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Relational Reasoning is in the Eyes of the Beholder: How Global Perceptual Groups Aid and Impair Algebraic Evaluations

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Relational Reasoning is in the Eyes of the Beholder: How Global Perceptual Groups Aid and Impair Algebraic Evaluations

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Motivation

Relational reasoning—reasoning that depends on the interactions of multiple elements, rather than on the intrinsic properties of the elements—is both ubiquitous and challenging. For example, children find it difficult to respond to relational commonalities when object-based similarities are present (Gentner & Rattermann, 1991). Since overt symbol systems such as algebra are external constructs, their terms can contain perceptual regularities. Models of symbolic reasoning, however, typically ignore perceptual regularities (Anderson, in press). It is reasonable to wonder whether people make use of available domain-general grouping processes when parsing mathematical structures.

The purpose of the experiments described here is to evaluate whether algebraic grouping is sensitive to visual grouping. If processing is strictly symbolic, then the manipulation of perceptual regularities should not affect judgments; however, if people use visual grouping to help them parse expressions, then they should make more errors in cases where the perceptual grouping gives an incorrect answer, and be more accurate when visual grouping supports the standard order of operations.

Experiment 1

Experiment 1 manipulated the spacing of simple presented expressions. 44 subjects saw 240 simple equations divided evenly between cases where the spacing was aligned with, opposed, or was neutral with regard to the order of operations rule (see Figure 1, 1-2). The subject's task was to judge whether the two sides of the equation were necessarily equal. Half of the equations of each type were in fact equal. On trials where grouping affected the correct response, the mean accuracies on congruent, neutral spacing, and incongruent trials were 94%, 81%, and 55% respectively. On trials where grouping does not affect the correct response, no significant effect of spacing was observed (95%, 94%, 93% for congruent, neutral, incongruent, $p>.1$ on a repeated measures ANOVA).

Experiment 1 establishes that spacing affects algebraic grouping judgments, but the effect might be limited to physical proximity. Experiment 2 explores another perceptual regularity, regular structure, in order to evaluate whether that also affects grouping.

Experiment 2

The materials were identical, except that the variables in the original expressions were replaced by complex terms (see Figure 1, 3-4). Instead of manipulating the spacing of the terms, we manipulated the similarities of these internal structures. In congruent cases, terms around the multiplications had similar structures, while in incongruent cases, terms around the additions had similar structures. Again, performance was significantly worse in incongruent cases only when grouping affected the correct answer (accuracies = 81.1%, 78.8%, and 75.8% on grouping-matters trials, 92.6%, 90.8%, 91.8% on grouping-irrelevant cases. An ANOVA analysis reveals that both congruent and incongruent cases when grouping matters are significantly different from neutral performance, ($n=28$, $p<.05$). No effects are significant in grouping-irrelevant cases.

These results support the sensitivity of algebraic computations on domain-general perceptual properties; understanding better the role of perception in reasoning with overt symbol-systems may help us better understand the character of internal symbolic processes.

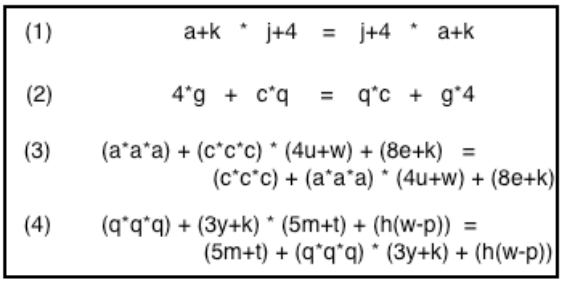

Figure 1: Sample stimuli. (1) incongruent, answer depends on the grouping. (2) congruent, answer does not depend on grouping. (3) incongruent. (4) congruent.

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