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Author

Cooper, W.S.

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W.S. Cooper

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Use of the Kalman Filter in Signal Processing to Reduce Beam
Requirements for Alpha-Particle Diagnostics*

William S. Cooper

Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

ABSTRACT

In principle, a probing beam of neutral atoms can be used to measure the velocity distribution of fast alpha particles confined in an ignited fusion plasma. In this article we discuss beam modulation techniques and a compatible signal processor based on a Kalman filter that together can provide, in one hypothetical case, a reduction of a factor of ten in the beam current required, as compared with that required for a straightforward measurement with a dc beam.

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INTRODUCTION

Twenty percent of the energy released in a D-T fusion reaction appears as the kinetic energy of a 3.5 Mev alpha particle. The next-generation magnetic fusion experiment will be designed to study the confinement and slowing down of these fast alphas, which contribute substantially to the heating and ultimate ignition of the plasma. A promising candidate diagnostic for measuring the velocity distribution of the alphas involves the injection of a fast probing beam of neutral hydrogen atoms into the plasma.¹ The alpha can capture an electron from the hydrogen atom to form an excited state of He^+ , which then radiates; information about the velocity distribution of the confined alphas is contained in the line radiation.

Because of the energy dependence of the electron capture cross section the technique will work best when the velocity difference between the alpha and the hydrogen atom is small. This sets an upper bound for the hydrogen beam energy of about 1 MeV. Post et al¹ have estimated that a current of 1 A or more of H atoms might be required; taken together, this set of numbers represents a formidable, but probably achievable, beam development goal. Such a beam would be produced by accelerating H^- ions to the required energy, then removing the electron in a gas target.

It is the purpose of this article to examine the possibility of using advanced signal detection techniques to reduce the beam current required. To do this, we need to find a beam modulation technique to code the signal, as well as a compatible signal processing algorithm. We assume that the signal-to-noise ratio has already been estimated for the case of a dc beam, and consider to what extent we can improve the situation.

BEAM MODULATION

It will be necessary to modulate the beam, so that the signal, which is proportional to the beam intensity, can be optimally extracted from the noise. Chopping the beam is an obvious approach, and would probably work up to frequencies of 10 kHz. A better method might be to modulate the velocity of a dc beam, in order to produce a stream of H atoms that "bunches" at the target without loss of beam. An energy modulation of a few percent at a frequency of ten MHz would cause substantial density modulation at a distance of 10 meters. Either technique requires a signal processor capable of extracting a signal with known time dependence but unknown amplitude from noise. In the next section we discuss a signal processing algorithm that is capable of dealing with both these cases.

SIGNAL PROCESSING

An optimal estimation technique known as the Kalman filter is well suited to this type of problem. It is the optimal (in the sense that it minimizes the sum of the squares of the estimated errors) recursive estimator for linear systems. It was devised by Kalman² and Kalman and Bucy,³ and has been thoroughly treated in the literature (see, for example, the book by Gelb⁴). I have discussed its application to data analysis and signal processing in another article.⁵

The Kalman filter uses new information (a new measurement) to generate a new optimal estimate of the state of the system being observed. In our case, the state of the system is represented by a vector $\underline{x}$ having two components, x_1 , the signal amplitude, and x_2 , the background. Both have a stochastic component $\underline{w}$ (the system noise); the system evolves according to

$$\underline{x}_k = \underline{x}_{k-1} + \underline{w}_{k-1}, \quad (1)$$

which represents a random walk. We measure a scalar quantity

$$z_k = x_{1,k} g_k + x_{2,k} + v_k, \quad (2)$$

where $x_{1,k}$ is the amplitude of the signal at the time of the k^{th} measurement, g_k is the value at the k^{th} measurement of a normalized function g_k containing the known time dependence of the signal, $x_{2,k}$ is the amplitude of the background, and v_k is a stochastic component (the measurement noise). This equation states that we measure the sum of a signal with known time dependence, a background, and noise. The amplitudes x_1 and x_2 are assumed to change slowly compared to the response time of the filter. The function g_k can be any time-varying but known function, such as a square wave or the calculated intensity variation of a bunched beam. The measurements are assumed to be periodic, but do not have to have the same period as g_k .

