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Journal

Water Resources Research, 43(7)

ISSN

00431397

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Publication Date

2007-07-01

DOI

10.1029/2006WR005774

Data Availability

The data associated with this publication are available upon request.

Peer reviewed

Jet and hydraulic jump near-bed stresses below a horseshoe waterfall

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Cite as: Pasternack, G. B., Ellis, C. R. and Marr, J. D. 2007. Jet and hydraulic jump near-bed stresses below a horseshoe waterfall, Water Resources Research 43, W07449, doi:10.1029/2006WR005774.

Abstract

Horseshoe waterfalls are a common but unexplained feature of bedrock rivers. As a step toward understanding their geomorphology, a 6-component force/torque sensor was used to evaluate clear-water scour mechanisms in a 0.91-m broad-crested horseshoe waterfall built into a 2.75-m wide flume to measure longitudinal, lateral, and temporal profiles of the near-bed spatially averaged stress vector. Jet impact caused moderate near-bed stress. Downstream of the impact point, the stress vector shifted into a much more forceful upthrust with peak stress beyond the boil crest. Instantaneous fluctuations in near-bed lift exceeded drag by 5-10x and occurred over a wider range of frequencies, suggesting a larger role for particle entrainment. Predictions of bed stress significantly underestimated actual conditions. These results show that horseshoe waterfalls have an extended region of high scour along their centerline corresponding with the 3D convergent dynamics associated with both jet impact and jump-enhanced lift fluctuations.

Keywords: hydraulic jumps, waterfalls, shear stress, bedrock rivers, fluvial geomorphology

1. Introduction

Waterfalls are among the most beautiful natural wonders of the world, but the adverse conditions they impose on even intrepid scientists has left them among the most poorly studied of natural physical phenomena on Earth. Even where an individual falls may be reached and probed on a mountain during accessible periods (e.g. dry periods with minimal snow accumulation), flow magnitude is often greatest during times when the waterfalls are inaccessible (e.g. early spring snowmelt when mountain roads are blocked by deep snow and intense summer rainstorms that turn dirt trails into impassable muds). There is also a wide diversity of 3D geometrical forms, multi-phase flows, and varying super-critical and sub-critical hydraulic regimes that together confound standard open-channel hydraulic and geomorphic equations, leaving many unanswered questions.

The world waterfall database (<http://www.world-waterfalls.com>) documents 949 waterfalls between ~100-1000 m high and ranging in estimated discharge from ~150-42500 m³/s. In addition to these rare spectacles are countless smaller river-bed steps with heights of 3-100 m found in all climate regions of the world. Waterfalls of diverse heights have been observed at all locations throughout drainage basins, from high peaks near drainage divides to cliffs at oceanic base level. Consequently, there is virtually no limit to the ratio of the height of waterfalls (P) to the specific energy of flow going over them (H). Even during the maximum probable flood that is extremely large relative to the contributing area for a given waterfalls, there is no reason to expect that a high falls with a small drainage area, such as one near a drainage divide, will ever become submerged. Thus, there exists an intriguing scientific conundrum regarding the physical processes associated with emergent waterfalls of all height to specific energy ratios.

A century of modern civil engineering research has been done on the hydraulic jump [e.g. *Ferriday, 1894; Ellms and Levy, 1927; Rouse et al., 1958; Rajaratnam, 1967; Liu et al., 2004*] and man-made overfalls [e.g. *Inglis and Joglekar, 1933; Moore, 1943; Rand, 1955; Elevatorski, 1959; Vischer and Hager, 1998*]. Transference of those classic foundations to the problem of understanding natural 3D waterfalls has valued but limited potential. The majority of hydraulic jump research has been performed using laboratory flumes in which an upstream sluice gate is used to create a uniform wall jet with a pre-determined Froude number and in which a downstream barrier is used to set the tailwater elevation, forcing a simple hydraulic jump in between that is unlike anything observed in mountain rivers [*Valle and Pasternack, 2002, 2006a, 2006b*]. Even interesting experimental variants of this set-up using uniformly sloped channels, uniform conduits with diverse cross-sectional shapes, and simple expansions or contractions [e.g. *Elevatorski, 1959; Rajaratnam, 1967*] do not approach observed natural hydraulic jumps.

In contrast to classic hydraulic jump research, studies of man-made overfalls and related hydraulic structures do provide at least a starting point for studying and understanding natural waterfalls [most notably *USBR, 1948a,b*]. Useful reviews of key hydraulic and scour processes associated with overfalls may be found in classic fluid mechanics textbooks [*Rouse, 1957; Henderson, 1966*] and more recent articles by *Mason and Arumugam [1985], Young [1985], Chanson [2002], and Pasternack et al. [2006]*. Enriching the current understanding of waterfalls would not only assuage hydrogeomorphic curiosity, but it could aid mitigation of gully erosion [*Hanson et al., 1997; Bennett et al., 2000; Stein and Latray, 2002*], yield mechanistic bedrock incision equations for process-based landscape evolution modeling [*Hancock et al., 1998; Dietrich et al., 2003*], and improve the engineering of hydraulic structures for fish passage [*Beach, 1984; Lauritzen et al., 2005*], whitewater parks, drown-proof weirs [*Leutheusser and*

85 *Birk, 1991*], stepped spillways [*Chanson, 2002*], bed sills [*Lenzi et al., 2002*], spillway stilling
86 basins [*Fiorotto and Rinaldo 1992*], dam removals, and river rehabilitation projects.

87 Although there are many interesting unanswered questions related to waterfalls, one of
88 the most important sources of uncertainty is the mechanism of scour below natural waterfalls.
89 Scour depends on the balance of substrate resistance and imposed hydraulic stresses [*Hayakawa*
90 *and Matsukura, 2003*] over a wide range of flows. Several approaches exist to measure the
91 former variable for a given substrate material present at a waterfall [*USBR, 1948a; Moore, 1997;*
92 *Sklar and Dietrich, 2001; Simon and Thomas, 2002*]. Also, several studies have bypassed
93 observing scour mechanisms directly and instead made process inferences based on
94 characterizing the geometry of scour holes in alluvial material below free overfalls [e.g. *Mason*
95 *and Arumugam, 1985; Bormann and Julien, 1991*]. Between these two endmembers lies an
96 important mechanistic gap involving actual measurements of drag and lift stresses under natural
97 waterfalls or even man-made overfalls that have not been made to our knowledge.

