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I. Introduction

Recent years have witnessed the development of accelerators of ever-larger current, both peak and average, as well as a proliferation of storage rings of ever-greater luminosity. Consequently, there is considerable interest in and growing concern with, the phenomena which limit beam currents and beam densities, namely, the collective modes of behavior of relativistic particle beams. Furthermore, it has been demonstrated that the collective behavior can be controlled, at least to some extent, turned to good advantage, and employed for collective acceleration in devices such as the electron ring accelerator.

Quite naturally then, almost every accelerator conference during the last five years has had a review paper on collective effects, while at the same time the number of original papers in this area now exceeds many hundreds. And thus I am faced with the dilemma of being unable to give a comprehensive and complete review (such a review, incidentally, would be most valuable; in my judgment the time is ripe for a comprehensive monograph on the subject.), and yet finding it difficult, in a brief review, to be comprehensible, balanced, and yet fresh.

I have resolved the delemma by firstly supplying sufficient references as to allow the interested reader to readily approach and efficiently attack the literature. Secondly, I take a few steps away from the details and the realities of the field and with the advantage of the broader view so gained, describe the basic many-body physics underlying the subject. Thirdly, I present a few examples of collective behavior, in part to make the general remarks concrete, but in large measure in order to illustrate the beauty of this kind of physics. Finally, I make some remarks on methods for control of undesirable collective behavior, and on the present state of understanding of the field.

II. The Literature

A comprehensive treatment of collective effects may be found in Ref. 1, where also may be found some 58 references to the original literature. A review of the instabilities of relativistic particle beams is given in Ref. 2, where the reader may find some 48 original-paper references. In Refs. 3, 4, and 5, the general subject of instabilities is approached from other points of view. Reference 6 is primarily concerned with longitudinal phenomena, while the text of Ref. 7 presents a unified approach to work prior to 1966. A recent review (8) is devoted to the influence of surroundings on collective behavior, and finally, a catalog of phenomena is presented in Ref. 9, which also includes a treatment of longitudinal phenomena.

In addition, much recent work is not yet referenced in the above-mentioned review articles, and attention, in particular, is directed to the important first-papers on the head-tail effect (10) and on ion-electron instabilities (11).

The use of collective fields for the acceleration of particles may conveniently be found in three review papers (12, 13, 14).

Finally, the reader who is interested in seeing how all this knowledge is brought to bear on an actual machine would be interested (for example) in the four papers listed in Refs. 15 and 16.

III. Basic Physics

Coherent and Incoherent Motion

The behavior of a beam of particles in an accelerator or in a storage ring may be described by a properly-relativistic Fokker-Planck equation. The diffusion terms arise from the scattering of

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beam particles on residual gas, the scattering of particles on each other, and (for electron beams) from the emission of photons. Gas scattering is well understood, and machines are usually designed so that the phenomenon is unimportant. Intra-beam scattering (ADA effect) can be important in stored intense beams of low energy, but again devices are normally designed so as to avoid the phenomenon. Radiation damping and the associated quantum fluctuations in the emission process are important phenomena in electron storage rings and this subject is well-understood.

For times which are short compared to the characteristic times associated with the above mentioned diffusion processes, the Fokker-Planck equation may be approximated by the collisionless Boltzmann- or Vlasov-equation. Consequently, the Vlasov equation is adequate for the analysis of the collective behavior of particle beams, provided--as we shall assume in the remainder of this paper--gas scattering, intra-beam scattering, and radiation phenomena are unimportant on the time scale under consideration.

In the Vlasov equation each particle experiences an external time-varying potential (which really arises from external fields and from other particles, but as far as any one particle is concerned is an external potential). Hence all particles, as a collection, satisfy Liouville's theorem (which is valid even with time-dependent potentials) in 6-dimensional phase space (which is a great reduction compared to the full 6N-dimensional space).

In a stationary state the external potential is time independent, and can be obtained by a self-consistent field calculation which is quite analogous to the Hartree approach to atomic structure.

