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Axelrod, A.

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Alan Axelrod

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FLAVOR CHANGING Z^0 DECAY AND THE TOP QUARK*

Alan Axelrod

Lawrence Berkeley Laboratory
and
Department of Physics
University of California
Berkeley, California 94720

ABSTRACT

The flavor changing neutral current decay of the Z^0 boson into charge $2/3$ quarks in the standard three generation $SU(2)_L \times U(1)$ theory of electroweak interactions has been studied. This process occurs first at one-loop order, where it has been calculated without approximation. The possibility of producing the as yet undiscovered top quark by this decay has been considered. The branching ratios are extremely small independent of the top quark mass and plausible quark mixing matrices if there are three generations, making it unlikely that the top quark will be produced by this mechanism. However, a massive fourth bottom quark could increase the rates.

*

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I. INTRODUCTION

The standard $SU(2)_L \times U(1)$ theory of electroweak interactions¹ appears thus far to have passed its experimental tests. One of its crucial untested predictions is that of the existence of a massive neutral vector particle, the Z^0 , whose mass and couplings are constrained by the theory together with experimental data at current energies.² It is expected to have a mass of nearly 100 GeV, and therefore to be within the range of the next generation of accelerators.

In the theory, charge $+2/3$ quarks appear in lefthanded doublets with their charge $-1/3$ partners. However, the charge $+2/3$ partner of the b quark, called the top quark, has not been seen. PETRA data indicates that its mass must be greater than about 20 GeV.³ Other considerations indicate that the top should not weigh more than a few hundred GeV.⁴ Analysis of the K^0 system indicates that a mass of less than 100 GeV is likely.⁵ Therefore, it is not unreasonable to assume that the top quark lies somewhere between about 20 GeV and the Z^0 mass. If $M_T < M_Z/2$, the Z^0 can decay into a $t\bar{t}$ pair via the usual tree graph. If however $M_T > M_Z/2$, this decay is energetically disallowed. In this case the decay $Z \rightarrow t\bar{c}$ or $t\bar{u}$ can occur as a weak radiative effect and could lead to the discovery of the top quark.

The process $Z \rightarrow t\bar{c}$ (and the related $\bar{t}c$, $t\bar{u}$, $\bar{t}u$, $c\bar{u}$, and $\bar{c}u$) is an example of a flavor changing neutral current interaction. These processes are generally highly suppressed and provide strong constraints on models.⁶ They have also led to accurate predictions for new particles.⁷

In the $SU(2)_L \times U(1)$ theory, the suppression of these processes occurs through the GIM mechanism.⁸ Not only are neutral current couplings connecting quarks of different generations absent in the tree Lagrangian, but they are suppressed even beyond the $O(g^3)$ one would expect from their appearance at one-loop order. This may be qualitatively understood as follows, using the $Z \rightarrow t\bar{c}$ one-loop amplitude (Figure 1) as a prototype. Its general form will be

$$\text{Amp1} = \sum_J U_{TJ}^{KM} (U_{Jc}^{KM})^\dagger f(\text{masses})_J, \quad (1.1)$$

where $(U^{KM})_{TJ}$ is the (top quark, j th bottom quark) matrix element of the unitary quark mixing matrix,⁹ and $f(\text{masses})_J$ is some function of the masses, including M_J , and external momenta and spins. Now suppose all the M_J 's were equal, eg. $M_d = M_s = M_b$ for three generations. Then $f(\text{masses})_J$ becomes independent of J , so we may remove its subscript and write

$$\text{Amp1} = f(\text{masses}) \sum_J U_{TJ}^{KM} (U_{Jc}^{KM})^\dagger = f(\text{masses}) \Pi_{Tc} = 0, \quad (1.2)$$

since any off-diagonal element of the identity matrix is 0. The amplitude will be nonvanishing only if there are nonzero quark mass differences, and so is naively proportional to $\Delta(M_J^2)$. As it is natural for the scale to be otherwise set by M_W , this will result in a huge suppression for ordinary quark mass differences.

I adopt the convention that $(Z \rightarrow \text{top})$ means $(Z \rightarrow t \text{ or } \bar{t} + X)$, where X is not a t or $\bar{t}$. Since for three generations ΔM_J^2 is dominated by M_b^2 , we are led to the estimate for the branching

ratio of

$$\begin{aligned} \text{B.R.} &= \frac{\Gamma(Z \rightarrow \text{top})}{\Gamma(Z \rightarrow \text{all})} \sim \left(\frac{g^3 \frac{M_b^2}{M_W^2} 10^{-1/2}}{g} \right)^2 \frac{1}{10} \\ &\sim 10^{-2} \frac{g^2 M_b^4}{M_W^4} \sim 10^{-9}, \end{aligned} \quad (1.3)$$

where $10^{-1/2}$ is from KM angles, and the extra $\frac{1}{10}$ takes rough account of the number of open channels for Z decay.

This branching ratio would be very small and probably not measurable, even with 10^8 Z 's per year at LEP. However, it is only an estimate; an exact calculation is required to see if the rate lies in a measurable range. This is what has been performed. Unfortunately, the results reported here are even a bit smaller than the above estimate.

The plan of this paper is as follows. Section 2 contains a description of the calculational scheme. Section 3 discusses the major checks on the calculation. Section 4 presents the results. In Section 5 I conclude and point out how the small rates might be increased. Appendices A through C contain some integration formulas, a series expansion for the Spence function, and some symmetries and Ward identities, respectively.

II. CALCULATIONAL SCHEME

I choose to work in the unitary gauge. The vector boson propagator is more complicated than in t'Hooft-Feynman gauge, but there are fewer diagrams. The four diagrams that need to be calculated for $Z \rightarrow t\bar{c}$ are in Figure 1; the ones for $\bar{t}c$, $t\bar{u}$, $\bar{t}u$, $c\bar{u}$, and $\bar{c}u$ follow from these by trivial substitution. Dimensional regularization is used to regulate the ultraviolet loop integral divergences that exist before all internal quarks and diagrams are summed over. It is sometimes said that the divergences in unitary gauge are more severe than in t'Hooft-Feynman or Landau gauge. Although true by naive power counting, this is somewhat irrelevant in the dimensional scheme when one performs Laurent expansion in N about physical 4 dimensions, since any divergence then appears as a pole in $N-4$ irrespective of its (even) degree.

As alluded to above, the sum over all internal quarks and diagrams is ultraviolet finite. In the dimensional scheme this means that the result for the amplitude is free of $\frac{1}{N-4}$ pole terms. This is expected in analogy with previous work on $K_L \rightarrow \mu\mu$.¹⁰

An immediate obstacle is the choice of what to use for γ_5 in N dimensions. There is still controversy in the literature on this point.¹¹ We adopt the definition of Ref.[12] of

$$\gamma_5^2 = 1; \{\gamma_5, \gamma_\mu\} = 0. \quad (2.1)$$

The authors in Ref. [12] found that this definition preserved the Ward identities, at least for one-loop graphs. Here, I rely on

the checks described in Section 3, in particular the Ward identities described in Section 3B and Appendix C, to verify that I am not in error.

The question arises of whether there is a legitimate approximation scheme. Previous calculations of flavor changing neutral current processes have often neglected internal masses, or external masses or momenta either completely or with respect to some large scale such as M_W . This cannot be justified for the large masses and momenta involved in $Z \rightarrow t\bar{c}$, as compared to the low energy processes that were studied previously. The on-shell decay rate, summed over fermion spins and averaged over boson polarizations, is a function of M_Z , M_W , M_A , M_B , and M_J , where A and B are the external quarks and J are the internal quarks. M_Z and M_W set the large scale, and if, say, $M_A = M_T$ then it is of the same order and cannot be neglected. If M_J is ignored completely, the decay rate is exactly zero by the GIM mechanism. It is possible to argue to neglect $M_B = M_c$, M_u and/or high powers of M_J . If one ignores M_B one also loses the possibility of computing $Z \rightarrow t\bar{c}$ and $\bar{t}c$ independently, and therefore loses the check of Section 3C that these should be equal. I chose the safest route and kept all masses and momenta without approximation.