98 The closest analog of actual measurement of hydraulic stresses below a natural waterfall
99 involved laboratory measurement of pressure differentials on the bed under the 2D classic
100 hydraulic jump (with no overfall) made using the Preston-tube technology of *Preston [1954]*
101 with improvements by *Head and Rechenberg [1962]* and *Patel [1965]*. In the studies, empirical
102 calibration was used to convert measured pressure differentials into estimates of local bed shear
103 stress [*Rouse et al., 1958; Rajaratnam, 1965; Rajaratnam, 1967; Wu and Rajaratnam, 1996*].
104 *Patel [1965]* carefully evaluated the accuracy of this method of bed shear stress estimation under
105 diverse conditions and reported that estimates were unreliable when there existed adverse
106 pressure gradients and a breakdown of the inner law of the wall. *Ackerman and Hoover [2001]*
107 noted that the method is limited in highly accelerating, recirculating, and/or separating flows as

well as where there is high fine-sediment load and/or air entrainment. All of these constraints are normal characteristics of the flow region downstream of man-made overfalls and natural waterfalls, making this technology limited for investigating scour mechanisms there.

Ackerman and Hoover [2001] summarized the pros and cons of 14 different methods for estimating wall shear stress, not including the new method used in this study. None of those methods is well suited for the complexity of waterfall hydraulics. Velocity-based estimates using propellers or acoustic sensors in particular do not work in the 3D aerated flow at the base of a falls. Furthermore, estimates of just skin friction are insufficient to the problem of understanding scour mechanisms, because hydraulic jets impinge on the bed at an angle, yielding a important vertical force component that would be important to determine [*Bormann and Julien, 1991*]. Also, pressure fluctuations known to exist under a hydraulic jump [*Fioroto and Rinaldo, 1992*] might produce large lift forces important to the dynamics.

Going beyond clear-water hydraulics, coarse bedload particles may serve as projectiles that are effective at scouring highly resistant bedrock where hydraulic force alone may be inadequate [*Sklar and Dietrich, 2004*]. The dominant effect and indicator of bedload on the scour of resistant, non-alluvial beds is the generation of sculpted bedforms over a range of spatial scales such as large inner channels and bedrock benches as well as small longitudinal grooves, potholes, flutes, and scallops [*Hancock et al., 1998; Wohl, 1998; Richardson and Carling, 2005*]. The presence of such bedforms in the vicinity of a waterfall (and especially on it) provides strong evidence of their importance there. Conversely, their absence strongly supports the dominance of clear-water scour, because clear-water scour at the base of a waterfall may occur at moderate flows for which the upstream channel may not entrain and transport bedload to the brink of the falls, assuming there is even a supply of bedload available. Horseshoe waterfalls that show no

bedload-induced bedforms have been observed in channels with cohesive-mud beds (e.g. waterfalls sculpted into Kilauea Stream at Morita Reservoir Dam ($22^{\circ}11'56.42''\text{N}$, $159^{\circ}23'20.51''\text{W}$), Kauai, Hawaii on March 12, 2006 after the failure of the upstream Ka Loko Reservoir Dam), horizontally layered sedimentary bedrock (Niagara Falls at $43^{\circ}4'37.60''\text{N}$, $79^{\circ}4'30.94''\text{W}$), uniform siliciclastic sedimentary bedrock (Shanghai Falls at $39^{\circ}5'27.22''\text{N}$, $121^{\circ}35'55.59''\text{W}$), and karst bedrock (seven waterfalls on the Krka River, Croatia in Krka National Park at $43^{\circ}48'17.11''\text{N}$, $15^{\circ}57'52.67''\text{E}$). The waterfalls at Morita, Krka, and Niagara are particularly illustrative because they receive minimal bedload due to their upstream reservoir/lake, leaving only clear water scour to initiate and/or maintain the horseshoe configuration in low-resistance bedrock, depending on the example.

In the experiments reported below, a significant new development for advancing the understanding of clear-water scour at waterfalls is presented in which direct measurements of near-bed drag and lift stresses were made at the base of a sizable experimental waterfall in a controlled facility. The experimental falls was designed to have a horseshoe brink configuration, which is thought to be the common form for waterfalls whose substrate is susceptible to scour by clear-water hydraulic stresses, thus requiring no ballistic coarse sediment [Pasternack et al, 2006]. Furthermore, flows were controlled to produce typical hydraulic jump conditions below a falls, since many waterfalls are never fully submerged, even during floods. Ideally, the hydraulics of a great many brink configurations and relative submergence conditions will need to be explored, and this first effort provides the foundation for such a future agenda.

The new data was collected using a 6-component force and torque sensor using a similar strain gage technology previously used to observe weak drag and lift forces over gravel beds [Nelson et al, 2001]. The specific objectives of this study were to 1) evaluate the feasibility of

measuring powerful hydraulic stresses using a submersible 6-component sensor, 2) determine the longitudinal and transverse patterns in instantaneous and mean bed stresses associated with the convergent hydraulic features induced by the horseshoe brink geometry, including the stresses at the key locations directly under the hydraulic jet at the toe of the falls, under the roller, and under the convergent boil, 3) assess stress time series for natural periodic oscillations and temporal stationarity, and 4) compare measurements against standard prediction methods. In the discussion, these observations are related to previous concepts for the man-made overfall analog to yield a clearer mechanistic understanding of bed scour below horseshoe waterfalls.

2. Experimental Setup and Methods

2.1 Experimental Design

The essential undertaking in this study involved measuring drag and lift stresses associated with the convergent hydraulics of a broad-crested horseshoe step with a plunging nappe and a hydraulic jump. These are conditions that we have frequently observed in mountain regions around the United States. The independent variables were non-dimensional energy $(H+P)/H$, and non-dimensional submergence h_d/H (see Fig. 1 for variable definitions). For these variables that were originally defined by USBR [1948b], lower values are associated with higher energy and submergence, respectively. So the variables are conceptually inverted, which is important to remember when considering the results. The former variable was set to values of 4.75 and 5.55 (equivalent to 0.477 and 0.357 m³/s), which were chosen to produce a plunging nappe with moderate energy. Based on observations of many waterfalls, these conditions are common in nature. Because $(H+P)/H$ is non-dimensional, the values chosen can represent high falls experiencing large floods or small falls experiencing base flows. The latter variable was set

to produce a near-optimal hydraulic jump along the centerline, which yielded values of 3.24 and 3.95 respectively for the two energy levels. Given the horseshoe shape of the step brink, this means that the jump was increasingly submerged with lateral distance from channel center, because the horseshoe brink protrudes downstream with distance from the center. For a broad-crested step, supercritical brink Fr is constant for all $(H+P)/H$, so both geometric and Froude scaling was achieved. Flume dimensions and step height were prototype scale for creeks. Aeration was present in both runs, indicating a sufficient step size to achieve prototype features typically not seen in small flumes with velocities under 0.3 m/s [Chanson, 2002].

