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Upper bound on the smuon mass from vacuum stability in the light of muon $g - 2$ anomaly

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Abstract

We derive an upper bound on the smuon mass assuming that the muon $g - 2$ anomaly is explained by the supersymmetric (SUSY) contribution. In the minimal SUSY standard model, the SUSY contribution to the muon $g - 2$ is enhanced when the Higgsino mass parameter is large. Then, the smuon-smuon-Higgs trilinear coupling is enhanced, which may destabilize the electroweak vacuum. We calculate precisely the decay rate of the electroweak vacuum in such a case. We include one-loop effects which are crucial to determine the overall normalization of the decay rate. Requiring that the theoretical prediction of the muon anomalous magnetic moment is consistent with the observed value at the 1 and 2σ levels (equal to the central value of the observed value), we found that the lightest smuon mass should be smaller than 1.38 and 1.68 TeV (1.20 TeV) for $\tan \beta = 10$ (with $\tan \beta$ being the ratio of the vacuum expectation values of the two Higgs bosons), respectively, and the bound is insensitive to the value of $\tan \beta$.

1 Introduction

The muon $g - 2$ measurements at the BNL and FermiLab experiments had a great impact on the study of particle physics. The value of the muon anomalous magnetic moment for these two combined is [1–4]

$$a_\mu^{(\text{exp})} = (11\,659\,206.1 \pm 4.1) \times 10^{-10}. \tag{1.1}$$

On the contrary, the standard-model (SM) predicts [5][#1]

$$a_\mu^{(\text{SM})} = (11\,659\,181.0 \pm 4.3) \times 10^{-10}. \tag{1.2}$$

These values give

$$\Delta a_\mu \equiv a_\mu^{(\text{exp})} - a_\mu^{(\text{SM})} = (25.1 \pm 5.9) \times 10^{-10}, \tag{1.3}$$

which shows 4.2σ discrepancy between the experimentally measured value of a_μ and the SM prediction (the so-called muon $g - 2$ anomaly). The discrepancy seems to strongly indicate the existence of a physics beyond the SM (BSM), which can be the origin of the muon $g - 2$ anomaly.

One of the attractive candidates of the BSM physics which can solve the muon $g - 2$ anomaly is the supersymmetry (SUSY). In particular, in the minimal SUSY SM (MSSM), the smuon-neutralino and sneutrino-chargino diagrams may contribute significantly to the muon anomalous magnetic moment [26–28]; the size of the SUSY contribution can be as large as Δa_μ to solve the muon $g - 2$ anomaly (for the recent studies about the MSSM contribution to the muon $g - 2$, see, for example, [29–54]). Because the superparticles are in the loops, the SUSY contribution to the muon $g - 2$ is suppressed as the superparticles become heavy. Thus, in order to explain the muon $g - 2$ anomaly, masses of (some of) the superparticles are bounded from above. A detailed understanding of the upper bound is important in order to verify the SUSY interpretation of the muon $g - 2$ anomaly with ongoing and future collider experiments [55–57].

The muon $g - 2$ anomaly can be explained in various parameter regions of the MSSM. If the masses of all the superparticles are comparable, the masses of superparticles are required to be of $O(100)$ GeV. Then, the muon $g - 2$ anomaly indicates that superparticles (in particular, sleptons, charginos, and neutralinos) are important targets of ongoing and future collider experiments. The SUSY contribution to the muon $g-2$ can be, however, sizable even if superparticles are much heavier. It happens when the Higgsino mass parameter (*i.e.*, the so-called μ parameter) is significantly large and the enlarged smuon-smuon-Higgs trilinear scalar coupling enhances the contribution to the muon $g - 2$. Such a trilinear coupling is, however, dangerous because it may make the electroweak (EW) vacuum unstable [58–61].

In this letter, we study the stability of the EW vacuum, paying attention to the parameter region of the MSSM where the muon $g-2$ anomaly is solved (or alleviated) by the SUSY

[#1]For more details about the estimation of the SM prediction, see Refs. [6–25].

contribution. Requiring that the SUSY contribution to the muon anomalous magnetic moment, denoted as $a_\mu^{(\text{SUSY})}$, be large enough to solve the muon $g-2$ anomaly, the smuon masses are bounded from above based on the observed longevity of the EW vacuum. Refs. [29, 55] have studied the vacuum stability bound using a tree-level analysis of the decay rate for the case where the SUSY breaking mass parameters of the left- and right-handed sleptons are degenerate. The tree-level analysis, however, has several inaccuracies. In particular, it cannot fix the dimensionful prefactor of the decay rate and receives the renormalization scale uncertainty. These difficulties cannot be avoided without performing the calculation at the one-loop level. We study the stability of the EW vacuum using the state-of-the-art method to calculate the decay rate of the false vacuum [62–64], with which a full one-loop calculation of the decay rate is performed. We also consider a wide range of the slepton mass parameters. Then, based on the accurate estimation of the decay rate, we derive an upper bound on the lightest smuon mass to explain the muon $g-2$ anomaly.

This letter is organized as follows. In Section 2, we briefly overview the SUSY contribution to the muon anomalous magnetic moment and discuss the importance of the stability of the EW vacuum. In Section 3, we introduce the effective field theory (EFT) we use in our analysis. In Section 4, we explain our procedure to calculate the decay rate of the EW vacuum. Our main results are given in Section 5. Section 6 is devoted for conclusions and discussion.

2   MSSM and muon $g-2$

We first overview the model we consider, which is the low energy effective theory obtained from the MSSM. (For the review of the MSSM, see, for example, Ref. [65].) We also explain why the stability of the EW vacuum is important in the study of the SUSY contribution to the muon $g-2$.