For the integrations, a modified version of the technique of Ref. [13] was used. I have put the details in Appendix A. It was desired to do as much of the work algebraically as possible. By using Appendix A, one can reduce all relevant integrals to algebraic sums of integrals with loop momenta in the denominators only. These can then be written analytically as sums of constants, logarithms, and Spence functions,¹⁴ combined together, and numerically

evaluated.¹⁵ A convenient expansion for the Spence function is listed in Appendix B. In this regard it is worth pointing out that it is important to keep track of the $i\epsilon$'s in the propagator denominators, as these specify which side of the cut of the complex logarithm or Spence function one is on.

Keeping all five masses variable and without approximation produces an algebraic explosion. A computer algebraic manipulator, MACSYMA, was essential in handling the algebra. The following guidelines were found useful in the calculation:

- (i) Expand in $(N-4)$, keeping only pole and finite parts, wherever convenient.
- (ii) Always throw out M_J - independent pieces. These contribute 0 due to GIM.
- (iii) Use Dirac equation repeatedly to eliminate 4-momenta in favor of masses.
- (iv) Utilize $\epsilon \cdot p_A = -\epsilon \cdot p_B$ for Z polarization vector ϵ and external quark momenta p_A, p_B .
- (v) In general, if a symmetry or constraining equation exists, it should be used to simplify or check. For example, if $M_W = M_Z \cos \theta_W$, then one of these should be eliminated in favor of the other two.

III. CHECKS ON CALCULATION

The calculation is very lengthy so that checks are indispensable to guard against errors. They are also useful in verifying the validity of certain calculational procedures.

Many checks suggest themselves at different stages of the work. These are too numerous to be discussed here. The checks listed below are the most global checks in the sense that they verify large amounts of work and/or important assumptions. Unfortunately, the algebraic derivations are far too long to be presented here.

A. Pole Cancellation

As noted earlier, the calculation is expected to be finite; all $\frac{1}{N-4}$ terms are expected to cancel. Vanishing of the poles with M_J (internal quark mass)-independent coefficients is rather trivial due to GIM. It is less trivial that the M_J -dependent pole coefficients sum to 0. These can come about from Dirac algebra as well as integrals such as¹⁶

$$\int d^N k \left(\frac{1}{k^2 - M_J^2 + i\epsilon} \right) \sim \frac{M_J^2}{N-4} + O(N-4)^0. \quad (3.1)$$

Exact pole cancellation was verified algebraically.

It is interesting to note that certain pole parts require the tree relation $M_W = M_Z \cos \theta_W$ in order to vanish. Any $O(g^2)$ correction to this relation is only relevant for the $O(g^5)$, i.e. two loop, $Z \rightarrow t\bar{c}$ amplitude and therefore irrelevant here. This proportionality is probably indicative of the dangers of calculating in unitary gauge off the mass shell, since M_Z^2 appears as the square

of the 4-momentum of the Z external leg.

B. Ward Identities

The relevant Z couplings to the order of interest are given in unitary gauge by the sum of a conserved vector current, and a partially conserved axial vector current. This leads to Ward identities that should be satisfied by the $Z \rightarrow t\bar{c}$, etc., decay amplitudes. These can be represented diagrammatically by Figure 2. Notice that checking the axial vector Ward identity requires computing an additional set of diagrams giving the "decay of a fictitious pseudoscalar" into $t\bar{c}$. Details are presented in Appendix C.

This is an important check, in particular for verifying the choice of γ_5 . Both the vector and axial vector Ward identities were found to be exactly satisfied algebraically.

C. CP Invariance

As noted in Section 4, the KM matrix is chosen to be real. Therefore, the calculation is performed in a CP-invariant theory, which requires that $\Gamma(Z \rightarrow t\bar{c}) = \Gamma(Z \rightarrow \bar{t}c)$, etc. Since $Z \rightarrow t\bar{c}$ and $Z \rightarrow \bar{t}c$ are computed independently from each other, this provides a numerical check that was verified.

D. Imaginary Part

The reality chosen for the KM matrix also insures that the theory is invariant under time-reversal. This means that the imaginary part of the $Z \rightarrow t\bar{c}$ decay amplitude is related to the absorptive part via

$$2\text{Im}\langle t\bar{c}|T|Z\rangle = \sum_n \langle n|T|t\bar{c}\rangle^* \langle n|T|Z^0\rangle, \quad (3.2)$$

where $S = 1 + iT$.

This check was confined to the simplest case of $M_T < M_W + M_J$, where the only cut that contributes is shown in Figure 3A. For M_T above the W-J threshold, several other cuts such as the one in Figure 3C will contribute, and I have not attempted to compute them. What has been calculated is the sum over the intermediate phase space of the product of the two diagrams shown in Figure 3B. As this obviously requires no regularization, it is another good check on our dimensional scheme, as well as on the algebra and numerics.

Even here the algebra is long, due to so many independent and variable masses. The methods of Appendix A are used to do the angular integrals, only using $O(3)$ tensors and δ_{ij} as a projector, instead of Lorentz tensors and $g_{\mu\nu}$. Although the integrals occurring here are trivial, I expect that this method is useful in computing very complicated phase space integrals.

The result is reduced to a similar form to that given for the amplitude iT in (4.1):

$$\begin{aligned} \text{Im}T = \sum_J U_{AJ}^{KM} (U_{JB}^{KM})^\dagger \bar{u}_A(p_A, s_A) \{ G_1 \epsilon \cdot p_A + G_2 \epsilon \\ + G_3 \epsilon \cdot p_A \gamma_5 + G_4 \epsilon \gamma_5 \} v_B(p_B, s_B), \end{aligned} \quad (3.3)$$

where $Z(\text{momentum } p_A + p_B, \text{polarization } \epsilon) \rightarrow A(p_A, \text{spin } s_A) + \bar{B}(p_B, s_B)$, and the G_i are functions of the masses. Their form is quite long and will not be reproduced here. However it should be noted that the phase space integrals produce logarithms whose arguments approach 1 in certain kinematic regions. In these cases, to retain

numerical precision these logarithms should be expanded by hand, as the high order terms in the expansions tend to cancel strongly with other similar terms.

Having now computed the right-hand side of (3.2), it can be compared to the left-hand side which is computed using the final expression for the total amplitude (4.1). This provides a last numerical check which was verified. When M_T rises above the W-J threshold, the agreement disintegrates as expected.

IV. RESULTS

The full algebraic result for the decay amplitude (sum of diagrams in Figure 1) is far too long to be reproduced here.*

Its general form is

$$i\mathcal{T} = \sum_J U_{AJ}^{KM} (U_{JB}^{KM})^\dagger \bar{u}_A(p_A, s_A) \{ F_1 \epsilon \cdot p_A + F_2 \epsilon$$

$$+ F_3 \epsilon \cdot p_A \gamma_5 + F_4 \epsilon \gamma_5 \} v_B(p_B, s_B), \quad (4.1)$$

where $Z(\text{momentum } p_A + p_B, \text{ polarization } \epsilon) \rightarrow A(p_A, \text{ spin } s_A) + \bar{B}(p_B, s_B)$. The F_i are functions of the five masses M_J (internal quarks), M_A , M_B , M_Z , and M_W ($\cos\theta_W$ was replaced by M_W/M_Z). They are in turn expressed as linear combinations of constants, logarithms, and Spence functions.