2.2 Flume Facility

All tests were done in a non-recirculating, non-tilting flume at University of Minnesota's St. Anthony Falls Laboratory (Minneapolis, MN, USA). This concrete and steel flume was 84-m long x 2.75-m wide x 1.8 m deep (Fig. 2). Water is supplied to it directly from the Mississippi River over an adjustable range of 0-8.5 m³ s⁻¹. A hollow-wood broad-crested step 4.28-m long x 2.75-m wide x 0.91-m high was bolted ~60 m downstream of the flume's inlet and coated with smooth paint. It was situated partly over a steel-plated, false-floor section with a glass sidewall. At the downstream end of the step, an additional 1.37-m section of joist-supported, 2-cm thick painted plywood was cantilevered out with a semi-circular area cut out yielding a U-shape (1.37 m radius = channel half-width). The ratio of brink length to channel width for this configuration was $\pi/2$. The horseshoe was also supported by a 10 cm x 10 cm wood pier at each downstream peripheral tip. Under the horseshoe, ventilation was provided to minimize nappe oscillations using a 2.54-cm dia. aluminum pipe through the floor. In nature, the multi-scalar roughness on a step locally disturbs the nappe or nappe-bank boundary providing ventilation. An adjustable

sharp-crested weir at the downstream end of the flume was used to set and maintain the desired tailwater depth, in this case to achieve optimal hydraulic jumps.

2.3 Positioning and Surveying System

Baseline data needed for the trials included discharge (Q), bed coordinates, and water surface coordinates. Full details of the positioning and surveying system are available in *Pasternack et al.* [2006]. In summary, the broad-crested step method was used with a discharge constant of 0.848 to measure and set Q for each trial [Ackers et al., 1978]. A large rolling trolley mounted over the flume served as the means for positioning instruments in the flow anywhere down the length of the flume. A triangular truss fixed level on the trolley was fitted with a small “rover” carriage to position instruments anywhere across the channel. A 2.565 m long x 2.54 cm diameter aluminum pole with a fine tip at the bottom and a surveying prism (1” accuracy glass) mounted on top was placed into a leveled bushing unit on the rover and operated up and down with a winch to accurately located the 3-D coordinate of any chosen point in the flume space.

A Topcon GTS-603 total station was used to measure bed and water surface topography. This unit had a 3-sec resolution with a distance (D) accuracy of $\pm(2 \text{ mm} + 2\text{ppm} \times D)$ mean square error. Control points were used to maintain accuracy, yielding a mean accuracy of $\pm 7.15 \text{ mm}$ ($\pm 3 \text{ mm SD}$) horizontal and $\pm 1.95 \text{ mm}$ ($\pm 1 \text{ mm SD}$) vertical. Centerline water surface profiles were surveyed using a feature-based approach with higher sampling density in areas of greatest change [e.g. Lane et al., 1994; Brasington et al., 2000]. A consistent approach to measuring the mean water surface on the jump roller was used [Pasternack et al., 2006].

2.4 Force, Stress, and Torque Vectors

The force vector ($\vec{F}$) applied to an object on the bed of a channel is defined as the integral of the infinitesimal stress over the surface of the object, neglecting components due to the object's buoyancy and atmospheric pressure. This vector is responsible for geomorphic channel change. Because an experimental object used to measure $\vec{F}$ may have a different geometry than random natural objects, any measured $\vec{F}$ may be divided by the experimental object's projected area (A) normal to the direction of $\vec{F}$ to obtain the spatially averaged stress vector ($\langle \vec{\tau} \rangle$). This vector enables force comparisons across different objects and flow conditions, and is the focus of this study.

Consider a Cartesian coordinate system defined with the X-axis parallel to a channel's longitudinal axis and positive in the downstream direction, the orthogonal Y-axis directed up in the positive direction, and the orthogonal Z-axis directed cross-channel and positive in the river-right direction (Fig. 3; selected to match the experimental design). In this system, the vernacular terms drag and lift refer to the F_x and F_y components of $\vec{F}$, and with F_z may collectively be written as F_i using index notation. Correspondingly, the terms “drag stress” and “lift stress” will be used to refer to the positive $\langle \tau \rangle_x$ and $\langle \tau \rangle_y$ components, respectively. Using index notation, the components of $\langle \vec{\tau} \rangle$ may be written as $\langle \tau \rangle_i$. When the components of A are written in matching index notation, then $\langle \tau \rangle_i = F_i/A_i$ may be used to calculate the stress components from measurements of F_i .

The torque vector ($\vec{M}$) is defined as the cross product $\vec{M} = \vec{r} \times \vec{F}$, where $\vec{r}$ is the position vector defined as the perpendicular distance from the force's line of application to the axis of rotation. According to the “right hand rule”, F_x applied to an object with a rotation point at the origin and extending along the positive Z-axis yields a positive torque (M_y). Similarly, F_y applied to the same object with a same orientation yields a positive torque (M_x), while F_z yields

no torque. For this object orientation, there is only 1 component of $\vec{r}$, which is r_z . The torque components may be collectively written as M_i in index notation. It is crucial to insure that one's conceptualization of the coordinate system (Fig. 3) is properly aligned according to the above definitions to enable proper right-hand rule visualization of vector directions and proper vector algebra.

2.5 Stress Component Measurement

The UDW3-100 force/torque sensor (Advanced Mechanical Technology, Inc., Watertown, MA) is suitable for simultaneous underwater measurement of all F_i and M_i as $\vec{F}$ changes direction and position through time. The sensor produces these six outputs based on foil strain gage technology beyond the scope of this article. The response time of the sensor is <1.66 ms, so it may be used to make measurements at up to 600 Hz, though care should be taken to eliminate any signal associated with the power supply itself (60 Hz). The effect of water pressure on the sensor is compensated for using a special bladder that equalizes internal and external pressures. Intense impacts by large sediment particles that impose forces beyond the margin of safety can damage the sensor, making it unsuitable for monitoring hydraulic forces during significant bedload transport. As no coarse sediment was used in the research reported herein, the sensor was ideal for measuring the hydraulic stresses associated with a horseshoe falls.

The UDW3-100 is designed to use levers to obtain the desired range and resolution of ambient hydraulic forces through measurement of torques. It can measure strong F_i of 0-222.4 N and M_i of 0-11.3 N·m along its X and Y axes, with corresponding sensitivities of $5.4 \mu\text{V}/(V_{\text{ex}} \cdot \text{N})$ and $265.5 \mu\text{V}/(V_{\text{ex}} \cdot \text{N} \cdot \text{m})$, respectively, where μV is the signal in microvolts and V_{ex} is excitation

voltage. The yield strength of strain elements is ~ 3 times the maximum design stress, but the actual point of irreversible sensor failure is not necessarily as high as that. Exact sensitivities are determined experimentally for each sensor in the factory. When $\vec{F}$ is applied to the sensor's built-in cylindrical lever in an arbitrary direction, an unwanted transfer of signals between communication channels (i.e. crosstalk) can cause error in the recorded F_i , and to a lesser extent M_i . To account for this effect, the manufacturer provides a crosstalk matrix based on carefully controlled calibrations.