In addition, the self-consistent field may have dynamic behavior. This is quite analogous to the Bohr-Mottleson approach to collective modes of atomic nuclei. Of course, dynamic behavior of the field can be described in terms of single particle motion, but it is usually easier to think of the self-consistent field as having associated degrees of freedom. The relation between these two approaches has been carefully studied in connection with atomic nuclei (the unified model) where the inter-play of collective modes (Bohr-Mottleson modes) and single-particle states (shell model states) is of great importance.

For accelerator beams we call the motion where the self-field is stationary, "incoherent", and where the self-field has dynamic behavior, "coherent". Since the coherent modes can, sometimes, lead to a rapid loss of the whole beam, we often describe such behavior as "an instability", but it must be remembered that incoherent collective motion can be just as effective in destroying a beam.

Calculational Techniques

There are two techniques which are commonly employed to evaluate the collective behavior of particle beams. A rather detailed discussion of these techniques is given in Ref. 2, from which Figs. 1 and 2 have been taken. These figures should be self-explanatory and hopefully, keeping the block diagrams in mind, will greatly ease the pain for someone fighting his way through the lengthy and detailed calculations which abound in the literature.

Self-Destructive Behavior

At the heart of our problem--and I mean the problem of the accelerator physicist--is the pernicious self-destructive behavior of particle beams. What inherent flaw makes beams destroy themselves? There are deep reasons which succinctly can be summarized with the remark that the system is not in thermodynamic equilibrium; in fact, with relativistic beams one could hardly be further from equilibrium. In statistical mechanics terms, the constant energy surface in 6N--perhaps 6×10^{13}--dimensional phase space is of very great extent, and the part corresponding to a working device is a tiny area over in one corner. Eventually, because of metric transitivity (ergodicity), all regions of the energy surface will be experienced, i.e., the beam will destroy itself.

But perhaps if the system is well-isolated from its surroundings, it will take a very long time before it comes to equilibrium. And certainly, we must isolate stored beams, for scattering from residual gas, noise in the current-supplies to the magnets, etc. will eventually lead to beam loss. We can readily calculate these relaxation rates and impose criteria which must be met in practice to ensure an adequate beam life. These relaxation phenomena provide an ultimate limit to beam lifetime. When the various other phenomena are contolled, one is still left with the limits imposed by imperfect beam isolation.

However, to return to the deep question of beam stability, even an isolated system may approach equilibrium. (If you pack a gas into one corner of an isolated box, it isn't going to stay there.) But, you say, look at a system like the solar system. It has been stable for 4.6×10^9 earth oscillations. Of course, I can respond that to hold a beam in a storage ring for one day corresponds to 3×10^{11} oscillations, and hence in designing a new storage ring I can't take any comfort from the present observations on the stability of the solar system. However, since we don't know how to scale from one system to the other without a theory, these are only flippant remarks.

Just such a theory was developed in the sixties. In fact, the last decade has witnessed remarkable progress on the classical problems of (1) ergodicity, metric transitivity, and the basis of statistical systems; and (2) the question of stability of conservative dynamical systems. Building on older work by Hopf, Hadamard, Krylov, and especially Kolomogorov (17), the new progress has been made, primarily, by Moser (18) and the Soviet (Moscow) school of mathematicians: Arnol'd, Sinai, Anosov, and Avez (19). The application of these new results to physical systems has been pushed first by Chirikov (20), and then by many others.

Consider a few of the new results. Firstly, the famous Kolmogorov-Arnol'd-Moser theorem considers the motion described by

$$H = \frac{1}{2} \sum_{k=1}^{N} (P_k^{\,2} + \omega_k^{\,2} Q_k^{\,2}) + \lambda [V_3 + V_4 + \cdots]$$

where ω_k are positive frequencies, N is the number of degrees of freedom, V_3 V_4 are cubic and quartic polynomials in P_k and Q_k, and λ is a measure of the nonlinearity. This describes, for example, motion near equilibrium in the solar system. The theorem (approximately stated) is that provided

(1) $\sum_k n_k \omega_k \neq 0 \quad \text{for} \quad \sum_k |n_k| \leq 4,$

(2) λ is sufficiently small (but not infinitesimal),

(3) V_3 is nonzero,

then except for a set of small measure the trajectories are quasiperiodic orbits lying on smooth N-dimensional integral surfaces embedded in the 2N-dimensional phase space.