The total decay width, averaged over Z polarizations and summed over A, B spins and momenta, is

$$\Gamma(Z \rightarrow A\bar{B}) = \left(\frac{P(A,B)}{16\pi M_Z^5} \right) \tau, \quad (4.2)$$

where $P(A,B)$ is a phase space suppression factor and τ is proportional to the trace. A factor of 3 for color has been included. Let

$$x_{1,2,3,4} = M_Z + M_B + M_A, \quad +-, -+, --$$

$$y_{1,2} = M_B + M_A, \quad M_B - M_A$$

$$F_{kR}, F_{kI} = \text{Re, Im of } \left(\sum_J U_{AJ} U_{JB}^\dagger F_k \right), \quad k = 1, 4. \quad (4.3)$$

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Then

$$P(A,B) = \sqrt{x_1 x_2 x_3 x_4} \quad (4.4)$$

$$\begin{aligned} \tau = \frac{1}{2} \{ & P^2(A,B) [x_1 x_4 (F_{1R}^2 + F_{1I}^2) + x_2 x_3 (F_{3R}^2 + F_{3I}^2) \\ & - 4y_1 (F_{1R} F_{2R} + F_{1I} F_{2I}) + 4y_2 (F_{3R} F_{4R} + F_{3I} F_{4I})] \\ & + x_2 x_3 (8M_Z^2 + 4y_1^2) (F_{2R}^2 + F_{2I}^2) + x_1 x_4 (8M_Z^2 + 4y_2^2) (F_{4R}^2 + F_{4I}^2) \}. \end{aligned} \quad (4.5)$$

Numerical results are presented in Figures 4 thru 7* and discussed below. Parameter choice is as follows:

(i) Quark masses - The (unknown) top quark mass is varied from 20 up to $M_Z - 1$ GeV. For the other quarks, current¹⁷ internal masses and constituent external ones were taken: $M_d = 7.5$ MeV, $M_s = 150$ MeV, $M_b = 4.75$ GeV, $M_u = 330$ MeV, $M_c = 1.5$ GeV. Varying M_b by ± 0.25 GeV causes the rates to change by $\pm \sim 20$ -25%. Small variations in the others make negligible difference. One could even take current external masses with the same effect, and constituent d and s quark masses tend to decrease the widths by only $\lesssim 2\%$. These statements are true except when the KM couplings to b are too small or one is right at a $Z \rightarrow AB$ decay threshold, in which cases the rates are completely¹⁸ insignificant anyway.

(ii) M_Z, M_W, θ_W : The central values of Ref. [18] are used: $M_W = 83.0$ GeV, $M_Z = 93.8$ GeV, $\sin^2 \theta_W = 0.217$ in the scheme where $\cos \theta_W = M_W/M_Z$. Varying these in the $\pm 2\sigma$ region of [18], i.e. $89.8 < M_Z < 98.7$, $0.245 > \sin^2 \theta_W > 0.189$, produces little

($\lesssim 30\%$ through $M_T = 80$ GeV) change, except near the endpoints since M_Z controls the position of the thresholds and of the zero in the phase space cutoff factor $P(A,B)$. But in this region the rates are dropping anyway and my conclusions are unaffected.

(iii) Z decay width: This varies with the top quark mass. For $20 < M_T < \frac{M_Z}{2}$ the Born amplitude into $t\bar{t}$ is phase space suppressed relative to the other fermions, and for $M_T > M_Z/2$ it drops out altogether. The formulas used were taken from Ref. [19]. For the current parameters $\Gamma(Z \rightarrow \text{all}) \approx 2.7$ GeV for $M_T = 20$ GeV, and ≈ 2.5 GeV for $M_T > M_Z/2$.

(iv) KM matrix: The conventions used for U^{KM} are as in [20]. In order to observe the effects of varying the mixing angles, a set of matrices consistent with the physics of u, d, and s quarks was generated by utilizing the analyses of Refs. [21, 22]. I chose to ignore the small CP-violation by taking the KM matrix to be real, i.e. $\cos \delta = \pm 1$. Next, $\cos \theta_1$ (θ_1 is often called the Cabibbo angle) was fixed at 0.9737, and $\sin \theta_3$ was varied between 0 and 0.5. Finally, $\sin \theta_2$ was generated using the constraining Equation (12) of Ref. [21]:

$$\tan \theta_3 = \frac{\cos \theta_1 (\sin^2 \theta_2 - a^2)}{(\sin \theta_2)(\cos \theta_2)(\cos \delta)}, \quad (4.6)$$

where the allowed values of a depend on M_T . For fixed M_T , a was varied by reading values off figure 2 of [21] that agree with the $K_L \rightarrow \mu^+ \mu^-$ data (upper band) and/or $K^0 - \bar{K}^0$ mixing for $0.2 < B < 0.6$ (B is a bag parameter). Widths were computed for all the matrices generated in this manner.

* More detailed numerical results are available to those interested.

In Figure 4 I have plotted the high, low, and average ($\equiv (\text{high} + \text{low})/2$) values varied over the KM matrices, of $\Gamma(Z \rightarrow \text{top}) = \Gamma(Z \rightarrow t\bar{c} + \bar{t}c + t\bar{u} + \bar{t}u)$ and the corresponding branching ratios $\text{BR}(Z \rightarrow \text{top})$ as functions of the top quark mass. In Figure 4, it was demanded that high or low values be generated by KM matrices satisfying, in addition to the above requirements from u, d, and s quark physics, the following constraints from b and c quark physics^{2,23}:

- a) $\left| \frac{U_{ub}}{U_{cb}} \right| \leq 0.3$ from $B \rightarrow K$'s, leptons;
- b) $|U_{cb}| \geq 0.04$ from B lifetime²⁴;
- c) $0.192 \leq |U_{cd}| \leq 0.34$ from $v + d \rightarrow c + u$;
- d) $|U_{cs}| \geq 0.6$ from $D \rightarrow K\bar{v}$. (4.7)

These constraints were not applied in (iv) because they are somewhat crude and model-dependent at this time. For example, a and b were derived using the spectator model for B decays, which is in some doubt. However, they seem reasonable and contain important information about the KM matrix. In any case, for the parameter set used in Figure 4 the value of $\Gamma(Z \rightarrow \text{top}, \text{high})$ was found, for all values of M_T checked, to be independent of whether the constraints (4.7) were applied or not, provided only that $Z \rightarrow t\bar{c}$ was energetically allowed (i.e., for $M_T < M_Z - M_c$). For M_T larger than this the rate is vanishingly small anyway. (4.7) does

tend to exclude extremely low values, but the average values are also relatively unaffected since there is usually about an order of magnitude's disparity between high and low values, implying that average $\cong \text{high}/2$.

The main feature of Figure 4 is how small the branching ratios are. The maximum over all top quark masses and KM matrices used is $\sim 7 \times 10^{-11}$, almost certainly unmeasurable with projected luminosities. The average value for potentially interesting values of $M_T (\geq 50 \text{ GeV})$ is $\lesssim 3 \times 10^{-11}$. There is, for the most part, a steady drop in all curves as M_T increases, and a sharp drop near the kinematic boundary (especially after the $t\bar{c}$ threshold is crossed, although this is not obvious from the graph). The exceptions to this steady drop are a shoulder at around 85 GeV that will be explained later, and a dip in $\Gamma(Z \rightarrow \text{top}, \text{low})$ at about 30 GeV which is due to fluctuations in which KM matrices agree with experimental constraints as M_T varies in that region.

