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Essays on Corruptible Markets, Strategic Certification and Online Peer  
Effects

by

Shubhranshu Ranjan Singh

A dissertation submitted in partial satisfaction of the  
requirements for the degree of  
Doctor of Philosophy

in

Business Administration

in the

Graduate Division  
of the  
University of California, Berkeley

Committee in charge:

Professor Ganesh Iyer, Chair  
Professor Shachar Kariv  
Professor Zsolt Katona  
Professor J. Miguel Villas-Boas

Spring 2013

Essays on Corruptible Markets, Strategic Certification and Online Peer  
Effects

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## Abstract

Essays on Corruptible Markets, Strategic Certification and Online Peer Effects

by

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Doctor of Philosophy in Business Administration

University of California, Berkeley

Professor Ganesh Iyer, Chair

Emerging markets offer significant business opportunities. However, local and foreign firms selling in these markets are often faced with corrupt agents. The first essay investigates the marketing strategy implications for firms competing for business in a corruptible market. We consider a setting in which a buyer (a firm or government) seeks to purchase a good through a corruptible agent. Supplier firms, that may or may not be a good fit, compete to be selected by the agent. Only the agent observes whether or not a firm is a good fit. Corruption arises due to incentive of the agent to select a non-deserving firm in exchange for bribes. Intuitively and as expected, a sufficiently large monitoring of the agent eradicates corruption. But the interesting point is that increasing the monitoring from an initial low level can backfire, making the agent more likely to select a non-deserving firm. As firms become reluctant to offer bribes in response to higher monitoring, it now becomes likely that the agent receives a bribe offer, in equilibrium, only from a non-deserving firm. This non-monotonic agent behavior makes it difficult to reduce corruption. The implication is that the buyer should choose either to be ignorant or to take drastic measures to limit corruption. Further, We show that unilateral anti-corruption controls, such as the Foreign Corrupt Practices Act of 1977, on a U.S. firm seeking business in a corrupt foreign market can actually increase the profits of the U.S. firm. This is because such a control on the U.S. firm puts pressure on the buyer to set monitoring at higher levels and reduces corruption.

The second essay describes how market forces create incentives for firms to seek product safety certifications. We consider a firm which makes the decision of whether or not to seek certification prior to selling the product. Consumers choose to be careful or negligent while using the product. The probability of an accident depends on both the consumer's effort and the product safety. We show that, even when both the firm and consumers have same beliefs about the product safety, the presence of

consumer moral hazard can create incentives for certification. Consumers' choice of effort may change as they update their beliefs upon observing a certification outcome. The consumer surplus that is thereby generated may be extracted by the firm through higher prices creating incentives for certification. Interestingly, if the certification decision is private information to the firm, the presence of consumer moral hazard may lead to more certification if safety and effort are substitutes but less certification if they are complements. If safety and effort are substitutes, a negligent product use hurts the consumer more when using a non-certified product compared to when using a certified product. This makes the certified product more valuable to the consumer. As a result, the certification equilibrium exists over a larger set of conditions. On the other hand, if safety and effort are complements, a negligent product use hurts the consumer more when using a certified product. The certified product becomes less valuable causing the certification equilibrium to exist over a smaller set of conditions.

The third essay empirically investigates the effect of consumers' product evaluations on the judgments of other consumers in an online setting. Consumers routinely get exposed to others' opinions, most often in the form of average of prior ratings, when reporting their own. It is not obvious if consumers incorporate these prominently displayed average ratings in their own evaluations. By use of movie ratings data from Net ix we find that consumers incorporate the average of the prior ratings displayed on screen in their evaluations. Simulations using the estimated parameter values indicate that this behavior changes the resulting pattern of the ratings. We also find that a small number of early extreme ratings has a long lasting impact on the average rating of a movie. Average ratings remain inflated even after 1000 periods when only the first 5 ratings are changed to the highest possible rating.

In loving memory of Amma

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# Chapter 1

## Competition in Corruptible Markets

### 1.1 Introduction

While the GDP growth in the more developed countries remains sluggish emerging economies are forecasted to continue growing at rates over 6%, as per IMF estimates. Increased purchasing-power of the sizable middle class consumers and public investments in infrastructure and defense in emerging markets has created significant business opportunities for domestic as well as foreign firms. U.S. firms, facing saturation in domestic markets, are increasingly counting on consumers, firms and governments in emerging markets as a source of future growth. One of the biggest challenges faced by firms competing for business in emerging markets is corruption. Bribery is considered a norm in many of these economies. Although more prevalent, corruption is by no means limited to emerging economies. Firms, when seeking business opportunities in such markets, may face corruption in many different forms. The following examples illustrate the type of corruption that we focus on in this paper.

No 3 Qiantang River Bridge, China procurement decision and construction took place between 1994 and 1997. Zhao Zhanqi, the chief of Zhejiang transportation

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<sup>1</sup>Foreign profits as a share of global profits for U.S. firms has increased from 21% in 2000 to 46% in 2010. (Sources: S&P 500: 2010 Global Sales report and U.S. Bureau of economic analysis)

<sup>2</sup>Transparency International's Perception of Corruption Index (2011) for major developing economies: Brazil 3.8, China 3.6, India 3.1, Mexico 3.0, Indonesia 3.0 and Russia 2.4.

<sup>3</sup>On average 750 government officials were convicted each year on charges of corruption (from 1992 to 2001) in the United States. Source: <http://www.justice.gov/criminal/pin/docs/arpt-2011-revised.pdf> ; Accessed 3/18/2013

department and deputy director of the No 3 Qiantang River Bridge project accepted more than 6 million yuan (\$930,000) in bribes from the contractor during the bidding process and construction of the bridge. The bridge went through a major repair in year 2005. A part of the bridge caved-in in year 2011 as a heavily loaded truck tried to cross the bridge<sup>4</sup>.

BP, like other large oil companies, charters oil tankers from shipping firms (such as Maersk and Frontline). Tanker chartering is an area that requires specialized skills and knowledge. Lars Dencker Nielsen, a senior executive in BP's tanker chartering division, allegedly received cash payments from a shipping magnate in return for giving multimillion pound contracts over a period of five years<sup>5</sup>. Bribery of private sector employees is illegal according to the Bribery Act 2010 in the UK.

DaimlerChrysler AG, between 1998 and 2008, paid at least \$56 million in improper payments to government officials in China, Russia, Indonesia and other countries. The company earned \$1.9 billion in revenue and at least \$90 million in illegal profits through these tainted sales transactions. Daimler paid \$185 million in fines to settle charges with the Securities and Exchange Commission (SEC) and the U.S. Department of Justice<sup>6</sup>.

There are some features, in the above examples, that we would like to highlight. The decision to select one firm over another is usually not straightforward. The selection of a firm that is best suited for a particular project requires expertise in the subject matter and information about the environment in which the product is to be used. The suppliers themselves may not know if they are best suited. The buyers, firms or governments, rely on agents such as experts or bureaucrats to make the selection decision on their behalf. This creates the scope of corruption. The agents, sometimes but not always, select a non-deserving firm in exchange for bribes. Buyers understand an agent's incentives to select a non-deserving firm. Corrupt agents sometimes get caught and punished. We capture these features in our model.

We analyze the firm's incentives for bribing an agent who is expected to select, on the behalf of the buyer, a deserving firm from two firms that are competing for a project. Only the agent knows if a particular firm is deserving or not. The agent is willing to select a non-deserving firm in the exchange for a bribe. We refer to the selection of a non-deserving firm as a dishonest agent behavior. A dishonest agent behavior hurts the buyer. The buyer understands an agent's incentives and randomly monitors the agent. Monitoring the agent is costly. Upon monitoring, the

<sup>4</sup>[http://www.chinadaily.com.cn/china/2011-07/18/content\\_12919771.htm](http://www.chinadaily.com.cn/china/2011-07/18/content_12919771.htm); Accessed 03/27/13

<sup>5</sup><http://www.telegraph.co.uk/nance/newsbysector/energy/oilandgas/9144236/BP-alerted-to-bribery-at-its-tanker-division.html>; Accessed 06/15/12

<sup>6</sup><http://www.sec.gov/news/press/2010/2010-51.htm>; Accessed 07/31/12

buyer learns if a non-deserving firm was selected. A dishonest agent, if caught, is punished. Firms compete in bribes to get selected by the agent. A sufficiently large monitoring eradicates corruption as firms find it unprofitable to offer large bribes that must be offered to compensate the agent's higher expected penalty. For any smaller monitoring corruption prevails. The bribe offer equilibrium is in mixed strategies. The profits of firms increase in the monitoring. This happens because when firms are required to pay higher bribes to be selected with certainty they become less willing to do so. The equilibrium bribes decrease and result in higher profits for the firms.

We also find that an increase in monitoring does not always result in more honest agent behavior. Interestingly, if bribery is prevalent, a small increase in the monitoring can make the agent more dishonest. The intuition is the following. If monitoring is small both firms offer bribes with probability one. The agent in this case often accepts the bribe offer of the deserving firm. If monitoring is increased firms become less likely to offer bribes. The agent is now faced with situations in which she receives a bribe offer only from a non-deserving firm. This forces the agent to select a bribe-offering, non-deserving firm more often. The agent becomes more dishonest as a result of higher monitoring. If the monitoring is sufficiently large the agent always selects the deserving firm. Various scholars have discussed this non-monotonic relationship between monitoring, or expected penalty, and honest behavior (see Akerlof and Dickens (1982) and Bénabou and Tirole (2006)). However, they draw upon the classic work on intrinsic motivation in psychology (see Deci (1972)). We present a rational agent model with no behavioral assumptions and show that an increase in monitoring can make the agent more dishonest. In our model, endogenous firm response to an increase in the monitoring makes the agent more dishonest.

The non-monotonic effect of the monitoring on the agent behavior makes it difficult for the buyer to reduce corruption. We find that, the buyer should either choose to be ignorant about corruption or commit to take drastic measures to limit it. A small monitoring only hurts the buyer.

Bribery of foreign government officials by U.S. firms competing in overseas markets was commonplace. During an investigation by the U.S. Securities and Exchange Commission, in mid-1970s, more than 400 U.S. companies admitted to having made questionable payments to foreign government officials. Congress enacted the Foreign Corrupt Practices Act (FCPA) of 1977 to bring a halt to the bribery of foreign officials and to restore public confidence in the integrity of the American business system.<sup>7</sup> This unilateral control on the U.S. firms seeking business in foreign markets has been a topic of debate. In the business community it is believed that the

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<sup>7</sup>See [www.justice.gov/criminal/fraud/fcpa/](http://www.justice.gov/criminal/fraud/fcpa/) (accessed 6/19/12) for the history and details of the Act.

Act puts American businesses at a competitive disadvantage in international business (see Kaikati and Label (1980) for a discussion). The evidence from a majority of empirical studies suggests that there is little or no disadvantage posed by the FCPA (see Graham (1984), Beck, Maher, and Tschoegl (1991), and Wei (2000)). On the contrary, James R. Hines (1995) suggests that the FCPA serves to weaken the competitive position of the U.S. firms.

We study the effect of a unilateral anti-corruption control, such as the FCPA, on a firm's profits. We show that the profits of the controlled U.S. firm can actually increase as a result of a unilateral anti-corruption control on it. The intuition is the following. A unilateral anti-corruption control on a firm reduces the bribe that the other firm must pay in order to get selected by the agent regardless of whether it is deserving or not. As a result, the agent selects a non-deserving firm with a higher probability. This hurts the buyer. The buyer may, therefore, strategically, set a higher monitoring to discourage bribery by the firm that is not controlled. Since higher monitoring results in higher profits for both firms, a unilateral control can lead to higher profits for the controlled firm. There is evidence of higher monitoring in the Middle East in the post-FCPA era presented in Gillespie (1987). She also concludes that the potential of the FCPA to hurt U.S. exports remains unproven.

Corruption has been studied extensively in many different contexts in the literature (See Jain (2001) for a review). Shleifer and Vishny (1993) study the implications of the structure of the corruption network on the level of corruption in government agencies. Mookherjee and Png (1995) study the optimal compensation policy for a corruptible inspector, charged with monitoring pollution from a factory. Hauser, Simester, and Wernerfelt (1997) look at bribery, or side payments, in the context of ratings given by salesforce to internal sales support.

There is relatively smaller literature on competition in the presence of corruption. Rose-Ackerman (1975) initiate this work by presenting a model in which corruption results in allocative inefficiency. The inefficiency in her model arises due to differences in the bribing capacity of competing firms or due to vague preferences of the government. Burguet and Che (2004) allow the agent to manipulate her quality evaluations in exchange for bribes. They find that if the agent has little manipulation power, corruption does not disrupt allocation efficiency but makes the efficient firm compete more aggressively. However, if the agent has substantial manipulation power, corruption facilitates collusion among competing firms and creates allocative inefficiency as bribery makes it costly for the efficient firm to secure a sure win. Compte, Lambert-Mogiliansky, and Verdier (2005) incorporate corruption in procurement auction through the possibility for bid readjustment that an agent may provide in exchange for a bribe. They show that corruption facilitates collusion in



price between firms and results in a price increase that goes beyond the bribe received by the bureaucrat. They also show that a unilateral anti-corruption controls on an efficient firm may restore price competition to some extent. Branco and Villas-Boas (2012) investigate the effect of the degree of competition on corruption in the context of a firm's investment in behaving according to the rules of the market.

This paper also relates to literature on strategic information transmission in the presence of a third party. Scharfstein and Stein (1990) examine behavior of an advisor who cares about his reputation for accuracy and show that advisor's incentive to say the expected thing can result in herd behavior. Durbin and Iyer (2009) study information transmission to a decision maker from an advisor that values his reputation for incorruptibility in the presence of a third party who offers unobservable bribes to influence the advice. They show that the advisor may send an inaccurate message in order to bolster his reputation for incorruptibility. Inderst and Ottaviani (2012) present a model of competition in which product providers compete to influence intermediaries' advice to consumers through hidden kickbacks or disclosed commissions. They study equilibrium commissions and welfare implications of commonly adopted policies such as mandatory disclosure and caps on commissions.

This work contributes to the literature on competition in the presence of corruption by presenting a model which captures the roles of corruption. Unlike Rose-Ackerman (1975), we assume firms to be symmetric and the government preference to be well defined. The scope of corruption arises as the buyer does not have the expertise or the information needed to make the purchase decision and delegates the decision to an agent. Compte, Lambert-Mogiliansky, and Verdier (2005) exogenously impose corruption. The buyer does not need an agent in their setup. Corruption arises endogenously in our setup, more like Burguet and Che (2004). The existing papers do not explicitly model both the agent and the buyer. Our agent is strategic. She understands the implications of dishonest behavior and does not always accept the higher bribe. The buyer is also strategic. She understands the incentives of the agent and tries to discipline her. By accommodating these features, which have been largely ignored in the existing literature, we are able to gain interesting new insights on competition in the presence of corruption. We show that an agent can become more dishonest as a result of increased monitoring using a rational agent model. We also provide a formal explanation for the disconnect between the common perception of the impact of the FCPA on the U.S. firms and the findings of the empirical studies. The findings of our work have important implications for firms doing business in international markets as well as governments. This paper also contributes to the growing literature in marketing on challenges that are relatively more pervasive in the emerging markets. Jain (2008) and Vernik, Purohit, and Desai (2011) investigate

digital piracy. Qian, Gong, and Chen (2012) examine the issue of counterfeits. We focus on another key challenge of bribery in this paper.

The rest of the paper is organized as follows. The next section presents the model where we discuss firms' decisions, agent's decision and buyer's decision in order. In Section 1.3 the analysis of the unilateral control setup and its comparison to the model discussed in Section 1.2 is presented. Section 1.4 summarizes our results.

## 1.2 Model

Consider a buyer that needs to buy a single, indivisible good. The suppliers of the product can be one of the two possible types, good  $t$  or bad  $t$ . Utility of the buyer who buys the product at price  $p$  is given by  $V(v_t; p) = v_t - p$ , where  $v_t = v$  if the product is bought from a supplier with good product  $t$  and  $v_t = 0$  if it is bought from a supplier with bad product  $t$ . The buyer does not have the expertise or the information needed to evaluate the product  $t$ . Therefore, firms are identical to the buyer. The reservation price of the buyer is  $v$ .

Two firms,  $i = 1, 2$ , compete to supply the product to the buyer. One of the two firm's product is a good  $t$ , the other's product is a bad  $t$ . The probability that firm  $i$ 's product is a good  $t$  is 0.5. Firms do not know if their product is a good  $t$  or not. This may happen because firms may not be aware of the intended use of the good, previous training received by buyer's staff, and the environment in which the good will be used or due to the firm's own lack of prior experience. Both firms have same cost of production which is assumed to be zero.

An agent, such as a bureaucrat, selects one of the two firms on behalf of the buyer. The agent, costlessly and privately, learns the  $t$  of the firm. This learning, while informative, is not perfect. The probability that a firm is actually a  $t$ , given the agent's signal is  $t$ , is  $> 0.5$ . The agent is expected to always select the firm for which she receives a  $t$  signal. The agent, in exchange for a bribe  $b_i$  ( $b_i > 0$ ) from firm  $i$  (firm  $j \in i$ ), can change her report and select a firm that she believes is mis  $t$ . Both firms simultaneously and privately submit price bids  $p_i$  and  $p_j$ , and bribe offers  $b_i$  and  $b_j$  to the agent. Price bids are observed by the buyer. From here onwards, we refer to the firm, for which the agent receives a  $t$  signal, as the deserving firm and the other firm as the non-deserving firm. The agent receives the bribe conditional on the firm receiving the order to supply the product. The buyer understands that the agent may accept the bribe and select a non-deserving firm. In order to discourage the agent from this behavior, the buyer monitors the agent with

probability  $\alpha \in [0; 1]$  after the good is purchased.<sup>8</sup> The cost of the monitoring  $c(\alpha)$  is assumed to be continuous and strictly convex with  $c(0) = 0$ . The buyer, as a result of monitoring, learns the signal that agent received and infers if the agent made a dishonest decision in exchange for a bribe. The buyer does not observe the bribe transfer. This is because in most cases no official record of the bribe transfer exists. In cases where a record does exist the buyer may not have access to those records.<sup>10</sup> A penalty  $P$  is imposed on the dishonest agent. We assume that  $\alpha > \frac{P}{2}$ .<sup>11</sup> All parties are risk neutral. The expected penalty imposed on the agent, if she makes a decision inconsistent with her signal, is simply  $P$ . The minimum bribe needed in order for the agent to select a non-deserving firm, therefore, is  $\frac{P}{2}$ .

Now we look at the agent's incentives in the selection process. The agent compares the two bribe offers  $b_i$  and  $b_j$ . If  $|b_i - b_j| > \frac{P}{2}$ , she selects and accepts the bribe from the firm which made the high bribe offer regardless of whether the firm is deserving or not. The agent, in this case, pays an expected penalty of  $\frac{P}{2}$ . If, however,  $|b_i - b_j| \leq \frac{P}{2}$  she selects the deserving firm and receives the bribe offered by that firm, even if it is lower than the bribe offered by the other firm. The agent, acting honestly for buyer, does not pay any penalty in this case. Here, the agent does not accept the non-deserving firm's bribe offer because the cost of doing so in the form of expected penalty ( $\frac{P}{2}$ ) is weakly higher than the benefit (a bribe higher by  $\frac{P}{2}$ ). We assume that the agent makes an honest decision if she is indifferent to either selecting the deserving or the non-deserving firm. The payoff function of the agent can, therefore, be written as

$$u_a(b_i; b_j) = \begin{cases} \max(b_i; b_j) - \frac{P}{2} & \text{if } |b_i - b_j| > \frac{P}{2} \\ b_{des} & \text{if } |b_i - b_j| \leq \frac{P}{2} \end{cases}$$

where,  $b_{des}$  is the bribe offered by the deserving firm.

<sup>8</sup>In our model monitoring captures many different things. It captures the efficiency of the legal system, extent of whistle-blower protection, and extent of control on media apart from the probability with which some department or agency verifies the selection decision of the agent.

<sup>9</sup>Bribes are typically paid in cash or as non-monetary benefits. Also, they are often transferred to foreign bank accounts of the agent or are received by relatives of the agent.

<sup>10</sup>Many foreign banks typically do not disclose information about their clients' accounts to the governments. According to different estimates Indians have \$500 billion to \$1.5 trillion of illegal money stashed in foreign banks. According to a Global Financial Integrity report China tops the list of highest illicit financial flows (2002-2006) from developing countries accompanied by Russia, India and Indonesia in the top 10 among others.

<sup>11</sup>This assumption ensures that the buyer can make the agent honest with probability one by making  $\alpha$  sufficiently large.

A particular firm gets selected by the agent with probability one (zero) if its bribe offer is higher (lower) than the bribe offer of the other firm by more than  $P$ . If the difference in bribes offered is weakly less than  $P$  the agent selects the firm only with probability 0.5, when it is a deserving firm. The expected profit of the firm  $i$  as a function of the bribes offered by firm  $i$  and firm  $j \in i$  can be written as

$$\pi_i(b_i; b_j) = \begin{cases} p_i - b_j & \text{if } b_i > b_j + P \\ \frac{1}{2}(p_i - b_j) & \text{if } |b_i - b_j| \leq P \\ 0 & \text{if } b_i < b_j - P \end{cases}$$

Figure 1.1 summarizes the timing of the actions. In the first stage, nature makes a draw of the  $t$ , from a distribution that is common knowledge, and assigns it to firms. The buyer then sets the probability with which the agent will be monitored after making the firm selection. Firms then submit simultaneous price and bribe bids to the agent. Next, the agent compares the bids and selects one of the two firms. The selected firm receives the accepted price and delivers the good to the buyer. In the next stage, the buyer randomly monitors the agent and imposes a penalty if a dishonest behavior is inferred. Finally, payoffs are realized. We look for Nash equilibrium in pure as well as mixed strategies. The computation of the mixed strategy equilibrium is similar to that of Varian (1980) and Narasimhan (1988).

The framework described above has two important features that are missing in the existing literature on competition in presence of corruption. First, our agent is strategic. She does not always accept the higher bribe. Also, she does not always change her report when she accepts a bribe. There are implications for a dishonest behavior and the agent takes them into account. The buyer is also strategic. She understands the incentives of the agent to cheat. There are, therefore, consequences for dishonest agent behavior. The buyer, in equilibrium, sets a monitoring which maximizes her payoffs.

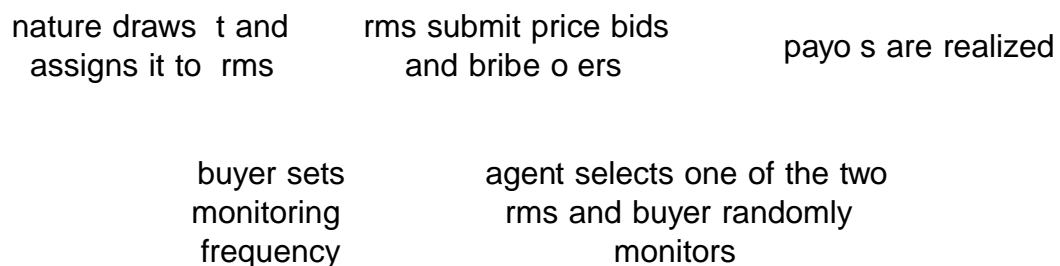


Figure 1.1: Timing

### 1.2.1 Price and Bribe Decisions

In this section the stages of the model that are relevant for the price and the bribe offer decisions of firms are described. A firm is selected by the agent either because it is deserving or because it offers a sufficiently large bribe. The agent can classify a non-deserving firm as a deserving firm<sup>12</sup>. Given this power of the agent, a firm can always benefit by increasing the price bid so long as the price is not rejected by the buyer. Both firms, therefore, submit price  $p$  as their price bid and compete in bribes to be selected by the agent. This leads us to the following result:

**Lemma 1.1** Both firms submit buyer's reservation price  $p$  as their price bid.

This high price bid is a typical result in the literature and has been interpreted as corruption facilitating collusion (see Compte, Lambert-Mogiliansky, and Verdier (2005)).