We assume the noise components of $\underline{w}$ are uncorrelated from one measurement to the next, and define $q_1 = E[w_1^2]$, $q_2 = E[w_2^2]$, and $r = E[v^2]$, where E denotes the expectation value, or mean. Values for these noise variances must be estimated either from the physics of the problem or from actual measurements.⁶

It is straightforward to derive,⁵ but unfortunately too long to quote in this short article, the recursion relations for this Kalman filter. The filter requires knowledge of the noise variances q_1 , q_2 , and r , and initial estimates for x_1 , x_2 , and the errors in these quantities. After each new measurement, improved estimates of the amplitudes and of the errors are generated. After a number of iterations, which depends on the noise variances, the filter reaches a steady state, and the estimated errors in the amplitudes do not change, although the amplitudes themselves may change.

We now specialize to the case of a square wave, with two measurements made per period, one with the beam on, and the other with the beam off. Figure 1 shows a computer simulation of the operation of such a filter, with $q_1 = 10^{-4}$, $q_2 = 10^{-2}$, and $r = 1$, and with signal amplitude = 1 and background = 10. After allowing 150 measurements to permit start-up transients to decay, the small square wave is switched on. The filter begins to respond, with a response time for x_1 of about 200 measurements (determined by the choices of noise variances), and will in this case ultimately estimate the amplitude of the square wave with an error of 14% and that of the background with an error of 3%.

Except in almost trivial applications of a Kalman filter, it is not possible to derive analytic expressions for the dependence of the estimated errors in the parameters being observed on the system and measurement noise characteristics. One can, however, use computer simulation of the operation of the filter to study this question experimentally. After the filter reaches steady state, and for the case of unit measurement noise, the estimated error in x_1 is about $1.3 \times (q_1)^{0.242}$, and that of x_2 is about $0.95 \times (q_2)^{0.226}$. These expressions are accurate to $\pm 30\%$ for $10^{-4} < q_1, q_2 < 1$.

The time response of the filter to a step function input is the sum of two time-varying terms that depend on the noise characteristics as well as the amplitudes of the signal and background. For the case of equal signal and background amplitudes, and again with $r = 1$, we show in Figure 2 the approximate "1/e" response times (determined by simulating the operation of the filter) in numbers of measurements for various choices of system noise characteristics. Because the responses are coupled, we show only the longer of the two response times.

AN EXAMPLE

Let us chop the beam at 10 kHz, and again make two measurements per period. The time between measurements is 50 μ sec. Suppose as before that the measurement noise has a variance of 1, the signal amplitude is 1, and the background is 10, and that both can change by 100% in 0.5 sec (10^4 measurements). Treating this as a random walk problem, according to Equation (1), we find that we require $q_1 = (1)^2/10^4 = 10^{-4}$, and $q_2 = (10)^2/10^4 = 10^{-2}$ to model these changes. This is the same set of signal and noise characteristics used in Figure 1. The time response, from Figure 2, is about $200 \times 50 \mu\text{sec} = 10^{-2}$ sec, adequately fast to follow the anticipated changes. The signal-to-noise ratio for a single measurement of the signal amplitude, if we take the background into account, is $1/\sqrt{2r} = 0.7$. The Kalman filter, however, provides a signal-to-noise ratio of $1/(\text{estimated error in } x_1) = 1/0.14 = 7.0$, an improvement of a factor of 10 in signal detecting capability, with a corresponding reduction of the current required in the hypothetical probing beam.

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FIGURE LEGENDS

Figure 1: Simulated operation of a square wave signal processor. Trace (a) is the simulated measurement, consisting of a square wave signal (when present), a background, and noise. Trace (b) is the baseline for (a), and shows also the square wave, turned on after 150 measurements. Trace (c) shows the estimate of the amplitude of the square wave and the theoretical response. Trace (d) is the estimate of the background.

Figure 2: Contours of constant response time, in numbers of measurements, versus system noise variances q_1 and q_2 with measurement noise variance $r = 1$.

Fig. 1

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Fig. 2

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