The simple physics estimate of (1.3) was not too far off, and even a bit high. That estimate assumed that the coupling to the b quark dominated the amplitude. Now, $Z \rightarrow t\bar{c}$ is generally Cabibbo enhanced relative to $Z \rightarrow t\bar{u}$. Therefore, it is expected that

$$\Gamma(Z \rightarrow \text{top}) \sim \Gamma(Z \rightarrow t\bar{c} + \bar{t}c)$$

$$\begin{aligned} \sim U_{Tb}^2 U_{cb}^2 &= (c_1 s_2 s_3 + c_\delta c_2 c_3)^2 (c_1 c_2 s_3 - c_\delta s_2 c_3)^2 \\ &= \frac{1}{4} \sin^2(2\theta_2 - 2c_\delta \theta_3) + 0(1 - c_1) \end{aligned} \quad (4.8)$$

(recall $c_\delta = \pm 1$ here).

To check this, $\Gamma(Z \rightarrow \text{top})$ is plotted in Figure 5 for all the KM matrices used (ignoring the constraints (4.7)), for four values of M_T . The solid lines represent what was actually computed, but the dashed lines are only their extension into the origin. In the neighborhood of the dashed lines, the computation produced a splatter of points with values generally higher than the line. As a further check, note that if b quark dominance holds,

$$\begin{aligned} \frac{\Gamma(Z \rightarrow t\bar{u})}{\Gamma(Z \rightarrow t\bar{c})} &\cong \frac{U_{ub}^2}{U_{cb}^2} = \left(\frac{s_1 s_3}{c_1 c_2 s_3 - c_\delta s_2 c_3} \right)^2 \\ &= \frac{s_1^2 s_3^2}{\sin^2(\theta_2 - c_\delta \theta_3)} + 0(1 - c_1). \end{aligned} \quad (4.9)$$

$\Gamma(Z \rightarrow t\bar{u})/\Gamma(Z \rightarrow t\bar{c})$ was plotted vs. $(U_{ub}/U_{cb})^2$ for all the same values as Figure 5 (not shown here), and all points were observed to lie on a straight line intersecting the origin, with slope 1. Thus, in the dashed line region of Figure 5, the coupling to b is still dominant but $\bar{u}$ has become important relative to $\bar{c}$.

Therefore, the KM couplings to b are the ones probed by the decay rates. The only exceptions to this noticed were the unusual cases when one of the KM matrix elements needed was ≈ 0 . For example, if $\sin\theta_3 = 0$ then $U_{ub} = 0$, and $Z \rightarrow t\bar{u}$ must go through the s or d quark. But in these cases the widths were seen to be negligibly small anyway.

As an additional check on the physics of (1.3), M_b was varied in the region around the physical b quark mass. As expected, the rates were observed to be roughly proportional to M_b^4 , with the above-mentioned exceptions.

Variation of $\Gamma(Z \rightarrow \text{top, high})$ with M_T can be traced to three factors, two of which can be inferred from Figure 5. Note that the high endpoints of the solid lines (the highest computed points for each mass) get lower with increasing M_T both because the slopes of the lines decrease, and the allowed maximum value of $(U_{Tb} U_{cb})^2$ tends to decrease. The former can be attributed to the drop in the phase space factor $P(t, c)$ of (4.4) as M_T increases.

To verify this, I have plotted $\frac{\Gamma(Z \rightarrow \text{top, high})}{(U_{Tb} U_{cb})^2 P(t, c)}$ vs. M_T in Figure 6. Notice that this quantity drops by only a factor of 4 from $M_T = 20$ to 91.8 GeV, while $\Gamma(Z \rightarrow \text{top, high})$ falls by about two orders of magnitude in the same range. Aside from this, Figure 6 is dominated by an enhancement peaked at around 86 GeV, that corresponds to the shoulder seen in Figure 4. This is due to the onset of the $t \leftrightarrow W$ -J thresholds; i.e., the W and J quark can propagate closer and closer to mass shell as M_T grows.

The remaining small variation with M_T not attributable to these three factors is then buried in the formula for the width.

I do not believe that the conclusions about unmeasurability are restricted by the choice of KM matrix. For example, if one takes the KM matrix giving the largest branching ratio into top ($\sim 7 \times 10^{-11}$) and for the sake of argument sets $U_{cb} = U_{Tb} = 1$ (the unitarity bound), the width would increase by only a factor of 4, raising the branching ratio to only 3×10^{-10} .

$\Gamma(Z \rightarrow u\bar{c} + \bar{u}c)$ was also calculated; its average values, defined as for $\Gamma(Z \rightarrow \text{top})$, are plotted in Figure 7. These are very close to $\frac{1}{2}$ the high values because the low values are down by five orders of magnitude. The variation seen with M_T is due to the variation of the

allowed KM matrices with M_T . The couplings through b are strongly Cabibbo suppressed, and so the high value for the branching ratio is never more than 2×10^{-12} , making this experimentally irrelevant.

V. CONCLUSION

The flavor changing neutral current decay processes $Z \rightarrow t\bar{c}$, $\bar{t}c$, $t\bar{u}$, $\bar{t}u$, $u\bar{c}$, and $\bar{u}c$ have been calculated. The exact expression for the decay rate was obtained as a function of the five independent masses, to one-loop order in the $SU(2)_L \times U(1)$ theory. Ward identities for the unitary gauge were derived to help check the result. The numerical results were then computed in the three generation model for realistic values of the parameters involved. The branching ratios such as $BR(Z \rightarrow \text{top})$ are at most $\sim 10^{-10}$, and for many of the allowed values of M_T and the quark mixing matrix are much worse. If LEP produces 10^8 Z's per year, then the prospects are for at most one top quark every 100 years by this process, making it a dismal experimental proposition.

Therefore, it is very unlikely that a massive top quark will be discovered by this mechanism. However, since the branching ratios are so small, they can provide an important theoretical flavor changing neutral current constraint on any alternative to the model studied here. Therefore, it will still be important to search for such events and set upper bounds on the rates.

One way these processes might be enhanced is by alteration of the Higgs sector. Additional Higgs doublets can induce larger flavor changing neutral currents because of off-flavor diagonal couplings in the tree Lagrangian.²⁵ Also, technicolor models are known to have difficulties suppressing flavor changing neutral current rates.²⁶

Another possibility for enhancement would be the existence of a massive fourth generation bottom quark, b' . It was noted earlier

that the widths were proportional to M_b^4 for bottom quark masses small compared to M_W . It is not obvious that this behavior continues for $M_b \gtrsim M_W$, or for $M_b \gg M_W$. However, it is suggested by earlier calculations of similar processes,²⁷ and preliminary numerical investigations I have made do show a strong increase with M_b . I will assume it here to get an idea of the possible effect of a b' .

It is natural to suppose that the b' contribution is doubly Cabibbo suppressed relative to the b . Then one estimates

$$\text{BR}(Z \rightarrow \text{top, with } b') \sim (\sin\theta_c)^4 \left(\frac{M_{b'}}{M_b} \right)^4 \text{BR}(Z \rightarrow \text{top, no } b')$$

$$\lesssim 10^{-16} M_b^4, \text{ (in GeV)}.$$

For $M_{b'} = 100 \text{ GeV}$, the right hand side is 10^{-8} . For $M_{b'} = 1 \text{ TeV}$, it is 10^{-4} . So a massive b' could make a significant difference. I hope to examine this issue in more detail in future work.