Consequently, a one-dimensional nonlinear system has a nonzero stable region around a linear stable equilibrium point (one-dimensional surfaces close-off the unstable zones), as is shown in Fig. 3. The one dimensional result is very interesting for application to beam problems in which there is negligible coupling between the three degrees of freedom. For N-dimensions, Arnol'd (very slow) diffusion is speculated to occur even when the theorem is satisfied.

Secondly, and conversely, when λ is large the motion is wildly unstable, i.e., the trajectories which are initially close, separate from each other at an exponential rate. The system is said to be strongly-mixing, or stochastic. This transition or stochasticity limit is a function of the initial amplitudes and of λ. Sinai proved the ergodicity and mixing in a hard sphere gas, thus solving a century-old mathematical problem (21). It seems that even a system of small dimensionality may obey statistical laws and thus there is provided a significant new basis for statistical mechanics.

Chirikov has suggested a method for estimating the stochasticity limit (20). Roughly speaking the criterion is that when the nonlinear resonances (computed in first-order) become so dense as to fill all available phase space, then the motion is stochastic. This simple criterion has been applied, with surprising success, to many different systems (20).

From this recent work we conclude that for a beam of particles to be stable, it is necessary that each particle be below the stochasticity limit, otherwise the beam will break up in an incoherent manner. As we shall see, beam self-fields can, at a certain current level, put particles over the stochasticity limit.

In electron storage rings it is probably sufficient to be below the stochasticity limit (since radiation damping should dominate Arnol'd diffusion) and thus provide long-time stability. For proton storage rings the importance of Arnol'd diffusion is presently moot; it may be involved in the ISR beam lifetime and experiments are being planned to try to find out.

Of course, stability of a beam against incoherent modes is not sufficient for beam stability. We know many examples from accelerators of coherent instabilities; in fact, most of the review articles on beam behavior have concentrated on coherent behavior, which is why I have emphasized here the incoherent effects.

Finally, I want to remark that whether or not the solar system is stable, is still an open question.

IV. Some Examples

Incoherent Weak-Strong Limit of Colliding Beams

In storage rings (for this phenomenon our experience to date, comes only from electron rings) a sufficiently intense beam will cause a weak beam (i.e., even a single particle) to blow-up. It is believed, as a result of extensive numerical studies (3, 22) and theoretical analysis (20, 23) that the blow-up may be explained in terms of the nonlinear fields associated with the intense beam, causing at a certain level of intensity, particles of the opposite beam to be over the stochasticity limit.

A convenient measure of the effect of the strong beam is the change it causes in the betatron oscillation frequency of a particle in the opposite beam. For a Gaussian beam of N particles, with rms transverse half-widths σ_x, σ_y, the change in the vertical ν-value (the number of betatron oscillations per revolutions) of a particle colliding head-on is:

$$\Delta \nu_y = \frac{r_e}{2\pi} \frac{N \beta_y}{\gamma \sigma_y(\sigma_x + \sigma_y)} ,$$

where r_e is the classical electron radius, γ is the particle energy in rest-mass units, and $2\pi\beta_y$ is the local wavelength (at the crossing point) of the betatron oscillation (24).

It is observed (on the Stanford rings, ADONE, ACO, the VEP's, and SPEAR) and it is computed, that when $\Delta \nu_y$ is only ≈ 0.02 there is a blow-up of the particle oscillation amplitude. (A computer-generated movie, by John Rees, delightfully illustrates this phenomenon and was presented at this juncture to the Conference.) In practice the strong-weak beam effect is a serious constraint on storage ring operation. In order to reach high luminosity it has been necessary to build machines with very low values of β_y, so that, in the face of a given $\Delta \nu_y$, one may achieve a large value of the beam density $N/\sigma_y(\sigma_x + \sigma_y)$.