Since $a_\mu^{(\text{SUSY})}$ is enhanced in the parameter region in which $\tan\beta$ is large [28], we concentrate on the large $\tan\beta$ case to obtain a conservative bound on the mass scale of the smuons. Importantly, $\tan\beta$ cannot be arbitrarily large if we require perturbativity. In particular, $\tan\beta$ is smaller than ~ 50 in the grand unified theory (GUT), which is one of the strong motivations to consider the MSSM. There, the coupling constants (in particular, the bottom Yukawa coupling constant) should be perturbative up to the GUT scale.

In order to study the behavior of $a_\mu^{(\text{SUSY})}$ in the large $\tan\beta$ case, it is instructive to use the so-called mass insertion approximation in which $a_\mu^{(\text{SUSY})}$ is estimated in the gauge-eigenstate basis and the interactions proportional to the Higgs vacuum expectation values (VEVs) are treated as perturbations. (In our following numerical calculation, $a_\mu^{(\text{SUSY})}$ is estimated more precisely by using the basis in which the sleptons, charginos, and neutralinos are in the mass eigenstates, as we will explain.) In Fig. 1, we show one-loop diagrams which may dominate the SUSY contribution to the muon $g-2$ in the large $\tan\beta$ limit. Because the superparticles are in the loop, $a_\mu^{(\text{SUSY})}$ is suppressed as the superparticles become heavier. For the case where the masses of all the superparticles are comparable, for example, the
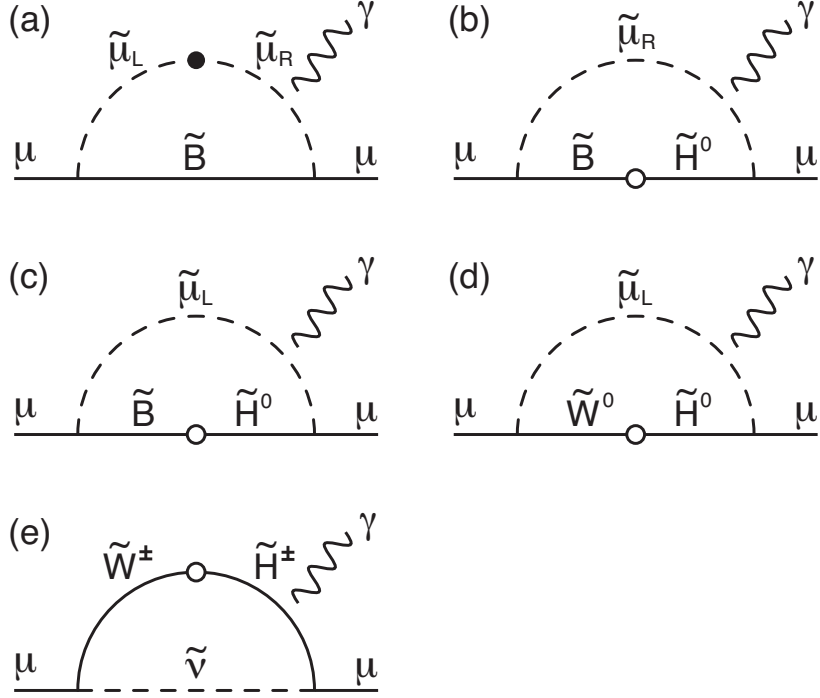
Figure 1: One-loop Feynman diagrams, which are enhanced in large $\tan\beta$ limit, giving rise to the SUSY contribution to the muon $g - 2$. Here, the mass insertion approximation is adopted. The black and white blobs are two-point interactions induced by the VEVs of Higgs bosons.

SUSY contribution to the muon anomalous magnetic moment is approximately given by $|a_\mu^{(\mathrm{SUSY})}| \simeq \frac{5g_2^2}{192\pi^2} \frac{m_\mu^2}{m_{\mathrm{SUSY}}^2} \tan\beta$, where g_2 is the gauge coupling constant of $SU(2)_L$, m_μ is the muon mass, and m_{SUSY}^2 is the mass scale of superparticles. (Here, the contributions of the diagrams that contain the Bino are neglected because they are subdominant.) Taking $\tan\beta \sim 50$, which is the approximate maximum possible value of $\tan\beta$ for the perturbativity up to the GUT scale, the superparticles should be lighter than ~ 700 GeV in order to make the total muon anomalous magnetic moment consistent with the observed value at the 2σ level.

Such an upper bound is significantly altered by the Bino-smuon diagram (Fig. 1 (a)). The other diagrams (i.e., Fig. 1 (b) $-$ (e)) have slepton, gaugino, and Higgsino propagators in the loop, and hence their contributions are suppressed when any of these particles is heavy. On the contrary, the Bino-smuon diagram has only the smuon and Bino propagators in the loop, and its contribution is approximately proportional to the Higgsino mass parameter μ. Thus, with a very large μ parameter, the contribution of the Bino-smuon diagram can be large enough to cure the muon $g - 2$ anomaly even if the smuon and/or Bino are much heavier than the upper bound estimated above.

In the following, we study the upper bound on the masses of superparticles in the light

of the muon $g-2$ anomaly, paying particular attention to the contribution of the Bino-smuon diagram. In the parameter region where the Bino-smuon diagram has a dominant contribution, a large μ parameter enhances the smuon-smuon-Higgs trilinear coupling. Such a large trilinear scalar coupling is dangerous because it may destabilize the EW vacuum. Consequently, the lifetime of the EW vacuum may become shorter than the present cosmic age [66]:

$$t_{\rm now} \simeq 13.8 \ {\rm Gyr.} \tag{2.1}$$

The parameter region predicting the too short lifetime of the EW vacuum is excluded.