Firm  $i$ , when confronted with a bribe offer  $b_j$  of the firm  $j$ , responds by making a bribe offer that can have three different implications. First, it can offer a bribe which is higher than  $b_j$  by more than  $P$  and be selected with probability one. Second, it can offer a bribe which is different from  $b_j$  by, at-most,  $P$  and be selected only if it is deserving. Lastly, it can offer a bribe which is lower than  $b_j$  by more than  $P$  and be selected with probability zero. However, we note that:

**Lemma 1.2** Firm  $i$  responds to a bribe offer  $b_j$  of firm  $j$  by offering a bribe  $b_i \geq b_j + P; \max(0; b_j - P)$ .

The intuition for this result is as follows. Any offer  $b_i > b_j + P$  is strictly dominated by an offer  $b_i = b_j + P$  for small enough  $P$ . Firm  $i$  still gets selected with probability one but offers a smaller bribe. Any bribe offer by firm  $i$  such that  $b_i + P < b_j - P$  is strictly dominated by the offer  $b_i = b_j - P$  as firm  $i$  still gets selected whenever it deserves but pays a smaller bribe. We do not consider negative bribes, as they are never accepted. Firm  $i$  therefore, responds to bribe offer  $b_j$  by offering one of the two bribes as specified in Lemma 1.2.

Now we look at the equilibrium in bribes (proofs are in the Appendix A).

**Proposition 1.1** If  $\frac{p}{2P}$  in equilibrium both firms offer no bribes and the

<sup>12</sup>This is consistent with general observation about the discretionary power of agents in corrupt countries. (See anti-corruption profiles for various countries at [www.trust.org/trustlaw](http://www.trust.org/trustlaw) for detailed information.)

agent selects the firm that is deserving. If  $\alpha < \frac{p}{2P}$  there is no Nash equilibrium in pure strategies.

The intuition behind this proposition is the following. Higher monitoring leads to higher expected penalty for the agent. A higher bribe, therefore, must be offered if a firm expects to be chosen even when it is non-deserving. A deviating firm's profits decrease with an increase in the monitoring. For sufficiently large monitoring ( $\alpha \geq \frac{p}{2P}$ ), gains from deviations are completely erased. In this region, a pure strategy Nash equilibrium exists and firms offer no bribes in equilibrium. Since firms offer no bribes and are selected when they are deserving the equilibrium profit for both firms is  $\frac{p}{2}$ . This profit does not depend on the monitoring chosen by the buyer so long as it is larger than  $\frac{p}{2P}$ .

If the monitoring is lower ( $\alpha < \frac{p}{2P}$ ), the equilibrium bribe offers are in mixed strategies. Firms respond to the other firm's bribe offer either by offering a higher bribe just enough to secure a sure win or by offering a lower bribe just enough to have the firm selected whenever it is deserving. The best response for a firm changes from a higher bribe offer to a lower bribe offer when the bribe offer of the other firm becomes high enough. This switching happens because profits on overbidding reduces faster than profits on underbidding with the increase in the bribe offer of the other firm. The best response bribe offers start increasing again and the switching happens for the other firm. The cycle continues.

We now characterize the equilibrium mixed strategies for  $\alpha < \frac{p}{2P}$ . If  $b_i > b_j + P$ , firm  $i$  is selected with probability one. However, if  $b_i \leq b_j + P$  then firm  $i$  is selected only when it is deserving. If  $b_i < b_j - P$  the agent does not select firm  $i$ . The profit of firm  $i$  is given by

$$\pi_i(b_i) = \text{prob}(b_i > b_j + P)(p - b_i) + \text{prob}(b_j - P \leq b_i \leq b_j + P) \frac{p - b_i}{2}$$

which can be written as

$$\pi_i(b_i) = [F_j(b_i - P) + F_j(b_i + P) - f_j(b_i - P)] \frac{p - b_i}{2} \quad (1.1)$$

where  $F_j(b_j)$  is the cumulative distribution function for firm  $j$ , and  $f_j(b_j)$  is the density at bribe  $b_j$ .

Since the equilibrium bribing strategies depend on the range of monitoring, we specify the mixed strategy equilibrium in two different parameter spaces.

Suppose that  $\alpha \geq \frac{p}{4P}$ . In this range, both firms prefer to offer bribes. Let firm  $i$ 's bribe offer be  $b_i$  such that  $p - b_i \leq P$ . Consistent with Lemma 1.2, firm  $j$

responds by making a bribe offer of either  $b_j + P$  or  $b_j - P$ . A bribe offer of  $b_j + P$  yields an expected profit of  $p - (b_j + P)$ , whereas a bribe offer of  $b_j - P$  yields an expected profit of  $\frac{1}{2}(p - (b_j - P))$  for firm  $j$ . A comparison of the profits in two options reveals that firm  $j$ , in response to  $b_j$ , is better off offering a bribe of  $b_j + P$  if  $b_j < p - 3P$  whereas it is better off offering  $b_j - P$  if  $b_j > p - 3P$ . Firm  $j$  is indifferent about overbidding or underbidding if firm  $i$  offers a bribe of exactly  $p - 3P$ . The same holds for firm  $i$ . Since both firms prefer to underbid in response to any bribe offer higher than  $p - 3P$ , a bribe higher than  $p - 2P$  will never be offered. Also, since both firms prefer to overbid in response to any bribe offer lower than  $p - 3P$ , a bribe lower than  $p - 4P$  will never be offered. The support of bribe offer distribution is, therefore,  $[p - 4P; p - 2P]$ . The bribing equilibrium, which is in mixed strategies, is described in Proposition 1.2.

Proposition 1.2 If  $\frac{p}{4P} < 1$ ,  
 (a) equilibrium bribing strategy for firm  $j$  is given by

$$F_j(b_j) = \begin{cases} \frac{3P}{p - b_j - P} & \text{if } p - 4P < b_j < p - 3P \\ \frac{3P}{p - b_j + P} & \text{if } p - 2P < b_j < p - 3P \end{cases} \quad (1.2)$$

and,

(b) both firms make profits of  $3P = 2P$ .

The equilibrium bribe distribution, for both firms, is continuous and has a mass point at the indifference point. In equilibrium, both firms offer positive bribes with probability one. The equilibrium is unique by construction. And it is straightforward to show, by contradiction, that the bribing strategies specified in Proposition 1.2 constitute Nash equilibrium.

It is of interest to look at how the equilibrium bribing strategies and profits respond to a small change in monitoring. An increase in monitoring requires that firms overbid their rivals by a larger amount if they wish to be selected with certainty. Given that both firms still offer bribes with probability one, it might appear counter-intuitive to see that profits are increasing in monitoring. The intuition for this result is the following. Since firms must overbid by a larger amount to get selected with probability one they become less willing to do so. Firms become indifferent to overbidding or underbidding the rival firm at lower bribes. As a consequence, lower bribes are offered in equilibrium which results in higher profits for both firms. We can express each point in the support of the distribution in equation (1.2) in the form  $p - aP$ , where  $2 < a < 4$ . This implies that the probability at each point in

the support of bribe distribution is independent of  $\beta$ .

We now look at the intermediate range of monitoring  $\frac{p}{4P} < \beta < \frac{p}{2P}$ . Given  $\frac{p}{2P}$ , both firms prefer to offer bribes if the other firm is not offering a bribe. Firms prefer to overbid by  $P$  on any rival firm's bid which is smaller than  $\frac{p}{2} - P$ . Since  $\frac{p}{4P}$ , the alternative strategy, as per Lemma 1.2, is to offer no bribes that yields lower profits. Here, firms also prefer to underbid on any rival firm's bid which is larger than  $P$ . Since firms prefer to overbid only in response to bribe offers smaller than  $\frac{p}{2} - P$ , a bribe higher than  $\frac{p}{2}$  is not offered. If a firm makes a bribe offer  $b \in [\frac{p}{2} - P; P]$  it must be in response to a bribe offer higher than  $\frac{p}{2}$ . However, since there are no bribe offers larger than  $\frac{p}{2}$  there are no bribe offers made in the interval  $[\frac{p}{2} - P; P]$ . The support of the bribe offer distribution therefore is  $0; \frac{p}{2} - P \cup [P; \frac{p}{2}]$ . The equilibrium bribing strategies and profits, in this range of monitoring, are given in Proposition 1.3.

**Proposition 1.3** If  $\frac{p}{4P} < \beta < \frac{p}{2P}$ ,  
 (a) equilibrium bribing strategy for firm  $j$  is given by

$$F_j(b) = \begin{cases} 0 & \text{if } 0 \leq b < \frac{p}{2} - P \\ \frac{p+2P}{2(p-b-P)} & \text{if } \frac{p}{2} - P \leq b < P \\ \frac{p+2P}{2(p-b+P)} & \text{if } P \leq b < \frac{p}{2} \\ 1 & \text{if } b \geq \frac{p}{2} \end{cases}$$

and,

(b) both firms make profits of  $\frac{p+2P}{4}$ .

The distribution is continuous in its support. There are two mass points for each firm, one at  $b = 0$  and the other at  $b = \frac{p}{2} - P$ . This equilibrium is also unique; it can be easily shown that the bribing strategies specified in Proposition 1.3 constitute Nash equilibrium. Firms do not offer bribes with probability one in this range of monitoring. As monitoring is increased, firms offer bribes with smaller probability. This happens in response to the higher amount by which firms must overbid their bribe in order to be selected with certainty. The firm profits are increasing in monitoring but at a smaller rate compared to the rate of increase in the  $\frac{p}{4P}$  case. The lower bound on bribes, at zero, causes profits to increase at a slower rate. We also note that there is no discontinuity in the bribe offer distribution or the firm profits at the boundaries of this parameter space.



### 1.2.2 Agent's Selection Decision

Having described the price and bribe offer decisions in the entire range of monitoring, we now look at agent behavior. We are interested in an agent's decision to select a non-deserving firm as it is this decision that hurts the buyer. The agent does not always select a non-deserving firm when she accepts a bribe. If the difference of bribes offered by the two firms is smaller than the expected loss that the agent incurs upon selecting a non-deserving firm, the agent simply selects and accepts the bribe from the firm that is deserving. Since firms do not know if they are deserving or not, they cannot condition their bribe payments on being non-deserving. The agent selects a firm with certainty only when the bribe offers are different by more than  $P$ . However, selecting a firm with certainty does not imply that the agent is selecting a non-deserving firm. Note that the firms are deserving with probability 0.5. We can write the probability  $P_r$  with which the agent selects a non-deserving firm as

$$P_r = \frac{1}{2} \text{prob}(b_j - b_i > P) \quad (1.3)$$

The probability  $P_r$  is computed using the equilibrium bribe distributions specified above. We obtain the following results.

**Proposition 1.4** The probability with which the agent selects a non-deserving firm

- (a) is strictly positive and independent of monitoring, if  $\mu$  is sufficiently small ( $\mu < \frac{P}{4P}$ ),
- (b) first increases and then decreases to zero at  $\mu = \frac{P}{2P}$ , when  $\mu$  is increased beyond  $\frac{P}{4P}$ , and
- (c) is zero if  $\mu > \frac{P}{2P}$ .

These results are also presented graphically in Figure 1.2. We note two observations that were discussed earlier. First, an increase in monitoring increases the cost of selecting a non-deserving firm to the agent. And firms respond to higher monitoring by offering smaller or no bribes. Yet an increase in monitoring has no effect on an agent's decision to select a non-deserving firm when monitoring is sufficiently small. Even more puzzling is the increase in  $P_r$  with monitoring in the intermediate range.

The intuition for the above results is the following. If monitoring is sufficiently small ( $\mu < \frac{P}{4P}$ ) both firms offer strictly positive bribes with probability one. When both firms offer bribes the agent often selects the deserving firm and accepts the

Figure 1.2: Probability with which the agent selects non-deserving firm as a function of monitoring

bribe offered by it. Since both firms offer bribes for sure in this range of monitoring the probability with which the agent selects a non-deserving firm does not change. If monitoring probability is increased beyond  $\frac{p}{4P}$ , firms become less likely to offer bribes. The agent is now faced with situations in which she receives a bribe offer only from a non-deserving firm. This makes the selection of a non-deserving firm more likely. As monitoring is further increased, firms become very unlikely to offer bribes. Therefore, the probability with which the agent selects a non-deserving firm also decreases. For a sufficiently large monitoring ( $\frac{p}{2P}$ ), firms do not offer bribes and, therefore, the agent does not select a non-deserving firm.

Insensitivity to or increase in dishonest behavior as a result of increased monitoring, or penalty, has been widely reported in various contexts. Mazar, Amir, and Ariely (2008) find the dishonesty of test takers insensitive to monitoring. Several studies originating from Deci (1972) show in experiments that an increase in the monitoring can result in more dishonest behavior. Most related to our work is a study reported by Schulze and Frank (2003) where they show that increase in monitoring can make an agent, making a procurement decision on behalf of a principal,

more dishonest as a result of monitoring. Akerlof and Dickens (1982) and Bénabou and Tirole (2006) draw on the behavioral literature and present models to derive these results. We present a rational agent model without any behavioral assumptions and show that dishonesty can be insensitive to or can even be increasing in the monitoring. This result has important implications for buyers as well as firms in markets where corruption is prevalent.

### 1.2.3 Monitoring Decision

If a firm is not the agent finds it deserving only with probability  $\alpha$ . Therefore, if the agent makes an honest decision to select the deserving firm she selects a firm only with probability  $\alpha$ . The payoff of the buyer is  $v$  with probability  $\alpha$  and zero with probability  $1 - \alpha$ . Similarly, if the agent selects a non-deserving firm the buyer gets a payoff of  $v$  with probability  $1 - \alpha$  and a payoff of zero with probability  $\alpha$ . The probability  $P$  with which the agent selects a non-deserving firm is discussed in the previous section. We can write the expected payoff of the buyer as

$$G = \Pr[(1 - \alpha)v] + (1 - P)\alpha v - p - c(\alpha) \quad (1.4)$$

We first look at  $G|_{\alpha=0}$ . The expressions of  $P$  as given in the proof of Proposition 4 are substituted in equation (1.4) to get

$$G|_{\alpha=0} = \begin{cases} \frac{8}{3} \ln \frac{9}{8} v - p & \text{if } \frac{p}{4P} \\ (4 - 1) \frac{v}{2} + \frac{p[(2 - 1)v - 4P]}{4P} - \frac{2(2 - 1)Pv}{p} & \text{if } \frac{p}{4P} < \frac{p}{2P} \\ \frac{(p+2P)^2(2 - 1)v}{4 \cdot 2P^2} \ln \frac{(p - P)(p+2P)}{p^2} & \text{if } \frac{p}{4P} > \frac{p}{2P} \end{cases} \quad (1.5)$$

Note that  $\alpha$  enters  $G|_{\alpha=0}$  only through the probability  $P$  with which the agent selects a non-deserving firm. It is now straightforward to understand how  $G|_{\alpha=0}$  changes with monitoring  $p$ . For  $\frac{p}{4P}$ , the probability  $P$  does not depend on  $p$ , therefore  $G|_{\alpha=0}$  also does not depend on  $p$ . For  $\frac{p}{4P} < \frac{p}{2P}$ , the probability  $P$  first increases and then decreases to zero, therefore  $G|_{\alpha=0}$  first decreases and then increases to maximum value at  $p = \frac{p}{2P}$ . For  $\frac{p}{4P} > \frac{p}{2P}$ , it stays at its maximum value which is  $v - p$ . These results are graphically presented in Figure 1.3. It is now simple to look at the buyer's choice of  $\alpha$  under cost of monitoring  $c(\alpha)$ . Buyer's payoff at zero monitoring  $G(\alpha=0)$  is  $v - p - (2 - 1)9 \ln \frac{9}{8} - 1 v$ . Since  $c(\alpha) > 0$  for every  $\alpha > 0$ , any monitoring  $p > 0$  for which  $G|_{\alpha=0} < G(\alpha=0)$  is not optimal.

Also since  $c'(\alpha) > 0$ , optimal  $\alpha$  cannot be larger than  $\frac{p}{2P}$ . There is no extra benefit of increasing  $\alpha$  beyond  $\frac{p}{2P}$  to the buyer as the agent behaves as desired at all  $\alpha \geq \frac{p}{2P}$ . Therefore, buyer chooses optimal  $\alpha$  from the set  $0; \alpha^* \geq \frac{p}{2P}$ , where  $\alpha^* \geq \frac{p}{4P}; \frac{p}{2P}$  is the monitoring at which  $G(\alpha^*) = G(\alpha = 0)$ . We see buyer as making one of the two choices. She either accepts corruption with zero anti-corruption enforcement or limits corruption, partially or fully, by setting  $\alpha \geq \alpha^* \geq \frac{p}{2P}$ . The buyer's choice to limit the corruption or not depends on  $\alpha^*$  which is defined as

$$\alpha^* = \alpha \text{ such that } (G(\alpha) - G(\alpha = 0)) = 0 \quad (1.6)$$

Further examination of the  $G(\alpha) = G(\alpha = 0)$  function leads us to the following proposition:

Figure 1.3: Buyer's payoff with monitoring (for costless monitoring)

Proposition 1.5 If  $c(\alpha) > 4 \delta \alpha \geq \frac{p}{2P}$ , the buyer allows corruption with zero anti-corruption enforcement ( $\alpha = 0$ ), else, she limits corruption by selecting a monitoring  $\alpha \geq \frac{p}{2P}$ .

The buyer sets the monitoring at either zero or at a sufficiently large value. A small monitoring does not make the buyer any better as the agent selects the non-deserving firm with the same or higher probability. A buyer selecting a positive monitoring expects the agent to select the non-deserving firm with a lower probability than she does when no monitoring is enforced. This happens only when  $\alpha > \frac{1}{2}$ . The buyer setting a non-zero monitoring incurs a cost. If the cost is higher than the benefit that the buyer enjoys due to more honest agent behavior, the buyer sets monitoring at zero. If not, the buyer sets monitoring sufficiently large and limits corruption. In the case when the buyer allows corruption with zero anti-corruption enforcement the firms make zero profits. All the surplus is transferred to the agent in the form of bribes. However, if the buyer limits corruption firms make expected profits of  $\frac{p+2}{4} \frac{P}{P}$ . If the buyer eliminates corruption by setting monitoring at  $\frac{p}{2P}$  the firms make expected profits of  $\frac{p}{2}$ , which is the highest profit that the firms can make in this symmetric set-up.

We next look at the role of  $c = \frac{p}{2P}$ , which is relevant to the analysis presented in the next section. Corruption is eradicated at the monitoring of  $\alpha = \frac{p}{2P}$ . The buyer eradicates corruption only if the cost at  $\alpha = \frac{p}{2P}$  is smaller than the increase in profit the buyer enjoys as a result of honest agent behavior compared to no monitoring. This gives

$$c = \frac{p}{2P} < (2 - \alpha) \ln \frac{9}{8} + 1 - v \quad (1.7)$$

The above equation, while necessary, is not sufficient for corruption eradication. There may be a  $\alpha_h \approx \frac{p}{2P}$  that dominates eradication. While the condition  $c = \frac{p}{2P} > (2 - \alpha) \ln \frac{9}{8} + 1 - v$  implies that corruption is not eradicated, equation (1.7) implies that the buyer limits the corruption, partially or completely.

Buyers either choose to be ignorant or commit to take drastic measures to limit corruption. It is never optimal for the buyer to set monitoring in the interval  $0; \frac{1}{2}$ . A small anti-corruption effort does not reduce corruption.<sup>13</sup> Singapore and Hong Kong were once corruption infested. However, as they implemented drastic measures to combat corruption, they have almost completely eradicated it. The efforts to limit corruption in many other emerging economies appear half-hearted. The impact on prevalence of corruption is therefore little, if any.

<sup>13</sup>Uslaner (2008) talks about some studies indicating evidence of stickiness of corruption and its effects.

### 1.3 Unilateral Control

In this section, we consider that one of the firms, say firm  $i$ , as required by the law in its home country, does not offer bribes to the agent. The structure and timing of the game is exactly as in the previous section. As before both firms make a price bid of  $p$ . The firm  $j$  either does not offer a bribe or it offers a bribe of  $P$ . We assume that if firm  $j$  is indifferent between offering and not offering a bribe it does not offer a bribe. If firm  $j$  does not offer any bribe the agent selects the firm that is deserving. Both firms make an expected profit of  $\frac{p}{2}$  in this case. However, if firm  $j$  offers a bribe of  $P$  the agent selects firm  $j$  with probability one. Any lower bribe does not make the agent select a non-deserving firm. Any higher bribe is strictly dominated by  $P$ . If firm  $j$  offers a bribe it makes a profit of  $p - P$  and firm  $i$  makes zero profit.

If  $\frac{p}{2P} > \frac{c}{v}$ , in equilibrium both firms offer no bribes. A possible deviation for the firm  $j$  is to offer a bribe of  $P$  and make a profit of  $p - P$ . However, given  $\frac{p}{2P} > \frac{c}{v}$  the deviation is not more profitable than the equilibrium strategy. Similarly, if  $\frac{p}{2P} < \frac{c}{v}$ , firm  $j$  offers a bribe of  $P$  in equilibrium. The buyer gets her valuation  $v$  with probability  $\frac{1}{2}$  if the agent selects the deserving firm, whereas she gets only with probability  $\frac{1}{2}$  if the agent selects firm  $j$  with certainty. The payoff of the buyer is

$$u_G = \begin{cases} v - p - c(\cdot) & \text{if } \frac{p}{2P} > \frac{c}{v} \\ \frac{1}{2}v - p - c(\cdot) & \text{if } \frac{p}{2P} < \frac{c}{v} \end{cases}$$

These payoffs, assuming  $c(\cdot) = 0$ , are shown in Figure 1.4. We can now look at the buyer's decision to set monitoring.

The buyer either eliminates corruption by setting monitoring at  $\frac{p}{2P}$  or sets monitoring at zero. Since  $c'(\cdot) > 0$ , the buyer does not set any other monitoring. The buyer's decision to set monitoring depends only on the cost of monitoring at  $\frac{p}{2P}$ . If  $c = \frac{p}{2P} < \frac{1}{2}v$ , the buyer sets monitoring at  $\frac{p}{2P}$  and eliminates corruption. However, if  $c = \frac{p}{2P} > \frac{1}{2}v$ , the buyer sets monitoring at zero. We ignore the equality case in which the buyer can mix between the two monitorings. A comparison of firm  $i$ 's profits with and without unilateral control leads us to the following proposition:

**Proposition 1.6** A unilateral control on bribing on a firm in a corrupt but

competitive market may increase its profits if

$$c > \frac{p}{2P} \left[ 9 \ln \frac{9}{8} - 1 \right] (2 - 1)v;$$

would definitely increase its profits if

$$9 \ln \frac{9}{8} - 1 \left[ (2 - 1)v \right] < c < \frac{p}{2P} < \frac{1}{2} (2 - 1)v;$$

and may decrease its profits if

$$c < \frac{p}{2P} - \frac{1}{2} (2 - 1)v;$$

Figure 1.4: Buyer's payoff with monitoring with and without unilateral anti-corruption control (for costless monitoring)

One striking result in the above proposition is that a unilateral anti-corruption control can actually benefit the firm that is being restricted from offering a bribe. A

unilateral control on one firm eliminates competition in bribes. The firm that is not controlled can offer just  $P$  and be selected with certainty. This makes the selection of a non-deserving firm by the agent more likely. The buyer may, strategically, set a higher monitoring to discourage bribery by the firm that is not controlled. This results in higher profits for a unilaterally controlled firm. A firm under unilateral control can be worse off as well. This happens if, in the absence of unilateral control, the buyer sets a non-zero monitoring, but with unilateral control the buyer sets zero monitoring. A steep increase in the cost before  $\frac{P}{2}$  can make the elimination of corruption unattractive for the buyer. If buyer's decision to set monitoring does not change as unilateral control is introduced, the profits of the controlled firm remain unchanged.