Coherent Azimuthal Behavior of a Coasting Beam

The very first instability of an accelerator beam to be studied was the negative mass instability (25) (a special case of coherent azimuthal behavior), and furthermore, it is the only instability to have been theoretically predicted. (Since calling experimentalist's attention to the possibility of instabilities, we theorists have never been able to catch up with them again!) However, the subject is not without current interest; we believe our present troubles with the electron ring accelerator are related to this instability.

Consider an azimuthally uniform beam of particles (mass m, charge e, number N) circulating on an orbit of radius R. A perturbation in beam current may be written in the form $I_n \exp i(n\theta - \omega t)$, where n is an integer (the mode number) and ω (which is close to $n\omega_0$, with ω_0 the circulating frequency) describes the time development of the collective mode. Associated with the perturbed current will be an azimuthal electric field, E_θ, of the form $E_n \exp i(n\theta - \omega t)$. The field, E_n, is related to I_n (via Maxwell's equations) and one is led to define (8, 26) a beam coupling impedance $Z_n = -2\pi R E_n/I_n$.

It can be shown (25, 27) that, approximately, the beam is stable provided

$$\frac{|Z_n|}{Z_0 n} \leq \frac{\gamma R}{2 r_e N} \left| \frac{1}{\nu^2} - \frac{1}{\gamma^2} \right| \left(\frac{\Delta E}{E} \right)^2 ,$$

where ν, γ and r_e are defined as before, $(\Delta E/E)$ is the full-width at half-maximum of the distribution of the beam in energy, and Z_0 is the impedance of free space. Note, then, that for a given coupling, sufficient dispersion in the beam will prevent the instability (25), which is an example of the general phenomenon of Landau damping.

Also, it is clear that the beam surroundings are important, as they strongly influence the value of Z_n. In fact, a succinct way to characterize one of the problems to be solved in developing an electron ring accelerator is to state that (for an accelerator having a reasonably high rate of energy gain) the device must be designed so that $(|Z_n|/n) \lesssim (Z_0/10)$, i.e., the coupling must be reduced significantly below its "natural value".

<u>Transverse Two-Stream Coherent Modes</u>

Rather recently, the theory of ion-electron coherent transverse oscillations has been developed (11) and extended (28), and seen to be relevant to electron ring accelerators (11, 28), the Bevatron (29), the ITEP 7 GeV accelerator (30), and possibly the CERN ISR (31).

Consider a beam of protons which is azimuthally uniform and not subject to clearing fields. In due course it will become somewhat neutralized by electrons produced in beam-background gas inelastic collisions. The electrons oscillate ("bounce") in the electrostatic potential well of the protons. An unstable--yet energy conserving--resonant coupling can occur between the (positive energy) coherent electron transverse motion and a (negative energy) slow-wave coherent transverse proton mode.

Let $x_k^{(p)}$ and $x_j^{(e)}$ be the transverse coordinates of the <u>k</u>th proton and the <u>j</u>th electron, and let ω_k be the circulating frequency and $\omega_k \nu_k$ the betatron oscillation frequency of the <u>k</u>th proton. The equations of motion for the <u>k</u>th proton and the <u>j</u>th electron are (ignoring self-species forces):

$$\left[\frac{\partial}{\partial t} + \omega_k \frac{\partial}{\partial \theta}\right]^2 x_k^{(p)} + \omega_k^2 \nu_k^2 x_k^{(p)} = \left(\frac{N_e}{N_p}\right)\left(\frac{m}{M\gamma}\right) \omega_e^2 \left[\bar{x}^{(e)} - x_k^{(p)}\right] ,$$

$$\frac{\partial^2 x_j^{(e)}}{\partial t^2} = \omega_e^2 \left[\bar{x}^{(p)} - x_j^{(e)}\right] .$$

Here ω_e is the electron bounce frequency and is given by

$$\omega_e^2 = \frac{N_p e^2}{\pi m R b^2} ,$$

where we have assumed N_p protons in a uniform beam of major radius R and minor radius b. The proton energy is γMc^2, the electron mass m, and $\bar{x}^{(e)}$ and $\bar{x}^{(p)}$ denote the positions of the centers of mass of the beams.