The purpose of this letter is to derive an upper bound on the smuon mass under the requirement that the muon $g-2$ anomaly be solved (or relaxed) by the SUSY contribution. We are interested in the case where the μ parameter is large so that the Bino-smuon diagram dominates $a_\mu^{\rm (SUSY)}$; hereafter, we consider the case where μ is much larger than the Bino and smuon masses. A large value of μ implies heavy Higgsinos. In addition, the stops are expected to be relatively heavy to push up the lightest Higgs mass to the observed value, *i.e.*, about 125 GeV, through the radiative correction [67–70]. On the contrary, in order to enhance $a_\mu^{\rm (SUSY)}$, slepton and Bino masses should be close to the EW scale. Based on these considerations, in this letter, we consider the case where the Bino $\tilde{B}$ and smuons are relatively light among the MSSM particles. These particles are assumed to have EW-scale masses comparable to the top pole mass M_t. The other MSSM constituents are assumed to have heavier masses, which are characterized by a single scale M_S. (For simplicity, masses of gauginos other than $\tilde{B}$ are assumed to be of $O(M_S)$.) We assume that there exists a significant hierarchy between M_t and M_S and thus the superparticles with the masses of $\sim M_S$ do not affect the physics of our interest. A comment on the case where some of the other SUSY particles are as light as the smuons will be given at the end of this letter.

A large value of μ suggests relatively large values of the soft SUSY breaking Higgs mass parameters for a viable EW symmetry breaking; the light Higgs mass (*i.e.*, the mass of the SM-like Higgs boson) is realized by the cancellation between the contributions of the μ and soft SUSY breaking parameters. The heavier Higgs doublet is expected to have masses of $O(M_S)$ which is comparable to μ. In such a case, the SM-like Higgs doublet, denoted as H, and the heavier doublet, H', are given by linear combinations of the up- and down-type Higgs bosons, denoted as H_u and H_d, respectively, as

$$\begin{pmatrix} H \\ H' \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} H_d \\ H_u \end{pmatrix}, \tag{2.2}$$

where $\tan\beta$ is the ratio of the vacuum expectation values (VEVs) of up- and down-type Higgs bosons.

In the case of our interest, the mass spectrum around the EW scale includes the second-generation sleptons and Bino $\tilde{B}$, as well as the SM particles. Hereafter, the sleptons in the second generation in the gauge eigenstate are denoted as $\tilde{\ell}_L$ and $\tilde{\mu}_R$; $\tilde{\ell}_L$ is an $SU(2)_L$ doublet

with hypercharge $\frac{1}{2}$, which is decomposed as

$$\tilde{\ell}_L = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{\mu}_L \end{pmatrix}, \tag{2.3}$$

while $\tilde{\mu}_R$ is an $SU(2)_L$ singlet with hypercharge -1.

3   Effective field theory analysis

We are interested in the case where there exists a hierarchy in the mass spectrum of the MSSM particles. To deal with the hierarchy, we resort to the EFT approach and solve the renormalization group (RG) equations with proper boundary conditions to evaluate the EFT coupling constants. Hereafter, we assume that the effects of possible CP-violating phases are negligible.

We adopt M_t and M_S as matching scales. For the renormalization scale $Q < M_t$ (with Q being the renormalization scale), we consider the QCD+QED that contains the SM gauge couplings and fermion masses as parameters. For $M_t < Q < M_S$, we consider an EFT with Bino and smuons as described below. At $Q = M_S$, the EFT is matched to the full MSSM, which imposes relations among EFT couplings. We choose M_S to be close to the Higgsino mass.

The Lagrangian of the EFT, which is relevant for the calculation of the decay rate of the EW vacuum and the muon $g - 2$, is given by

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \Delta \mathcal{L}_{\text{kin}} + \Delta \mathcal{L}_{\text{mass}} + \Delta \mathcal{L}_{\text{Yukawa}} - V, \tag{3.1}$$

where $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian without the Higgs potential, and the additional kinetic terms, mass terms, and Yukawa couplings are described by

$$\Delta \mathcal{L}_{\text{kin}} = |D_\mu \tilde{\ell}_L|^2 + |D_\mu \tilde{\mu}_R|^2 - i\tilde{B}\sigma^\mu \partial_\mu \tilde{B}^\dagger, \tag{3.2}$$

$$\Delta \mathcal{L}_{\text{mass}} = -\frac{1}{2} M_1 \tilde{B}\tilde{B} + \text{h.c.}, \tag{3.3}$$

$$\Delta \mathcal{L}_{\text{Yukawa}} = Y_L \tilde{\ell}_L^\dagger \ell_L \tilde{B} + Y_R \tilde{\mu}_R^\dagger \mu_R \tilde{B}^\dagger + \text{h.c.}, \tag{3.4}$$

where ℓ_L and μ_R are the second generation left-handed lepton doublet and right-handed lepton, respectively. We use the two-component Weyl notation for fermions. The scalar potential V is given by

$$V = V_2 + V_3 + V_4, \tag{3.5}$$

with

$$V_2 = m_H^2 |H|^2 + m_L^2 |\tilde{\ell}_L|^2 + m_R^2 |\tilde{\mu}_R|^2, \tag{3.6}$$

$$V_3 = -T H^\dagger \tilde{\ell}_L \tilde{\mu}_R^\dagger + \text{h.c.}, \tag{3.7}$$

$$V_4 = \lambda_H |H|^4 + \lambda_{HL} |H|^2 |\tilde{\ell}_L|^2 + \lambda_{HR} |H|^2 |\tilde{\mu}_R|^2 + \kappa (H^\dagger \tilde{\ell}_L)(\tilde{\ell}_L^\dagger H)$$
$$+ \lambda_L |\tilde{\ell}_L|^4 + \lambda_R |\tilde{\mu}_R|^4 + \lambda_{LR} |\tilde{\ell}_L|^2 |\tilde{\mu}_R|^2, \tag{3.8}$$

5

where T is the trilinear scalar coupling constant.