The firm that is not controlled benefits from the unilateral control on the other firm. For the controlled firm the benefit comes as a result of the buyer setting higher monitoring. The firm that is not controlled also benefits when the buyer sets zero monitoring. Without unilateral control all the surplus was transferred to the agent in the process of competitive bribing. But with unilateral control on the other firm, a firm makes higher profits as it offers a bribe of only  $P$ . We think that claims about competitive disadvantage faced by the controlled firm originate from this comparison where the buyer sets monitoring at zero. If the buyer sets monitoring at zero the firm that is not controlled is selected by the agent and makes higher profits than the controlled firm, which makes zero profits. However, what is not taken into consideration is that even if the unilateral control is not there the firm would still make zero profits.

The higher profits for the controlled firm results due to the buyer's choice of higher monitoring. There is some evidence of such increased monitoring. Gillespie (1987) finds evidence of this in the Middle East after 1977. It comes to us as no surprise that most empirical studies find no evidence of competitive disadvantage posed by the FCPA of 1977. We also note that the enforcement of the FCPA has increased drastically in recent years<sup>14</sup> It is interesting to observe recent anti-corruption efforts in BRIC<sup>15</sup> countries. Brazil enacted the Freedom of Information Law of 2011, which is a step forward in the direction of reducing corruption. Russia signed the OECD's Anti-Bribery Convention in 2012. An anti-corruption movement started in India in year 2010 that seeks strong legislation and enforcement against corruption. China implemented a stricter anti-bribery law in 2011. We have no reason to believe that

<sup>14</sup>According to a report published by Shearman & Sterling LLP (2012), the corporate FCPA cases increased from 14 between 2002 and 2006 to 70 between 2007 and 2011.

<sup>15</sup>Brazil, Russia, India and China, the major emerging economies, are collectively referred to as BRIC.



these efforts are only in response to the increased FCPA enforcements. However, we believe that this increase in the FCPA enforcements will make the U.S. firms better off as foreign governments take measures to limit corruption.

## 1.4 Conclusion

This paper studies competition in a corrupt market. The buyer lacks the expertise or the information needed to evaluate firms. An agent selects the firm for the buyer. This creates scope of corruption. Sometimes, the agent selects a non-deserving firm in exchange for bribes. Both the buyer and the agent are strategic. The competitive bidding behavior of the ex-ante symmetric firms is examined. A pure strategy Nash equilibrium in bribes exists only if the monitoring of the agent is sufficiently large. The expected penalty to the agent is so large that firms find it unprofitable to offer such a large bribe. The agent selects the deserving firm.

If monitoring is not sufficiently large the bribe offer equilibrium is in mixed strategies. The agent selects a non-deserving firm if its bribe offer is sufficiently larger than the bribe offer of the deserving firm. Otherwise, the agent accepts the bribe offer of the deserving firm and selects it. We find that an increase in the monitoring does not always result in more honest agent behavior. It sometimes backfires. This agent behavior originates due to the endogenous bribe offers made by firms.

The non-monotonic agent behavior in response to changes in the monitoring, or anti-corruption efforts, makes it difficult for the buyer to reduce corruption. If bribery is prevalent, a small change in the monitoring does not reduce corruption. The buyer must take drastic measures if she wishes to curb corruption.

We find that a unilateral anti-corruption control on a firm, such as the FCPA of 1977, can result in higher profits for the controlled firm. A direct effect of the anti-corruption control is that it makes the foreign government worse off by making the selection of a non-deserving firm by the agent more likely. The foreign government may strategically set a higher monitoring. Profits of the controlled firm may increase as a result. We resolve the disconnect between the prevailing perception about the FCPA in the business community and findings of the empirical studies. Higher monitoring set by the foreign government in response to the FCPA is the key to higher profits of a controlled firm. There is some evidence of increase in anti-corruption enforcements by foreign governments in response to the FCPA.

The findings of this work have important implications for firms conducting business in emerging markets, buyers in these markets and the US government. US firms should note that the debate about the competitive disadvantage posed by the FCPA may be misplaced. Also, while governments in the emerging economies may be

disinterested in reducing corruption, it is in the interest of firms to support the anti-corruption efforts. Buyers should either ignore corruption or take drastic measures to limit it. Implication for the US government is that the unilateral anti-corruption control should be aggressively enforced as it not only reduces corruption but may also increase profits of US firms. The model can be applied to various settings where an agent makes a decision, such as awarding certification, issuing permit, law enforcement or procurement, on behalf of a principal and the principal lacks the expertise or the information to make the same decision.

## Chapter 2

# Voluntary Product Safety Certification

### 2.1 Introduction

Product safety is an attribute of the product that leads to an accident and injury to the consumer upon failure. The product safety has become increasingly important to the consumers in the recent years. In a survey of 1,000 American consumers conducted by Thomson West Research in the year 2007 it was found that 61% of the consumers were worried or very worried about the safety of the products they used; 55% of the consumers reported that they were more worried than what they were a year ago. In the light of the recent product recalls<sup>1</sup> and safety mishaps<sup>2</sup>, consumers are probably more concerned about the product safety than ever. Although concerned about the product safety the consumers are typically not aware of the safety level of the products they buy in the market.

In fact, in most cases even the firms are not aware of the safety levels of their products. This is because the product safety depends upon the integration of the design and the manufacturing of the product and the environment in which it is used in a complex manner. The firms, in most cases, do not have the specialized equipments required to test the safety of their products. Safety certification is one way that the firms can learn about the safety of their products, if they do not already

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<sup>1</sup>Some of the major recalls of 2010: Over 7 million vehicles by Toyota, about 10 million products by Fisher Price, 2.2 million electric heaters by Walmart, 2 million strollers by Grace, 1.7 million dishwashers by Maytag.

<sup>2</sup>BP Gulf of Mexico oil spill (April-July 2010) and Massey Energy's Upper Big Branch mine disaster (April 2010) resulted from respective firm's ignorance about safety.

know it, and communicate it to the consumers. A firm may contact a certification agency<sup>3</sup> and request its product to be safety certified. Certification agencies upon receiving a certification request test the products to see if they meet a given set of standards. If the product does meet the standards it gets certified. The firm then informs the consumers about this certification usually by putting a sticker or a seal on the product which claims the product to be certified by a particular certification agency. In the real world we observe that while some firms go through the tedious and costly certification process and sell safety certified products, others decide to sell their products without such certifications. In many cases, firms market products to consumers even without getting mandatory certifications. We make an attempt to understand under what conditions a firm finds it optimal to go for certification and under what conditions it does not.

We consider a model in which a monopolist firm that does not know the safety level of its product makes a decision of whether or not to get the product certified by a certification agency. The firm then sets a price and sells the product in a homogeneous consumer market. The consumers observe the certification decision and the outcome of the certification as well as the price and make the purchase decision.<sup>4</sup> The product fails with some probability in the hands of the consumer and leads to injury. The probability with which the accident occurs depends on the safety of the product. The injured consumer can go to the court and get partially compensated by the firm for the losses she suffered in the accident.

Two important points to be noted, in this context, are the presence of an injury and the liability that is imposed on the firm by the court. A product failure that involves an injury to the consumer is treated very differently in the court compared to a product failure that does not involve an injury. While the latter is subjected to the law of contract warranties, the former is subjected to the law of torts. A firm can limit or extend its liability for the product failures that do not involve injury to the consumers by writing contracts. Such product failures have been extensively studied in the literature (see Lutz (1989); Padmanabhan and Rao (1993)). A firm, however, as dictated by the law of torts, cannot limit its liability for the losses incurred by the consumer while using a defective product. This liability, referred to as the 'strict liability', is imposed on the firm by the court. It is strict because it is applied even if

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<sup>3</sup>American National Standards Institute (ANSI), ASTM International, Safe Toys USA, Underwriters Laboratories (UL) are examples of such agencies.

<sup>4</sup>Euro NCAP, Better Business Bureau, Insurance Institute for Highway Safety and Consumer Reports are examples of the organizations that publish the information about the products even when the products fail to meet the standards. Rejection of FDA approval for drugs also becomes news.

the firm was not negligent and made every possible effort to make the product safe or the defect was not known to the firm. The consumer only needs to prove in the court that an injury was caused by the use of a defective product manufactured by the firm. In this paper, we only consider product failures that lead to injury to the consumer, and therefore, are subjected to the strict liability.<sup>5</sup> While the firm is held strictly liable for damages, the consumer is often not fully compensated. Even if the court requires the firm to compensate for all the physical losses, pain, suffering and the legal fees<sup>6</sup> remain greatly uncompensated.

We find that in the setup described above the firm finds it optimal not to invest in certification at any positive certification cost. This result arises due to the symmetric information structure and the ability of the firm to extract all the consumer surplus given the homogeneous consumer market. If, however, the probability of an accident depends not only on the product safety but also on the effort or care taken by the consumer in using the product, the firm finds it optimal to go for certification under certain market conditions. The negligence on the part of the consumer can be used as defense by the firm in the court. There is no consensus on how the losses resulting from an accident from a negligent use of a defective product should be shared by the firm and the consumer. We use a commonly adopted approach<sup>7</sup> of reducing the negligent consumer's damages to reflect her fault in causing the accident. In our setup the consumer chooses between using the product with care at some cost or be negligent, which is assumed to be costless.

We find that if a firm can influence the choice of effort made by the consumer by going to certification, the firm may go for certification. The ability of the firm to influence the effort choice of the consumer depends on how the product safety and the consumer effort jointly determine the probability of accident-free use of the product. If they are independent the firm would not be able to influence the effort choice, and therefore, the firm would find it optimal not to go for certification. If the product safety and the consumer effort are substitutes or complements the firm finds it optimal to go for certification unless the effect of consumer effort on the probability of the accident is too little or too much. If a negligent use of the product by the

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<sup>5</sup>We abstract away from the issues of 'negligence' by the firm and 'punitive damages' and focus only on 'strict liability' by assuming that the products meet the minimum safety standards, if any, as stipulated by government. We also assume that firm gets all the mandatory certifications.

<sup>6</sup>Under the American Rule, which is the default rule controlling assessment of attorneys' fees in the USA, each party pays its own attorney's fee.

<sup>7</sup>Most states, with the exception of North Carolina which treats negligence by the consumer as a full defense to a product liability claim and 12 others which use strict liability, use some form of comparative negligence which reduces the negligent consumer's damages to reflect her fault in causing the accident (Glannon, 2010).

consumer drastically increases the probability of the accident she would be careful in using the product regardless of the certification decision made by the firm or the outcome of certification. Also, if a negligent use of the product by the consumer leaves the probability of the accident almost unchanged the consumer would want to be negligent regardless of the certification decision or the outcome. In this setup, where the certification decision of the firm is common knowledge, we find that the presence of the consumer moral hazard can create incentive for certification.

The certification decision of the firm is not always common knowledge. In many cases the certification agencies publish only the information about products that meet the standards.<sup>8</sup> The consumer while buying the product sometimes observes a safety certification sticker on the product and infers that the firm went for certification. If she does not observe the certification sticker she does not know if it is because the firm was denied the certification sticker or it did not try to get certified. In this setup we find that the presence of the consumer moral hazard leads to more certification if the product safety and the consumer effort are substitutes but leads to less certification if they are complements.

The present analysis describes the role of the consumer moral hazard and the information asymmetry about certification decision in creating incentives for the product safety certifications.

Product safety is one of the areas that has received limited attention in literature. Focus of the earlier work on product safety was on the producer liability rules (see Oi (1973), Spence (1977), Polinsky and Rogerson (1983), Png (1987) and Polinsky and Rubinfeld (1988)). The consumer was assumed to know the safety of the product, underestimate the product risk or not know the safety level of the product. Daughety and Reinganum (1995) assumed the safety of the product to be a random outcome of an R&D process which was known only to the firm. They presented a signaling model in which the firm signaled its safety level to the consumers using price. Daughety and Reinganum (2008) compared signaling with disclosure as a means of communicating product quality to the consumers. Disclosure in their model was achieved by making a credible direct claim. They noted that such a disclosure can be made via the use of an independent auditing process with public announcement of what quality was found to exist, or advertising in the presence of truth-in-advertising laws with high penalty for misrepresentation. They argued that a firm would choose disclosure as a means of communication over signaling with the price if the cost of disclosure is lower than the signaling cost. In their model, disclosure is perfect, that is, upon disclosure the consumer learns the quality with certainty. If the disclosure is made

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<sup>8</sup>American National Standards Institute (ANSI), ASTM International, Safe Toys USA and Snell Foundation are examples of such certification agencies.

even slightly noisy a firm would be unwilling to pay any positive amount for such a disclosure. De and Nibar (1991) show that a pool of high-quality sellers opt for third party certification and the remaining low-quality sellers prefer to remain uncertified when selling a product of unobserved quality in a homogeneous consumer market.

While the consumer effort clearly affects the probability of an accident-free use of the product and, therefore, affects the profit of the firm, we did not come across any paper that models consumer effort with the product safety. The works of Cooper and Ross (1985) and Lutz (1989) tell us how the product warranties can be used by the firm to influence the choice of effort. The literature says little about the situation in which the firm as required by the law cannot write such contracts.

This paper contributes to the product safety literature by analyzing the incentive for certification in presence of the consumer moral hazard. It also looks at the role of the consumer heterogeneity and the firm's private information about the product safety on the certification decision.

The rest of the paper is organized as follows. The next section presents the model and the analysis in the setup where certification decision is common knowledge. In section 2.3 the analysis of the private certification setup is presented. Section 2.4 discusses the effect of consumer heterogeneity and firm's private information about its type on certification decision. Section 2.5 summarizes our results and suggests some extensions.

## 2.2 Model

Consider a monopolist firm that produces a product with the safety attribute. The product safety is captured by a parameter  $\theta \in [0, 1]$ ;  $\theta_H > \theta_L > 0$ . The product safety is interpreted as the probability that the product would work without an accident when used by the consumer. We will refer to  $\theta$  as the product type or the firm type. Nature moves first and makes a draw of the firm type from the distribution of  $\theta$  and assigns it to the firm. The probability that the firm is of product safety  $\theta_H$  is  $\alpha$ . The firm is of product safety  $\theta_L$  with complementary probability. The probability distribution from which nature draws the firm type is common knowledge. However, the realized value of  $\theta$  is neither revealed to the firm nor to the consumer. In section 4, we consider a case in which nature reveals the draw to the firm but not to the consumer. We assume that both the high safety and the low safety products meet the minimum safety standards, if any, as stipulated by the government. The firm can sell the product to consumers either with or without safety certification. We limit our attention to the voluntary certification in this paper. We represent the certification decision of the firm by  $C \in \{C, N\}$  where

C represents 'certification' and NC represents 'no certification'. The certification is done by a third party at a fee  $c > 0$  which does not depend on the type of the firm or the outcome of certification. The process of certification is efficient but not perfect. A firm of a given type may correctly be classified as its own type with probability  $\alpha > 0.5$  and incorrectly classified as the other type with probability  $1 - \alpha$ .<sup>9</sup> The outcome of certification which is either a 'sticker' if the firm is found to be of type  $\theta_h$  and 'no sticker' if the firm is found to be of type  $\theta_l$  by the certification agency is represented by  $\sigma \in \{S, NS\}$ . We do not allow the firm to reapply for certification. The firm then sets the price  $p$  for the product. We assume that the marginal cost of production does not depend on the product type and is zero.

There is mass 1 of consumers. Each consumer buys at most one unit of the product. The valuation of the product is  $v$  regardless of its type. The valuation  $v$  is large enough such that the consumer buys the product at a positive price even if she believes with probability one that the product is of type  $\theta_l$ .<sup>10</sup> Consumers observe the certification decision of the firm, the outcome of certification, if the firm went for the certification, and the price before making the purchase decision. We relax the assumption of the certification decision being common knowledge in section 3. The consumer makes her purchase decision  $\sigma \in [0, 1]$  based on her belief  $(c; \alpha; p)$  about the firm type, price and the expected uncompensated loss in the case of an accident. We represent by  $L$  the expected loss to the consumer upon an accident. Both types of the products inflict the same expected loss to the consumer in case of an accident.

Accidents caused by the defective products are subjected to the law of torts. In this analysis the type of accidents that we consider are those caused by the defects in the manufacturing process or in the product design. We do not consider accidents caused by the inappropriate warnings given by the firm. The strict liability is applied by the law of torts in the case of the accidents resulting from the use of the defective products even if the firm was not negligent in making the product. Under strict liability which is enforced by the court a firm cannot limit its liability by means of any contract. We represent by  $L_f$  the transfer that the court requires the firm to make to the consumer in case of an accident. Also, even if the court requires the firm to fully compensate for the physical losses there remain uncompensated losses

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<sup>9</sup>If the certification decision of the firm is common knowledge,  $\alpha < 0.5$  can be interpreted as switching the identity of type  $\theta_h$  and as type  $\theta_l$  certification. If the certification decision is made privately by the firm and  $\alpha < 0.5$ , the firm would be better off without a certification sticker and therefore would always choose no certification. If  $\alpha = 0.5$ , consumers would ignore certification outcome. We, therefore, focus on the more interesting case of  $\alpha > 0.5$ .

<sup>10</sup>If  $v$  is such that the consumer buys a product only if it has a safety sticker, the firm would go for certification for small enough cost. By assuming  $v$  to be sufficiently large, we eliminate this simple reason for the firm to seek certification.



to the consumer. We assume that  $L_f < L$ . The uncompensated loss  $s_c > 0$  to the consumer can be thought of as resulting from the legal fees, pain and suffering. We assume  $L_c = L - L_f$ .

The equilibrium concept used is the sequential equilibrium. The equilibrium consists of the belief of the consumer  $(c; o; p)$ , purchase decision of the consumer  $s(c; p)$ , certification decision of the firm  $c$  and the price set by the firm  $p(c; o)$  such that i) the purchase decision of the consumer maximizes her expected payoff, given her beliefs ii) certification decision and the price maximizes firm's expected payoff, given consumer's purchase decision and iii) beliefs are justifiable as coming from some set of totally mixed strategies that are a small perturbation of the equilibrium strategies.

A summary of the timing of the model follows:

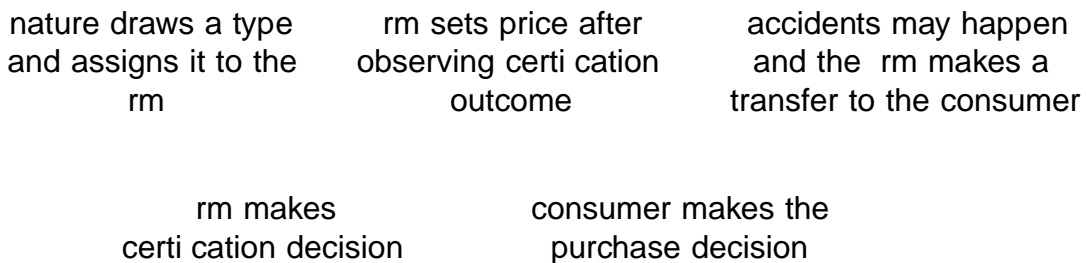


Figure 2.1: Timing

### 2.2.1 The uninformed firm

Now we assume that nature draws the product type and does not reveal it to both the firm and the consumer. Both the firm as well as consumers have same beliefs about the product safety<sup>11</sup>. The firm decides whether or not to go for certification. The decision to go for certification as well as the outcome of certification is observed by the consumer. If the firm goes for certification it gets certified as type  $h$  with probability  $\theta + (1 - \theta)(1 - \alpha)$ . With probability  $(1 - \theta) + (1 - \theta)\alpha$  the

<sup>11</sup>This assumption is consistent with the observation that The Consumer Product Safety Improvement Act (CPSIA) of 2008 requires that all consumer product companies must report potentially unsafe, hazardous, or non-compliant products to the CPSC or face substantial civil penalties. We assume firms to be law abiding.

certification agency classifies the firm to be of type  $\theta_1$ . The beliefs of the consumer about the firm type can be written using Bayes' rule as follows<sup>12</sup>

$$p(c; o; \theta) = \frac{\pi_{\theta_1} \frac{1}{1 + (1 - \theta_1)(1 - \theta_2)}}{\pi_{\theta_1} \frac{1}{1 + (1 - \theta_1)(1 - \theta_2)} + \pi_{\theta_2} \frac{1}{1 + (1 - \theta_1)(1 - \theta_2)}} \quad \text{if } c = C; o = S$$

$$= \frac{\pi_{\theta_1} \frac{1}{1 + (1 - \theta_1)(1 - \theta_2)}}{\pi_{\theta_1} \frac{1}{1 + (1 - \theta_1)(1 - \theta_2)} + \pi_{\theta_2} \frac{1}{1 + (1 - \theta_1)(1 - \theta_2)}} \quad \text{if } c = C; o = NS$$

$$= \pi_{\theta_1} \quad \text{if } c = NC$$

The expected surplus of the consumer when buying the product depends on her belief about the firm type, the price set by the firm and the uncompensated loss that she expects to bear in case of an accident. The firm sets the price to extract all the consumer surplus. The price set by the firm depends on the outcome of certification. It is given by

$$p(c; o) = v - \left[ \pi_{\theta_1} p(c; o; \theta_1) + (1 - \pi_{\theta_1}) p(c; o; \theta_2) \right] (1 - \theta_1) L_c$$

The expected profit of the firm depends on the price it sets and the transfer it is required to make in case of an accident. It is straightforward to show that the expected profit of the firm if it decides to go for certification is

$$\pi_C = v - \left[ \pi_{\theta_1} (1 - \theta_1) + (1 - \pi_{\theta_1}) (1 - \theta_2) \right] (L_c + L_f) - k$$

The expected profit of the firm in the case of no certification can be written as

$$\pi_{NC} = v - \left[ \pi_{\theta_1} (1 - \theta_1) + (1 - \pi_{\theta_1}) (1 - \theta_2) \right] (L_c + L_f)$$

**Lemma 2.1** For any certification cost  $k > 0$ , the firm decides against safety certification.

The profit of the firm if it decides against the certification is exactly same as the full information profits.<sup>13</sup> This is because the information structure of the game is symmetric and the firm extracts all the consumer surplus in both the cases through prices. In fact, even in the case of certification the information structure is symmetric

<sup>12</sup>The consumer ignores price in forming her belief about the firm type as it does not convey any additional information.