Interestingly enough, despite the possibility of Landau damping in the proton motion (due to a spread in frequencies arising from the spreads in ω_k and in $\nu_k \omega_k$ values), the above equations have unstable solutions when the parameters are such that the average values $\bar{\nu}_k$ and $\bar{\omega}_k$ satisfy $(n - \bar{\nu}_k)\bar{\omega}_k \approx \omega_e$, and n is any integer larger than $\bar{\nu}_k$. However, the proton potential well is not perfectly harmonic, and thus there is also a spread in the bounce frequency ω_e, which might more properly be written as ω_{ej}. The condition for stability is (11, 28):

$$(\Delta \omega_e)\, \Delta\left[(n - \nu_k)\omega_k\right] \gtrsim \left(\frac{N_e}{N_p}\right)\left(\frac{m}{M\gamma}\right) \frac{\omega_e^3}{\bar{\nu}_k\, \bar{\omega}_k} ,$$

where Δ denotes the spreads in the appropriate electron and proton frequencies.

Analysis similar to that outlined above has been applied to electron ring accelerators where one recalls the mutual ion-electron interaction is central to the concept. It is found, theoretically (11, 28), that the instability limits the range of performance capabilities of the device, but to date no experimental information is available to either confirm or deny these conclusions. In the Bevatron, on the other hand, the instability has been observed (29), and then removed by the simple and definitive method of clearing the electrons from the beam (32).

<u>Transverse Emittance Growth in Linacs</u>

As a final example, consider the phenomenon of transverse beam size growth in the early sections of a proton linear accelerator. Much theoretical effort has been devoted to this subject, but because of its difficulty the numerical simulation studies have to date, provided more insight than the analytical studies.

In particular, it has been shown (33) that the blow-up is not due to longitudinal-transverse coupling through the rf fields in the gap between drift tubes, but rather, is due to nonlinear space charge forces. A convenient parameter is the beam brightness, B, defined by

$$B = \frac{I \cdot 10^6}{\frac{1}{2} \epsilon^2} ,$$

where I is the current in mA and ϵ is the normalized emittance in cmrad. It was found that (33), in a linac with an injection energy of 750 keV, beams with $B \gtrsim 10^9$ underwent considerable blow-up.

Subsequent studies (34), partially motivated by a desire to understand the blow-up phenomenon, have explored the coherent modes of oscillation of a beam and the thresholds for instability of these modes. It is not yet clear what relation, if any, these coherent modes have to the blow-up studies in Ref. 33. Alternatively, one can conjecture that the phenomenon studied in Ref. 33 is the result of nonlinear space charge forces causing particles to be above the stochasticity limit. It would be most illuminating to undertake analyses, analogous to those in Ref. 20 and 23, so as to confirm or disprove this conjectured explanation of emittance growth.

V. General Remarks

Methods of Control

The collective behavior of particle beams can be characterized as either coherent or incoherent. In an incoherent mode the self-consistent field is stationary, while the coherent patterns have time-varying self-consistent fields and hence can be observed through their associated ac macroscopic fields.

In order to control the self-destruction of a beam through an incoherent mechanism, one must change the parameters upon which the phenomenon depends. A practical example of such a means is the use of low-β in colliding beam devices to reduce the effect of the intense beams upon particles in the oppositely directed beam.

In electron machines, in contrast to proton devices, radiation damping helps to control incoherent as well as coherent behavior.

Self-destuction of a beam through a coherent mode, i.e., an instability, may be controlled in at least three different ways, namely by (1) Landau damping, by (2) control of the environment, and by (3) feedback.