The key to successful application of the UDW3-100 for measuring hydraulic stress is to add a lever sufficiently long to yield M_i measurements within the sensor's safe operational range, ideally taking advantage of the full range of sensitivity available. To achieve this, it is necessary to estimate the expected F_i or $\langle \vec{\tau} \rangle_i$, calculate the resulting M_i for a range of levers with different lengths, and select the one that gives the best match, with an extra margin of safety in case the spatially averaged stress component estimates were too low. It is always possible to add longer, more sensitive levers after preliminary measurements. To add such levers, a female-threaded cylindrical adapter was machined with the same diameter as the built-in lever, and it was bolted to the built-in lever to enable male-threaded cylindrical levers to be screwed into it. As the built-in lever, adapter, and additional levers have unique lengths (L_{zk}), rectangular projected areas (A_{ik}), and position vectors (r_{zk}), M_i must be computed as $\sum r_{zk} \cdot F_{ik}$, where the subscript k denotes the contribution of each section (sensor, adapter, and lever) to the total torque component.

The approach taken in this study to determine the desired lever arm length was to 1) guestimate the expected step-toe peak flow velocity (U), 2) calculate $F_{xk} = 0.5 \cdot C_x \rho_w U^2 A_{xk}$ using an estimated drag coefficient (C_x) of 0.5 appropriate for cylinders and a water density (ρ_w) of 998 kg/m³, and 3) calculate the torque component (M_y). The above guestimation approach assumes

that F_x exceeds F_y , but if the opposite is expected than an analogous approach could be used to estimate F_y and M_x , recalling that the operational range is identical for F_x and F_y . Since all lever sections are cylindrical, $A_{xk} = A_{yk}$. Without any lever, the sensor's exposed L_{zk} and A_{xk} were 5.08 cm and $2.91 \cdot 10^{-3} \text{ cm}^2$, respectively. Because the sensor's axis of rotation is partially encapsulated in a water-tight housing, the sensor's lever is actually 5.555 cm long, not 5.08 cm. Assuming that hydraulic forces are applied equally along the exposed length of the sensor, $r_{zk} = (5.555 - 5.08) + 0.5 \cdot (5.08) = 3.02 \text{ cm}$. The lever adaptor had a L_{zk} and A_{xk} of 2.60 cm and $2.91 \cdot 10^{-3} \text{ cm}^2$, respectively. Again assuming that hydraulic forces are applied equally along the length of the adaptor, the adaptor's $r_{ik} = 5.555 + 0.5 \cdot (2.60) = 6.86 \text{ cm}$. Levers were made to have a diameter of 1.172 cm, any desired length, and thus a corresponding $r_{ik} = 5.555 + 2.6 + 0.5 \cdot L_{zk}$. Assuming one can estimate $\langle \tau \rangle_i$ for the measurement location of interest, the useful range of the sensor for measuring M_i using different levers is shown in Figure 4, with the relation for the 42.3 cm rod used in this study given double thickness.

2.6 Torque Measurement Validation

Despite the high quality of the factory calibration, calibrations specific to this application using additional levers were performed and are reported to evaluate the accuracy of the sensor for use in water resources applications. In the tests, the sensor was positioned level on a table with a long lever over the edge of the table. A precision calibration weight was suspended from the sensor at the location along the lever necessary to achieve the desired torque between 0-10 N·m for one axis in isolation (Fig. 5a, b). The test was repeated for each axis, X and Y. The resulting raw error of measured torque versus actual torque averaged 26.1 and 7.6 % for M_x and M_y , respectively (Fig. 5c,d). A regression equation was fit to the data (reported in Fig. 5a,b) and

the adjusted error associated with using the regression equation computed. Calibrated M_x error for 0.1-10.0 N·m averaged 1.51%, while that for M_y error was 1.85 % (Fig. 5c,d).

Since the calibrations were done using only the primary sensitivity values provided by the manufacturer, they did not include an evaluation of crosstalk. Using the raw μV outputs of the sensor for all F_i and M_i along with the factory-provided crosstalk matrix, cross-talk error estimates of M_x and M_y were calculated. They averaged 1.27 and 0.36 %, respectively, with minimal variation as a function of the magnitude of the torque. Thus, no need for applying cross-talk corrections was deemed necessary for this application.

As an additional test, F_x and F_y sensor outputs were compared to the known forces applied. The test showed that direct force measurement had significant errors when using the desired levers. Specifically, raw errors for F_x and F_y averaged 25 and 20 %, respectively. Calibration regressions did not improve these errors significantly. Correcting for cross-talk did not improve the estimates of F_x , but did reduce the average error of F_y to 10 %. These evaluations indicate that it is not recommended to use direct F_x and F_y measurements when adding levers onto the sensor.

The only assumption used in the application of the UDW3-100 is that the force applied to each section of the sensor is applied uniformly at any instant in time. This assumption is not directly testable under a waterfall, but is likely to not hold exactly. The theoretical worst-case overestimate would result if $\vec{F}$ was actually only applied at the end of the additional lever of the whole assembly through the entire period of measurement. It was calculated that for the set-up used in this study this error would be a 29% overestimate of the actual stress regardless of the strength of the force applied. The situation for underestimate is worse as strong forces applied very closely to the rotation point would yield small moments. If $\vec{F}$ was actually only applied at

the midpoint of the sensor's built-in lever but assumed to occur uniformly over the three lever sections, then a 92% underestimate in actual force would result. These two theoretical worst-case scenarios cannot happen in natural flows where force is applied in a distributed manner that shifts around through time, but they provide certain conservative limits on measurement uncertainty.

Overall, the outcome of these evaluations is that the UDW3-100 sensor is best used by aligning it with its X axis in line with the channel and Z axis cross-channel (Fig. 3), and then using levers to obtain torques corresponding with drag (M_y) and lift (M_x). When used in this way, accurate torque measurements can be made. These torques can be used to calculate F_i and $\langle \tau \rangle_i$. The results below report $\langle \tau \rangle_i$ that may be fairly compared to $\langle \tau \rangle_i$ values for any hydrodynamic system. The centimeter-scale height and decimeter-scale length of the sensor relative to the meter-scale height and width of the test channel insure that the obtained data is spatially averaged to provide a robust measure of hydraulic stress insensitive to bed-roughness effects. This makes the sensor of high value for use in the field at real waterfalls, especially those at the decameter height and width scales. Further investigations into the applications and limitations of the UDW3-100 are warranted, but its use appears adequately validated for the purpose of this study.

