shape of the peaks consider the variation of the 2PR given by Eq. (19) with the magnetic field. When the magnetic field is such that the filling factor at the center of the wire is unity, the central incompressible strip is, typically, wide.⁶ Correspondingly, l_1 is very big (bigger than L) and the resistance R assumes a quantized value h/e^2 [Fig. 2(a)]. With lowering of the magnetic field, the central strip breaks in two, and a narrow compressible strip appears at the center. However, even a strip of a half the original width corresponds to $l_1 \gg L$, and R is still very close

to the plateau [Fig. 2(b)]. With a further decrease in the magnetic field, the two incompressible strips shrink. The equilibration length l_1 rapidly decreases and becomes smaller than L . The resistance grows and becomes much bigger than the quantized value [Fig. 2(c)]. At the same time, the filling factor at the center increases and approaches two. A new incompressible strip and a new equilibration length l_2 emerge and grow. The increase of R slows down [Fig. 2(d)]. Due to the sharp exponential dependence of the equilibration lengths on the widths of the corresponding strips, the denominator in Eq. (19) is determined by the $l_i^\pm$ of the edge channel pair separated by the widest strip. When the width of the central strip reaches the width of peripheral ones, the resistance should exhibit a cross over, since the new strip is expanding, while the two existing ones are shrinking. Therefore, R is at its peak value [Fig. 2(e)] at this magnetic field. The central incompressible strip expands tremendously fast, and very soon R assumes the next quantized value [Fig. 2(f)] and then the cycle is repeated again.

In the experiment the low-field sides are, indeed, much steeper than the high-field ones, but not as abrupt as in the simulation. One possible explanation is the averaging of the individual peaks of several hundred distinct wires. A puzzling feature of experimental data is that a *nonzero slope* of R in the plateau areas was observed. We do not have an interpretation of this phenomenon at present.

V. CONCLUSION

We suggested a model for the dissipative transport in a long narrow wire with low disorder and at relatively high temperatures. Only incompressible strips significantly contribute to the transport. Nonequilibrium currents carried by them are determined by the differences of the electrochemical potentials of neighboring compressible strips in a given cross section of the wire. These differences set in as a result of inelastic equilibration, which counteracts the buildup of Hall voltage across the width of the wire. The electrochemical potential difference in a pair of adjacent compressible strips is the more pronounced the bigger their equilibration length is. The equilibration length is proportional to the tunneling resistance per unit length of the separating incompressible strip, and exponentially increases with its width.

Owing to this sharp dependence, we were able to find the two-probe resistance of the wire as a function of magnetic field by introducing a single fit parameter and without knowing details of electron tunneling and subsequent relaxation in the compressible liquid. This fit parameter affects only the heights of the peaks but not their shape. It is important to note also that we used geometrical parameters describing a clean wire and did not assume anything about the disorder potential besides its relative smallness.

The two-probe resistance in our model proved to be distinctly nonmonotonous with high peaks between successive plateaus of quantized resistance. The shape of the Shubnikov–de Haas peaks is calculated explicitly for the quantum Hall regime. We obtained the strong asym-

metry of the peaks and predicted the linear scaling of their heights with the length of the wire. Both agree with the experiment. However, we were unable to interpret the observed temperature dependencies. We assume that proper treatment of this issue as well as a more accurate description of inter-edge-channel tunneling should be based on the understanding of the equilibration and localization processes in the strips of the compressible liquid.

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