(1) Landau Damping. This relies on a spread in the frequency of particles partaking in the coherent motion. Thus, energy spread stabilizes the negative mass instability, and octopoles, which produce frequency dependence upon betatron oscillation amplitude, stabilize the transverse resistive wall instability. Because of Landau damping there exists a threshold current for each instability, and thus, by use of adequate amounts of damping, an instability can be prevented.

(2) Control of the Environment. The driving terms of the instabilities always depend on the beam surroundings. One can reduce an instability by reducing the driving term, for example by reducing the coupling impedance in the negative mass instability. Thus, one can make growth times very long and/or raise the threshold for the instability. (Conversely, one must be very careful not to inadvertently make the impedance too high. At ADONE, they experienced great beam difficulties, but when the clearing electrodes were removed they were able to store 10 times as much current.) Because the environment is so important (for example, reistive instabilities are present only as a result of the beam surroundings.), and because it is more under the machine designer's control than any other factor, much attention must be devoted when designing a machine to the role of the environment in beam instabilities.

(3) Feedback. "If it is coherent, it can be cured," has sometimes been said. That is far from true, but it refers to the fact that when coherent modes can be detected, feedback may be used provided one has adequate bandwidth, etc. to force the mode down. It is routinely used on the ZGS, for example, to keep the transverse resistive wall instability under control at a current level 10 times the threshold value.

State of Understanding

Generally, despite the insight which we have gained during the last decade into the collective behavior of particle beams, we have not been able to predict in which manner a new high current beam will be limited. With each small step into novel regimes we have encountered a new instability (Cosmotron and Bevatron-negative mass instability, MURA 40 MeV model-resistive wall instability, Stanford electron rings-incoherent weak-strong beam instability, PS-longitudinal effects at transition, VEP-2 (Novosibrisk)-beam-cavity effects, ADA-Toushek effect, ISR-coherent ion-electron instability, ADONE-head-tail instability, etc.).

On the other hand, with each step we have been able to understand the newly-encountered problem so as to avoid the same trouble in future machines. (Thus, the ISR was successfully designed to avoid longitudinal and transverse-resistive instabilities, and SPEAR immediately reached currents which took years to achieve in the first storage ring.)

So much work remains to be done both on the theoretical and on the experimental side. And it is important that such work be done, for deeper understanding is needed in order to be able to build more efficient devices, design convenient and economical collective-field accelerators, and safely take that exciting step to the next generation of colliding-beam devices.