Next, we describe the matching conditions of coupling constants at the threshold scales. All the SM parameters including the Higgs quartic coupling $\lambda_H^{(\text{SM})}$ and the mass squared parameter $m_H^{2(\text{SM})}$ are determined at $Q = M_t$. Importantly, the top Yukawa coupling, the gauge couplings, the Higgs quartic coupling, and the Higgs mass parameter are subject to the possibly large weak-scale threshold corrections. We use the results of [71] to fix these parameters with using physical parameters $\alpha_3(M_Z) = 0.1179$, $M_t = 172.76\,\text{GeV}$, $M_W = 80.379,\text{GeV}$, and $M_h = 125.25\,\text{GeV}$ [66]. As for the light fermion couplings, we calculate the running of their masses with the one-loop QED and three-loop QCD beta functions [72–74] to determine the corresponding Yukawa couplings at $Q = M_t$.

For other parameters, we mostly adopt the tree-level matching between the SM and the EFT at $Q = M_t$, but take into account some of the one-loop corrections which can be sizable. The corrections to the Higgs quartic coupling and the mass term are given by

$$\lambda_H = \lambda_H^{(\text{SM})} + \Delta\lambda_H, \tag{3.9}$$

$$m_H^2 = m_H^{2(\text{SM})} + \Delta m_H^2, \tag{3.10}$$

with

$$(16\pi)^2 \Delta\lambda_H = \left(\lambda_{HL}^2 + \lambda_{HL}\kappa + \frac{1}{2}\kappa^2\right) B_0(m_L^2, m_L^2) + \frac{1}{2}\lambda_{HR}^2 B_0(m_R^2, m_R^2)$$
$$+ (\lambda_{HL} + \kappa)T^2 C_0(m_L^2, m_L^2, m_R^2) + \lambda_{HR}T^2 C_0(m_R^2, m_R^2, m_L^2)$$
$$+ \frac{1}{2}T^4 D_0(m_L^2, m_R^2, m_L^2, m_R^2), \tag{3.11}$$

$$(16\pi)^2 \Delta m_H^2 = (2\lambda_{HL} + \kappa) A_0(m_L^2) + \lambda_{HR} A_0(m_R^2) + T^2 B_0(m_L^2, m_R^2), \tag{3.12}$$

where A_0, B_0, C_0, and D_0 are the Passarino-Veltman one-, two-, three-, and four-point functions without momentum inflow, respectively [75]. In determining the muon Yukawa coupling in the EFT, we also take account of the one-loop correction [76, 77] because it may significantly affect the vacuum decay rate and $a_\mu^{(\text{SUSY})}$. The correction Δy_μ is given by

$$(16\pi)^2 \Delta y_\mu = Y_L Y_R T M_1 J(M_1^2, m_R^2, m_L^2), \tag{3.13}$$

with

$$J(a, b, c) \equiv -\frac{ab\ln(a/b) + bc\ln(b/c) + ca\ln(c/a)}{(a-b)(b-c)(c-a)}. \tag{3.14}$$

The muon Yukawa coupling constant in the EFT, y_μ, and that in the SM, $y_\mu^{(\text{SM})}$, are related as $y_\mu = y_\mu^{(\text{SM})} + \Delta y_\mu$. In the present case, the sign of Δy_μ is correlated with that of $a_\mu^{(\text{SUSY})}$ and is negative. These corrections can be sizable due to the hierarchy of the scales M_1, m_L, $m_R \ll M_S$. Concerning the trilinear coupling T, the value is determined so that an input value of $a_\mu^{(\text{SUSY})}$ is realized.

In the MSSM, the scalar potential is completely determined only by the parameters of the superpotential, $i.e.$, the Yukawa and gauge couplings. Accordingly, we impose the matching conditions on the EFT couplings at the matching scale $Q = M_S$. At the tree level, these conditions are given by

$$Y_L = \frac{1}{\sqrt{2}}g_Y = \sqrt{\frac{3}{10}}g_1, \quad Y_R = -\sqrt{2}g_Y = -\sqrt{\frac{6}{5}}g_1, \tag{3.15}$$

where g_Y and g_1 are the $U(1)_Y$ gauge coupling constant and its $SU(5)$-normalized value, respectively, and

$$\lambda_R = \frac{3}{10}g_1^2, \tag{3.16}$$

$$\lambda_L = \frac{1}{8}g_2^2 + \frac{3}{40}g_1^2, \tag{3.17}$$

$$\lambda_{LR} = \frac{y_\mu^2}{\cos^2\beta} - \frac{3}{10}g_1^2, \tag{3.18}$$

$$\lambda_{HR} = y_\mu^2 - \frac{3}{10}g_1^2\cos 2\beta, \tag{3.19}$$

$$\lambda_{HL} = \left(\frac{1}{4}g_2^2 + \frac{3}{20}g_1^2\right)\cos 2\beta, \tag{3.20}$$

$$\kappa = y_\mu^2 - \frac{1}{2}g_2^2\cos 2\beta. \tag{3.21}$$

The T-parameter is related to the MSSM parameters as

$$T = y_\mu\mu\tan\beta + A_\mu\cos\beta, \tag{3.22}$$

with A_μ being the soft SUSY breaking trilinear scalar coupling of smuon. For simplicity, we assume that the SUSY breaking contribution to the scalar trilinear coupling, T, is negligible. We expect that this assumption is valid when $\mu\tan\beta$ is much larger than the typical smuon masses, which is the case in our following discussion. Notice that, as we see below, the SUSY contribution to the muon $g-2$ and the decay rate of the EW vacuum are both dependent on the MSSM parameters through the T-parameter. Thus, the upper bound on the smuon mass, which will be derived in the following Sections, will be almost unchanged even if the effect of A_μ on T is sizable. Notice that we use Eq. (3.22) only to evaluate μ.