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APPENDIX A - INTEGRATION SCHEME

This appendix is somewhat similar in content to Appendices D and E of Ref. [13]. However, it is presented here since

- (i) the formulas are valid for arbitrary N (assuming N is such that the integrals converge), allowing algebraic reduction of all integrals to linear combinations of scalar loop integrals only;
- (ii) the metric of Ref. [16] is used; and
- (iii) some extra formulas are given, including some useful in unitary gauge.

The scheme is as follows. Let the integration "measure" be

$$d^N k = \mu^{3(4-N)/2} \frac{d^N k}{(2\pi)^N}, \quad (\text{A.1})$$

where $g_\mu^{(4-N)/2}$ is the dimensionful coupling constant in N dimensions. Let

$$I = (k^2 - m_1^2); \quad II = ((k + \ell)^2 - m_2^2); \quad III = ((k + \ell + s)^2 - m_3^2) \quad (\text{A.2})$$

where ℓ and s are some external momenta and m_i are masses.

It is desired to express all relevant loop integrals as algebraic linear combinations of

$$A_0(m_1) = \int d^N k \left(\frac{1}{I} \right); \quad B_0(\ell, m_1, m_2) = \int d^N k \left(\frac{1}{I II} \right);$$

$$C_0(\ell, s, m_1, m_2, m_3) = \int d^N k \left(\frac{1}{I II III} \right). \quad (\text{A.3})$$

For the current calculation, the only scalar integrals that appear are (p_A, p_B are quark A and B external momenta, respectively, and J stands for internal quarks)

$$A_0(M_W), A_0(M_J);$$

$$B_0(p_A, M_W, M_J), B_0(p_B, M_W, M_J), B_0(p_A + p_B, M_W, M_J), B_0(p_A + p_B, M_J, M_J);$$

$$C_0(p_A, p_B, M_W, M_J, M_J), C_0(p_A, -(p_A + p_B), M_W, M_J, M_J). \quad (A.4)$$

These are then written in terms of constants, logarithms, and Spence functions,¹⁴ combined together algebraically with coefficients from Dirac algebra, and evaluated numerically.¹⁵ By maximizing the amount of work done algebraically, it was hoped that problems of nasty cancellations between terms would be reduced.

The following formulas are derived using Lorentz invariance and index symmetry, and utilizing vector momenta and $g^{\mu\nu}(g^{\mu\nu}g_{\mu\nu} = N)$ as "projectors" until enough equations are derived to solve for all scalar factors. For later use define

$$s_1 = \ell^2 + m_1^2 - m_2^2$$

$$s_2 = s^2 + 2\ell s + m_2^2 - m_3^2. \quad (A.5)$$

Then, for example, "projecting" B_{μ_1} below with ℓ^{μ_1} and using $\ell k = \frac{1}{2}(II - I - s_1)$ gives the result for B_1 . The formulas below then define an iterative ladder in which the ones on top are defined in

terms of the ones on the rung below, etc., with A_0, B_0 , and C_0 at the bottom.

$$\tilde{B}_0; B_{\mu_1}; \tilde{B}_{\mu_1}; B_{\mu_1\mu_2}(\ell, m_1, m_2) = \int d^N \tilde{k} \frac{k^2; k_{\mu_1}^2; k_{\mu_1}^2 k_{\mu_1}^2; k_{\mu_1} k_{\mu_2}}{I II} \quad (A.6)$$

$$B_{\mu_1} = \ell_{\mu_1} B_1(\ell, 1, 2)$$

$$\tilde{B}_{\mu_1} = \ell_{\mu_1} \tilde{B}_1$$

$$B_{\mu_1\mu_2} = \ell_{\mu_1} \ell_{\mu_2} B_{21} + g_{\mu_1\mu_2} B_{22}, \quad (A.7)$$

where

$$\tilde{B}_0 = A_0(m_2) + m_1^2 B_0$$

$$B_1 = \frac{1}{2\ell^2} [A_0(m_1) - A_0(m_2) - s_1 B_0]$$

$$\tilde{B}_1 = m_1^2 B_1 - A_0(m_2)$$

$$B_{21} = \frac{1}{2\ell^2(N-1)} [(N-2)A_0(m_2) - 2m_1^2 B_0 - N s_1 B_1]$$

$$B_{22} = \frac{1}{2(N-1)} [A_0(m_2) + 2m_1^2 B_0 + s_1 B_1].$$

$$\tilde{C}_0; \tilde{C}'_0; C_{\mu_1}; \tilde{C}_{\mu_1}; C_{\mu_1\mu_2}; \tilde{C}_{\mu_1\mu_2}; C_{\mu_1\mu_2\mu_3}(\ell, s, m_1, m_2, m_3) = \int d^N \tilde{k} \frac{k^2; (k^2); k_{\mu_1}^2; k_{\mu_1}^2 k_{\mu_1}^2; k_{\mu_1} k_{\mu_2}; k_{\mu_1}^2 k_{\mu_1} k_{\mu_2}; k_{\mu_1} k_{\mu_2} k_{\mu_3}}{I II III} \quad (A.8)$$

$$\begin{aligned}
C_{\mu_1} &= \ell_{\mu_1} C_{11}(\ell, s, 1, 2, 3) + s_{\mu_1} C_{12}(\ell, s, 1, 2, 3) \\
\tilde{C}_{\mu_1} &= \ell_{\mu_1} \tilde{C}_{11} + s_{\mu_1} \tilde{C}_{12} \\
C_{\mu_1 \mu_2} &= \ell_{\mu_1} \ell_{\mu_2} C_{21} + s_{\mu_1} s_{\mu_2} C_{22} + \{\ell s\}_{\mu_1 \mu_2} C_{23} + g_{\mu_1 \mu_2} C_{24} \\
\tilde{C}_{\mu_1 \mu_2} &= \ell_{\mu_1} \ell_{\mu_2} \tilde{C}_{21} + s_{\mu_1} s_{\mu_2} \tilde{C}_{22} + \{\ell s\}_{\mu_1 \mu_2} \tilde{C}_{23} + g_{\mu_1 \mu_2} \tilde{C}_{24} \\
C_{\mu_1 \mu_2 \mu_3} &= \ell_{\mu_1} \ell_{\mu_2} \ell_{\mu_3} C_{31} + s_{\mu_1} s_{\mu_2} s_{\mu_3} C_{32} + \{\ell \ell s\}_{\mu_1 \mu_2 \mu_3} C_{33} \\
&\quad + \{\ell s s\}_{\mu_1 \mu_2 \mu_3} C_{34} + \{\ell g\}_{\mu_1 \mu_2 \mu_3} C_{35} + \{s g\}_{\mu_1 \mu_2 \mu_3} C_{36}, \quad (A.9)
\end{aligned}$$

where $\{ \}$ $\mu_1 \dots \mu_n$ means the product of all tensors inside, totally symmetrized over all indices shown.