References

1. A. A. Kolomenskii and A. N. Lebedev, Proc. Soviet Conference on Charged-Particle Accelerators, Moscow, 1968 (Viniti, Moskva, 1970), p. 285.
2. A. M. Sessler, Proc. Fifth International Conference on High-Energy Accelerators, Frascati, 1965 (CNEN, Rome, 1966), p. 319.
3. E. D. Courant, Proc. First National Particle Accelerator Conference, 1965, IEEE Trans. on Nucl. Sci. NS-12, #3 (June 1965), p. 550.
4. V. K. Neil, Proc. U. S. National Particle Accelerator Conference, 1967, IEEE Trans. on Nucl. Sci. NS-14, #3 (June 1967), p. 522.
5. E. D. Courant, Annual Review of Nuclear Science 18 (Annual Reviews, Inc., Palo Alto, Calif., 1968), p. 435.
6. E. D. Courant, Proc. of the Seventh International Conference on High Energy Accelerators, Yerevan, 1969, II (Academy of Sciences of Armenia SSR, 1970), p. 224.
7. H. Bruck, Accélérateurs Circulaires de Particules (Presses Universitaires de France, 1966), p. 277-302.
8. A. M. Sessler, Proc. 1971 Particle Accelerator Conference, IEEE Trans. on Nucl. Sci. NS-18, #3 (June 1971), p. 1039.
9. H. G. Hereward, Proc. of the Eight International Conference on High-Energy Accelerators, 1971 (CERN, Geneva, 1971), p. 548.
10. C. Pellegrini, Il Nuovo Cimento LXIV A N.2, 447 (1969).
11. D. G. Koshkarev and P. R. Zenkevich, Particle Accelerators 3, 1 (1972).
12. M. S. Rabinovich, "Collective Methods of Acceleration," Lebedev Physics Institute Preprint No. 36 (1969).
13. A. M. Sessler, Proc. of the Seventh International Conference on High Energy Accelerators, Yerevan, 1969, II (Academy of Sciences of Armenia SSR, 1970), p. 431.
14. J. M. Peterson, "The Use of Collective Fields in the Acceleration of Particles," Lawrence Berkeley Laboratory Report LBL-704 (Feb. 1972).
15. K. Hubner, E. Keil, and B. Zotter, Proc. Eight International Conference on High-Energy Accelerators, CERN, 1971 (CERN, Geneva, 1971), p. 295; E. Keil, ibid, p. 372.
16. D. Möhl, "Intensity Limitations in the Present and Future PS," CERN Report MPS/DL Note 72-6 (Feb. 1972); R. R. Fullwood, "Design Study for a Medium-Energy, Intense, Proton-Storage Ring," Los Alamos Scientific Laboratory Report LA-4946 (July 1972).
17. E. Hopf, J. Math. and Physics 13, 51 (1934); see also A. S. Wightman in Statiscal Mechanics at the Turn of the Decade, Edited by D. Cohen (Marcel Dekker, New York, 1971), p. 1; N. S. Krylov, Papers on Foundations of Statistical Physics, Izd. Akad. Nauk. SSSR (1950); A. N. Kolmogorov, Dokl. Adad. Nauk. SSSR 98, 527 (1954).
18. J. Moser, Nachr. Akad. Wiss Gotingen, II Math-Physik Kl, No. 1 (1962); J. Moser, Math. Annalen 169, 136 (1967).
19. V. I. Arnol'd, Russian Math. Surveys 18, No.6, 85 (1963); V. I. Arnol'd and A. Avez, Ergodic Problems of Classical Mechanics (W. A. Benjamin, Inc., New York, 1968).
20. B. V. Chirikov, Research Concerning the Theory of Non-Linear Resonance and Stochasticity, CERN Transl. 71-40 (1971); G. M. Zaslavskii and B. V. Chirikov, Soviet Physics Uspekhi 14, 549 (1972).
21. Ya G. Sinai, Izv. Akad. Nauk. Mat. 30, 1275 (1966).
22. M. Bassetti, Proc. Fifth International Conference on High-Energy Accelerators, Frascati, 1965 (CNEN, Rome, 1966), p. 708; J. R. Rees, Proc. of the International Symposium on Electron and Positron Storage Rings, Saclay, 1966 (Presses Universitaires de France, 1967), p. VI-7; E. Keil, private Communication, CERN.

23. E. Keil, "Non-Linear Space Charge Effects II," CERN Report in preparation (1972); E. Keil, "Arnol'd Diffusion Lifetime in Storage Rings," CERN Report ISR-TH/EK/cb (July 1972).

24. M. Sands, "The Physics of Electron Storage Rings," Stanford Linear Accelerator Report SLAC-121 (Nov. 1970), p. 65.

25. C. E. Nielsen, A. M. Sessler, and K. R. Symon, Proc. of the International Conference on High-Energy Accelerators, CERN, 1959 (CERN, Geneva, 1959), p. 239; A. A. Kolomenskij and A. N. Lebedev, ibid, p. 115.

26. R. J. Briggs and V. K. Neil, Plasma Physics 8, 255 (1966); A. M. Sessler and V. Vaccaro, "Longitudinal Instabilities of Azimuthally Uniform Beams in Circular Vacuum Chambers with Walls of Arbitrary Electrical Properties," CERN Report 67-2 (Feb. 1967).

27. K. Hubner and V. G. Vaccaro, "Dispersion Relations and Stability of Coasting Particle Beams," CERN Report ISR-TH70-44 (1970).

28. L. J. Laslett, A. M. Sessler, and D. Möhl, "Transverse Two-Stream Instability in the Presence of Strong Species-Species and Image Forces," Lawrence Berkeley Laboratory Report LBL-1072 (Sept. 1972).