Although we determine λ_H at $Q = M_t$, there is also the SUSY relation between λ_H and other couplings. Considering only the stop contribution to the threshold correction at $Q = M_S$, we obtain the one-loop matching condition [78],

$$\lambda_H = \left(\frac{1}{8}g_2^2 + \frac{3}{40}g_1^2\right)\cos^2 2\beta + \delta\lambda_H, \tag{3.23}$$

with

$$(16\pi^2)\delta\lambda_H \simeq \frac{3}{2}y_t^2 \left[y_t^2 + \left(\frac{1}{2}g_2^2 - \frac{1}{10}g_1^2 \right) \cos 2\beta \right] \ln \frac{m_{Q3}^2}{Q^2}$$

$$+ \frac{3}{2}y_t^2 \left(y_t^2 + \frac{2}{5}g_1^2 \cos 2\beta \right) \ln \frac{m_{U3}^2}{Q^2}$$

$$+ \frac{\cos^2 2\beta}{200} \left[(25g_2^4 + g_1^4) \ln \frac{m_{Q3}^2}{Q^2} + 8g_1^4 \ln \frac{m_{U3}^2}{Q^2} + 2g_1^4 \ln \frac{m_{D3}^2}{Q^2} \right], \qquad (3.24)$$

where y_t is the top-quark Yukawa coupling constant while m_{Q3}, m_{U3}, and m_{D3} are the mass parameters of the third generation left-handed squark, right-handed up-type squark, and right-handed down-type squark, respectively. For simplicity, we neglect the threshold correction to the parameters of the superpotential, *i.e.*, the top Yukawa coupling and the gauge couplings, and use their values in the EFT at $Q = M_S$ to evaluate the size of $\delta\lambda_H$. Once the value of λ_H at the matching scale M_S is obtained, we can solve (3.23) against the stop mass $m_{\tilde{t}}$ assuming the universality $m_{\tilde{t}} \equiv m_{Q3} = m_{U3} = m_{D3}$. Requiring the observed Higgs mass to be realized, we have checked that difference between $|\mu|$ and $m_{\tilde{t}}$ is within one or two orders of magnitude in the region with small enough decay rate of the EW vacuum.[#2]

For $M_t < Q < M_S$, we solve the RG equations of the EFT. We use the two-loop RG equations [79] augmented by some important three-loop contributions calculated in [71] for the SM-like couplings. On the other hand, Bino and smuon contributions to the beta functions of the SM-like couplings and the beta functions of the couplings specific to the EFT are calculated at the one-loop level. Since all the SM parameters are fixed at $Q = M_t$ and below, while the other couplings are determined at $Q = M_S$, we iteratively solve the RG evolution in $M_t < Q < M_S$ to obtain consistent solutions.

Next, we explain how we calculate the SUSY contribution to the muon $g - 2$. Because we are interested in the case where the masses of Bino and smuons are much lighter than Higgsino (and other superparticles), the EFT parameters introduced above are used.

The mass matrix of the smuons is given by

$$\mathbf{M}_{\tilde{\mu}}^2 = \begin{pmatrix} m_L^2 + (\lambda_{HL} + \kappa)v^2 & -Tv \\ -Tv & m_R^2 + \lambda_{HR}v^2 \end{pmatrix}, \qquad (3.25)$$

where $v \simeq 174$ GeV is the vacuum expectation value of the SM-like Higgs. The mass matrix can be diagonalized by a 2×2 unitary matrix U as

$$\mathrm{diag}(m_{\tilde{\mu}_1}^2, m_{\tilde{\mu}_2}^2) = U^\dagger \mathbf{M}_{\tilde{\mu}}^2 U, \qquad (3.26)$$

and the gauge eigenstates are related to the mass eigenstates, denoted as $\tilde{\mu}_A$ ($A = 1, 2$), as

$$\begin{pmatrix} \tilde{\mu}_L \\ \tilde{\mu}_R \end{pmatrix} = U \begin{pmatrix} \tilde{\mu}_1 \\ \tilde{\mu}_2 \end{pmatrix} \equiv \begin{pmatrix} U_{L,1} & U_{L,2} \\ U_{R,1} & U_{R,2} \end{pmatrix} \begin{pmatrix} \tilde{\mu}_1 \\ \tilde{\mu}_2 \end{pmatrix}. \qquad (3.27)$$

[#2]In some case, $m_{\tilde{t}}$ becomes one or two orders of magnitude smaller than $|\mu|$ and it may induce a color breaking minimum where stops acquire VEVs. We do not discuss the instability due to such a color breaking minimum because it depends on various fields and parameters that are not included in our EFT.