<sup>13</sup>In the full information setup, the expected utility of a consumer buying a type  $\theta$  product and paying the price  $p^l$  can be written as  $U^l = v - p^l (1 - \theta) L_c$ . The firm sets the price  $p^l$  to extract all the consumer surplus. Expected profit of a type  $\theta$  firm which sets price  $p^l$  to extract all the consumer surplus becomes  $\pi^l = v - p^l (1 - \theta) (L_c + L_f)$ . The full information ex-ante expected profit of the firm is therefore given by:  $\pi^l = v - \left[ \pi_{\theta_1} (1 - \theta_1) + (1 - \pi_{\theta_1}) (1 - \theta_2) \right] (L_c + L_f)$

and the firm extracts all the consumer surplus but now it must also incur the cost of certification. The expected profit if the firm goes for certification is, therefore, lower than the profit if the firm does not go for certification by the certification cost  $k$ . The firm therefore decides against certification for any  $k > 0$ . For the certification cost of zero the firm would be indifferent between certification and no certification.

## 2.2.2 The consumer moral hazard

In this section, we introduce the consumer moral hazard and study its effects on the incentive for certification. The consumer can either be negligent in using the product which is costless or use the product with care at cost  $c > 0$ . We refer to the negligent use of the product by the effort  $e_n$  and the careful use of the product by the effort  $e_c$ . The probability of an accident depends on the effort choice of the consumer apart from the product safety. If the consumer uses a type product with care accidents happen with probability  $1 - \alpha$ . The consumer effort and the product safety could interact in different ways to determine the probability of the accident. We examine firm's incentive for certification under three different setups. In the first setup we consider that the consumer effort and the product safety are independent in deciding probability of the product working safely. The probability of accident given effort  $e_n$  is given by  $1 - \alpha + \beta$  in this setup. In the second setup, we consider that the consumer effort and the product safety are substitutes in deciding the probability with which the product works without an accident. The probability of accident when the consumer takes the effort  $e_n$  in this setup is assumed to be given by  $(1 - \alpha)(1 + \beta)$ . The last setup is that of the complements and the probability of accident given effort  $e_n$  is assumed to be  $1 - \alpha + \beta$ . The parameter  $\beta > 0$  can be interpreted as the effectiveness of consumer effort. A small  $\beta$  means that the consumer effort does not significantly change the probability of accident.

If the consumer uses the product with care the court requires the firm to make a transfer  $L_f$  to the consumer using the strict liability rule in the law of torts. The assignment of liability when the consumer is also negligent has been a topic of debate in the law of torts (see Glannon (2010)). We use a commonly practised liability assignment rule called 'the law of comparative negligence' in this model. When using the law of comparative negligence the firm's liability for the accident is reduced to reflect the negligence on the part of the consumer in causing the accident.<sup>14</sup>

<sup>14</sup>As an example, in the famous *Liebeck v. McDonald's Restaurants, P.T.S., Inc.* (No. D-202 CV-93-02419, 1995 WL 360309) case the jury in its Aug 18, 1994 verdict awarded Liebeck US\$200,000 in damages but reduced it to US\$160,000 using the law of comparative negligence to reflect the 20% fault of Liebeck in causing the accident.

The liability of the firm for an accident in case where the product safety and the consumer effort are independent, substitutes and complements is given by  $\frac{1}{1+\alpha}L_f$ ,  $\frac{1}{1+\beta}L_f$  and  $\frac{1}{1+\alpha+\beta}L_f$  respectively. The timing of the game is same as before except that now the consumer decides between using the product negligently or with care after the purchase.

Consider first that the product safety and the consumer effort are independent in deciding the probability of accident. Suppose the firm decides not to go for certification and sets a price  $p_{NC}^l$ . The consumer after buying the product decides if she wants to take the effort  $e_h$  or  $e_l$  while using the product. The expected utility of the consumer from taking the effort  $e_h$  can be written as:

$$U_h = v - p_{NC}^l \left[ (1 - \theta_h)(L - L_f) + (1 - \theta_l)(1 - \theta_l)(L - L_f) \right]$$

Similarly the expected utility of the consumer from taking the effort  $e_l$  is given by:

$$U_l = v - p_{NC}^l \left[ (1 - \theta_h + \alpha) L - \frac{L_f (1 - \theta_h)}{1 - \theta_h + \alpha} \right] \\ + (1 - \theta_l)(1 - \theta_l + \beta) L - \frac{L_f (1 - \theta_l)}{1 - \theta_l + \beta}$$

The consumer takes effort  $e_h$  if her expected utility from taking effort  $e_h$  exceeds her expected utility from taking the effort  $e_l$ .

This leads us to the condition  $\alpha > \beta L$ . Note that the condition for the consumer to choose between effort  $e_h$  and  $e_l$  does not depend on  $\theta_h$  or  $\theta_l$ .

Now suppose that the firm decides to go for certification. As above we can solve for the conditions for the consumer to take effort  $e_h$  in the case when firm is classified as  $\theta_h$  and in the case when firm is classified as type  $\theta_l$  by the certification agency. It turns out that the consumer takes effort  $e_h$  when  $\alpha > \beta L$  regardless of the certification decision of the firm or the outcome of certification.<sup>15</sup> The intuition is simple. The certification outcome changes the belief of the consumer about the firm type. If the consumer effort and the product safety are independent the marginal return of the effort does not depend on the product safety. Therefore, the choice of the effort does not depend upon the belief of the consumer about the firm type. As

<sup>15</sup>We ignore the knife-edge case in which the consumer is indifferent between taking effort  $e_h$  and effort  $e_l$ .

a result the certification outcome does not influence the effort choice of the consumer.

**Lemma 2.2** If the product safety and the consumer effort are independent in deciding the probability of no accident, the firm decides against safety certification for any certification cost  $k > 0$ .

The result follows directly from the comparison of the profit of the firm if it goes for certification and the profit if it does not go for certification. As in the case without the consumer moral hazard the firm extracts all the expected consumer surplus using the price regardless of its certification decision. The expected consumer surplus that the firm extracts is same but it incurs a cost of certification if it decides to go for certification. The firm, therefore, does not choose certification if product safety and consumer effort are independent.

If, however, the product safety and the consumer effort are substitutes, the consumer upon observing that the firm did not go for certification puts effort  $e_h$  if  $\theta > \theta^*$  and  $e_l$  if  $\theta < \theta^*$  where  $\theta^*$  is defined as

$$\theta^* = \frac{L}{L[(1 - \theta_h) + (1 - \theta_l)]}$$

If the firm decides to go for certification and gets classified as type  $h$  the consumer chooses  $e_h$  if  $\theta > \theta_S$  and  $e_l$  if  $\theta < \theta_S$  whereas if the firm gets classified as type  $l$  the consumer chooses  $e_h$  if  $\theta > \theta_{NS}$  and  $e_l$  if  $\theta < \theta_{NS}$ . We define  $\theta_S$  and  $\theta_{NS}$  as

$$\theta_S = \frac{L[(1 - \theta_h) + (1 - \theta_l)]}{L[(1 - \theta_h) + (1 - \theta_l)](1 - \theta_l)}$$

$$\theta_{NS} = \frac{L[(1 - \theta_h) + (1 - \theta_l)]}{L[(1 - \theta_l)(1 - \theta_h) + (1 - \theta_l)](1 - \theta_l)}$$

Note that since

$$\frac{L[(1 - \theta_h) + (1 - \theta_l)]}{L[(1 - \theta_h) + (1 - \theta_l)](1 - \theta_l)} > > \frac{(1 - \theta_l)}{(1 - \theta_h) + (1 - \theta_l)}$$

we have

$$\theta_{NS} < \theta^* < \theta_S$$

It is useful at this point to understand how the effort choice of the consumer changes with the effectiveness of her effort. If the choice of effort has very small effect on the probability of accident ( $\theta < \theta_{NS}$ ), the consumer uses the product

negligently regardless of the certification decision or the outcome of certification. If the effectiveness of her effort is increased beyond some threshold  $\bar{\theta}_S$ , the consumer still uses the product negligently if the firm chooses no certification or it gets classified as type  $\theta_h$  but she uses the product with care if the firm gets classified as type  $\theta_l$  by the certification agency. The consumer switches to careful use upon negative certification outcome first because the belief of the consumer about the firm type  $\theta_h$  is lowest when the firm gets classified as type  $\theta_l$ . If the effectiveness of effort is further increased ( $\theta > \bar{\theta}$ ) the consumer starts using the product with care when the firm does not go for certification as well. If the consumer effort is highly effective ( $\theta > \bar{\theta}_S$ ) the consumer uses the product with care regardless of the certification decision or the outcome of certification. The consumer uses the product with care upon positive certification outcome only if the effort is highly effective because the consumer effort and the product safety are substitutes and the consumer's belief about the firm type  $\theta_h$  is highest upon a positive certification outcome.

A comparison of the profits of the firm if it decides to go for certification and if it decides not to go for certification provides us the following results. The incentive for certification is defined as the difference in the profit of the firm upon certification and the profit upon no certification. Proofs of all the propositions are given in the Appendix B.

**Proposition 2.1 (Substitutes case)** If the product safety and the consumer effort are substitutes, the incentive for certification is positive in an intermediate range of the effectiveness of the consumer's effort ( $\bar{\theta}_{NS} < \theta < \bar{\theta}_S$ ). It first increases (when  $\bar{\theta}_{NS} < \theta < \bar{\theta}$ ) and then decreases (when  $\bar{\theta} < \theta < \bar{\theta}_S$ ) with  $\theta$  in this range.

The range in which the incentive for certification is positive, the firm can influence the consumer into taking a different effort by choosing to go for certification. If the effectiveness of effort is sufficiently small ( $\theta < \bar{\theta}_{NS}$ ), the consumer uses the product negligently regardless of the certification decision or the outcome of certification. The firm does not gain from certification in this range as the surplus it extracts from consumers with or without certification is same but it incurs a cost when it goes for certification.

In the range  $\bar{\theta}_{NS} < \theta < \bar{\theta}$ , the firm makes the consumer switch from using the product negligently to using the product with care if it gets classified by the certification agency as a low safety firm in the certification process by choosing to get certification. This creates higher expected surplus for the consumer. Incentives for certification is created as firm extracts this surplus through prices. In this range of effectiveness of effort, the incentive for certification increases with increase in the

effectiveness of effort. This is because consumers get more out of their careful use of the product upon a denied certification while incurring the same cost. A higher  $\alpha$  results in a lower incentive for certification due to lower likelihood of the firm being denied certification. A higher cost of effort reduces incentive for certification because this cost is incurred only if the firm decides to seek certification.

Now we look at the range  $\alpha < \alpha^*$ . In this range of the effectiveness of effort, the firm makes the consumer switch from using the product with care to using it negligently if it gets classified as high safety firm by choosing to go for certification. This again creates higher expected surplus for the consumer and, as a result, incentive for the firm to seek certification. However, in this range the incentive for certification gets smaller as effort becomes more effective. This is because the consumer sacrifices more by using the product negligently. The consumer does not enjoy the full benefit of increased effectiveness of effort when the firm seeks certification as she uses the product negligently when certification outcome is positive. An increase in  $\alpha$  makes the likelihood of the firm getting certified more likely if it seeks certification. This increases incentive for certification. A higher cost of effort increases incentive for certification as the consumer saves more when she chooses to be negligent.

If the effectiveness of effort is sufficiently large ( $\alpha > \alpha^*$ ), the consumer takes care in using the product regardless of the certification decision or the outcome of certification. The firm does not gain from going to certification and, therefore, optimally decides not to go for certification.

At this point, it is useful to examine how the range of  $\alpha$  in which the firm has positive incentive for certification reacts to the changes in the market conditions. This range expands with an increase in the cost of consumer effort. The intuition is simple. An increase in the cost of the effort makes the consumer more unwilling to take care when using the product. However, given the product safety and the consumer effort are substitutes the effect is more pronounced if the firm gets certified as high safety firm compared to if the firm gets certified as low safety firm. As a result the range of the effectiveness of consumer effort under which the incentive of certification is positive increases. The range first increases and then decreases with increase in  $\alpha$ . This happens because when the uncertainty about the firm type is small the certification provides little information to the consumer. However, if the uncertainty about the firm type is large the certification outcome has large effect on the beliefs of the consumer about the firm type. We also note that if the precision of the certification is increased the incentive for certification is positive over a wider range of  $\alpha$ . The intuition is straightforward.

Now we look at the case in which the product safety and the consumer effort are complements. As in the case of substitutes we can define the threshold  $\alpha^*$

and  $\lambda_{NS}$  where,  $\lambda_S < \lambda < \lambda_{NS}$ . If the choice of the consumer effort has a very small effect on the probability of the accident ( $\epsilon < \lambda_S$ ), the consumer uses the product negligently regardless of the certification decision or the outcome of certification. As the effectiveness of consumer effort is increased beyond a threshold, the consumer still uses the product negligently if the firm does not go for certification or it gets classified as low safety firm but she uses the product with care if the firm gets classified as high safety firm by the certification agency. The consumer switches to careful use upon positive certification outcome first because the belief of the consumer about the firm type  $\theta_h$  is highest when the firm gets classified as a high safety firm. If the effectiveness of effort is further increased ( $\epsilon > \lambda$ ), the consumer starts using the product with care when the firm does not go for certification as well. If the consumer effort is highly effective ( $\epsilon > \lambda_{NS}$ ), the consumer uses the product with care regardless of the certification decision or the outcome. The consumer uses the product with care upon negative certification only if the consumer effort is highly effective because the consumer effort and the product safety are complements and the belief of the consumer about firm type  $\theta_h$  is lowest upon negative outcome of certification.

A comparison of the profit of the firm if it decides to go for certification and if it decides to not go for certification leads us the following results:

**Proposition 2.2 (Complements case)** If the product safety and the consumer effort are complements, the incentive for certification is positive in an intermediate range of the effectiveness of the consumers effort ( $\lambda_S < \epsilon < \lambda_{NS}$ ). It first increases (when  $\lambda_S < \epsilon < \lambda$ ) and then decreases (when  $\lambda < \epsilon < \lambda_{NS}$ ) with  $\epsilon$  in this range.

As in the substitutes case, the certification helps the firm only when it is able to induce the consumer to take a different effort from what she would have taken if the firm did not go for certification. In the range  $\lambda_S < \epsilon < \lambda$ , the firm makes the consumer switch from using the product negligently to using the product with care when it gets classified as a high safety firm by choosing to go for certification. Whereas in the range  $\lambda < \epsilon < \lambda_{NS}$ , the firm makes the consumer switch from using the product with care to using the product negligently when the firm gets classified as low safety firm by choosing to go for certification. However, if the consumer effort is highly ineffective ( $\epsilon < \lambda_S$ ) or it is highly effective ( $\epsilon > \lambda_{NS}$ ), the firm certification decision or outcome of certification has no impact on the consumers' choice of effort. Therefore, the incentive for certification is positive in an intermediate range of the effectiveness of the consumers effort ( $\lambda_S < \epsilon < \lambda_{NS}$ ).

In the range  $\lambda_S < \epsilon < \lambda$ , the incentive for certification increases with the effec-



tiveness of consumer effort and  $\beta$  but decreases with the consumer's cost of careful product use. An increase in the effectiveness of effort increases incentive for certification since the consumer uses the product with care only if the firm gets a positive certification. A higher prior probability about the firm being high safety makes it more likely for the firm to get classified as a high safety firm in the certification process. This results in higher incentive for certification. If the cost of effort is increased the effect is felt only when the firm goes for certification and gets classified as high safety firm. This reduces the incentive for certification. In the range  $\beta^H < \beta < \beta^{NS}$ , the incentive for certification decreases with increase in the effectiveness of effort and  $\beta$  but increases with increase in cost of careful product use. The intuition is similar to the case of  $\beta^S < \beta < \beta^H$ .

The range of  $\beta$  in which the firm has positive incentive for certification increases with the increase in the cost of effort. As in the case of substitutes, an increase in the cost of the effort makes the consumer more unwilling to take care when using the product. However, given the product safety and the consumer effort are complements the effect is larger if the firm gets certified as low safety firm compared to if the firm gets certified as high safety firm. This result in an wider range of the effectiveness of consumer effort under which the incentive of certification is positive. The responses to a change in the prior probability of the firm being of high safety as well as to a change in the precision of certification are same as in the case of substitutes.

## 2.3 Private Certification

The setup of the model here is same as in Section 2.3 with the exception that now the certification decision of the firm is its private information. As the certification decision is not observed by the consumer, she can not tell if the absence of the safety certification sticker on the product is due to a no certification decision made by firm or it is because the firm got classified as low safety firm by the certification agency. The consumer, therefore, must form belief about the certification decision of the firm when a certification sticker is not present on the product. The consumer makes her purchase decision based on her belief about the certification decision, which affects her beliefs about the firm type, and the price set by the firm. In equilibrium, the belief of the consumer about the certification decision is consistent with the certification decision of the firm.

As in the case where the certification decision was common knowledge, we consider three different possibilities for the product safety and the consumer effort interaction in deciding the probability of accident-free use of the product. We first look at the possibility that the product safety and the consumer effort are independent in

deciding the probability of the accident. In this setup, we get the following result:

Proposition 2.3 (Independent case) If the product safety and the consumer effort are independent, the equilibrium involves the firm going for certification for

$$k < \frac{(1 - \alpha)(2 - \alpha)(h - l)L_c}{(1 - \alpha) + (1 - \alpha)}$$

the firm not going for certification for

$$k > (1 - \alpha)(2 - \alpha)(h - l)L_c$$

and the firm mixing between certification and no certification in the region of parameter space where both certification and no certification equilibria exist.

Unlike in the setup where the consumers observed the certification decision, the no certification equilibrium does not exist for small enough certification cost. If the consumers believe that the firm did not go for certification, the firm for a small enough certification cost would deviate to certification and make higher profits than upon no certification. This happens because upon positive certification the firm would be believed to have gone for certification and set a higher price. Whereas upon negative certification the firm sets the same price as it would have set in the case of no certification. For low enough certification costs the firm in equilibrium goes for certification. This firm would find a deviation from the certification equilibrium profitable only if the cost of certification for the firm is sufficiently high. There exists an intermediate range of parameter space in which both certification and no certification is equilibrium. In fact there also exist mixed strategy equilibria in the region where both certification and no certification are equilibria. The prior probability with which the firm is believed to be a high safety firm also plays a role. As  $\alpha$  approaches zero or one the price difference upon positive and negative certification approaches zero. The equilibrium for the extreme consumer beliefs, therefore, involves no certification.

Also the equilibrium does not depend on the choice of effort. This happens because the consumer's choice of effort does not depend upon her beliefs about the firm type given the consumer effort and the product safety are independent. The certification plays a role by changing the beliefs of the consumer about the firm type. Since beliefs do not affect the choice of effort the equilibrium does not depend on the effort choice or the effectiveness of consumer effort.

Next we look at the certification equilibrium in a setup where the product safety and the consumer effort are substitutes. The equilibrium analysis leads us to the

following result:

Proposition 2.4 (Substitutes case) If the product safety and the consumer effort are substitutes, the equilibrium involves

(1) the firm going for certification for

$$k < \frac{\frac{\delta}{w} \frac{(1-\alpha)(2-\alpha)(h-l)(L_c + \alpha L)}{(1-\alpha) + (1-\alpha)}}{(1-\alpha)(2-\alpha)(h-l)L_c + K_s} \quad \text{if } \alpha < \alpha_{NS}$$

$$\frac{\delta}{w} \frac{(1-\alpha)(2-\alpha)(h-l)L_c}{(1-\alpha) + (1-\alpha)} \quad \text{if } \alpha_{NS} < \alpha < \alpha_S$$

$$\frac{\delta}{w} \frac{(1-\alpha)(2-\alpha)(h-l)L_c}{(1-\alpha) + (1-\alpha)} \quad \text{if } \alpha > \alpha_S$$

where  $K_s = (\alpha + (1-\alpha)(1-\alpha)) \alpha L [ (1-h) + (1-\alpha)(1-\alpha)(1-l) ]$

(2) the firm not going for certification for

$$k > \frac{\delta}{w} \frac{(1-\alpha)(2-\alpha)(h-l)(L_c + \alpha L)}{(1-\alpha) + (1-\alpha)} \quad \text{if } \alpha < \alpha_{NS}$$

$$\frac{\delta}{w} \frac{(1-\alpha)(2-\alpha)(h-l)L_c + K_s}{(1-\alpha) + (1-\alpha)} \quad \text{if } \alpha_{NS} < \alpha < \alpha_S$$

$$\frac{\delta}{w} \frac{(1-\alpha)(2-\alpha)(h-l)L_c}{(1-\alpha) + (1-\alpha)} \quad \text{if } \alpha > \alpha_S$$

and

(3) the firm mixing between certification and no certification in the region of parameter space where both certification and no certification equilibria exist.

These equilibrium conditions are also graphically presented in Figure 2.2. Suppose the market conditions are such that the consumer uses the product negligently regardless of the certification decision or the outcome of certification ( $\alpha < \alpha_{NS}$ ). In an equilibrium in which the firm goes for certification, the maximum cost of certification for which the equilibrium exists increases with the effectiveness of consumer effort. An increase in the effectiveness of effort hurts the consumer more when the firm gets classified as low safety firm compared to when it gets classified as high safety firm. Therefore, the difference between the price the firm sets upon getting classified as a high safety firm and upon getting classified as a low safety firm increases with the effectiveness of consumer effort. This makes a deviation to no certification more costly since the firm sets the same price when deviating as it sets upon getting classified as a low safety firm. Therefore, the maximum certification cost for which the certification equilibrium exists increases with the effectiveness of consumer effort. It remains unchanged with an increase in the cost of the consumer effort since the consumer does not incur this cost in this region of parameter space. Now we move to the region of parameter space where the consumer uses the product negligently when she buys a product classified by the certification agency as a high safety product but

Figure 2.2: Equilibrium conditions: C, NC, C-NC represent certification, no certification and mixed strategy equilibrium respectively;  $\bar{c}$  represents certification when certification decision is common knowledge. Parameter values are  $\alpha=0.9$ ,  $\beta_h=0.7$ ,  $\beta_l=0.3$ ,  $L_c=2$ ,  $L_f=2$ ,  $\gamma=0.4$ ,  $\delta=0.5$ .

uses the product with care when she buys a product classified as low safety product ( $\beta_{NS} < \beta < \beta_S$ ). An increase in the effectiveness of effort hurts the consumer only when the firm gets classified as high safety firm and not when it gets classified as low safety firm. This results in a decrease in maximum cost of certification that supports the certification equilibrium with increase in the effectiveness of consumer effort. It increases with the cost of effort as the consumer incurs this cost only when the firm gets classified as low safety firm. In the region of parameter space where the consumer uses the product with care regardless of certification outcome ( $\beta > \beta_S$ ), this maximum certification cost does not change with the cost of effort or the effectiveness of effort. The relationship with the prior belief of the consumer about firm type is as in the case where product safety and the effort are independent. We do not provide intuitions for the no certification equilibrium and the mixed strategy equilibrium here. They are similar.

We now look at the certification equilibrium in a setup in which the product safety and the consumer effort are complements. We get the following result:

Proposition 2.5 (Complements case)

If the product safety and the consumer effort are complements, the equilibrium involves

(1) the firm going for certification for

$$k < \frac{\frac{(1-\alpha)(2-\alpha)(h_i)(L_c - L)}{(1-\alpha)+(1-\alpha)}}{\frac{(1-\alpha)(2-\alpha)(h_i)L_c}{(1-\alpha)+(1-\alpha)}} + K_c \quad \begin{array}{l} \text{if } \alpha < \alpha_S \\ \text{if } \alpha_S < \alpha < \alpha_{NS} \\ \text{if } \alpha > \alpha_{NS} \end{array}$$

where

$$K_c = \frac{(\alpha + (1-\alpha)(1-\alpha))}{\alpha + L \left[ (1-\alpha)(1-h_i) + (1-\alpha)(1-\alpha) \right]} \frac{+(1-\alpha)(1-\alpha)}{(1-\alpha)+(1-\alpha)}$$

(2) the firm not going for certification for

$$k > \frac{\frac{(1-\alpha)(2-\alpha)(h_i)(L_c - L)}{(1-\alpha)+(1-\alpha)}}{\frac{(1-\alpha)(2-\alpha)(h_i)L_c}{(1-\alpha)+(1-\alpha)}} + K_c \quad \begin{array}{l} \text{if } \alpha < \alpha_S \\ \text{if } \alpha_S < \alpha < \alpha_{NS} \\ \text{if } \alpha > \alpha_{NS} \end{array}$$

and

(3) the firm mixing between certification and no certification in the region of parameter space where both certification and no certification equilibria exist.