$$\text{Let } X^{-1} = \begin{pmatrix} \ell^2 & \ell s \\ \ell s & s^2 \end{pmatrix}^{-1} = \frac{1}{\ell^2 s^2 - (\ell s)^2} \begin{pmatrix} s^2 & -\ell s \\ -\ell s & \ell^2 \end{pmatrix}. \text{ Then}$$

$$\tilde{C}_0 = B_0(s, 2, 3) + m_1^2 C_0$$

$$\tilde{C}_0^* = \tilde{B}_0(s, 2, 3) - 2(\ell s) B_1(s, 2, 3) + \ell^2 B_0(s, 2, 3) + m_1^2 \tilde{C}_0$$

$$\begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = \frac{1}{2} X^{-1} \begin{pmatrix} B_0(\ell+s, 1, 3) - B_0(s, 2, 3) - s_1 C_0 \\ B_0(\ell, 1, 2) - B_0(\ell+s, 1, 3) - s_2 C_0 \end{pmatrix}$$

$$\tilde{C}_{11} = -B_0(s, 2, 3) + m_1^2 C_{11}$$

$$\tilde{C}_{12} = B_1(s, 2, 3) + m_1^2 C_{12}$$

$$\begin{pmatrix} C_{21} \\ C_{23} \end{pmatrix} = \frac{1}{2} X^{-1} \begin{pmatrix} B_1(\ell+s, 1, 3) + B_0(s, 2, 3) - s_1 C_{11} - 2C_{24} \\ B_1(\ell, 1, 2) - B_1(\ell+s, 1, 3) - s_2 C_{11} \end{pmatrix}$$

$$\begin{aligned}
C_{22} &= \frac{1}{2[\ell^2 s^2 - (\ell s)^2]} \{ -\ell s [B_1(\ell+s, 1, 3) - B_1(s, 2, 3) - s_1 C_{12}] \\
&\quad + \ell^2 [-B_1(\ell+s, 1, 3) - s_2 C_{12} - 2C_{24}] \}
\end{aligned}$$

$$C_{24} = \frac{1}{2(N-2)} [B_0(s, 2, 3) + 2m_1^2 C_0 + s_1 C_{11} + s_2 C_{12}]$$

$$\tilde{C}_{21} = B_0(s, 2, 3) + m_1^2 C_{21}$$

$$\tilde{C}_{22} = B_{21}(s, 2, 3) + m_1^2 C_{22}$$

$$\tilde{C}_{23} = -B_1(s, 2, 3) + m_1^2 C_{23}$$

$$\tilde{C}_{24} = B_{22}(s, 2, 3) + m_1^2 C_{24}$$

$$\begin{pmatrix} C_{31} \\ C_{33} \end{pmatrix} = \frac{1}{2} X^{-1} \begin{pmatrix} B_{21}(\ell+s, 1, 3) - B_0(s, 2, 3) - s_1 C_{21} - 4C_{35} \\ B_{21}(\ell, 1, 2) - B_{21}(\ell+s, 1, 3) - s_2 C_{21} \end{pmatrix}$$

$$\begin{pmatrix} C_{34} \\ C_{32} \end{pmatrix} = \frac{1}{2} X^{-1} \begin{pmatrix} B_{21}(\ell+s, 1, 3) - B_{21}(s, 2, 3) - s_1 C_{22} \\ -B_{21}(\ell+s, 1, 3) - s_2 C_{22} - 4C_{36} \end{pmatrix}$$

$$\begin{pmatrix} C_{35} \\ C_{36} \end{pmatrix} = \frac{1}{2} X^{-1} \begin{pmatrix} B_{22}(\ell+s, 1, 3) - B_{22}(s, 2, 3) - s_1 C_{24} \\ B_{22}(\ell, 1, 2) - B_{22}(\ell+s, 1, 3) - s_2 C_{24} \end{pmatrix}.$$

Other equations were derived, but these were always found to be equivalent to one of the above when both were expressed in terms of A_0 , B_0 , and C_0 functions.

A $k_\mu k_\nu$ numerator factor is more difficult to work out than if its indices are known to be contracted into k^2 , because the number of available Lorentz tensors increases rapidly with the number of tensor indices. If one looks at the unitary gauge expressions appearing in this calculation naively, it appears that one needs to compute objects as bad as

$$C_{\mu_1 \mu_2 \mu_3 \mu_4 \mu_5 \mu_6} = \int d^4 k \frac{k_{\mu_1} k_{\mu_2} k_{\mu_3} k_{\mu_4} k_{\mu_5} k_{\mu_6}}{I II III}, \quad (A.10)$$

which would be very tedious due to the large number of available tensors. However, if one first multiplies terms out and does Dirac algebra, simplifications occur and only the integrals evaluated above are seen to be needed. As no box diagrams appear here, integrals with four denominator factors are not necessary.

APPENDIX B - SPENCE FUNCTION SERIES

Recall the definition of the Spence function, or dilogarithm

$$Sp(z) = - \int_0^z \frac{\log(1-t)}{t} dt. \quad (B.1)$$

Reference [14] gave the following algorithm for numerical computation: use Spence function identities to map the argument into the unit disk with real part less than $\frac{1}{2}$, and then use

$$Sp(z) = \sum_{n=0}^{\infty} B_n \frac{[-\log(1-z)]^{n+1}}{(n+1)!}, \quad (B.2)$$

where B_n are the Bernoulli numbers.

Here a modified version of the series (B.2) was utilized, obtained by rewriting the Bernoulli numbers in terms of the Riemann zeta function:

$$Sp(z) = -\log(1-z) - \frac{1}{4} [\log(1-z)]^2 + 4\pi \sum_{m=1}^{\infty} \frac{(-1)^m \zeta(2m)}{(2m+1)} \left[\frac{\log(1-z)}{2\pi} \right]^{2m+1}, \quad (B.3)$$

where the Riemann zeta function is given by

$$\zeta(n) = \sum_{k=1}^{\infty} \frac{1}{k^n}. \quad (B.4)$$

The advantages of series (B.3) are that

(i) $\zeta(2m)$ are easy to compute. Additionally, beyond a certain m , a computer will not be able to distinguish it from 1, whereupon it can be set equal to 1 for the remainder of the series. These properties

are useful for iterative high precision computation.

(ii) It is obvious what the expansion parameter for the series is, since the 2π denominator has been made explicit ($\zeta(2m)/(2m+1)$ is monotonic decreasing as m increases). In fact, in the region of interest

$$|\operatorname{Re} \log(1-z)| \leq \log 2.$$

$$|\operatorname{Im} \log(1-z)| < \frac{\pi}{3}$$

$$\Rightarrow \left| \frac{\log(1-z)}{2\pi} \right| < \frac{\sqrt{(\log 2)^2 + \pi^2/9}}{2\pi} \cong 0.2. \quad (\text{B.5})$$

APPENDIX C - WARD IDENTITIES

The Lagrangian for unitary gauge is not long and is included to clarify the later discussion:

$$\begin{aligned} \mathcal{L} = & \sum_i \left\{ \bar{t}_i (i\not{\partial} - m_{t_i} - \frac{m_{t_i}}{v} H) t_i + \bar{b}_i (i\not{\partial} - m_{b_i} - \frac{m_{b_i}}{v} H) b_i \right. \\ & + e A_\mu \left[\frac{2}{3} \bar{t}_i \gamma^\mu t_i - \frac{1}{3} \bar{b}_i \gamma^\mu b_i \right] \\ & + \frac{g}{4c_W} Z_\mu \left\{ \bar{t}_i \gamma^\mu (-1 + \frac{8}{3} s_W^2 + \gamma_5) t_i + \bar{b}_i \gamma^\mu (1 - \frac{4}{3} s_W^2 - \gamma_5) b_i \right\} \\ & + \frac{g}{2\sqrt{2}} \sum_{ij} \{ W_\mu^+ \bar{t}_i \gamma^\mu (1 - \gamma_5) U_{ij} b_j + W_\mu^- \bar{b}_j \gamma^\mu (1 - \gamma_5) (U^\dagger)_{ji} t_i \} \\ & - \frac{1}{4} | \partial_\mu A_\nu - \partial_\nu A_\mu - ie(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) |^2 \\ & - \frac{1}{4} | \partial_\mu Z_\nu - \partial_\nu Z_\mu + igc_W(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) |^2 \\ & - \frac{1}{2} | \partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - W_\nu^+ A_\mu) - igc_W(W_\mu^+ Z_\nu - W_\nu^+ Z_\mu) |^2 \\ & + \frac{1}{2} \partial_\mu H \partial^\mu H - \frac{\mu^2}{2} (v + H)^2 - \frac{\lambda}{4} (v + H)^4 + \frac{1}{2} Z_\mu^2 Z^\mu + M_W^2 W_\mu^+ W^{\mu-} \\ & + \frac{1}{8} (2vH + H^2) \left[\left(\frac{g}{c_W} \right)^2 Z_\mu^2 Z^\mu + 2g^2 W_\mu^+ W^{\mu-} \right]. \quad (\text{C.1}) \end{aligned}$$

The notation is:

W_μ, Z_μ = weak bosons; A_μ = photon; H = physical Higgs;

t_i = charge +2/3 quarks; b_i = charge -1/3 quarks;

$g=SU(2)_L$ coupling, $e=U(1)_{QED}$ coupling;

$s_W, c_W = \sin$ and $\cos$ of Weinberg angle; $U = KM$ quark mixing matrix;

$\mu, \lambda =$ quadratic and quartic couplings of Higgs potential; and

$v =$ Higgs vacuum expectation value.