29. H. Grunder and G. Lambertson, Proc. of the Eighth International Conference on High Energy Accelerators, CERN, 1971 (CERN, Geneva, 1971), p. 308.

30. L. L. Goldin, Private communication, ITEP.

31. E. Keil, private communication, CERN.

32. H. Grunder, private communication, Lawrence Berkeley Laboratory.

33. R. Chasman, Proc. of the 1968 Proton Linear Accelerator Conference (Brookhaven National Laboratory, Upton, N. Y., 1968), p. 372.

34. R. L. Gluckstern, Proc. of the 1970 Proton Linear Accelerator Conference (National Accelerator Laboratory, Batavia, 1970), Vol. 2, p. 811; R. L. Gluckstern, R. Chasman, and K. Crandall, ibid, p. 823.

Block diagram of the Single Particle Motion approach to self-field phenomena

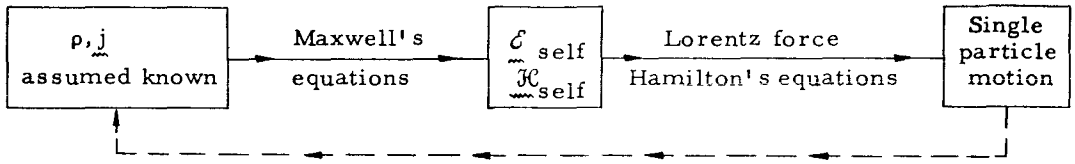

Fig. 1

Block diagram of the Collisionless Boltzmann Equation approach to self-field phenomena

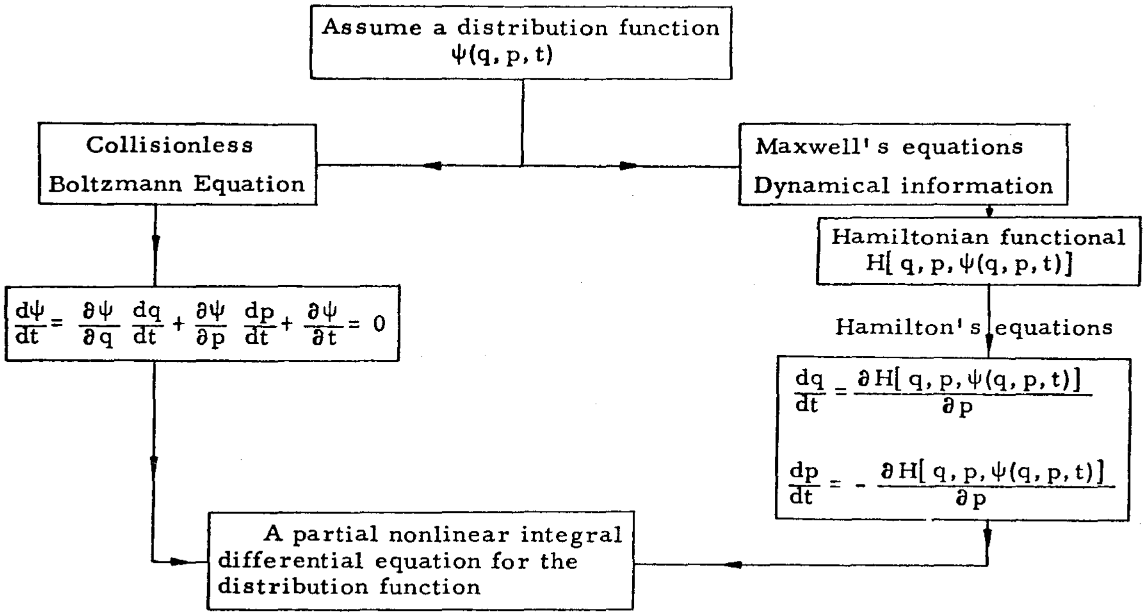

Fig. 2

Fig. 3. Phase plane for one dimensional motion (with a time-dependent periodic Hamiltonian).