At the one-loop level, the Bino-smuon loop contributions to the muon anomalous magnetic moment is given by [28]

$$a_\mu^{(\text{SUSY, 1-loop})} = \frac{m_\mu^2}{16\pi^2} \sum_{A=1}^{2} \frac{1}{m_{\tilde{\mu}_A}^2} \left[-\frac{1}{12}\mathcal{A}_A f_1(x_A) - \frac{1}{3}\mathcal{B}_A f_2(x_A) \right], \qquad (3.28)$$

where $x_A \equiv M_1^2/m_{\tilde{\mu}_A}^2$,

$$\mathcal{A}_A \equiv Y_L^2 U_{L,A}^2 + Y_R^2 U_{R,A}^2, \quad \mathcal{B}_A \equiv \frac{M_1 Y_L Y_R U_{L,A} U_{R,A}}{m_\mu}, \qquad (3.29)$$

and the loop functions are given by

$$f_1(x) \equiv \frac{2}{(1-x)^4}(1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x), \qquad (3.30)$$

$$f_2(x) \equiv \frac{3}{(1-x)^3}(1 - x^2 + 2x \ln x). \qquad (3.31)$$

In the MSSM, some of the two-loop contributions to the muon anomalous magnetic moment may become sizable. One important contribution is the non-holomorphic correction to the muon Yukawa coupling constant [76,77]. In the limit of large $\tan\beta$ (or, large T), such an effect can be significant. In the present setup, such a non-holomorphic correction to the muon Yukawa coupling constant is taken into account when the EFT parameters (in particular, y_μ) are matched to the MSSM parameters at the SUSY scale. Another is the photonic two-loop correction [80, 81]. Such a contribution includes large QED logarithms and can affect the SUSY contribution to the muon $g - 2$ by ~ 10 % or more. The full photonic two-loop correction relevant for our analysis is given by [81]

$$a_\mu^{(\text{SUSY, photonic})} = \frac{m_\mu^2}{16\pi^2} \frac{\alpha}{4\pi} \sum_{A=1}^{2} \frac{1}{m_{\tilde{\mu}_A}^2} \left[16 \left\{ -\frac{1}{12}\mathcal{A}_A f_1(x_A) - \frac{1}{3}\mathcal{B}_A f_2(x_A) \right\} \ln \frac{m_\mu}{m_{\tilde{\mu}_A}} \right.$$
$$\left. - \left\{ -\frac{35}{75}\mathcal{A}_A f_3(x_A) - \frac{16}{9}\mathcal{B}_A f_4(x_A) \right\} + \frac{1}{4}\mathcal{A}_A f_1(x_A) \ln \frac{m_{\tilde{\mu}_A}^2}{Q_{\text{DREG}}^2} \right], \qquad (3.32)$$

where α is the fine structure constant, Q_{DREG} is the dimensional-regularization scale, and

$$f_3(x) \equiv \frac{4}{105(1-x)^4}[(1-x)(-97x^2 - 529x + 2) + 6x^2(13x + 81)\ln x$$
$$+ 108x(7x + 4)\text{Li}_2(1-x)], \qquad (3.33)$$

$$f_4(x) \equiv \frac{-9}{4(1-x)^3}[(x+3)(x\ln x + x - 1) + (6x+2)\text{Li}_2(1-x)]. \qquad (3.34)$$

In our analysis, the SUSY contribution to the muon anomalous magnetic moment is evaluated as

$$a_\mu^{(\text{SUSY})} = a_\mu^{(\text{SUSY, 1-loop})} + a_\mu^{(\text{SUSY, photonic})}, \qquad (3.35)$$

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using the EFT parameters evaluated at the renormalization scale $Q = M_t$. We note that the above prescription gives a good estimation of the SUSY contribution to the muon anomalous magnetic moment in the parameter region we consider in the following discussion. In particular, for the case of our interest, the effect of the Bino-Higgsino-smuon diagrams (*i.e.*, Fig. 1 (b) and (c)) is estimated to be $O(0.1)$ % or smaller relative to $a_\mu^{(\text{SUSY})}$ given above. The Wino-Higgsino-slepton diagrams (*i.e.*, Fig. 1 (d) and (e)) become irrelevant in the decoupling limit of the Winos.

4   Decay rate of electroweak vacuum

Using the method proposed by Callan and Coleman [82, 83], the vacuum decay rate can be written in the following form:

$$\gamma = \mathcal{A}e^{-\mathcal{B}}, \tag{4.1}$$

where $\mathcal{B}$ is the so-called bounce action and $\mathcal{A}$ is a prefactor with mass-dimension four. Previous tree-level analyses naively estimated the prefactor $\mathcal{A}$ based on a typical energy scale of the bounce. It has been pointed out that $\mathcal{A}$ may deviate significantly from the naive estimation in particular when there are many particles that couple to the bounce [84] and hence the precise calculation of $\mathcal{A}$ is important for the accurate determination of the allowed parameter space. The prefactor has been first evaluated for the SM in [85] and it has been reevaluated recently with the correct treatment of zero modes in [86–88] using the prescription proposed in [62, 63]. The prescription has been generalized to a multi-field bounce in [64], which enabled the calculation of precise decay rates in a more complex setup like the one in this letter. All the coupling constants used below should be understood as those in the EFT at the renormalization scale of $Q = M_t$.

The bounce is a spherical object in four-dimensional Euclidean space. We parameterize the bounce as

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \rho_h(r) \end{pmatrix}, \quad \tilde{\ell}_L = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \rho_L(r) \end{pmatrix}, \quad \tilde{\mu}_R = \frac{1}{\sqrt{2}}\rho_R(r), \tag{4.2}$$

where ρ_I ($I = h, L, R$) are real fields and r is the radius in the four-dimensional Euclidean space. Notice that the upper component of H can be taken to be 0 without loss of generality due to the $SU(2)_L \times U(1)_Y$ symmetry. The directions of the other fields are chosen such that the trilinear interaction, $\tilde{\mu}_R^\dagger H^\dagger \tilde{\ell}_L$, becomes non-vanishing. Then, the bounce configuration is a solution of the Euclidean equations of motion:

$$\partial_r^2 \rho_I + \frac{3}{r}\partial_r \rho_I = \frac{\partial V}{\partial \rho_I}, \tag{4.3}$$

satisfying the following boundary conditions:

$$\rho_h(\infty) = \sqrt{2}v_{\text{EFT}}, \quad \rho_L(\infty) = \rho_R(\infty) = 0, \quad \partial_r \rho_I(0) = 0, \tag{4.4}$$

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where v_{EFT} is the Higgs VEV at the false vacuum in the EFT. We obtain the bounce solution by numerically solving Eq. (4.3) using a modified version of the gradient flow method [89–91].