Leptons and the question of gauge fixing in the QED sector do not concern us here so we ignore them.

Of course, this theory is invariant under the local $U(1)$ gauge symmetry of QED, and therefore under global $U(1)$ QED. Only the global $U(1)$ is required to show electric charge conservation. Now although the local $SU(2)_L \times U(1)_Y$ gauge symmetry of the original Lagrangian has been destroyed by fixing the unitary gauge, there are residual continuous symmetries left associated with the Z couplings.

Consider the global $U(1)$ transformations, with real parameters α and β , given by

$$\begin{aligned} t_i &\rightarrow e^{i\alpha r} t_i & \frac{U^V(1)}{e^{i\beta u \gamma_5 t_i}} \\ b_i &\rightarrow e^{i\alpha s} b_i & \frac{U^A(1)}{e^{-i\beta u \gamma_5 b_i}} \\ W_\mu^+ &\rightarrow e^{i\alpha(r-s)} W_\mu^+ & e^{-2i\beta u} W_\mu^+, \end{aligned} \quad (C.2)$$

where $r = -\frac{g}{4c_W}(-1 + \frac{8}{3}s_W^2)$, $s = -\frac{g}{4c_W}(1 - \frac{4}{3}s_W^2)$, and $u = -\frac{g}{4c_W}$. $\mathcal{L}$ is invariant under $U^V(1)$; $U^A(1)$ is broken only by quark masses and Yukawa couplings to Higgs. The associated currents are conserved in the case of $U^V(1)$ and partially conserved in the case of $U^A(1)$, and are given by Noether's theorem:

$$\begin{aligned} J_V^\mu &= \sum_i \left(\frac{g}{4c_W} \right) \left[\left(-1 + \frac{8}{3}s_W^2 \right) \bar{t}_i \gamma^\mu t_i + \left(1 - \frac{4}{3}s_W^2 \right) \bar{b}_i \gamma^\mu b_i \right] \\ &+ \left\{ i \left(\frac{g}{2c_W} - g c_W \right) W_\nu^+ [(\partial^\mu W^{\nu-} - \partial^\nu W^{\mu-}) - ie(W^{\mu-} A^\nu - W^{\nu-} A^\mu) \right. \\ &\left. + ig c_W (W^{\mu-} Z^\nu - W^{\nu-} Z^\mu)] \right\} + \{\text{herm. conj. of vector term}\} \quad (C.3) \end{aligned}$$

$$\partial_\mu J_V^\mu = 0 \quad (C.4)$$

$$\begin{aligned} J_A^\mu &= \sum_i \left(\frac{g}{4c_W} \right) [\bar{t}_i \gamma^\mu \gamma_5 t_i - \bar{b}_i \gamma^\mu \gamma_5 b_i] \\ &+ \{\text{same vector term} + \text{h.c. as } J_V^\mu \text{ except } (\frac{g}{2c_W} - g c_W) \rightarrow (-\frac{g}{2c_W})\} \quad (C.5) \end{aligned}$$

$$\partial_\mu J_A^\mu = \frac{ig}{2c_W} \sum_i \left\{ \bar{t}_i \left(m_{t_i} + \frac{m_{t_i}}{v} H \right) \gamma_5 t_i - \bar{b}_i \left(m_{b_i} + \frac{m_{b_i}}{v} H \right) \gamma_5 b_i \right\} \equiv i P^5. \quad (C.6)$$

The sum of these two currents gives almost the full current coupling to the Z , but not quite. For one thing, it misses the triple vector term $\sim \partial Z(WW - WW)$. However, integration by parts gives terms $\sim Z\partial(WW - WW)$, leading one to define

$$J_{\text{deriv}}^\mu = ig c_W \partial_\nu (W^{\nu+} W^{\mu-} - W^{\mu+} W^{\nu-}) \quad (C.7)$$

$$\partial_\mu J_{\text{deriv}}^\mu = 0, \quad (C.8)$$

where (C.8) follows from index symmetry. Now one can define

$$\tilde{J}_V^\mu = J_V^\mu + \left(1 - \frac{1}{2c_W^2} \right) J_{\text{deriv}}^\mu$$

$$\tilde{J}_A^\mu = J_A^\mu + \frac{1}{2c_W^2} J_{\text{deriv}}^\mu$$

$$J_Z^\mu = \tilde{J}_V^\mu + \tilde{J}_A^\mu, \quad (C.9)$$

then

$$\partial_\mu \tilde{J}_V^\mu = 0; \quad \partial_\mu \tilde{J}_A^\mu = i p^5. \quad (C.10)$$

J_Z^μ is now the entire source current for the Z except for the ZZH and ZZHH couplings. The equation of motion for the Z field is

$$(\square + M_Z^2)Z^\mu - \partial^\mu(\partial \cdot Z) = -J_Z^\mu - \frac{1}{4}(2vH + H^2) \frac{g}{c_W} Z^\mu \equiv S^\mu. \quad (C.11)$$

The above equations imply Ward identities obeyed by the $Z \rightarrow t\bar{c}$, etc., decay amplitudes, generically $Z(p, \epsilon) \rightarrow A(p_A, s_A) + \bar{B}(p_B, s_B)$ with notation as in the text. The decay amplitude is

$$S_{fi} = (-iz_3^{-1/2}) (-iz_2^{A-1/2}) (iz_2^{B-1/2}) \int d^4x d^4y_A d^4y_B e^{-i(px - p_A y_A - p_B y_B)} \\ \bar{u}_A(p_A, s_A) \overrightarrow{(-i\gamma_{y_A} - M_A)} \langle 0 | T(\epsilon_\mu(p) S^\mu(x) \bar{\psi}_B(y_B) \psi_A(y_A)) | 0 \rangle \\ \overleftarrow{(-i\gamma_{y_B} - M_B)} v_B(p_B, s_B), \quad (C.12)$$

where the Z_j 's are the wave function renormalizations.

Now ignore the Higgs term in S^μ ; to one loop order it is irrelevant. Ward identities then can be derived by replacing ϵ_μ by p_μ in (C.12), integrating by parts, noting the vanishing of the equal time commutator terms since on-shell $A \rightarrow B$ propagation is energetically not allowed, and using (C.9) and (C.10):

Vector Ward Identity

$$\{S_{fi} \text{ with } \epsilon_\mu S^\mu + p_\mu (-\tilde{J}_V^\mu)\} = 0 \quad (C.13)$$

Axial Ward Identity

$$\{S_{fi} \text{ with } \epsilon_\mu S^\mu + p_\mu (-\tilde{J}_A^\mu)\} = \{S_{fi} \text{ with } \epsilon_\mu S^\mu + (-P^5)\}, \quad (C.14)$$

where P^5 has been defined in (C.6). These identities are represented diagrammatically in Figure 2.