The equilibrium conditions as presented in the proposition are graphically depicted in Figure 2.3. The maximum certification cost for which the certification equilibrium exists responds to changes in the effectiveness of effort in the opposite manner compared to the substitutes case except for the region where the consumer uses the product with care regardless of the certification decision or the outcome of certification. In that region because the consumer never uses the product negligently the condition for equilibrium is same as when the effort and the product safety are independent. As the intuition for the change in the condition for equilibrium is similar to the substitutes case, we do not present them here.

A comparison of the equilibria in the substitutes and the complements case implies that while the presence of consumer moral hazard results in product safety certification over a larger set of market conditions in the case of substitutes it leads

Figure 2.3: Equilibrium conditions: C, NC, C-NC represent certification, no certification and mixed strategy equilibrium respectively  $\bar{c}$  represents certification when certification decision is common knowledge. Parameter values are  $\alpha=0.9$ ,  $\beta_h=0.7$ ,  $\beta_l=0.3$ ,  $L_c=2$ ,  $L_f=2$ ,  $\delta=0.4$ ,  $\gamma=0.5$ .

to certification over a smaller set of conditions in case of complements.

The intuition is as follows: If the product safety and the consumer effort are substitutes, the consumer is more willing to use the product with care when she buys a non-certified product whereas if they are complements she is more willing to use the product with care when she buys a certified product. Therefore, a negligent use of the product hurts the consumer more when the firm is classified as a low safety firm in the substitutes case whereas it hurts the consumer more when the firm is classified as a high safety firm in the complements case. Since the maximum certification fee for which the certification equilibrium exists depends on the difference in the prices the firm sets upon getting classified as a high safety firm and upon getting classified as a low safety firm, the equilibrium exists for larger certification fee in case of substitutes but smaller certification fee in the case of complements compared to the case when there is no consumer moral hazard.

## 2.4 Extensions

### 2.4.1 Heterogeneous consumers

In this section, I assume that the consumers are heterogeneous in their valuations of the product. A fraction  $\alpha$  of the consumers have high valuation  $v_h$  for the product whereas the remaining  $1-\alpha$  have low valuation  $v_l$ . We assume that the low valuation consumer gets positive surplus from using a low safety product. The valuation of the product is private information of the consumer. The firm when setting a price must decide if it wants to sell only to high valuation consumers or to all the consumers. This decision depends on relative size of the two segments. The thresholds  $\tilde{p}_s$ ,  $\tilde{p}_{NS}$  and  $\tilde{p}_{NS}$  in Figure 2.4 are defined in the appendix.

Figure 2.4: Equilibrium conditions: C represents certification and NC the no certification equilibrium, c represents certification when certification decision is common knowledge. Parameter values are  $\alpha=0.9$ ,  $\alpha_h=0.7$ ,  $\alpha_l=0.3$ ,  $L_c=2$ ,  $L_f=2$ ,  $v_h=5.5$ ,  $v_l=4$ ,  $\beta=0.5$ .

The heterogeneity leads to certification if the certification decision is common knowledge. The incentive for certification for the firm is positive in the intermediate region. In this region, the certification decision changes the demand for the firm under one of the two outcomes of certification. A change in the demand creates the incentive for certification through the price and the expected liability payment made by the firm.

If the certification decision is private information, consumer heterogeneity can lead to less certification. To understand the intuition we consider a situation in which the market is fairly homogeneous and almost all the consumers are high valuation consumers. Because almost all the consumers are high valuation consumers, the firm would choose to sell to only high valuation consumers regardless of the certification decision or outcome. If we make the market slightly more heterogeneous by increasing the proportion of low valuation consumers the profit of the firm would decrease. The cost of certification, however, does not depend on the demand. Therefore, as the market becomes more heterogeneous the firm would be more willing to deviate to no certification.

## 2.4.2 Informed firm and uninformed consumers

Here we assume that nature after making the draw of the firm type reveals it to the firm but not to the consumer.<sup>16</sup> The firm then makes the decision of whether or not to go for certification privately. We assume that the cost of certification  $k$  is less than  $(v_h - v_l)L_c$ , which is the maximum benefit a firm can possibly enjoy by going to certification. The firm discloses the outcome of certification if the certification agency classifies it as high safety and then sets a price. The consumers buy the product based on their belief about the firm type and the price set by the firm. As the certification decision of the firm is made privately, the belief of the consumer about firm type only depends on the presence or the absence of the safety certification sticker and the price.

The equilibrium conditions under which different equilibria exist are presented in Figure 2.5. If the cost of certification is very small, both types of firm in equilibrium go for certification. The high safety firm benefits from certification by charging a higher price when it is correctly classified by the certification agency as a high safety firm whereas the low safety firm benefits from certification by charging a higher price when it is incorrectly classified as a high safety firm. The maximum cost of certification for which this equilibrium exists depends on the difference in price the firm sets upon being classified as a high safety firm and the price it sets upon being classified as a low safety firm. As the prior belief about the firm type approaches zero or one the prices the firm sets upon getting the sticker or not converge to same point therefore, the cost of certification for which both types of firm go for certification

<sup>16</sup>A more general case can be worked out in which the firm knows more about the safety of the product than the consumer but is not perfectly informed. This adds complexity in the model and does not provide additional interesting results. The general model converges to the model presented here in the limit.



also approaches zero. It is maximum at  $\theta=0.5$ . Note that whether it is optimal for a firm type to go to certification or not at a given certification cost depends on what equilibrium is being played.

Figure 2.5: Equilibria in  $k$ -space. C,C-NC indicates high safety firm goes for certification and low safety firm is indifferent between certification and no certification. Parameter values are  $\theta=0.7$ ,  $\theta_h=0.9$ ,  $\theta_l=0.7$ ,  $L_c=2$ .

If the cost of certification is increased we get less certification in equilibrium. The low safety firm switches to no certification first. For sufficiently large certification cost, both types of firm decide against certification. An interesting observation to be made here is that in the equilibrium in which the high safety firm goes for certification or it is indifferent but the low safety firm does not go for certification as belief of the consumer about firm type  $\theta_h$  approaches zero, the maximum certification cost for which the equilibrium exists keeps increasing. This happens because the price that the high safety firm sets upon getting a sticker remains constant as in this case consumer knows for sure that it is a type  $\theta_h$  firm but the price it sets on no sticker keeps decreasing with decrease in the prior belief of the consumer about the firm type  $\theta_h$ . Therefore, as  $\theta_h$  gets smaller the high safety firm will be even more willing to go for certification. In the equilibrium in which the high safety firm goes for certification but the low safety firm is indifferent, the maximum certification cost for which the equilibrium exists can increase or decrease as  $\theta_h$  approaches zero depending on the probability with which the low safety firm goes for certification. If in equilibrium both types do not go for certification as  $\theta_h$  reduces it becomes more difficult to sustain

the equilibrium as the high safety firm finds it more tempting to deviate.

Also note that when the prior belief of the consumer approaches one, the incentive for certification approaches zero no matter which equilibrium is being played. This happens because it is being perceived as high safety firm correctly or incorrectly that creates incentives for certification. When  $\beta$  approaches one the firm is believed to be of type  $\theta_h$  even when it does not go for certification. Therefore, the maximum certification fee for which the firm would not want to go for certification approaches zero.

## 2.5 Conclusion

Almost all the consumer products that are legally sold in the market meet a set of minimum safety standards specified by the government. Some firms, however, decide to go for voluntary safety certifications for which the safety standards are usually much harsher. For examples, automobile manufacturers go for Euro NCAP certification, toy manufacturers go for American National Standards Institute or ASTM International certification and bike helmet manufacturers go for Snell Certification. The objective of this paper is to understand why some firms go for such voluntary safety certifications and others do not. The market characteristics such as consumer heterogeneity, consumer moral hazard and information asymmetry about the certification decision as well as about the safety of the product create incentives for certification by the firm.

In the model presented above, a firm which does not know how safe is its product when sells in a homogeneous consumer market finds it optimal not to go for certification if the certification decision is common knowledge. We relate this finding to the pharmaceutical industry. We can think of the market to consist homogeneous consumers especially for drugs marketed for life threatening diseases. The certification decisions are common knowledge as the firms announce the positive as well as negative certification decisions given the impact of such decisions on investors. We observe that in this industry there is no voluntary certification activity. In fact, the firms even try to sell without mandatory FDA approval. One implication of this result is that a regulator requiring mandatory certification in such an industry should be concerned about enforcement mechanisms as well.

The presence of the consumer moral hazard creates incentives for certification when the certification decision is common knowledge. It is the ability of the firm to influence the choice of effort by the consumer that creates the incentive for certification. We relate this result to the automobile industry. Rear impact safety and the consumer effort can be thought of as independent in deciding probability of an

accident. We observe that the rear impact crash test is traditionally not done as a part of voluntary crash testing certifications. Euro NCAP started rear impact testing only recently in year 2009. A regulator should be more concerned about the safety features for which the consumer effort and product safety are independent. Voluntary certification would be more common for features when product safety and the consumer effort are not independent. A car equipped with an automatic braking system, Volvo S60 for example, can be thought of as a product for which the product safety and the consumer effort are substitutes. Volvo tested the product in front of the press.

If the certification decision of the firm is not common knowledge, the presence of the consumer moral hazard leads to more certification when the product safety and the consumer effort are substitutes in deciding the probability of accident-free use but leads to less certification when they are complements. This is because using a product negligently in the substitutes case is more painful for the consumer if it is certified as low safety product but in the case of complements using negligently is more painful if the product is certified as high safety product.

In many settings, it can be argued that a firm does not know how safe is its product before selling it in the market but learns it privately after consumers buy, use and report the failures back to the firm. We can think Toyota Prius brake safety issue as an example of this setup. How does this ex-post learning by the firm create incentives for ex-ante certification? Next, we would like to answer this question. In addition, the present analysis focuses on the case of a monopoly, the setting with competition which can yield interesting results is yet to be done.

## Chapter 3

# The Impact of Others' Opinions on Online Ratings <sup>1</sup>

### 3.1 Introduction

The internet has become a popular source of information gathering as well as information sharing. Most websites such as amazon.com, net ix.com and target.com that sell or rent products, also let consumers share their consumption experience. Even when the retailers do not offer rating services, websites such as imdb.com and yelp.com let consumers share their consumption experiences about virtually any product and service. Consumers routinely check the information available on these websites before making a purchase decision for a product with uncertain quality regardless of whether the purchase is made online or through other channels. The same consumers often return to their favorite websites to share their product consumption experiences.

Online ratings and reviews have become a popular way of reporting a product consumption experience on the internet. The following observations give us an idea of how common the use of online ratings is: in the first 4 weeks of the movie *Glorious Basterds* received over 60,000 ratings on imdb.com, *Harry Potter and the Deathly Hallows* has received 3,410 reviews on amazon.com and *The Venetian Las Vegas* has received 1,321 reviews on priceline.com. <sup>2</sup> When reporting these ratings and/or reviews consumers try to incorporate all the information about the quality of the product that they have acquired from various sources including, but not limited to, their own product consumption experience.

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<sup>1</sup>This chapter is a joint work with Pedro Gardete.

<sup>2</sup> As of 10/01/2009.

Websites vary in how they let consumers report their consumption experience. Some websites such as [elp.com](http://www.elp.com) and [target.com](http://www.target.com) require users to write reviews together with ratings; others such as [imdb.com](http://www.imdb.com) and [walmart.com](http://www.walmart.com) let consumers decide whether they want to write a review or just rate the product on an ordinal scale. One major difference between reviews and ratings, apart from the fact that ratings take far less time to report, is that while ratings are mostly anonymous reviews are not. This might have implications on how consumers behave when entering online ratings compared to writing online reviews. We focus our attention on the online ratings in this paper.

Extensive research has been carried out in the study of information transmission. According to this research, information transmitters report extreme attitudes (Cohen, 1961), look for supportive information and ignore inconsistencies (Brock and Fromkin, 1968). Transmitters are also shown to modify their communication in the presence of self-presentation concerns. Schlosser (2005) demonstrated through experiments that "posters" (those communicating their experiences to others) appearing later in a sequence of communications get influenced by earlier negative opinions (but not positive opinions) and adjust their public attitude downwards when responding in public because of self-presentation concerns. Those who reported their opinions privately were less influenced by negative opinions as they had no self-presentation concerns.

It is common for consumers to get exposed to the average of the prior ratings, prominently displayed on screen, immediately before reporting their own rating. The literature on information transmission may be more inclined towards the possibility that consumers will ignore the average rating. Based on (Cohen, 1961), we expect consumers to report extreme ratings, which makes it less likely that their ratings would be affected by the average of the prior ratings. Consumers might find the average of the prior ratings to be inconsistent with their opinion in some cases and consistent with their opinion in other cases. Following Brock and Fromkin (1968) we expect consumers to disregard the average ratings that are inconsistent with their opinion. Finally, since there are little self-presentation concerns in online ratings (because they are usually anonymous), the results by Schlosser (2005) suggest that those arriving later in the sequence would not change their ratings upon observing the average of the prior ratings.

There are several ways in which consumers could use the average of the prior ratings to arrive at their final ratings. One possibility is that consumers might have an objective of guiding the audience to the same opinion as their own. This would require them to enter extreme low ratings if the average of the prior ratings is higher than their opinion, and vice-versa.

Another possibility is that in order to arrive at the final ratings consumers might start with the displayed average ratings and then adjust until they reach a value they think is acceptable.<sup>3</sup> Tversky and Kahneman (1974) refer to this mechanism as the anchoring and adjustment heuristic. Further research (e.g. Plous, 1989; Jacowitz and Kahneman, 1995; Mussweiler and Strack, 2004) has shown that final estimates provided by subjects are influenced by initial anchor values. For example, subjects estimated that Gandhi lived to be 67 years old when they first answered whether he died before or after the age of 140, but they estimated that he lived to be only 50 years old when they first answered whether he died before or after the age of 9 (Strack and Mussweiler, 1997). Although anchoring and adjustment is helpful in most cases, adjustments tend to be insufficient, leaving people's final estimates biased towards the starting values (see also Epley and Gilovich, 2006).

Yet another possibility is that consumers might consider the average of the prior ratings as relevant information and intentionally incorporate it in their evaluations in the process of arriving at their final ratings. Individual consumers might not consider themselves experts in evaluating all the aspects of the products. This might prompt them to incorporate the information present in the average of the prior ratings in their own evaluations in the hope of providing better evaluations of that product. Some examples in which a decision maker's action is influenced by those of previous decision makers follow. People choose to dine in restaurants where they see a higher fraction of seats occupied (Becker, 1991); voters are known to be influenced by opinion polls to vote for the party that is expected to win (Cukierman, 1991); The Discipline of Market Leaders remained selling as best seller for some time once authors secretly bought 50,000 copies of the book to make the New York Times bestseller list (see Bikhchandani et al., 1998); researchers are known to work on topics that are hot (see Banerjee, 1992); Patients adjust their quality perception about a kidney for transplant based on refusals by patients ahead in the waiting line (Zhang, 2010).

By use of movie ratings data from [rottentomatoes.com](http://www.rottentomatoes.com), we investigate whether exposure to the average of the prior ratings displayed on screen leads individuals, intentionally or unintentionally, to incorporate it in their own ratings. We are also interested in the impact of such behavior on the overall pattern of the ratings. Ariely et al. (2003) reported in their working paper that subjects were willing to pay \$11.62 for a wine with a wine advocate rating of 86 compared to \$17.42 for the wine with a wine advocate rating of 98. This suggests that consumers' willingness to pay should be higher for a product with a higher average rating. To the extent that this result is generalizable, firms may try to take advantage by including inflated ratings. The fact that [imdb.com](http://www.imdb.com) has adopted a weighted average scheme to offset such effects

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<sup>3</sup> This is especially true in the presence of uncertainty.

highlights the importance of this issue.<sup>4</sup>We therefore also examine to which extent it is possible to artificially inflate the average rating of a movie in our dataset.

We use an ordered logit model to test our hypothesis since our dependent variable, ratings, is on an ordinal scale. We treat the average of the prior ratings and movie dummies as the independent variables and let the unobserved factors follow a logistic distribution. The rating given by an individual might also depend on the media reports, on the release of competing titles and on other marketing efforts by movie studios, which are not observed. These shocks might have a carryover effect on consumers who rate the movie in subsequent periods. Not accounting for this carryover effect could bias the coefficient. We therefore add a movie specific error term and allow it to be serially correlated. We note that estimation of limited dependent variable models with autocorrelated errors has received relatively little attention probably due to computation complexity of obtaining the maximum likelihood estimator. We use a Bayesian approach in order to implement a feasible estimation method.

In terms of methodology, this paper draws heavily from Albert and Chib (1993) and Allenby and Lenk (1994) in order to estimate an ordered logit model with autocorrelated errors. Albert and Chib (1993) propose a Bayesian method for modeling categorical response data using an ordered probit model. Applications of this approach in marketing include Manchanda and Gupta (1999), Jedidi and Manchanda (2003) and Chung and Rao (2003). Allenby and Lenk (1994) apply a logistic normal regression with autocorrelated errors using Bayesian procedures to scanner panel data of ketchup purchases. McCulloch and Rossi (1994) also discuss in short a binary multiperiod probit model with autoregressive errors.

We find that the coefficient of the average rating is positive and significant. This implies that when reporting anonymous online ratings, individuals incorporate the average ratings displayed on the screen in their evaluations of the movie. Our findings are in contrast with those of Schlosser (2005), who reports that in an anonymous setting, feedback is not affected by previous responses. One possible explanation could be that in Schlosser's setting, subjects were experts and they might not see the value of incorporating others' feedback in their own rating. However, a typical consumer might not consider herself an expert and may treat the average of the

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<sup>4</sup> From imdb.com (10/08/09) on the weighted average rating scheme: "The objective of the scheme is to present a more representative rating which is immune from abuse by subsets of individuals who have combined together with the aim of influencing (either up or down) the ratings of specific movies. This includes people involved in the production of a movie and their friends or fans trying to unduly raise the rating of a movie far above that of where the typical IMDb users would rate it."

previous ratings as a relevant piece of information.

Simulations indicate that incorporating the average of the prior ratings in one's own evaluations changes the overall pattern of ratings for a representative movie. We also found that a few early extreme ratings, good or bad, change the ratings that a movie receives in subsequent periods. This effect creates a lasting impact on the average rating of the movie. Therefore, the final ratings of a movie may critically depend on a handful of initial ratings.

The rest of the paper is organized as follows. Section 3.2 summarizes the data. Section 3.3 describes the ordered logit model with serially correlated errors. Section 3.4 presents the insights into rating behavior of individuals from the model and from the estimation procedure. Section 3.5 summarizes the findings.

## 3.2 Data

The use of online ratings is widespread. Due to the availability of the data we chose online movie ratings for this study. People typically refer to one or more movie websites like [rotentomatoes.com](http://rotentomatoes.com), [imdb.com](http://imdb.com) or [movies.yahoo.com](http://movies.yahoo.com) to look at movie ratings, among other things, before making a decision to watch movies. Many of them return to rate the movies they watch. Most movie review websites let their registered users rate movies. While they use an ordinal scale, the number of possible ratings varies from website to website: [imdb.com](http://imdb.com) allows for 10 possible ratings from 1 to 10; [movies.yahoo.com](http://movies.yahoo.com) allows for 13 possible ratings from F to A+; and [rotentomatos.com](http://rotentomatos.com) allows for 11 possible ratings from 0% to 100%.

For this study we use the [netix.com](http://netix.com) (an online DVD rental website) dataset.<sup>5</sup> Movies received an average of over 5,000 ratings during this period. For this study, a sample of 60 movies was randomly chosen from a subset of movies that received on average at least one rating per day. These 60 movies were released between 1934 and 2004 and represent all genres.

We also note that [netix.com](http://netix.com) users are often exposed to the average of the prior ratings but not necessarily to the number of ratings. Although the latter information is available, the user has to make an effort if she wants to know the number of ratings based on which the average is reported.

Cumulative average ratings were calculated for each movie for each period using the individual ratings data. We round the average ratings to the first decimal in

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<sup>5</sup> Data description by Netix: "The movie rating files contain over 100 million ratings from 480 thousand randomly-chosen, anonymous Netix customers over 17 thousand movie titles. The data were collected between October, 1998 and December, 2005 and reflect the distribution of all ratings received during this period. The ratings are on a scale from 1 to 5 (integral) stars."



order to be consistent with how they appeared on screen. Average ratings are as low as 1.9 and as high as 4.8 in the working sample. Only 200 periods of data are included in the study as the average ratings typically do not change after 200 periods. Figure 3.1 presents average ratings over 200 periods for three movies in the sample. It can be seen that while average ratings changed frequently in early periods, they become stable after about 150 periods. We therefore restrict the data to only 200 periods.

Figure 3.1: Average ratings changing over time for three randomly chosen movies.

It has been observed that most individuals choose to report their ratings either when they are highly satisfied or when they are highly dissatisfied with the product. This might create bias in our results. However, we did not observe this bimodal distribution in Netflix data. Distributions of ratings for five randomly chosen movies are shown in Figure 3.2. One of the reasons for this fact could be that since Netflix users get movie recommendations based on the ratings they give to other movies, they have an incentive to report all experiences; not just the extreme ones. This may offset the urge to influence others' opinions through the use of extreme ratings.

### 3.3 The Model

Suppose that all the individuals rating the movie at time  $t$  receive the same signal. This is a simplifying assumption and may be relaxed in a future version of the paper. We transform all the ratings given to movie  $m$  in period  $t$  to one rating on the same ordinal scale of 1 to 5, by taking the average and rounding it

Figure 3.2: Distribution of ratings for 5 randomly chosen movies.

in the following fashion. We assume that if the average of the ratings given on a particular day for a movie is 2.3 for example, a representative consumer would give a rating of 2 with probability 0.7 and a rating of 3 with probability 0.3.<sup>6</sup> Suppose  $r_{m,t}$  denotes the rating for movie  $m$  at period  $t$  given by a representative consumer. This rating could take one of the five possible values (from 1 to 5) represented by  $j \in \{1, 2, \dots, J\}$ . Let  $y_{m,t}$  be the latent utility for movie  $m \in \{1, 2, \dots, M\}$  at rating occasion  $t \in \{1, 2, \dots, T_m\}$ .