Next, we explain how we can obtain the prefactor, $\mathcal{A}$, which takes care of the one-loop effect on the decay rate. The prefactor is obtained by the functional determinant of the fluctuation matrix which is given by the second-order functional derivative of the total action (containing the total scalar potential given in Eq. (3.5)). The prefactor can be expressed as

$$\mathcal{A} = 2\pi \mathcal{J}_{\text{EM}} \frac{\mathcal{B}}{4\pi^2} \mathcal{A}^{(A,\varphi,c\bar{c})} \mathcal{A}^{(\psi)}, \tag{4.5}$$

where $A^{(A,\varphi,c\bar{c})}$ ($\mathcal{A}^{(\psi)}$) is the effect of gauge bosons, scalar bosons and Faddev-Popov ghosts (fermions), and $\mathcal{J}_{\text{EM}}$ is the Jacobian in association with the zero-mode due to the electromagnetic symmetry breaking. In calculating $\mathcal{A}$, we take into account the effects of the smuons and the Bino as well as the $SU(2)_L$ and $U(1)_Y$ gauge bosons, the Higgs boson, the muons, and the top quark. $\mathcal{A}^{(A,\varphi,c\bar{c})}$ and $\mathcal{A}^{(\psi)}$ are given by the ratios of functional determinants for the partial waves:

$$\mathcal{A}^{(A,\varphi,c\bar{c})} = \frac{\det \mathcal{M}_0^{(c\bar{c})}}{\det \widehat{\mathcal{M}}_0^{(c\bar{c})}} \left(\frac{\det' \mathcal{M}_0^{(S\varphi)}}{\det \widehat{\mathcal{M}}_0^{(S\varphi)}} \right)^{-1/2} \left(\frac{\det' \mathcal{M}_1^{(SL\varphi)}}{\det \widehat{\mathcal{M}}_1^{(SL\varphi)}} \right)^{-2} \prod_{\ell=2}^{\infty} \left(\frac{\det \mathcal{M}_\ell^{(SL\varphi)}}{\det \widehat{\mathcal{M}}_\ell^{(SL\varphi)}} \right)^{-\frac{(\ell+1)^2}{2}}, \tag{4.6}$$

$$\mathcal{A}^{(\psi)} = \prod_{\ell=0}^{\infty} \left(\frac{\det \mathcal{M}_\ell^{(\psi)}}{\det \widehat{\mathcal{M}}_\ell^{(\psi)}} \right)^{\frac{(\ell+1)(\ell+2)}{2}}, \tag{4.7}$$

where the prime indicates the subtraction of zero modes, $\mathcal{M}_\ell$'s indicate fluctuation matrices around the bounce, and $\widehat{\mathcal{M}}_\ell$'s indicate those around the false vacuum.

A general procedure to calculate the decay rate of the false vacuum, including the prescription for the zero-mode subtraction and the renormalization, is given in Refs. [62–64]. We follow the procedure given in these articles to calculate the decay rate of the EW vacuum in the model of our interest. A more detailed explanation of the calculation of the decay rate of the EW vacuum in the present model will be given elsewhere [91].

5 Numerical results

Now we are at the position to show the constraints from the stability of the EW vacuum. In order to investigate how large the slepton mass can be, we do not take into account the constraints from other considerations, like the collider and dark matter constraints. These constraints depend on the detail of the model; for example, if the R-parity is violated, these are relaxed considerably.

We first calculate the required value of T to realize a given value of $a_\mu^{(\text{SUSY})}$ for given values m_L, m_R, and M_1 (as well as other MSSM parameters).[#3] In Fig. 2, we show the

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[#3]When the Bino mass is relatively large, $|\Delta y_\mu|$ may become larger than the SM muon Yukawa coupling
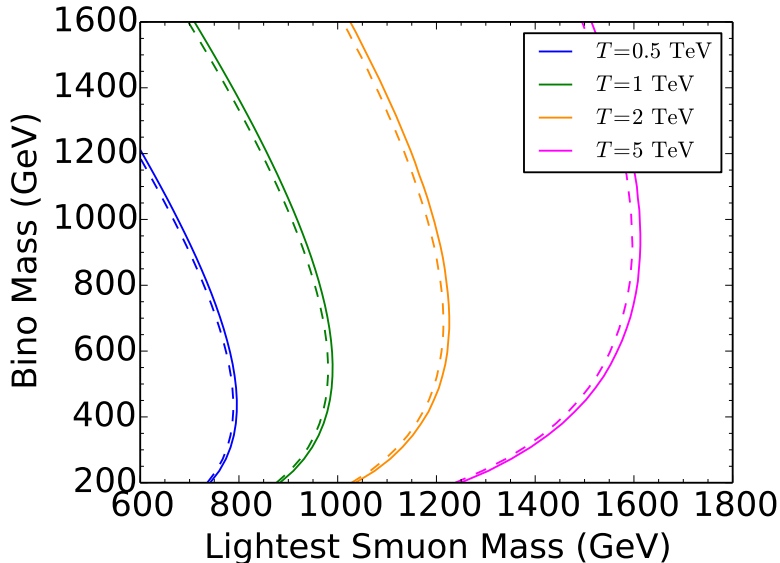
Figure 2: Contours of constant T for the case of $a_\mu^{(\mathrm{SUSY})} = 25.1 \times 10^{-10}$ and $m_R = m_L$. The $\tan\beta$ parameter is taken to be 10 (solid) and 50 (dashed). The blue, green, orange, and magenta lines are for $T = 0.5$, 1, 2, and 5 TeV, respectively.