2.5 Data Acquisition Procedure

A consistent method for drag and lift data acquisition was used for both runs. Prior to data collection, the sensor was equilibrated to the cold Mississippi River water for over an hour in a large barrel suspended from the flume wall. This enabled establishment of the calm-water reference state of the force system canceling out atmospheric pressure and the sensor's

buoyancy. To obtain a longitudinal bed-stress profile, the flume trolley and truss rover carriage were first positioned to locate the sensor on the bed under the step toe. The sensor was then locked into place on the bed using heavy weights and clamps on the truss and rover carriage to preclude any movement. Bed-stress conditions were sampled at 10 Hz for 2 minutes, with the 1-Hz mean and standard deviations logged using a Campbell Scientific CR23X data logger running PC208W software. Because the longitudinal variation in bed stress was not visible, a uniform sampling strategy was used in which the trolley was advanced by one-quarter turn of its wheels, equaling 8.68 cm, and then the 2-minute data acquisition was repeated. The trolley was advanced until the sensor had moved beyond the hydraulic boil and out of the step's immediate domain of influence. 10-Hz fluctuations were assessed using the means and standard deviations calculated each second.

A stationary process is one whose statistical properties, such as mean, variance, and autocorrelation structure, do not vary with time. Even though the UDW3-100 sensor's response time can enable fast sampling at 600 Hz, evaluating temporal stationarity of near-bed stress and the possibility of channel-controlled periodic fluctuations at the time scale of seconds to minutes would be highly relevant for improving the understanding of scour processes below waterfalls. Thus, a 1 Hz time series was collected for a duration of 23932 s ($>2^{14}$ s) for the $(H+P)/H=5.55$ trial. This time series was obtained at the position of maximum 2-minute mean drag after quickly processing the longitudinal profile data for the run and re-positioning the sensor to this location. Stationarity in drag and lift time series was assessed by calculating trends, autocorrelations, and moving averages. To evaluate the independence of the two stress components, both time-based simple correlation analysis and frequency-based coherence analysis [Rabiner and Gold, 1975; Pasternack and Hinoov, 2003] were used. Power spectral

analysis was performed on both raw time series as well as de-trended time series with means removed to identify periodic fluctuations. Power is reported as spectral power density (data variance/frequency) versus frequency.

To obtain a lateral bed-stress profile, the sensor was positioned at the centerline location of maximum drag and then incremented by 10-cm at a time toward the flume-left wall. Given that the horseshoe was symmetrical, there was no need to measure both halves of the channel. At each location bed-stress was measured using the same method as for the longitudinal profile.

2.6 Erosive Stress Prediction

Three methods for estimating bed stress were evaluated relative to the observed bed stress values. *Hayakawa and Matsukura* [2003] proposed that the erosive stress of a waterfall on an area of impact could be expressed as

$$\left[\frac{Q\sqrt{\rho_w}}{A_{\text{impact}}} \right]^2 \quad (1)$$

where Q is discharge [L^3/T], ρ_w is water density [M/L^3], and A_{impact} is area that the falling water impacts [L^2]. They used this equation along with estimates of bedrock resistance to explain estimated rates of waterfall recession. Two predictions using equation (1) were tested against observed bed stress. First, A_{impact} was measured as channel width times the length of bed between the step toe and the boil crest, which represents the larger domain of jet impact and convergence. Second, A_{impact} was defined as the narrow region of direct jet impact to compare against the observed bed stress of the jet impact. This area was measured as the length along the step toe times the thickness of the jet.

The third approach for predicting bed stress that was tested was to use the system of classic fluid mechanic bed stress equations [Robertson and Crowe, 1993]:

$$\tau_b = \frac{f}{4} \rho_w \frac{U_{toe}^2}{2} \quad \text{and} \quad U_{toe} = \frac{Q}{h_{toe} B_{toe}} \quad (2), (3)$$

$$\frac{1}{\sqrt{f}} + 0.8 - 2 \text{Log}(Re \sqrt{f}) = 0 \quad \text{and} \quad Re = \frac{U_{toe} h_{toe}}{\nu_w} \quad (4), (5)$$

where f is the resistance coefficient, U_{toe} is velocity of the free jet at the point of contact with the bed (i.e. the step toe), Q is flume discharge, h_{toe} is flow depth at the step toe, B_{toe} is the jet length along the horseshoe step toe, Re is Reynolds number, and ν_w is the kinematic viscosity of water at the ambient water temperature.

3. Results

Non-dimensional centerline water surface profiles captured the essential flow features associated with the horseshoe step (Fig. 6). The profiles may be divided into three distinct regions- the free-falling nappe, the hydraulic jump, and the tailwater. At the two energy levels used in this study, centerline nappe profiles were unaffected by flow convergence and were well predicted using Rouse's [1957] semi-empirical profile and ballistic equation for a 2D overfall [Pasternack et al., 2006]. When both nappe profiles were collapsed to the same brink datum, they overlapped very closely, showing ideal non-dimensionality. In contrast, the profile for the hydraulic jump region (beginning at the nappe toe and ending at the slope inflection on the back of the boil) showed deviations from non-dimensionality (Fig. 6b) caused by the differing degrees of flow convergence resulting from each flow's momentum over the drop. Lower energy flow (i.e. higher $(H+P)/H$) over the horseshoe falls enabled greater streamline conformity with

horseshoe topography, leading to stronger convergence in the channel center, and thus a steeper hydraulic jump with a higher boil. No approach has yet been developed to non-dimensionalize the flow convergence and divergence effects associated with the horseshoe falls.

3.1 Lumped Stresses

Near-bed drag and lift stresses along the centerline downstream of the toe of a 0.91-m high horseshoe falls were remarkably strong and variable for even relatively low energy levels (Fig. 7). Lumping all centerline data from a trial together, drag stress was predominantly directed downstream, whereas lift stress included both upthrusts (positive lift) and downthrusts (negative lift). The mean (μ) ± 1 standard deviation (σ) shear stresses for $(H+P)/H$ equal to 5.55 and 4.75 were 525 Pa (± 185 Pa) and 875 Pa (± 365 Pa), respectively. In comparison, the corresponding values of μ ($\pm 1 \sigma$) for lift stress were 44 Pa (± 188 Pa) and 75 Pa (± 357 Pa), respectively. The values for lift are misleading, because they average upthrusts and downthrusts. Taking the absolute value of lift, the μ ($\pm 1 \sigma$) was 144 Pa (± 128 Pa) and 268 Pa (± 247 Pa), respectively. Even these averages mask the most significant finding that 1-s peak lift stresses exceeded 1-s peak shear stresses. The 1-s peak upthrust and downthrust for $(H+P)/H=5.55$ equaled 1010 and -1316 Pa, respectively, in comparison to the 1-s peak drag of 1003 Pa. For $(H+P)/H=4.75$, the 1-s peak upthrust and downthrust was 1799 Pa and -1381 Pa, respectively, in comparison to the 1-s peak drag of 1581 Pa.