There exist cut-off points  $\{k_0, k_1, \dots, k_J\}$  which divide the space of  $y_{m,t}$ , the real line, in  $J$  intervals, satisfying

$$1 < k_0 < k_1 < \dots < k_J < +1$$

such that

$$r_{m,t} = j \text{ if } k_{j-1} < y_{m,t} < k_j$$

We decompose the latent utility as

$$y_{m,t} = \alpha (R_{m,t}) + (1 - \alpha) (\mu_m d_m + \epsilon_{m,t} + \eta_{m,t}) \quad (3.1)$$

where  $\alpha$  is the weight that individuals put on the average of the prior ratings,  $R_{m,t}$  represents the average of the prior ratings for movie  $m$  at period  $t$  and  $d_m$  rep-

<sup>6</sup> We also used a more general method of drawing the ratings by using the daily empirical distribution of the ratings and obtained similar parameter estimates.

resents the movie dummy. The error term  $\epsilon_{m,t}$  is distributed logistic with cumulative distribution

$$F(\cdot) = \frac{\exp(\cdot)}{1 + \exp(\cdot)}$$

and  $\epsilon_{m,t}$  is an error term, potentially serially correlated. These errors allow the model to accommodate systematic changes in opinions of movies. For example, the public opinion for a movie might be favorable at a specific moment in time, leading the average rating to increase. Ignoring that these opinions may be correlated over time could lead us to attribute a disproportionate influence of the average rating to the next period's ratings.

The errors  $\epsilon_{m,t}$  are independent across different movies. The error terms within movie  $m$  are autocorrelated as:

$$\epsilon_{m,t} = \rho_m \epsilon_{m,t-1} + \eta_{m,t}$$

for  $t = 1; 2; \dots; T_m$ .

The factor  $\rho_m$  represents the autocorrelation parameter, and is between -1 and 1.  $\eta_{m,t}$  is independent and identically distributed  $N(0, \sigma^2)$ . The error term in period  $t = 0$ ,  $\epsilon_{m,0}$ , is independent and identically distributed  $N(0, 1)$ .

The function  $f(\cdot)$  transforms the average of the prior ratings to utilities.  $f(R_{m,t})$  is given by the inverse logistic cumulative distribution

$$f(R_{m,t}) = s_m \log \frac{R_{m,t} + 1}{5 - R_{m,t}}$$

where  $s_m$  is the scale parameter of the logistic distribution that is used to transform the average of the prior ratings to the utilities. The parameter  $s_m$  is restricted such that the variance of this logistic distribution is same as the variance of the latent utility in equation (3.1).

The probability of rating  $j$  being given to movie  $m$  at time  $t$  simplifies to

$$p_{m,t}(j) = \frac{\exp\left(\frac{k_j}{s_m} y_{m,t}\right)}{1 + \exp\left(\frac{k_j}{s_m} y_{m,t}\right)} \frac{\exp\left(\frac{k_{j-1}}{s_m} y_{m,t}\right)}{1 + \exp\left(\frac{k_{j-1}}{s_m} y_{m,t}\right)}$$

for  $j = 1; 2; \dots; J$ ;  $t = 1; 2; \dots; T_m$  and  $m = 1; 2; \dots; M$ , where  $y_{m,t}$  is given by

$$y_{m,t} = \frac{1}{s_m} f(R_{m,t}) + \rho_m d_m + \epsilon_{m,t}$$

Hereafter, we refer to the coefficient  $\beta_j$  as  $\beta_j$  and to the vector formed by  $\beta_1$  and  $\beta_m$  as  $\beta$ .

### 3.3.1 The Gibbs Sampler

We estimate the autoregressive ordered logit model by generating random draws from the model's posterior distribution. This distribution does not have a convenient form from which we can take random draws. We use the Gibbs sampling, a Markov Chain Monte Carlo (MCMC) method for taking draws from the joint posterior. Gibbs sampling proceeds by drawing iteratively from the conditional posterior densities. As the number of iterations approach infinity, the process converges to draws from the joint posterior distribution.

Using the convenient notation  $[A|B]$  for representing the conditional distribution of A given B introduced by Gelfand and Smith (1990), we can write the joint distribution of data and parameters as

$$[R; Y; \beta; \gamma; \sigma^2; K] / [R|K; Y|\beta; \gamma; \sigma^2][\beta|\gamma; \gamma|\sigma^2][\sigma^2|K]$$

where  $R_{m;t}$  and  $\{y_{m;t}\}$  are stacked to obtain  $R$  and  $Y$ . All observations for a movie appear in chronological order followed by all observations for the next movie.  $\beta_{m;t}$  is a vector of  $\beta_{m;0}$  and  $\gamma$  is a vector of  $\gamma_m$  in the same order as movies appear in  $R$  and  $Y$ .  $K$  is a vector of cut-off parameters  $\{k_0; k_1; \dots; k_J\}$  where  $k_0 = 1$  and  $k_J = 1$ . To ensure that parameters are identifiable, it is necessary to impose one restriction on the cut-off parameters; without loss of generality we assume  $k_2 = 0.5$ . The posterior conditional distribution from which random draws are taken is described in detail in the Appendix C. They are presented in the same order in which draws were taken in each iteration.

### 3.3.2 Simulation

To test our algorithm, we consider a simulated example. We also want to look at the time it takes to run an iteration and if the sampler is sensitive to initial conditions. We generated the data assuming a set of parameter values,  $(\beta; \gamma; \sigma^2) = (\beta_1; \beta_2; \beta_3; \gamma_1; \gamma_2; \gamma_3; \sigma^2) = (1; 1; 1; 1; 1; 0.5; 1)$ . (For simplicity we assume the rating can take only three values instead of five.)

As shown in Figure 3.3, the Gibbs sampler converged quickly. The convergence plots are shown for the Gibbs sampler with starting values  $\beta = (0.5, 0.5, 0, 0.5)$ . The parameter was fixed at -1 for identification. The parameter values recovered from the sampler were  $\beta = (1.03, 0.91, 0.53, 0.95)$  and standard errors were  $(0.02, 0.03, 0.18,$

0.07) respectively. We verified the sensitivity of the sampler using different sets of starting values and got very similar results to the one shown in Figure 3.3.

Figure 3.3: Convergence plots using simulated data for (a)  $\mu_1$  (b)  $\sigma^2$  (c)  $k_2$  and (d)

### 3.4 Estimation Results

We start with the basic model (Model 1) with fixed effects and independent error terms and allow for serially correlated error terms in our full model (Model 2). Both the models were estimated with 75,000 iterations each. 50,000 iterations were used for burn-in and the last 25,000 iterations were used to calculate the mean and standard error for each run. In the posterior analysis based on the Markov Chain Monte Carlo method, we need to satisfy two conditions in order to check if the posterior distribution is accurately described. The first is that the chain should become stationary. Figure 3.4 plots draws of the coefficient of the cumulative average rating, error variance, cut-off parameters and autocorrelation coefficients (for movie id 662) across iterations for Model 2. For this chain, movie fixed effect coefficients and autocorrelation parameters are started at 0, the coefficient of the average of the prior ratings is started at 1, the variance parameter  $\sigma^2$  is started at 0.5 and the

cut-off parameters are started at  $(k_1, k_2; k_3) = (1; 0.5; 1.5)$ : The plots indicate that the Gibbs sampler reaches a stationary distribution rapidly.

If the starting values for cut-off parameters are too different from the true values, the algorithm takes significantly longer to converge. This is due to the conditional distribution for cut-off parameters, which allows smaller steps to be taken in each iteration as the sample size increases. As suggested by Albert and Chib (1993), we proxy for the starting values of the cut-off parameters with the ones estimated by MLE with iid errors. The mean of the posterior draws in the stationary region provides the point estimates, and the standard deviation provides the standard errors.

Figure 3.4: Convergence plots using simulated data for (a)  $\beta_1$  (b)  $\beta_2$  (c)  $k_2$  and (d) for movie id 662

Stationary Markov chains are necessary but not sufficient to gauge the performance of Gibbs sampler, because they may converge at a local mode and ignore other areas of higher posterior probability. One way to increase our confidence in the Gibbs sampler is to start chains from different regions in the parameter space. This will satisfy the second requirement if the estimated parameters are similar across different starting values.

We investigate the convergence of the Gibbs sampler by starting with different points in the parameter space. We compare the estimated posterior means and standard deviations for three chains. Chain 1 initializes the movie specific effect coefficients and autocorrelation parameters at 0, coefficient of average of the

prior ratings at 1 and the variance parameter at 0.5. Chain 2 initializes the movie specific fixed effect coefficients and the coefficient of the average of prior ratings at 0.5, autocorrelation parameters at 0.4 and the variance parameter at 1, while chain 3 initializes movie specific fixed effect coefficients at 1, the coefficient of the average of the prior ratings at 0, autocorrelation parameters at 0.7 and the variance parameter at 0.1. The movie specific fixed effects are measured relative to the movie 'Inventing the Abbotts (1997)'. Movie-specific fixed effects capture the unobserved heterogeneity across movies.

The Gibbs sampler ran 75,000 iterations for each of the three chains while posterior means and standard deviations were calculated using the last 25,000 iterations. The estimated posterior means and standard deviations for the three chains are close to each other, providing evidence that the Markov chain generates draws from a region of the sample space with highest posterior probability. Results for chain 1 are presented in Table 3.1 and will be used for the analysis later. Although all the tests were done using the sample of 60 movies as described earlier, only the results for five movies are reported in Table 3.1 to save space.

Table 3.2 presents the posterior means and standard deviations of the fixed effects for Model 1 and Model 2. We note that the results of the two models are very close to one another. This is possibly because Model 2 reveals that for only 5 movies in the sample have an autocorrelation coefficient significantly different from zero. This implies that for most movies there is no movie specific error that is autocorrelated over time. Figure 3.4(d) displays a typical convergence plot showing that autocorrelation parameter is not significantly different from zero.

We now turn our attention to the interpretation of  $\beta_1$ . We get a positive and significant value of 1.18 using Model 2. The corresponding value of  $\beta_0$  is 0.54. This means that individuals put roughly half of the weight on the average of the prior ratings and the other half on their private evaluations in order to arrive at their final ratings.

Table 3.2: Posterior mean and standard error of fixed effects for two models.

Parameters		Model 1	Model 2
Intercepts	$\beta_m$ ( movie id 662)	0.21 (0.15)	0.41 (0.26)
	$\beta_m$ ( movie id 1,642)	1.70 (0.24)	2.15 (0.31)
	$\beta_m$ (movie id 1,788)	1.13 (0.22)	1.59 (0.25)
	$\beta_m$ (movie id 2,136)	0.58 (0.15)	0.79 (0.17)
	$\beta_m$ (movie id 2,380)	0.99 (0.15)	1.19 (0.17)
Variable	$\beta_1$	1.48 (0.18)	1.18 (0.23)

Table 3.1: Posterior means and standard errors for Model 2 using three different Gibbs chains.

Parameters		Estimates (std. errors)
Intercepts	$\mu_m$ (movie id 662)	0.41 (0.26)
	$\mu_m$ (movie id 1,642)	2.15 (0.31)
	$\mu_m$ (movie id 1,788)	1.59 (0.25)
	$\mu_m$ (movie id 2,136)	0.79 (0.17)
	$\mu_m$ (movie id 2,380)	1.19 (0.17)
Variable	1	1.18 (0.23)
Error Variance	2	0.57 (0.10)
Autocorrelation coef.	$\rho_m$ (movie id 662)	0.23 (0.26)
	$\rho_m$ (movie id 1,642)	0.15 (0.30)
	$\rho_m$ (movie id 1,788)	-0.01 (0.25)
	$\rho_m$ (movie id 2,136)	0.21 (0.26)
	$\rho_m$ (movie id 2,380)	0.19 (0.36)
Cutto Parameters	$k_1$	-1.45 (0.16)
	$k_2$	0.79 (0.09)
	$k_3$	2.08 (0.22)

We also tested if  $\alpha$  depends on the time since release of the title. We found that  $\alpha$  decreases (p-value=0.08) as movies get older. This indicates that individuals may put lower weight on the average of the prior ratings for older movies. This is probably because they consider themselves relatively more informed about older movies than about recent releases.

### 3.5 Sensitivity analysis

We start with a question: Does the pattern of ratings change because of  $\alpha$  not being equal to zero? We simulated ratings for 1,000 rating occasions using all the parameter estimates from Model 2. In an additional condition, we used all the estimates from Model 2 except the coefficient for the average of the prior ratings ( $\mu_1$ ) which was set to zero. The pattern of the ratings obtained in the two conditions is shown in Figure 3.5. While the means of the ratings are very similar in the two



conditions, the variance of the ratings in the first condition is smaller (0.90 compared to 2.22). The tendency of individuals to incorporate the information presented in the average ratings changed the pattern of ratings.

Figure 3.5: Simulated ratings showing the effect of incorporating cumulative average rating in evaluations.

We also checked the effect of receiving early positive or negative shocks to the ratings on the overall pattern of ratings. For example, this shock could be a result of ratings given in the beginning, with the intention of affecting subsequent ratings upwards or downwards. In the first case, we restrict five of the first 6 ratings to be equal to one. In the second case, we proceed in a similar fashion by restricting five of the first six ratings to be equal to five. We look at the resulting pattern of the first 1,000 ratings. Figure 3.6 shows the effect of early low or high ratings on the subsequent evaluations. The average of 1,000 ratings is 2.98 in the second case,

compared to 2.72 in the first case. We note that the difference in the cumulative average ratings persists even after 1,000 ratings.

Figure 3.6: Simulated ratings showing the effect of early shock on subsequent evaluations.

### 3.6 Conclusion

The internet has created new ways of gathering as well as communicating information. Anonymous online ratings are a quick and popular way to communicate information about a product by summarizing one's overall experience on an ordinal scale. This paper explores the process of reporting online ratings. In particular we examine if individuals incorporate the average of the prior ratings reported on the screen in their evaluations when they visit a particular website, in order to report their own experiences through anonymous online ratings.

We find the coefficient of cumulative average ratings to be significant and positive, suggesting that users incorporate the average of previous ratings displayed on screen in their own evaluations of the movie quality. The magnitude of the coefficient suggests that individuals put roughly half of the weight on the average of the prior ratings and the other half of the weight on their own evaluations in order to arrive at their final ratings. We can also infer that netfix.com users do not report ratings with the objective of influencing others' opinions, given the distribution of ratings we observe. We also find that for most movies the movie specific unobserved factors do not seem to be serially correlated. The value of the autocorrelation parameters was not significantly different from zero for 55 out of 60 movies.

Simulations using the estimated parameter values suggest that the tendency of individuals to incorporate the average rating in their own evaluations changes the pattern of the ratings significantly. We also found that early positive or negative shocks to the ratings get carried over to future rating occasions compared to the case when individuals ignore the average ratings. This suggests that the introduction of a few high ratings at the start could significantly affect the ratings upwards. Hence, ratings appear to be susceptible to manipulation.

Future work can focus on the inclusion of individual level and movie specific characteristics. In addition, trying to identify the mechanism responsible for the inclusion of the average ratings as information may be a worthwhile challenge for future research.

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## Appendix A

### Appendix to Chapter 1

#### Proof of Proposition 1.1

Profit of each firm in equilibrium is  $\frac{p}{2}$ . The best possible deviation for a firm is to make a bribe offer of  $P$  and get selected with probability one. Profit of the firm under this deviation is  $p - P$ . However, deviation is not profitable given  $\frac{p}{2P}$ . The agent also has no profitable deviations. Hence, no bribes are offered in equilibrium.

Now, we show that there is no pure strategy Nash equilibrium for  $\frac{p}{2P}$ .

Suppose  $b_1; b_2$  is a pair of Nash equilibrium strategies. Then there is no other  $b_i$  ( $i = 1; 2$ ) such that  $\pi_i(b_i; b_j) > \pi_i(b_1; b_2)$ . We show that such  $b_i$  exists.

If  $b_1 = b_2$ , any of the firms can strictly benefit by cutting the bribe offer by small  $\epsilon$ .

If  $b_1 > b_2$  (the proof for  $b_1 < b_2$  is analogous),

Case (1)  $b_1 > b_2 + P$

Since firm 1 gets selected with probability one,  $\pi_1(b_1; b_2) = p - b_1$ .

$\exists \epsilon$  such that  $b_1 = b_2 + \epsilon$  and  $b_1 > b_2 + P$ . Firm 1 can make larger profits by offering  $b_2$ .

Case (2)  $b_1 = b_2 + P$

Agent picks firm 1 only when it is deserving (i.e. with probability 0.5). Equilibrium profits in this case are  $\pi_1(b_1; b_2) = \frac{1}{2}(p - b_1)$ .

$\exists \epsilon$  such that  $b_1 = b_2 + \epsilon$  and firm 1 is selected by agent whenever it is deserving. This generates strictly higher profits for the firm 1.

Therefore, there is no Nash equilibrium in pure strategies.

## Proof of Proposition 1.2

We prove Proposition 1.2 in following steps.

Step 1 If firm  $i$  offers  $p - 3P$  firm  $j$  would be indifferent between overbidding and underbidding.

Suppose firm  $i$  bids  $\hat{b}_i$ . Firm  $j$  can bid  $\hat{b}_i + P$  (+ infinitesimally small  $\epsilon$ ) and get selected with probability one or bid  $\hat{b}_i - P$  and get selected with probability  $\frac{1}{2}$ . Firm  $j$  would be indifferent if

$$\frac{1}{2} p(\hat{b}_i - P) = p(\hat{b}_i + P)$$

$$\hat{b}_i = p - 3P$$

If  $b_j > p - 3P$  firm  $j$  offers  $b_j - P$ , whereas if  $b_j < p - 3P$  firm  $j$  offers  $b_j + P$ .

Step 2  $f(b) = 0$  for  $b_j > p - 2P$  and for  $b_j < p - 4P$ .

Suppose  $b_j > p - 2P$ . This can happen only if firm  $i$  offers  $b_j + P$  in response to  $b_j > p - 3P$ . However, for any  $b_j > p - 3P$ , as established in Step 1, firm  $i$  responds by offering  $b_j - P$ . Now suppose  $b_j < p - 4P$ . This implies that firm  $i$  must be offering a bribe of  $b_j - P$  in response to  $b_j < p - 3P$ . But, following Step 1, firm  $i$  should be offering  $b_j + P$ . Both cases lead to contradiction.

Step 3 The equilibrium bribing strategy sets  $S_i$  and  $S_j$  are convex.

We prove this by contradiction. Suppose there is an interval  $I = [b^k; b^h]$ , where  $p - 4P < b^k < b^h < p - 2P$  and firm  $i$  offers  $b_j \in I$  with probability zero.

Claim 1 Firm  $j$  offers  $b_j \in (I + P) \cup (I - P)$  with probability zero.

(a) If  $b^h < p - 3P$ ,  $f(b) = 0$  for  $b_j \in I - P$  (from Step 2)

Suppose firm  $j$  offers  $b_j \in I + P$  with positive probability. Since  $f(b) = 0$  for  $b_j \in I$ , firm  $j$  can offer  $\inf(I + P)$  and make higher profit than offering any  $b_j \in I + P$ . Therefore,  $f(b) = 0$  for  $b_j \in (I + P) \cup (I - P)$ .

(b) Now if  $b^k > p - 3P$ ,  $f(b) = 0$  for  $b_j \in I + P$  (from Step 2)

Suppose firm  $j$  offers  $b_j \in I - P$  with positive probability. Since  $f(b) = 0$  for  $b_j \in I$ , firm  $j$  can offer  $\inf(I - P)$  and make higher profit than offering any  $b_j \in I - P$ . Therefore,  $f(b) = 0$  for  $b_j \in (I + P) \cup (I - P)$ .

(c) Lastly, if  $b^h > p - 3P > b^k$ , firm  $j$  will be better off offering  $\inf(I + P)$  instead of any  $b_j \in (I + P)$ . If  $b_j \in (I - P)$ , since  $f(b) = 0$  for  $b_j < p - 4P$ , it must be that  $b_j \in [p - 4P; b^h - P)$ . Note that, if  $b_j < b^k$  firm  $j$  prefers to make a bribe offer  $b_j = b_j + P > p - 3P$ , and if  $b_j > b^h$  firm  $j$  prefers to make a bribe offer  $b_j = b_j - P > b^h - P$ . Therefore, given  $b_j \notin I$  and firm  $j$  offers  $b_j \in [b^h - P; b^k + P)$ .

Therefore,  $f(b) = 0$  for  $b \in (l + P) \cup (l - P)$ .

Claim 2 Firm  $i$  offering  $b \in I$  with probability zero and firm  $j$  offering  $b \in (l + P) \cup (l - P)$  with probability zero constitutes a contradiction.

Let us represent  $b = b^h$ .

Using equation (1.1), we can write

$$\begin{aligned} \pi_i(b) &= \int_{l-P}^h F_j(b+P) + F_j(b-P) - \int_j(b-P) \frac{p(b)}{2} \\ \pi_i(b^k) &= \int_{l-P}^h F_j(b^k+P) + F_j(b^k-P) - \int_j(b^k-P) \frac{p(b^k)}{2} \end{aligned}$$

But since

$$F_j(b+P) = F_j(b^k+P) ;$$

$$F_j(b-P) = F_j(b^k-P)$$

and

$$\int_j(b^k-P) = 0$$

profits of firm  $i$  when offering  $b^k$  is strictly higher than offering  $b$  contradicting the assumption of an equilibrium.

Step 4 There can be a mass point in the bribe distribution of a firm only at  $b = p - 3P$ .

Suppose firm  $j$  has a mass point at  $b \in [p - 4P; p - 3P)$  equal to  $\epsilon$ .

We can write

$$\begin{aligned} \pi_i(b + P + \epsilon) &= [F_j(b + 2P + \epsilon) + F_j(b + \epsilon) - \int_j(b + \epsilon)] \frac{p(b + P + \epsilon)}{2} \\ \pi_i(b + P - \epsilon) &= [F_j(b + 2P - \epsilon) + F_j(b - \epsilon) - \int_j(b - \epsilon)] \frac{p(b + P - \epsilon)}{2} \end{aligned}$$

Subtracting 2nd equation from 1st (for small enough  $\epsilon > 0$ ), we get

$$\pi_i(b + P + \epsilon) - \pi_i(b + P - \epsilon) > 0$$

and firm  $i$  by shifting some density from bottom to top of  $b + P$  can be strictly better off. So there can not be a mass point at  $b \in [p - 4P; p - 3P)$ .

Now suppose firm  $j$  has a mass point at  $b \in (p - 3P; p - 2P]$ .

As before, we get

$$\pi_i(b - P + \epsilon) - \pi_i(b - P - \epsilon) > 0$$

for small enough". Therefore, there cannot be a mass point in this range as well.

From above it is clear that  $\mu_j$  (and by the same argument  $\mu_i$  also) can have mass point only at  $b = p - 3P$ .

Step 5 Equilibrium profits for both firms are  $\frac{3P}{2}$ .

Firm  $i$  is playing a mixed strategy so it must be indifferent between offering any bribe in its support including  $b_i = p - 3P$ . Profit for firm  $i$  can be written using equation (1.1) as

$$\mu_i(p - 3P) = [F_j(p - 2P) + F_j(p - 4P) - F_j(p - 4P)] \frac{p - (p - 3P)}{2}$$

which simplifies to  $\mu_i = \frac{3P}{2}$ . Proof for firm  $j$  is similar.

Step 6 Both firms have mass points at  $b = p - 3P$ .

Suppose  $\mu_j(p - 3P) = 0$ . Using equation (1.1), we can write

$$\begin{aligned} \mu_i(p - 2P) &= [F_j(p - P) + F_j(p - 3P) - F_j(p - 3P)] \frac{p - (p - 2P)}{2} \\ &= [1 + F_j(p - 3P)] P \end{aligned}$$

But from Step 5,  $\mu_i = \frac{3P}{2}$ . Therefore,  $F_j(p - 3P) = \frac{1}{2}$ .

Now again using equation (1.1), we write

$$\begin{aligned} \mu_i(p - 4P) &= [F_j(p - 2P) + F_j(p - 5P) - F_j(p - 5P)] \frac{p - (p - 4P)}{2} \\ &= F_j(p - 3P) 2P \end{aligned}$$

Substituting  $F_j(p - 3P)$ , we get  $\mu_i = P$ . We got a contradiction. The proof for firm  $i$  is identical.