The above considerations are sufficient to one-loop order for the decay amplitude, the subject of this paper. All orders merits a brief comment. One should keep separate track of the Higgs term in S^μ . Also, eventually the leptons should be included. Their charges are deduced from their couplings to Z:

	$\frac{U^V(1)\text{charge}}{c_W}$	$\frac{U^A(1)\text{charge}}{4c_W}$	
$\ell_1(e, \mu, \tau)$	$\frac{g}{c_W} (s_W^2 - \frac{1}{4})$	$\frac{g}{4c_W}$	
$\nu_1(\nu_e, \nu_\mu, \nu_\tau)$	$\frac{g}{4c_W}$	$-\frac{g}{4c_W}$	(C.15)

and the fermion sectors of the currents expand to include the leptons.

REFERENCES

1. S. L. Glashow, Nucl. Phys. 22, (1961) 579;
S. Weinberg, Phys. Rev. Lett. 19, (1967) 1264;
A. Salam, Proc. 8th Nobel Symposium, Stockholm, 1968 (Almqvist and Wiksells, Stockholm, 1968) 367.
2. For example, see J. Ellis, Lectures presented at the Les Houches Summer School, August 1981, CERN preprint TH-3174-CERN (September, 1981).
3. J. Burger, Recent Results from PETRA: Search For New Particles, DESY preprint 81-074 (October, 1981).
4. M. K. Gaillard and L. Maiani, New Quarks and Leptons, Cargese Summer Institute on Quarks and Leptons, Cargese, 1979 (Plenum Press, New York, 1980) 433;
C. T. Hill, Phys. Rev. D24, (1981) 691;
D. V. Nanopoulos and D. A. Ross, Phys. Lett. 108B, (1982) 351.
5. A. J. Buras, Phys. Rev. Lett. 46, (1981) 1354;
V. Barger, W. F. Long, E. Ma, and A. Pramudita, Phys. Rev. D25, (1982) 1860;
R. J. Oakes, Constraints on Charge 2/3 Quark Masses, Fermilab preprint FERMILAB-Pub -82/13-Thy (January, 1982).
6. For example, S. L. Glashow and S. Weinberg, Phys. Rev. D15, (1977) 1958;
M. Chanowitz, J. Ellis, and M. K. Gaillard, Nucl. Phys. B128, (1977) 506;
J. Ellis, Technicolour-Oasis or Mirage?, CERN preprint TH-3139-CERN (August, 1981).

7. M. K. Gaillard and B. W. Lee, Phys. Rev. D10, (1974) 897.
8. S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D2, (1970) 1285.
9. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, (1973) 652.
10. M. K. Gaillard, B. W. Lee, and R. E. Shrock, Phys. Rev. D13, (1976) 2674;
E. Ma and A. Pramudita, Phys. Rev. D22, (1980) 214;
T. Inami and C. S. Lim, Prog. Theor. Phys. 65, (1981) 297.
11. G. Bonneau, Phys. Lett. 96B, (1980) 147;
G. Bonneau, Nucl. Phys. B177, (1981) 523;
B. A. Ovrut, Axial Vector Ward Identities and Dimensional Regularization, Institute for Advanced Study preprint (February, 1981); and references contained therein.
12. W. A. Bardeen, R. Gastmans, and B. Lautrup, Nucl. Phys. B46, (1972) 319;
M. Chanowitz, M. Furman, and I. Hinchliffe, Nucl. Phys. B159, (1979) 225.
13. G. Passarino and M. Veltman, Nucl. Phys. B160, (1979) 151.
14. G. 't'Hooft and M. Veltman, Nucl. Phys. B153, (1979) 365.
15. M. Veltman, Radiative Corrections to Weak and Electromagnetic Processes, Cargese Summer Institute on Quarks and Leptons, Cargese, 1979 (Plenum Press, New York, 1980) 1.
16. I follow the metric and conventions of C. Itzykson and J. B. Zuber, Quantum Field Theory (McGraw-Hill, U.S.A., 1980).
17. S. Weinberg, in A Festschrift for I. I. Rabi (New York Academy of Sciences, New York, 1977) 185.

18. A. Sirlin and W. J. Marciano, Nucl. Phys. B189, (1981) 442.
19. D. Albert, W. J. Marciano, D. Wyler, and Z. Parsa, Nucl. Phys. B166, (1980) 460;
W. J. Marciano and Z. Parsa, Z^0 Decay, Proc. of the Cornell Z^0 Theory Workshop, Cornell, 1981, Cornell preprint CLNS-81-485 (February, 1981) 127.
20. Particle Data Group, Review of Particle Properties, Rev. Mod. Phys. 52, II, (April 1980) S41.
21. V. Barger, W. F. Long, E. Ma, and A. Pramudita in Ref. [5].
22. R. E. Shrock and L. L. Wang, Phys. Rev. Lett. 41, (1978) 1692;
R. E. Shrock, S. B. Treiman, and L. L. Wang, Phys. Rev. Lett. 42, (1979) 1589.
23. S. Pakvasa, S. F. Tuan, and J. J. Sakurai, Phys. Rev. D23, (1981) 2799;
J. J. Sakurai, The Structure of Charged Currents, Neutrino 81 Proc. Vol. II, Hawaii 1981 (Univ. of Hawaii, Honolulu, 1981) 457.
24. JADE Collaboration, Upper Limit on Beauty Lifetime and Lower Limit on Weak Mixing Angles, DESY preprint DESY-82-014 (March, 1982).
25. H. E. Haber, G. L. Kane, and T. Sterling, Nucl. Phys. B161, (1979) 493.
26. J. Ellis in Ref. [6].
27. For example, T. Inami and C. S. Lim in Ref. [10].

FIGURE CAPTIONS

- Figure 1. Diagrams giving the one-loop $Z \rightarrow t\bar{c}$ amplitude in unitary gauge.
- Figure 2. Diagrammatic representation of the Ward identities of Appendix C. (A) Vector Ward identity; (B) Axial Ward identity.
- Figure 3. (A) Cut giving the absorptive part of the one-loop $Z \rightarrow t\bar{c}$ amplitude for $M_T < M_W + M_J$. (B) Product of diagrams computed for the imaginary part check. (C) An additional cut contributing for $M_T > M_W + M_J$.
- Figure 4. High, low, and average values of $\Gamma(Z \rightarrow \text{top})$ and $\text{BR}(Z \rightarrow \text{top})$, as defined in the text, vs. top quark mass.
- Figure 5. $\Gamma(Z \rightarrow \text{top})$ vs. KM matrix elements $(U_{Tb} U_{cb})^2$, for four top quark masses. Computed points follow the solid lines, but deviate from the dashed lines which are only the extensions of the solid lines to the origin. The high endpoint of each solid line gives the largest computed Γ for that M_T .
- Figure 6. High value of $\Gamma(Z \rightarrow \text{top})$ (as in Fig. 4), divided by product of KM matrix elements $(U_{Tb} U_{cb})^2$ and phase space factor $P(t, c)$ (see Eq. (4.4)).
- Figure 7. Average values of $\Gamma(Z \rightarrow u\bar{c} + \bar{u}c)$ and $\text{BR}(Z \rightarrow u\bar{c} + \bar{u}c)$, as defined in the text.

FIGURE 1

XBL 827 - 868

FIGURE 2

FIGURE 3

FIGURE 4

XBL 827 - 863

FIGURE 5

FIGURE 6

X8L 827 - 862

FIGURE 7

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