3.2 Shear stress Profiles

Centerline profiles of mean shear stress demonstrate characteristic features of how the flow below a falls scours and transports sediment. Mean shear stress was found to be at its

minimum under the toe of the falls where the water impacted the bed: 131 Pa for the lower energy trial with more flow convergence and 85 Pa for the higher energy trial with less flow convergence (Figs. 8a, 9a). At this location, the coefficient of variation (σ/μ) was ~ 1 , so peak instantaneous drag was somewhat higher. Moving downstream through the roller for $(H+P)/H=5.55$, shear stress was observed to rise to a peak of 686 Pa and then fluctuate with no further trend until it rose sharply beyond the boil crest to a maximum of 806 Pa (Fig. 8a). For $(H+P)/H=4.75$, shear stress trended up through the entire roller and peaked on the boil crest (Fig. 9a). At this energy level, stresses were too high beyond the boil to use the 42.3 cm lever arm, so measurements were stopped. Though not measured, qualitative observations suggested that drag increased sharply as flow accelerated down the backside of the boil. For both trials, the coefficient of variation of drag was steady at 0.1 through the roller and boil.

3.3 Lift Stress Profiles

Centerline profiles of mean lift stress did not correspond with the pattern observed for shear stress (Figs. 8b, 9b). The falling nappe flow impacted the bed causing a forceful downthrust of -133 Pa for the lower energy trial and -79 Pa for the higher energy trial, with the difference accounted for by convergence differences and trajectory of descent. At this location, the lift stress coefficient of variation was ~ 1 . Over a very short distance ($<0.25 X/H$), the downthrust became an upthrust whose strength increased to a maximum under the midpoint of the roller. The peak for the lower energy trial was 137 Pa and for the higher energy trial was 185 Pa. The corresponding coefficients of variation were 0.5 and 1.6. After the roller's midpoint, lift trended downward through the boil and beyond. For $(H+P)/H=4.75$, the coefficient of variation of lift steady increased over this domain (Fig. 9b). Instantaneous upthrusts on the top of the boil

exceeded the measurable range, suggesting upthrusts of >3078 Pa. Unfortunately, the experimenter only monitored mean lift and drag values during data collection, and it turned out that the M_x strain element of the sensor used to measure lift was inadvertently damaged downstream of the boil by instantaneous upthrusts during the higher energy run. Given the factor of safety built into the sensor, it is estimated that >1 -Hz upthrusts of >9234 Pa were occurring. Had a shorter lever been used, the damage would not have occurred, emphasizing the importance of doing preliminary trials with conservatively small levers prior to optimizing the lever length for final data collection.

3.4 Cross-channel Stresses

The cross-channel stress profile was only observed for $(H+P)/H=5.55$. The observed profile, located at the position of peak drag midway through the roller, showed a decrease in drag with distance from the centerline (Fig. 10a). In contrast, lift increased to a peak 1/3 of the way toward the flume wall. Between the centerline and this location, the coefficient of variation decreased from 4 to 2. Close to the wall, lift switched from upthrust to downthrust as the sensor moved under the falling nappe near the periphery of the horseshoe step.

3.5 Time Series Analysis

Statistical analysis of the 1-Hz dataset collected for $(H+P)/H=5.55$ at the mid-roller location with peak drag showed that instantaneous $\langle \tau \rangle_i$ time series fit Gaussian distributions over the central 95% and 90% of their respective distributions for drag and lift, respectively. Drag- and lift-stress time series did not have equal variances, so a Wilcoxon matched pairs test was used instead of a student t test to evaluate the relation between the two distributions. The test

yielded a p-value <0.0001 , demonstrating that the two distributions are statistically different beyond the 99.99% confidence level. No meaningful correlation was found between drag and lift stresses. In the frequency domain, coherence analysis revealed that 95% of frequencies had an $r^2 < 0.4$, corroborating the simpler finding in the time domain. The few coherent fluctuations were scattered throughout the frequency domain and had low spectral density. Thus, the values observed for drag versus lift stress components at any instant in time were not merely a result of $\langle \vec{\tau} \rangle$ being oriented off-axis. The two series were independent.

Both stress components were found to be non-stationary. Very small linear temporal trend with slopes of -0.001 and 0.0007 were observed for drag and lift, respectively. Lag-1 autocorrelations were 0.405 and -0.222 for drag and lift, respectively. The moving average with a 100 s window yielded ranges of 561 to 681 Pa and -21.7 to 108 Pa for drag and lift, respectively.

Power spectral analysis of the first 2^{14} points from each $\langle \tau \rangle_i$ time series yielded spectra with significant differences (Fig. 11). In the 0 - 0.16 Hz frequency range, the spectral density for lift stress was 2.5 times that for drag stress, and in the 0.24 - 0.5 Hz range it was 18.5 times higher. Drag stress showed a $1/f^\alpha$ noise spectra, with f defined as frequency (Hz) and $\alpha = -0.543$. Lift stress showed two discrete white noise spectra, with the mean spectral density for the 0.24 - 0.5 Hz range 2.3 times that for the 0 - 0.16 Hz range. It is possible that there was inadvertent clipping or filtering of lift stress somehow. The four highest peaks for drag stress in decreasing order of spectral density occurred with periods of 107 , 98 , 50 , and 59 s. For lift stress they had periods of 2.7 , 2.9 , 2.06 , and 2.1 s. Given that the duration of measurement was in excess of 4.5 hours and that these peaks were more than 5 standard deviations higher than their local average background noise level, the cycles they represent are highly statistically significant. Taken

together, these results demonstrate that fluctuations in lift and drag stresses occurred at significantly different frequencies.

3.6 Erosive Stress Prediction

When eq. (1) was applied to the two flume trials reported herein, the spatially averaged erosive stress estimates obtained were 99 Pa for $(H+P)/H=5.55$ and 156 Pa for $(H+P)/H=4.75$. In comparison, the corresponding spatially averaged drag stresses over the same domain were 525 and 875 Pa, which are larger by factors of 5-6. Even assuming the worst-case overestimate error of 29% for the sensor, this suggests that eq (1) underestimates average bed stress by a factor of 4. When A_{impact} was restricted to the jet impact area, then eq (1) predicted stresses of 1802 and 3218 Pa, respectively, compared against observed downthrusts of 133 and 79 Pa, respectively. In this comparison, erosive stresses were overestimated by factors of 13.5 and 40.7, respectively. Further, the trend in bed stress as a function of input energy was backwards, because higher discharges yield less direct impacts on the bed, and equation 1 does not account for that mechanism.