Step 7 Both firms have point mass of  $\frac{1}{4}$  at  $b = p - 3P$ .

Using equation (1.1), we can write

$$\begin{aligned} \mu_i(p - 2P) &= [1 + F_j(p - 3P) - F_j(p - 3P)] \frac{p - (p - 2P)}{2} \\ &= [1 + F_j(p - 3P) - F_j(p - 3P)] P \end{aligned}$$

similarly,

$$\mu_i(p - 4P) = F_j(p - 3P) 2P$$

Using the result from Step 5, we solve two equations to get  $F_i(p - 3P) = \frac{3}{4}$ , and  $F_j(p - 3P) = \frac{1}{4}$ . The proof for firm  $i$  is identical.

Step 8 Equilibrium bribing strategy for firm  $j$  is given by

$$F_j(b) = \begin{cases} < \frac{3P}{p - b - P} & \text{if } p - 4P < b < p - 3P \\ \frac{3P}{p - b + P} & \text{if } p - 2P < b < p - 3P \end{cases}$$

Using equation (1.1) and Step 5, we can write

$$[F_j(b - P) + F_j(b + P) - F_j(b - P)] \frac{p - b}{3P} = 1$$

Using above equation and the results from Step 2 and Step 7, we can write

$$\begin{aligned} F_j(b + P) &= \frac{3P}{p - b} & \text{if } b_i < p - 3P \\ F_j(b - P) &= \frac{3P}{p - b} - 1 & \text{if } p - 2P > b_i > p - 3P \\ F_j(b - P) &= \frac{3}{4} & \text{if } b_i = p - 2P \end{aligned}$$

Applying appropriate transformations to above three equations proves Step 8.

### Proof of Proposition 1.3

Since the proof of Proposition 1.3 is similar to that of Proposition 1.2, we only provide the steps here.

Step 1 If a firm makes a bribe offer of  $\frac{p}{2} - P$  the other firm will be indifferent between offering a bribe higher by  $P$  and offering no bribe. It prefers to overbid on smaller offers. This holds given  $\frac{p}{4P}$ .

Step 2 If a firm makes a bribe offer  $b - P$  the other firm prefers to offer  $b - P$ . This also holds given  $\frac{p}{4P}$ .

Step 3  $f(b) = 0$  for  $b > \frac{p}{2}$  and for  $b < \frac{p}{2} - P$ ;  $P$ .

Step 4 There are no holes in the interval  $0; \frac{p}{2} - P$  and in the interval  $P; \frac{p}{2}$ .

Step 5 There is no density at  $b = P$  for both firms. Because, if there is a firm can strictly benefit by moving density from  $b = P$  to  $b = \frac{p}{2} - P$ .

Step 6 Both firms make profits of  $\frac{p+2P}{4}$ . This is obtained by evaluating equation (1.1) at  $b = \frac{p}{2} - P$ .

Step 7 There is a mass point of  $\frac{4P - p}{2(p - P)}$  at  $b = 0$  and a mass point of  $\frac{p - 2P}{2p}$  at  $b = \frac{p}{2} - P$  for both firms.

Step 8 Using equation (1.1) and Step 6 we can write

$$[F_j(b - P) + F_j(b + P) - f_j(b - P)] \frac{2(p - b)}{p + 2P} = 1$$

Using Step 2, Step 7, above equation and applying appropriate transformations we get the cdf given in Proposition 1.3.

### Proof of Proposition 1.4

(a)  $\frac{p}{4P}$  case

Using equation (1.3), we can write  $Pr$  as

$$\begin{aligned} Pr &= \frac{1}{2} \int_{p-4P}^{p-3P} [1 - F_j(b + P)] f_i(b) db + \frac{1}{2} \int_{p-3P}^{p-2P} [F_j(b - P)] f_i(b) db \\ &= \frac{1}{2} \int_{p-4P}^{p-3P} \left[ 1 - \frac{3P}{p-b} \right] \frac{3P}{(p-b-P)^2} db \\ &\quad + \frac{1}{2} \int_{p-3P}^{p-2P} \frac{3P}{p-b} \left[ 1 - \frac{3P}{(p-b+P)^2} \right] db \end{aligned}$$

where  $f_i(b)$  is the pdf of the bribe offer and is obtained by differentiating the cdf described in Proposition 1.2. Simplifying the above equation we get  $Pr = 9 \ln \frac{9}{8} - 1$ .

(b)  $\frac{p}{4P}$   $\frac{p}{2P}$  case

Using equation (1.3), we can write  $Pr$  as

$$\begin{aligned} Pr &= \frac{1}{2} \int_0^2 f_i(0) [1 - F_j(P)] + \int_0^{\frac{p}{2}-P} [1 - F_j(b + P)] f_i(b) db \\ &\quad + \int_P^{\frac{p}{2}} [F_j(b - P)] f_i(b) db \\ &= \frac{1}{2} \int_0^2 \frac{4P-p}{2(p-P)} \left[ 1 - \frac{p+2P}{2p} \right] + \int_0^{\frac{p}{2}-P} \frac{p+2P}{2(p-b)} \left[ 1 - \frac{p+2P}{2(p-b+P)^2} \right] db \\ &\quad + \int_P^{\frac{p}{2}} \frac{p+2P}{2(p-b)} \left[ 1 - \frac{p+2P}{2(p-b+P)^2} \right] db \end{aligned}$$

where pdf is obtained by differentiating the cdf described in Proposition 1.3. Simplifying the above expression gives

$$Pr = \frac{(p - 2P)(p + 4P)}{2pP} + \frac{(p + 2P)^2}{2 \cdot 2P^2} \ln \frac{(p + 2P)(p - P)}{p^2} \quad (A.1)$$

It is straightforward to check that

$$Pr \Big|_{p=0} = \frac{p}{4P} = 9 \ln \frac{9}{8} - 1 ; \quad Pr \Big|_{p=2P} = \frac{p}{2P} = 0$$

and,

$$\frac{\partial Pr}{\partial p} \Big|_{p=\frac{p}{4P}} > 0 ; \quad \frac{\partial Pr}{\partial p} \Big|_{p=\frac{p}{2P}} < 0$$

The maxima of the Pr function is numerically calculated. It is found to be at  $p = \frac{p}{3P}$ .

(c)  $\frac{p}{2P}$  case

Firms do not offer bribes if monitoring is sufficiently large ( $\frac{p}{2P}$ ). The agent, therefore, does not select a non-deserving firm. The probability Pr is zero in this range.

### Proof of Proposition 1.5

From equation (1.5),  $\Delta$  is zero for  $\frac{p}{4P}$ . If  $\frac{p}{4P} < \frac{p}{2P}$ , substituting the expressions of  $G_j(\alpha)=0$  and  $G(\alpha=0)_j(\alpha)=0$  from equation (1.5) to equation (1.6) we get

$$\Delta = \frac{[p^2 - 8 \cdot 2P^2 + 2pP (18 \ln(\frac{9}{8}) - 1)]}{4pP} - \frac{(p+2P)^2}{4 \cdot 2P^2} \ln \frac{(p-P)(p+2P)}{p^2} - (2 - 1)v$$

The difference  $\Delta$  is negative for all  $\frac{p}{4P} < \frac{p}{2P}$ . It is zero at  $\frac{p}{4P} = \frac{p}{2P}$  and increases to  $9 \ln \frac{9}{8} - 1 - (2 - 1)v$  at  $\frac{p}{4P} = \frac{p}{2P}$ .

In the range  $\frac{p}{4P} < \frac{p}{2P}$ , it can be shown using equation (1.5) that  $\Delta$  stays at  $9 \ln \frac{9}{8} - 1 - (2 - 1)v$ . The buyer would, therefore, set  $\alpha$  only from the set  $\{0; \frac{p}{2P}\}$ . Since  $\Delta$  the extra benefit of setting a non-zero  $\alpha$ , a cost of monitoring higher than  $\Delta$  would discourage the buyer from setting that  $\alpha$ . If it is the case for all  $\frac{p}{4P} < \frac{p}{2P}$  the buyer sets the monitoring at zero. If  $\Delta < 0$  for some  $\frac{p}{4P} < \frac{p}{2P}$ , the buyer sets a non-zero  $\alpha$  which maximizes her payoff.



## Proof of Proposition 1.6

Note that the difference in buyer's payoff, when setting  $c = \frac{p}{2P}$  and when setting  $c = 0$ , is given by  $9 \ln \frac{9}{8} - 1 - (2 - 1)v$  if there is no unilateral control on firm  $i$ . It is given by  $\frac{1}{2}(2 - 1)v$  if the firm  $i$  is unilaterally controlled. We now look at the three cases.

(a)  $c = \frac{p}{2P} - 9 \ln \frac{9}{8} - 1 - (2 - 1)v$  case

The profits of firm  $i$  under unilateral control is  $\frac{p}{2}$ . In the absence of the unilateral control firm  $i$ 's profit is  $\frac{p+2}{4}P$ , where  $2 \sim \frac{p}{2P}$ . The maximum profit of firm  $i$  in absence of unilateral control could be  $\frac{p}{2}$  if  $c = \frac{p}{2P}$ . Therefore, in this range of cost curves the profit for firm  $i$  as a result of unilateral control on bribes will either not change or increase. It can not decrease.

(b)  $9 \ln \frac{9}{8} - 1 - (2 - 1)v < c = \frac{p}{2P} < \frac{1}{2}(2 - 1)v$  case

The firm  $i$  still makes  $\frac{p}{2}$  under unilateral control since buyer eliminates corruption. However, if there is no unilateral control in this range of cost curves the buyer does not find it optimal to completely eliminate corruption resulting in profits of strictly lower than  $\frac{p}{2}$ . Here, the controlled firm strictly benefits as a result of unilateral control.

(c)  $c = \frac{p}{2P} - \frac{1}{2}(2 - 1)v$  case

The firm  $i$  makes zero profits under unilateral control. In the absence of the unilateral control the buyer does not find it optimal to completely eliminate corruption. However, for cost curve that become very steep as they approach  $\frac{p}{2P}$  the buyer might find it optimal to set a  $2 \sim \frac{p}{2P}$ . Therefore, in this case the controlled firm will either make the same or lower but not higher profits compared to if it was not controlled.

## Appendix B

### Appendix to Chapter 2

#### Proof of Proposition 2.1

The product safety and the consumer effort are substitutes. We compare the profit of the firm in the case of certification and in the case of no certification to arrive at the following results for the given range of:

(a) if  $\beta < \beta_{NS}$

The consumer takes effort  $e_i$  regardless of firm's certification decision or outcome. The profits of the firm if it goes for certification and if it does not go for certification are given by

$$C = V \left[ (1 - \beta_h) + (1 - \beta) (1 - \beta_l) \right] (1 + \beta) L - k$$

$$NC = V \left[ (1 - \beta_h) + (1 - \beta) (1 - \beta_l) \right] (1 + \beta) L$$

Comparing the profit of the firm upon certification and upon no certification, we get the incentive for certification

$$K = k$$

The firm optimally chooses not to go for certification for any  $k > 0$ .

(b) if  $\beta_{NS} < \beta < \beta_{\sim}$

The consumer takes effort  $e_i$  if the firm chooses of no certification or if the firm gets classified as type  $h$  by the certification agency but takes effort  $e_h$  if the firm gets classified as type  $l$  by the certification agency. A comparison of the profit of the firm upon certification and upon no certification gives the incentive for certification

$$K = [ (1 - \alpha) + (1 - \beta) ] + [ (1 - \alpha)(1 - \beta_h) + (1 - \alpha)(1 - \beta_l) ] "L \quad k$$

The firm optimally chooses certification for small enough  $k$ .

Also,

$$\frac{\partial K}{\partial \alpha} = [ (1 - \alpha)(1 - \beta_h) + (1 - \alpha)(1 - \beta_l) ] L > 0$$

$$\frac{\partial K}{\partial \beta} = [ (1 - \alpha) + (1 - \beta) ] < 0$$

$$\frac{\partial K}{\partial \beta_h} = (2 - 1) + [(1 - \alpha)(1 - \beta_h) - (1 - \alpha)(1 - \beta_l)] "L < 0$$

$$\frac{\partial K}{\partial \beta_l} = (2 - 1) - [ (1 - \alpha)(1 - \beta_h) - (1 - \alpha)(1 - \beta_l) ] "L > 0$$

(c) if  $\beta < \beta_S$

The consumer takes effort  $e_h$  if the firm chooses of no certification or if the firm gets classified as type  $l$  by the certification agency but takes effort  $e_l$  if the firm gets classified as type  $h$  by the certification agency. A comparison of the profit of the firm upon certification and upon no certification gives the incentive for certification

$$K = [ (1 - \alpha) + (1 - \beta)(1 - \beta_h) ] + [ (1 - \alpha)(1 - \beta_h) + (1 - \alpha)(1 - \beta_l) ] "L \quad k$$

The firm optimally chooses certification for small enough  $k$ .

Also,

$$\frac{\partial K}{\partial \alpha} = [ (1 - \alpha)(1 - \beta_h) + (1 - \alpha)(1 - \beta_l) ] L < 0$$

$$\frac{\partial K}{\partial \beta} = (1 - \alpha) + (1 - \beta)(1 - \beta_h) > 0$$

$$\frac{\partial K}{\partial \beta_h} = (2 - 1) - [ (1 - \alpha)(1 - \beta_h) - (1 - \alpha)(1 - \beta_l) ] "L > 0$$

$$\frac{\partial K}{\partial \beta_l} = (2 - 1) - [ (1 - \alpha)(1 - \beta_h) - (1 - \alpha)(1 - \beta_l) ] "L > 0$$

(d) if  $\beta > \beta_S$

The consumer takes effort  $e_h$  regardless of the certification decision or the outcome of certification. A comparison of the profit of the firm upon certification and upon no certification gives the incentive for certification

$$K = k$$

The firm optimally chooses not to go for certification for any  $k > 0$ .

## Proof of Proposition 2.2

The product safety and the consumer effort are complements. In this case, the analysis proceeds exactly as in the case where they are substitutes therefore we only report the results.

The incentive for certification in different ranges of  $\theta$  is given by

$$K = \begin{cases} \frac{8}{k} & \text{if } \theta < \theta_S \\ \frac{v}{k} \left[ \frac{v + (1 - \theta)(1 - \theta)}{v + (1 - \theta_h)(1 - \theta_l)} \right] + \frac{v}{k} \left[ \frac{v + (1 - \theta_h)(1 - \theta_l)}{v + (1 - \theta_h)(1 - \theta_l)} \right] L & \text{if } \theta_S < \theta < \theta^* \\ \frac{v}{k} \left[ \frac{v + (1 - \theta)(1 - \theta)}{v + (1 - \theta_h)(1 - \theta_l)} \right] + \frac{v}{k} \left[ \frac{v + (1 - \theta_h)(1 - \theta_l)}{v + (1 - \theta_h)(1 - \theta_l)} \right] L & \text{if } \theta^* < \theta < \theta_{NS} \\ \frac{8}{k} & \text{if } \theta > \theta_{NS} \end{cases}$$

If  $\theta_S < \theta < \theta^*$ , we get  $\frac{\partial K}{\partial \theta} < 0$ ,  $\frac{\partial K}{\partial v} < 0$ ,  $\frac{\partial K}{\partial L} > 0$  and  $\frac{\partial K}{\partial \theta} > 0$  whereas if  $\theta^* < \theta < \theta_{NS}$ , we get  $\frac{\partial K}{\partial \theta} > 0$ ,  $\frac{\partial K}{\partial v} > 0$ ,  $\frac{\partial K}{\partial L} < 0$  and  $\frac{\partial K}{\partial \theta} > 0$ .

## Proof of Proposition 2.3-2.5

Here, we provide the proof of the case in which the product safety and the consumer effort are substitutes (Proposition 2.4). The proofs of the independent case (Proposition 2.3) and the complements case (Proposition 2.5) are similar.

### Certification equilibrium

We first look at the equilibrium in which the firm goes for certification. The belief of the consumer about the certification decision of the firm is consistent with the certification decision of the firm. The conditions under which equilibrium exist can be derived for the different ranges of parameters as:

(a) if  $\theta < \theta_{NS}$

The consumer takes the effort regardless of the certification outcome. The firm sets price  $p_S$  upon getting classified as type  $h$  and price  $p_{NS}$  upon getting classified as type  $l$ , which extracts all the consumer surplus in respective situations. These prices are given by

$$p_S = v \frac{\theta \left[ \frac{(1 - \theta_h) + (1 - \theta)(1 - \theta_l)}{v + (1 - \theta_h)(1 - \theta_l)} \right]^\#}{v + (1 - \theta_h)(1 - \theta_l)} \left[ (1 + \theta) L - L_f \right]$$

$$p_{NS} = v \frac{(1 - \alpha_h)(1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_l)}{(1 - \alpha_h) + (1 - \alpha_l)} [(1 + \alpha) L_c - L_f]$$

The equilibrium profit of the firm can be written as

$$\pi_c = v \left[ (1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_l) \right] (1 + \alpha) L_c - k$$

If the firm deviates to no certification it would set the same price as it would have set upon getting classified as type<sub>l</sub> by the certification agency. If it sets a higher price upon getting classified as type<sub>l</sub> consumers, who believe that the firm has gone for certification, would not buy the product. Any lower price is dominated by the equilibrium no sticker price. We can write the profit of the firm if it deviates to no certification as

$$\pi_d = v \frac{(1 - \alpha_h)(1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_l)}{(1 - \alpha_h) + (1 - \alpha_l)} [(1 + \alpha) L_c - L_f]$$

$$[ (1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_l) ] L_f$$

In order for the equilibrium to exist, the deviation must not be more profitable. This gives

$$k < \frac{(1 - \alpha_h)(2 - \alpha_l)(\alpha_h - \alpha_l)(L_c + \alpha L)}{(1 - \alpha_h) + (1 - \alpha_l)}$$

(b) if  $\alpha_{NS} < \alpha < \alpha_S$

The consumer takes effort<sub>h</sub> if the product is classified as type<sub>h</sub> and takes the effort<sub>h</sub> if the product is classified as type<sub>l</sub> by the certification agency. The equilibrium prices and profits as well as profits under deviation to no certification can be written as before. A comparison of firm profits under equilibrium and under deviation to no certification gives the condition

$$k < \frac{(1 - \alpha_h)(2 - \alpha_l)(\alpha_h - \alpha_l) L_c}{(1 - \alpha_h) + (1 - \alpha_l)} + K_s$$

where

$$K_s = \left( (1 + \alpha)(1 - \alpha) \right) \alpha L \left[ (1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_l) \right]$$

(c) if  $\alpha > \alpha_S$

The consumer takes effort<sub>h</sub> regardless of the certification outcome.

The firm sets a price  $p_S$  upon getting classified as type<sub>h</sub> and a price  $p_{NS}$  upon getting classified as type<sub>l</sub>, which extracts all the consumer surplus in respective situations. These prices are given by

$$p_S = v \frac{(1 - \theta_h) + (1 - \theta_l)(1 - \theta_i)}{(1 - \theta_h) + (1 - \theta_l)} L_c$$

$$p_{NS} = v \frac{(1 - \theta_l)(1 - \theta_h) + (1 - \theta_i)(1 - \theta_l)}{(1 - \theta_l) + (1 - \theta_i)} L_c$$

The equilibrium profit of the firm can be written as

$$\pi_c = v \left[ (1 - \theta_h) + (1 - \theta_l)(1 - \theta_i) \right] L_c - k$$

If the firm deviates to no certification, it would set the same price as it would have set upon getting classified as type<sub>l</sub> by the certification agency. If it sets a higher price upon getting classified as type<sub>l</sub> consumers, who believe that the firm has gone for certification, would not buy the product. Any lower price is dominated by the equilibrium no sticker price. We can write the profit of the firm if it deviates to no certification as

$$\pi_d = v \frac{(1 - \theta_l)(1 - \theta_h)}{(1 - \theta_l) + (1 - \theta_i)} + \frac{(1 - \theta_l)(1 - \theta_l)}{(1 - \theta_l) + (1 - \theta_i)} L_c$$

In order for this equilibrium to exist, the deviation must not be more profitable. This gives

$$k < \frac{(1 - \theta_l)(2 - \theta_l)(1 - \theta_h - \theta_i) L_c}{(1 - \theta_l) + (1 - \theta_i)}$$

### No certification equilibrium

Now we look at the equilibrium in which the firm does not go for certification and the consumer believes that the firm has not gone for certification. If a firm deviates to certification from this equilibrium, the consumers upon observing the sticker would believe that the firm has gone for certification but upon not observing the sticker they would still believe that the firm has not gone for certification. They would, therefore, be willing to pay a higher price than if they could observe that the firm has actually been denied a sticker. The firm benefits from this information advantage but incurs a cost of certification if it deviates. A comparison of profits under equilibrium and deviation to certification reveals that the equilibrium exists if

$$k > \begin{cases} \frac{\theta_l}{\theta_h} (1 - \theta_l)(2 - \theta_l)(1 - \theta_h - \theta_i)(L_c + \theta_l L) & \text{if } \theta_l < \theta_h \\ (1 - \theta_l)(2 - \theta_l)(1 - \theta_h - \theta_i) L_c + K_s & \text{if } \theta_l < \theta_h < \theta_s \\ (1 - \theta_l)(2 - \theta_l)(1 - \theta_h - \theta_i) L_c & \text{if } \theta_l > \theta_s \end{cases}$$

### Mixed strategy equilibrium

Suppose in equilibrium, the firm goes for certification with probability  $\alpha$ . Upon getting classified as type  $h$  the firm sets a price  $p_S$  and upon getting classified as type  $l$  the firm sets a price  $p_{NS}^0$  that extracts all the consumer surplus. The beliefs of the consumer can be written as

$$\begin{aligned} (\alpha; S; p_S) &= \frac{\alpha(1-\beta_h)(1-\beta_l)}{\alpha(1-\beta_h)(1-\beta_l) + (1-\alpha)(1-\beta_h)(1-\beta_l)} \\ (\alpha; NS; p_{NS}^0) &= \frac{(1-\alpha)(1-\beta_h)(1-\beta_l)}{(1-\alpha)(1-\beta_h)(1-\beta_l) + \alpha(1-\beta_h)(1-\beta_l)} \end{aligned}$$

The consumer after buying a product with no safety certification sticker puts effort  $e_h$  if  $\alpha > \alpha^0$  where

$$\alpha^0 = \frac{h}{L} \frac{(1-\beta_h)(1-\beta_l)}{(1-\beta_h)(1-\beta_l) + \alpha(1-\beta_h)(1-\beta_l)}$$

We derive the conditions for this equilibrium for different range of parameters.

(a) if  $\alpha < \alpha_{NS}$

The equilibrium prices  $p_S$  and  $p_{NS}^0$  are given by

$$\begin{aligned} p_S &= v \int_h (\alpha; S; p_S)(1-\beta_h) + (1-\alpha)(1-\beta_h)(1-\beta_l) (L_f + "L) \\ p_{NS}^0 &= v \int_l (\alpha; p_{NS}^0)(1-\beta_h) + \alpha(1-\beta_h)(1-\beta_l) (L_c + "L) \end{aligned}$$

The firm must be indifferent between certification and no certification. Equating the profits under certification and no certification, we get the condition for equilibrium as

$$k = [\alpha + (1-\alpha)(1-\beta_l)] p_S - p_{NS}^0$$

where  $p_S$  and  $p_{NS}^0$  are as given above. We note that when  $\beta_l = 0$  the condition reduces to  $k = (1-\beta_h)(2-\beta_h)(1-\beta_h)(1-\beta_l) ("L + L_c)$  whereas when  $\beta_l = 1$  the condition reduces to  $k = \frac{(1-\beta_h)(2-\beta_h)(1-\beta_h)(1-\beta_l) ("L + L_c)}{(1-\beta_h)(1-\beta_l)}$ . The mixed strategy equilibria exist in the same region where both certification and no certification equilibria exist.

(b) if  $\alpha_{NS} < \alpha < \alpha^0$ , we must distinguish between  $\alpha > \alpha^0$  when consumer takes effort  $e_h$  upon buying a product with no safety sticker and  $\alpha < \alpha^0$  when consumer takes effort  $e_l$  upon buying a product with no safety sticker. Although the calculations are a little more involved than previous case, it can be shown that the mixed strategy equilibria exist in the same region where both certification and no certification equilibria exist.

(c) if  $\alpha < \alpha_S$  the analysis is similar to above case. The consumer takes effort upon buying a product with no safety sticker in this range. As in the previous case, we find that the mixed strategy equilibria exist in the same region where both certification and no certification equilibria exist.

(d) if  $\alpha > \alpha_S$  the analysis and the findings are similar to above cases.

## Heterogeneous Consumer Market

The firm sets one of the two candidate prices  $p_h$  and  $p_l$ . When the firm sets the high price  $p_h$ , it extracts all the surplus of the high valuation consumers but loses the demand of the low valuation consumers. When it sets the low price corresponding to expected surplus of the low valuation consumers it gets all the demand but fails to extract all the surplus. The high valuation consumers get positive surplus. We first consider the case of no certification. The two candidate prices corresponding to surplus of the high valuation consumers and low valuation consumers can be written as

$$p_h = v_h \left[ (1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_i) \right] L_c$$

$$p_l = v_l \left[ (1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_i) \right] L_c$$

Profit of the firm if it sets  $p_h$  and sells only to high valuation consumers is given by

$$\pi_h = [v_h \left[ (1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_i) \right] (L_c + L_f)]$$

Profit of the firm if it sets  $p_l$  and sells to all the consumers can similarly be written as

$$\pi_l = v_l \left[ (1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_i) \right] (L_c + L_f)$$

Firm will set  $p_h$  if its profit from setting  $p_h$  and selling to only high valuation consumers is higher than the profit from charging  $p_l$  and selling to all the consumers else it will set  $p_l$ . A comparison of  $\pi_h$  and  $\pi_l$  implies that the firm sets  $p_h$  when  $\alpha > \tilde{\alpha}$  and sets  $p_l$  when  $\alpha < \tilde{\alpha}$  where  $\tilde{\alpha}$  is defined as

$$\tilde{\alpha} = \frac{v_l \left[ (1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_i) \right] (L_c + L_f)}{v_h \left[ (1 - \alpha_h) + (1 - \alpha_l)(1 - \alpha_i) \right] (L_c + L_f)} \quad (\text{B.1})$$

We now consider the case when firm decides to go for certification. With probability  $\alpha + (1 - \alpha)(1 - \alpha_i)$  the firm gets classified as type  $h$ , and with probability



$(1 - \alpha) + (1 - \beta)$  the firm gets classified as type  $I_1$ . The firm in each of the situations decides if it wants to sell only to high valuation consumers or to all the consumers. The analysis in this case is similar to the case of no certification presented above. We find that when the firm is classified as type  $h$  by the certification agency, it sets a high price and sells only to high valuation consumers when  $\tilde{v}_S > \tilde{v}_{NS}$  and sets a low price and sells to all the consumers when  $\tilde{v}_S < \tilde{v}_{NS}$ . We can define  $\tilde{v}_S$  similar to  $\tilde{v}$  by replacing  $\tilde{v}$  in equation (B.1) by  $\frac{(1 - \alpha)(v_h) + (1 - \beta)(v_l)}{(1 - \alpha) + (1 - \beta)}$ . When the firm is classified as type  $I_1$  by the certification agency, it sets a high price and sells only to high valuation consumers when  $\tilde{v} > \tilde{v}_{NS}$  and sets a low price and sells to all the consumers when  $\tilde{v} < \tilde{v}_{NS}$ . We can define  $\tilde{v}_{NS}$  by replacing  $\tilde{v}$  in equation (B.1) by  $\frac{(1 - \alpha)(v_h) + (1 - \beta)(v_l)}{(1 - \alpha) + (1 - \beta)}$ .

Note that since

$$\frac{(1 - \alpha)(v_h) + (1 - \beta)(v_l)}{(1 - \alpha) + (1 - \beta)} > \tilde{v} > \frac{(1 - \alpha)(v_h) + (1 - \beta)(v_l)}{(1 - \alpha) + (1 - \beta)}$$

we have

$$\tilde{v}_S > \tilde{v} > \tilde{v}_{NS}$$

Now we look at the firm's incentive to go for certification,  $K$ , which is calculated by subtracting expected profit of the firm without certification from firm's expected profit when it goes for certification.

### Certification common knowledge

The certification decision as well as the certification outcome is observed by the consumer. We calculate the incentive for certification by subtracting the firm's profit if it decides not to go for certification from the profit if it decides to go for certification in different ranges of  $\tilde{v}$ .

$$K = \begin{cases} k & \text{if } \tilde{v} < \tilde{v}_{NS} \\ \frac{\alpha}{\alpha + \beta} [(1 - \alpha)(v_h) + (1 - \beta)(v_l)](v_h - v_l) + L_1^e k & \text{if } \tilde{v}_{NS} < \tilde{v} < \tilde{v}_S \\ \frac{\beta}{\alpha + \beta} [(1 - \alpha)(v_h) + (1 - \beta)(v_l)](v_l - v_h) - L_2^e k & \text{if } \tilde{v} < \tilde{v} < \tilde{v}_S \\ k & \text{if } \tilde{v} > \tilde{v}_S \end{cases}$$

where  $L_1^e = \frac{\alpha}{\alpha + \beta} [(1 - \alpha)(v_h) + (1 - \beta)(v_l)](1 - \alpha)L$

and  $L_2^e = \frac{\beta}{\alpha + \beta} [(1 - \alpha)(v_h) + (1 - \beta)(v_l)](1 - \beta)L$ .

Note that  $\frac{\partial K}{\partial \tilde{v}} > 0$  for  $\tilde{v}_{NS} < \tilde{v} < \tilde{v}_S$  and  $\frac{\partial K}{\partial \tilde{v}} < 0$  for  $\tilde{v} < \tilde{v}_{NS}$  or  $\tilde{v} > \tilde{v}_S$ .

### Certification private information

The proof is similar to Proposition 2.4. We only provide results here. The equilibrium involves the firm going for certification for

$$k < \frac{\frac{\delta}{\omega} \frac{(1-\alpha)(2-\alpha)(\theta_h - \theta_l)L_c}{(1-\alpha)+(1-\alpha)}}{\frac{\delta}{\omega} \frac{(1-\alpha)(2-\alpha)(\theta_h - \theta_l)L_c}{(1-\alpha)+(1-\alpha)}} + [\alpha + (1-\alpha)(1-\alpha)](v_l - v_h) L_2^e \quad \begin{array}{l} \text{if } \alpha < \tilde{\alpha}_{NS} \\ \text{if } \tilde{\alpha}_{NS} < \alpha < \tilde{\alpha}_S \\ \text{if } \alpha > \tilde{\alpha}_S \end{array}$$

and the firm not going for certification for

$$k > \frac{\frac{\delta}{\omega} \frac{(1-\alpha)(2-\alpha)(\theta_h - \theta_l)L_c}{(1-\alpha)+(1-\alpha)}}{\frac{\delta}{\omega} \frac{(1-\alpha)(2-\alpha)(\theta_h - \theta_l)L_c}{(1-\alpha)+(1-\alpha)}} + [\alpha + (1-\alpha)(1-\alpha)](v_l - v_h) L_2^e \quad \begin{array}{l} \text{if } \alpha < \tilde{\alpha} \\ \text{if } \tilde{\alpha} < \alpha < \tilde{\alpha}_S \\ \text{if } \alpha > \tilde{\alpha}_S \end{array}$$

There also exist mixed strategy equilibria in which the firm randomizes between certification and no certification. Such equilibria exist only in the region where both certification and no certification equilibria exist.

### Informed Firm and Uninformed Consumers

First of all, we explore the possibility of a separating equilibrium. Suppose the type  $\theta_h$  firm goes for certification and sets a price  $p_h$  whereas the type  $\theta_l$  firm does not go for certification and sets price  $p_l$ . We write the consumer's expected utility from buying a product as:

$$U = [v - p - [b(S; p)(1 - \theta_h) + (1 - b(S; p))(1 - \theta_l)]L_c]s \quad (\text{B.2})$$

where  $b(S; p)$  is the belief of the consumer about the firm type  $\theta_h$  given the certification outcome  $S$  and the price  $p$  set by the firm. The consumer's strategy is to choose a probability  $s$  with which she purchases the product. In a separating equilibrium using equation (B.2), the prices set by type  $\theta_h$  and type  $\theta_l$  firm can be written as:

$$p_h = v - (1 - \theta_h)L_c$$

$$p_l = v - (1 - \theta_l)L_c$$

The consumer buys with probability one at price  $p_l$ . However, at a price  $p_h$  she must buy with probability lower than one in order to stop the type  $i$  firm from mimicking type  $h$  firm. We represent by  $s_h^S$  and  $s_h^{NS}$  the probability with which the consumer buys upon observing price  $p_h$  set by a firm which is certified as type  $h$  and by a firm which is not certified as type  $h$  respectively. Now consider two possible deviations: 1) type  $h$  firm mimics  $p_l$  whenever certification agency classifies it as type  $i$ ; and 2) type  $i$  firm deviates to  $p_h$ . There exists no strategy for the consumer that can stop the firm from making both the deviations, therefore, such an equilibrium can not exist.

In fact, there does not exist any separating equilibrium. The intuition is that when the consumer reduces the purchase probability it is the type  $h$  firm which gets hurt more. We note that even semi-separating equilibrium in which one type sets a particular price and the other type is indifferent between setting two prices does not exist. A strategy that makes one type indifferent between setting the two prices makes the other type strictly prefer deviation.

We now look at the equilibria involving pure pooling in prices.

(i) Consider an equilibrium in which the firm goes for certification regardless of its type and sets price  $p_S$  if it gets a certification sticker and price  $p_{NS}$  in absence of certification sticker. The consumer buys with probability one. As the firm of type  $h$  gets a sticker with probability  $\alpha$  and a firm of type  $i$  gets the sticker with probability  $1 - \alpha$ , we can write beliefs of the consumer using Bayes' rule. The belief of the consumer about firm being of type  $h$  on observing the sticker is  $b(S) = \frac{\alpha}{\alpha + (1 - \alpha)(1 - \beta)}$ , and upon not observing the sticker is  $b(NS) = \frac{(1 - \alpha)\beta}{(1 - \alpha)\beta + (1 - \alpha)}$ . The equilibrium price in presence of the sticker must be in the interval  $[(1 - \alpha)L_f; p_S]$  and in the absence of the sticker must be in the interval  $[(1 - \alpha)L_f; p_{NS}]$  in order for the consumer to be willing to buy and for the firm to be willing to sell in equilibrium. The price  $p_S$  is given by

$$p_S = v - b(S)(1 - \alpha)L_c - [1 - b(S)](1 - \alpha)L_c$$

and  $p_{NS}$  is given by

$$p_{NS} = v - b(NS)(1 - \alpha)L_c - [1 - b(NS)](1 - \alpha)L_c$$

It may be noted that technically any price in the interval  $[v - (1 - \alpha)L_c; p_S]$  with certification sticker and in the interval  $[v - (1 - \alpha)L_c; p_{NS}]$  when sticker is not available survives D1 refinement and can be supported as an equilibrium. This is because the set of mixed strategy best responses that make a type  $h$  firm prefer a deviation to its equilibrium strategy is a proper subset of the set of mixed strategy best responses that make a type  $i$  firm prefer the same deviation to its equilibrium strategy.

The deviations are, therefore, believed to be coming from type  $i$  with probability one and are rejected. However, we hereafter pick only the most efficient equilibrium corresponding to the highest price for the analysis.

In equilibrium both types of the firm (in particular the type  $i$  firm) must also prefer certification to no certification. However, since the certification decision is private if a firm deviates to no certification from this equilibrium it must set the same price as set by a firm upon being denied a certification sticker. This leads us to the condition  $k = (1 - \alpha)(p_S - p_{NS})$ . Substituting for  $p_S$  and  $p_{NS}$ , we get

$$k = \frac{\alpha}{\alpha + (1 - \alpha)(1 - \alpha)} = \frac{(1 - \alpha)}{(1 - \alpha) + (1 - \alpha)} (1 - \alpha)(\alpha - \beta)L_c$$

It can be shown that there are other equilibria, given below, that exist depending on the market conditions. The proofs are similar to one provided above.

(ii) The firm of type  $h$  goes for certification whereas the type  $i$  is indifferent between certification and no certification. Suppose, the type  $i$  firm goes for certification with probability  $\alpha_1$ . This equilibrium exists for

$$k = \frac{\alpha}{\alpha + (1 - \alpha)\alpha_1(1 - \alpha)} = \frac{(1 - \alpha)}{(1 - \alpha) + (1 - \alpha)(\alpha_1 + 1 - \alpha_1)} (1 - \alpha)(\alpha - \beta)L_c$$

(iii) The type  $h$  firm goes for certification but type  $i$  firm does not. This equilibrium exists for

$$k = \frac{(1 - \alpha)(\alpha - \beta)L_c}{(1 - \alpha) + 1}$$

(iv) The type  $h$  firm is indifferent between certification and no certification whereas type  $i$  firm does not go for certification. Suppose, type  $h$  firm goes for certification with probability  $\alpha_2$ . This equilibrium exists for

$$k = \frac{(1 - \alpha)(\alpha - \beta)L_c}{1 + [\alpha_2(1 - \alpha) + 1 - \alpha_2]}$$

(v) The firm does not go for certification regardless of its type. We assume that the out of equilibrium beliefs in this case are that if the consumer observes a sticker she believes with probability one that the firm is high type. This equilibrium exists for

$$k = (1 - \alpha)(\alpha - \beta)L_c$$

<sup>1</sup>[21] also choose the highest price equilibrium from a price interval when equilibria corresponding to all of them survive D1 refinement.

## Appendix C

### Appendix to Chapter 3

The Gibbs sampler proceeds by drawing iteratively from the conditional distributions described below, in the same order as they are presented.

#### 1. Generating $Y$

The posterior distribution for  $Y$  given the data and other parameters is

$$p(Y; R, \mu, \sigma^2; K) \propto \prod_{j=1}^K [Y_j | \mu, \sigma^2]$$

Because  $y_{m,t}$  are conditionally independent for different movies, we can generate  $y_{m,t}$  independently for each movie. Within a movie,  $y_{m,t}$  have a hierarchical structure due to their autocorrelation:

$$p(y_{m,t} | y_{m,0}) = \prod_{t=2}^{T_m} p(y_{m,t} | y_{m,t-1})$$

The vector  $Y$  is sequentially generated from  $y_{m,1}$  to  $y_{m,T_m}$ . Define

$$y_{m,t} = X_{m,t} \mu + e_{m,t} \quad (C.1)$$

and

$$e_{m,t} = y_{m,t} - X_{m,t} \mu \quad \text{for } t = 1, \dots, T_m \quad (C.2)$$

where  $\mu$  and  $y_{m,t}$  are the most recent random draws from the Gibbs sampler.

The conditional distribution of  $y_{m,t}$  for  $t = 1, \dots, T_m - 1$  is

$$\frac{[y_{m,t} | r_{m,t}; y_{m,1}; \dots; y_{m,t-1}; y_{m,t+1}; \dots; y_{m,T_m}]}{p_{m,t}(r_{m,t}) [y_{m,t} | y_{m,t-1}] [y_{m,t+1} | y_{m,t}]} \exp \left[ -\frac{1}{2} (e_{m,t} - e_{m,t-1})^2 V_m^{-1} \right] \quad (C.3)$$

where

$$e_{m,t} = V_m \frac{m}{2} (e_{m,t-1} + e_{m,t+1})$$

and

$$V_m = \frac{2}{1 + \frac{2}{m}}$$

For the last period,  $T_m$ , the conditional distribution is

$$\frac{[y_{m,T_m} | r_{m,T_m}; y_{m,1}; \dots; y_{m,T_m-1}]}{p_{m,T_m}(r_{m,T_m})} \exp \left[ -\frac{1}{2} (e_{m,T_m} - e_{m,T_m-1})^2 \right] \quad (C.4)$$

Rejection sampling is used to generate  $y_{m,t}$ . Starting a period  $t = 1$ , a candidate for  $y_{m,t}$  is generated from the normal density in equation (C.3) or (C.4), and the test function  $p_{m,t}(r_{m,t})$  is computed. If the test function is greater than a uniform random number, then the candidate is accepted and the residual  $e_{m,t}$  is updated. Then  $t$  is increased by one until  $y_{m,1}$  to  $y_{m,T_m}$  are generated. When generating  $y_{m,t}$  for next period the updated  $e_{m,t}$  from previous period is used. Logistic errors  $(e_{m,t})$  are also generated corresponding to each  $y_{m,t}$ , and saved for generating  $K$  later.

## 2. Generating $\theta_{m,0}$

The posterior distribution of  $\theta_{m,0}$  given the other parameters is

$$[\theta_{m,0} | R; Y; \dots; \sigma^2; K] \propto \exp \left[ -\frac{1}{2\sigma^2} \theta_{m,0}^2 \right] \exp \left[ -\frac{2}{\sigma^2} \theta_{m,0} e_{m,1} \right]$$

where  $e_{m,1}$  is defined by equations (C.1) and (C.2), and  $\sigma^2 = 1$ .  $\theta_{m,0}$  is generated from  $N(u; V)$  where

$$u = \frac{V_m e_{m;1}}{2}$$

and

$$V = \frac{2}{m} + \frac{1}{\sigma^2} :$$

### 3. Generating

The posterior distribution of  $\beta$  given the remaining parameters is

$$[\beta | R; Y; \sigma^2; K] / [Y | \sigma^2; K] :$$

For movie  $m$  let

$$y_{m;1} = y_{m;1} - \beta_{m;0}$$

and

$$X_{m;1} = X_{m;1}$$

for the first period, and

$$y_{m;t} = y_{m;t} - \beta_{m;t-1}$$

and

$$X_{m;t} = X_{m;t} - \beta_{m;t-1}$$

for  $t = 2; 3; \dots; T_m$ .

The likelihood of  $\beta$  is proportional to

$$\exp \left\{ -\frac{1}{2} (Y - X\beta)' (Y - X\beta) \right\}$$

which gives rise to Zellner's seemingly unrelated regression equation (SURE) model. If the prior for  $\beta$  is  $N_p(a_0; A_0)$ , then we generate from  $N_p(a; A)$  where  $p$  is equal to the dimension of  $\beta$  and

$$A = \frac{1}{2} X' X + A_0^{-1}$$

and

$$a = A^{-1} X^0 X^{-1} a + A_0^{-1} a_0$$

where

$$a = \left( \frac{1}{2} X^0 X^{-1} \right)^{-1} \left( \frac{1}{2} X^0 Y \right) :$$

If the prior for  $a$  is non-informative, then  $a$  can be generated from

$$N_p \left( a; \left( \frac{1}{2} X^0 X^{-1} \right)^{-1} \right) :$$

#### 4. Generating

The posterior distribution of  $a$  given the other parameters is

$$\begin{aligned} & [ jR; Y; \sigma; \beta; \gamma; \delta; K ] / [ Y_j | \sigma; \beta; \gamma; \delta ] \\ & / \exp \left( - \frac{1}{2} \sum_{m=1}^M \sum_{t=1}^T (e_{m,t} - \beta e_{m,t-1})^2 \right) \\ & / \exp \left( - \frac{1}{2} (a - \gamma)^T W (a - \gamma) \right) \prod_{j=1}^M I(0 < \beta_j < 1) \end{aligned}$$

where

$$W = \frac{1}{2} \begin{bmatrix} \sum_{t=1}^T e_{1,t}^2 & & 0 \\ \vdots & \ddots & \vdots \\ 0 & & \sum_{t=1}^T e_{M,t}^2 \end{bmatrix}$$

and

$$= W^{-1} \begin{bmatrix} \sum_{t=1}^T e_{1,t} \cdot e_{1,t-1} \\ \vdots \\ \sum_{t=1}^T e_{M,t} \cdot e_{M,t-1} \end{bmatrix}$$

Finally,

$$e_{m;0} = \beta_{m;0}$$

and



$$e_{m,t} = y_{m,t} - X_{m,t} \beta_m \quad \text{for } t = 1, \dots, T_m$$

The prior for  $\beta_m$  is given by  $U(-1; 1)$ :

As a result,  $\beta_m$  is generated from a truncated multivariate normal distribution with support  $[-1; 1]^M$ . (That is, we keep drawing from the distribution until all the coordinates are between -1 and 1.)

## 5. Generating $\sigma^2$

The posterior distribution of  $\sigma^2$  given the other parameters is

$$\frac{[\sigma^2]^{-\frac{N}{2}} \prod_{m=1}^M \prod_{t=1}^{T_m} \exp\left(-\frac{1}{2\sigma^2} X_{m,t} X_{m,t}' e_{m,t}^2\right)}{\int \prod_{m=1}^M \prod_{t=1}^{T_m} \exp\left(-\frac{1}{2\sigma^2} X_{m,t} X_{m,t}' e_{m,t}^2\right) d\beta_m}$$

where

$$e_{m,0} = \beta_{m,0}$$

$$e_{m,1} = y_{m,1} - X_{m,1} \beta_{m,1}$$

$$e_{m,t} = (y_{m,t} - \beta_{m,t-1} X_{m,t}) - (X_{m,t} - \beta_{m,t-1} X_{m,t-1}) \quad \text{for } t = 2, \dots, T_m$$

and  $N$  is the total number of observations in the dataset.

The prior distribution of  $\sigma^2$  is  $IG(a_0; b_0)$  where  $a_0 = 3$  and  $b_0 = 0.001$  (as in Allenby and Rossi, 1999). Thus,  $\sigma^2$  is generated from the posterior density  $IG(a_1; b_1)$  where

$$a_1 = a_0 + \frac{N}{2}$$

$$b_1 = b_0 + \sum_{m=1}^M \sum_{t=1}^{T_m} e_{m,t}^2$$

## 6. Generating K

The posterior distribution of  $k_j$  given the remaining parameters is

$$[k_j | R; Y; \theta; \sigma^2]$$

$$/ \prod_{m=1}^{Q_M} \prod_{t=1}^{Q_{T_m}} I(r_{m;t} = j) k_{j-1} < y_{m;t} < k_j + I(r_{m;t} = j+1) k_j < y_{m;t} < k_{j+1} :$$

This conditional distribution is equivalent to the uniform distribution on the interval

$$\max_{m,t} y_{m;t} : r_{m;t} = j ; k_{j-1} ; \min_{m,t} y_{m;t} : r_{m;t} = j+1 ; k_{j+1}$$

where

$$y_{m;t} = y_{m;t} + \sigma_{m;t} :$$

Consequently,  $k_j$  is generated from  $U(m; n)$  where

$$m = \max_{m,t} y_{m;t} : r_{m;t} = j ; k_{j-1}$$

and

$$n = \min_{m,t} y_{m;t} : r_{m;t} = j+1 ; k_{j+1} :$$