Observed values of Q , h_{toe} , B_{toe} for each trial were used in eqs (2)-(5) to predict bed stress and compare against observed values. The resulting stress predictions were 13 and 10 Pa for $(H+P)/H=5.55$ and $(H+P)/H=4.75$, respectively. When compared with downthrust stress or drag stress at the point of impact, these estimates showed the correct decreasing trend, but were too low by a factor of ~ 10 . When considered as an estimate of spatially averaged bed shear stress for the hydraulic jump region, they underestimated observed values by factors of 40 and 88 for the two trials, respectively. In calculating these estimates, the most sensitive variable was h_{toe} , because supercritical flow depth was < 5 cm. Surveying accuracy checks against the control

network showed only ± 0.2 cm deviations, but precisely locating the point-gage tip at the surface of the jet could have additional error. Assuming the highly unlikely situation that depth was overestimated by 5σ (1 cm), bed-stress estimates only increase to 21 and 13 Pa, respectively.

4. Discussion

4.1 Hydraulic Stress Measurement

Most of the time, open-channel bed skin friction is estimated from velocity measurements using a variety of methods with large uncertainties, and bed lift stress is totally ignored [Carstens, 1966; Ackerman and Hoover, 2001]. To answer the central question limiting the understanding of waterfall mechanics, an industrial strain gage sensor was modified with a lever arm to amplify hydraulic force components and yield accurate near-bed measurements of spatially averaged stress vector components. Direct measurement of clear-water hydraulic stress components under a waterfall was found to be possible using the UDW3-100 sensor. Although there is no other method available for independently measuring $\langle \tau \rangle_i$ under a waterfalls to evaluate the accuracy of the sensor under actual field conditions with spatially distributed infinitesimal stress components, a lab-based evaluation showed that the sensor measures M_i with known $\bar{\tau}$ accurately to within ± 1.5 -2%. Accounting for uncertainty in $\bar{\tau}$ likely adds another 5-20% error in most cases. Given the moderate size of the sensor and lever arm, the resulting observations are representative of processes on the same centimeter to decimeter spatial scale as is relevant for transport of sediment grains commonly moving through bedrock rivers.

4.2 Erosive Stress Prediction

Some studies [e.g. *Hayakawa and Matsukura, 2003; Bormann and Julien, 1991*] use equations to predict the bed stress available to do erosive work below overfalls. Comparison of those equations against observations revealed differences ranging from factors of >4 -6 for the spatially averaged bed stress over the hydraulic jump domain using eq. (1) to factors >40 for the local jet impact stress using eqs. (2)-(5). In both cases equations predicted lower values than observations, except where eq. (1) overpredicted local jet impact stress by orders of magnitude. Since the maximum possible time-averaged overestimate for the sensor is 29 %, the explanation for the differences must be inherent inadequacies with the equations. That the observations were made for a horseshoe falls instead of a 2D rectangular falls had no effect on the jet-impact predictions made in which local variables at the step toe were used in the predictions. For the use of eq. (1) to predict average centerline stresses over the length of the hydraulic jump, flow convergence promoted by the horseshoe brink configuration amplifies stress, thereby likely contributing to underpredictions.

Another useful reference for appreciating the magnitude of the observed peak drag stresses is to consider the value associated with the subcritical tailwater region, which can be estimated using eqs (2)-(5), substituting tailwater depth (h_{tail}) for h_{toe} , and substituting channel width for step toe length. Note that this is the tailwater associated with the optimal jump condition, and in nature it would vary as a function of the downstream hydraulic geometry. These equations yield average tailwater drag stresses of 0.382 and 0.487 Pa for the $(H+P)/H=5.55$ and $(H+P)/H=4.75$ trials, respectively. The resulting non-dimensional peak drag stresses are therefore 2110 and 2747, respectively. This demonstrates that for a given horseshoe geometry, drag stress does not scale independently of non-dimensional input energy, and that can be explained by the observed pattern of flow straightening over the step as energy increases.

4.3 Waterfalls Scour Concepts

The conventional concept of clear-water scour below overfalls with plunging water jets and erodible substrates focuses on three specific mechanisms. The first scour mechanism is thought to be direct impact of the free jet of plunging water on the bed, which attacks the bed surface. The classic experimental study done by *USBR* [1948a] showed that even highly resistant concrete slabs are erodible by the direct impact of a hydraulic jet, with the rate of scour diminishing as the angle of attack deviates from perpendicular. The second scour mechanism is the bed drag stress immediately downstream of the jet impact that is conceived to occur after the flow from the impinging jet becomes a wall jet. This stress is thought to be responsible for moving detached bed particles out of the growing scour hole [*Bormann and Julien, 1991*]. The third scour mechanism, though not widely considered in past overfall studies, is severe pressure fluctuations associated with the energy dissipation in hydraulic jumps. Even in studies where the overfall is partially to fully submerged, there was often still a hydraulic jump present. Such pressure fluctuations are thought to be responsible for lifting blocks of material up off the bed [*Fiorotto and Rinaldo, 1992; Fiorotto, V., Salandin, 2000*].

Accepting some uncertainty in the measurement technology, the observations of bed stresses below a horseshoe waterfalls revealed some remarkable characteristics that clarify the conceptual model of how hydraulically erodible channels change below falls. First, the magnitude of $\langle \tau \rangle_i$ for the jet impact on the bed was significantly lower than that for either drag or lift downstream of the impact point. This outcome may be related to the emerged state of the jet and jump controlled by the tailwater depth. Emergent jets and jumps are very common in bedrock rivers, even during floods. The implication is that if jet impact does play a significant

role in bed scour, then the fact that it is perpendicular to the bed must be of great significance. Consider that the domain of jet impact is limited to a very small spatial region of length $<0.35H$, and that it is immediately followed by a much larger region of length $\sim 8H$ with a 5-15 time stronger force vector pointing at an angle up and away from the bed. Lowering tailwater depth to its minimum reveals the limited extent of jet impact and illustrates the strength of the converging flow under the hydraulic jump (Fig. 12).

Second, the magnitude of $\langle \tau \rangle_i$ for drag stress along the bed is not at its greatest in the zone of the scour hole, as one might have expected, though it is significant there. The high drag stress recorded downstream of the impinging jet reasonably conforms to the pre-existing paradigm of the presence of a wall jet there. However, shear stress increases through the hydraulic jump and reaches a maximum downstream of the boil (Fig. 12). This suggests that any equilibrium scour hole should extend beyond the location of the boil crest observed in this study. Also, it suggests that the eventual transition from a wall jet to a wall-bounded open channel flow must occur a significant distance downstream in a uniform or constricting channel, with the actual distance strongly dependent on the tailwater depth imposed by the downstream hydraulic control.

Third, the magnitude of $\langle \tau \rangle_i$ for lift is less than that for drag on average, but at any given instant can be an order of magnitude higher. Also, high-amplitude lift-stress fluctuations operate over the full range of frequencies whereas those for drag stress are limited to frequencies <0.1 -Hz. As a result, drag stress may not be primarily responsible for moving particles out of the scour hole as quantified in the predictive model of *Bormann and Julien* [1991] that they applied to a free jet impinging on a bed. Their experimental illustrations include hydraulic jumps, but their computations do not account for them. Instead, high-frequency lift-stress fluctuations are

much more likely to pluck out particles faster than drag stress fluctuations can have an effect. This is especially the case for an alluvial bed, such as that used in studies of gully headcut migration. These results and interpretations corroborate the insights of *Fiorotto and Rinaldo* [1992] based on their observations of bed-pressure fluctuations regarding the ability of lift to extract large blocks of material out of the bed. The best explanation is that mean drag stress provides an incessant stressful condition upon which the high-frequency lift fluctuations do the heavy lifting. Once particles are lifted into the water column, drag could then transport them away downstream or lift fluctuations could smash particles or blocks up and down on the bed repeatedly further scouring the bed. Thus, the wall-jet paradigm used exclusively to explain scour downstream of overfalls in many past studies is deficient in its lack of consideration of lift stress and the role of the hydraulic jump, which is present in any plunging overfall or bed-supported slide, as long as those structures are not totally submerged. *USBR* [1948b] indicates that the absence of a hydraulic jump would require a degree of submerged of $h_d/H < 0.15$, which is quite low. Many waterfalls never become that submerged. Hydraulic jumps are commonly found at small-scale gully knickpoints in cohesive beds, man-made hydraulic structures, and large natural waterfalls.

The final concept to address is the relative erosive potential of the observed clear-water hydraulic stresses versus the stress induced by sediment-particle impacts. If the 2.75-m wide x 0.91-m high horseshoe falls observed in this study was located in a real step-pool or cascade alluvial stream, then drag stress below the falls would be high enough to move <2.8-m diameter boulders, assuming a critical Shields stress of 0.03 and a friction coefficient of 0.1 (Fig. 8), and that is with <0.5 m³/s. In addition to falls over alluvial steps, waterfalls occurring in channels with weak bedrock composed of hardpan soils, siliciclastic sedimentary rock, or carbonates

would also be expected to be susceptible to clear-water erosion. Given the magnitude of lift fluctuations, clear-water scour could be significant in even a highly resistant bedrock channel if it was fractured enough to enable plucking. Finally, as demonstrated in the work of *USBR* [1948a], a hydraulic jet acting on resistant rock over a long enough time will cause degradation. Whereas bedload-induced scour is only possible when the channel is transporting bedload, the results of this study suggest that hydraulic scour is possible much of the time, creating a classic frequency-magnitude competition. Morphologically, the number and size of potholes and longitudinal grooves formed on and downstream of a waterfall relative to the size of the hydraulic scour pool are indicative of the relative roles of clear-water scour and sediment-induced scour. Even where sediment-induced scour predominates, an understanding of fluid mechanics is essential to predicting waterfall morphodynamics.

5. Conclusions

A sizable laboratory flume was used to study the fluid mechanics of the ballistic hydraulic jet and hydraulic jump associated with horseshoe-waterfall scour. The UDW3-100 force/torque sensor was found to be a useful tool for measuring clear-water hydraulic stress below a waterfalls. Observed stresses were in the 100-1500 Pa range. These observations were significantly higher than predicted using two standard approaches. The lift component of stress not only played a key role in jet-induced scour, but also fluctuated significantly under the hydraulic jump. Its fluctuations occurred over a wide range of temporal frequencies, including high-energy fluctuations at a higher frequency than drag stress, enabling lift to potentially pluck pieces out of the bed faster than drag stress can abrade the bed. These results suggest that the mechanics of the ballistic jet impact on the bed alone should not be used to predict equilibrium

scour depth and length. A more complete model of clear-water scour must account for both drag and lift dynamics.

6. Acknowledgements

This study is based on work supported in part by the STC Program of the National Science Foundation under Agreement number EAR-0120914, in part by the Hydrology Program of the National Science Foundation under Agreement number EAR-0207713, and in part by private funding by the lead PI- Greg Pasternack. We thank Jon Hansberger, Sara Johnson, Kyle Leier, Omid Mohseni, Gary Parker, Mike Plante, Jared Roddy, Alfredo Santana, and Jeremy Schultz for assistance with experimental setup and data collection. We also thank Prof. Wes Wallender (UC Davis) for discussions on vector algebra and continuum mechanics, Prof. Josef Ackerman (U. British Columbia) for discussions on Preston tube technology and applications, and Noel Bormann for discussion on jet-scour mechanisms.

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Figure Captions.

Figure 1. Definition sketch of longitudinal flow profile over a broad-crested, ventilated step. Not to scale.

Figure 2. Experimental broad-crested step with semi-circular horseshoe brink.

Figure 3. Definition sketch of force/torque sensor spatial configuration relative to a defined coordinate system. Not to scale.

Figure 4. Illustration of the range versus resolution trade-off for hydraulic stress measurement using the UDW3-100 sensor in 4 different configurations. The thick line indicates the configuration used in this study.

Figure 5. Evaluation of the accuracy of the UDW3-100 sensor showing measured versus actual A) drag and B) lift as well as the error reduction after sensor calibration for C) drag and D) lift.

Figure 6. Centerline profiles for the two trials showing key hydraulic features for A) complete profiles and B) hydraulic jump regions.

Figure 7. Box and whisker distributions for all 1-Hz $\langle \tau \rangle_i$ measurements for each run. Box boundaries denote upper and lower quartiles, box mid-lines denote median values, whiskers denote the range for points whose values are within 1.5 times the interquartile distance, and points denote outliers. N is the number of points for each dataset.

Figure 8. Centerline profiles for lower energy trial showing 2-minute time-averaged $\langle \tau \rangle_i$ and its coefficient of variation for A) drag and B) lift. The peak coefficient of variation that is offscale had a value of 218. As a reference drag stress, an estimate of the critical shear stress

necessary to entrain 1-m boulders from an unconsolidated bed of such particles is shown assuming a bed friction coefficient of 0.1 and a critical Shields stress of 0.03.

Figure 9. Centerline profiles for higher energy trial showing 2-minute time-averaged $\langle \tau \rangle_i$ and its coefficient of variation for A) drag and B) lift. As a reference drag stress, an estimate of the critical shear stress necessary to entrain 1-m and 2.8-m boulders is shown using the same assumptions as in Figure 8.

Figure 10. Cross-channel profiles from centerline to flume-left wall for lower energy trial taken at the centerline position of peak drag showing 2-minute time-averaged $\langle \tau \rangle_i$ and its coefficient of variation for A) drag and B) lift.

Figure 11. Power spectra of 1-Hz $\langle \tau \rangle_i$ measurements over 2^{14} s with mean and linear trend subtracted for A) drag with best-fit power function shown in grey and B) lift with average white noise spectral density shown in grey.

Figure 12. Photo of $(H+P)/H=4.75$ with minimal tailwater depth to illustrate the localization of the jet impact and the predominance of the flow convergence zone with peak bed stress at the rooster tail (i.e. boil when submerged).