contours of constant T parameter on the $m_{\tilde\mu_1}$ vs. M_1 plane, assuming $a_\mu^{(\mathrm{SUSY})} = 25.1 \times 10^{-10}$. Here we take $m_R/m_L = 1$ and $\tan\beta = 10$ and 50. We can see that the required value of T to realize $a_\mu^{(\mathrm{SUSY})} \sim \Delta a_\mu$ is insensitive to the value of $\tan\beta$. We can also see that the T parameter is required to be significantly larger than the smuon masses for the case of heavy sleptons. Such a choice of T, required to solve the muon $g-2$ anomaly, gives rise to a deeper minimum of the potential in addition to the EW vacuum. In such a minimum of the potential, which we call a charge breaking minimum, the smuons acquire vacuum expectation values. The longevity of the EW vacuum is not guaranteed for the case with the charge breaking minimum.

We calculate the decay rate of the elecroweak vacuum with the procedure explained in the previous Section. We parameterize the decay rare per unit volume as

$$S_{\mathrm{eff}} \equiv -\ln\left(\frac{\gamma}{1\ \mathrm{GeV}^4}\right). \tag{5.1}$$

Then, requiring that the bubble nucleation rate within the Hubble volume, $\frac{4}{3}\pi H_0^{-3}$, be smaller than t_{now}^{-1}, we obtain

$$S_{\mathrm{eff}} > 386. \tag{5.2}$$

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constant $\tilde{y}_\mu$. In such a case, the EFT muon Yukawa coupling constant y_μ is negative. (Notice that $\Delta y_\mu < 0$.) We have checked that our main result, Fig. 8, is unchanged even if we consider only the parameter region with $y_\mu > 0$.
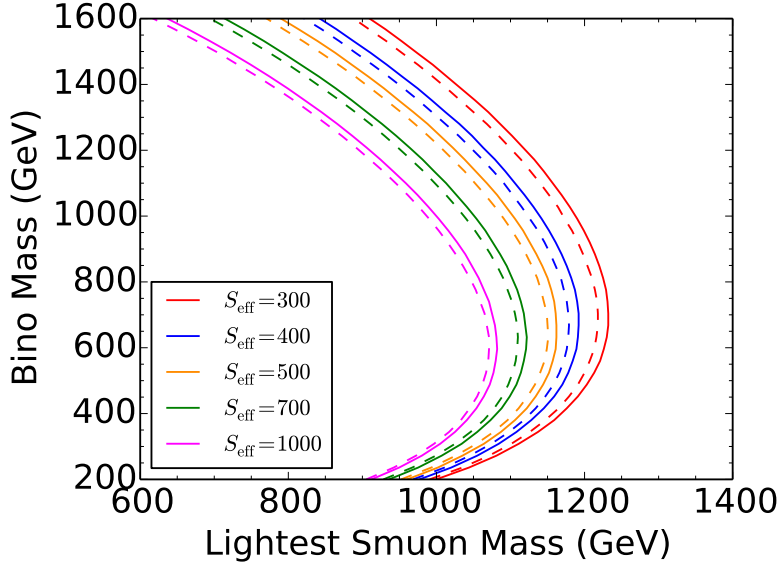
Figure 3: Contours of constant S_{eff}, taking $a_\mu^{(\text{SUSY})} = 25.1 \times 10^{-10}$ and $m_R = m_L$. The red, blue, orange, green, and magenta lines are for $S_{\text{eff}} = 300$, 400, 500, 700, and 1000, respectively. The solid and dashed lines are for $\tan\beta = 10$ and 50, respectively.
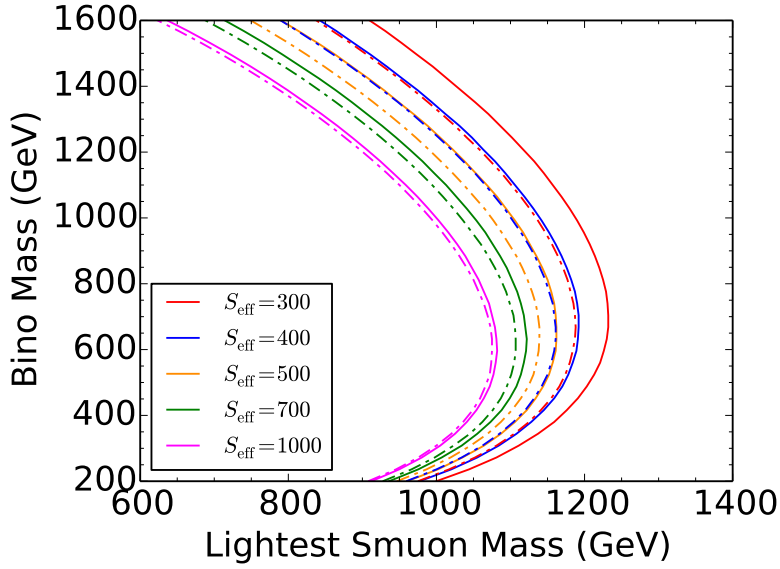

Figure 4: Contours of constant S_{eff} (solid) and $S_{\text{eff}}^{(\text{tree})}$ (dashdotted), taking $a_\mu^{(\text{SUSY})} = 25.1 \times 10^{-10}$, $m_R = m_L$ and $\tan\beta = 10$. The red, blue, orange, green, and magenta lines show the contours on which S_{eff} or $S_{\text{eff}}^{(\text{tree})}$ is equal to 300, 400, 500, 700, and 1000, respectively.

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In Fig. 3, we show the contours of constant S_{eff} on the lightest smuon mass vs. Bino mass plane with fixing the T parameter by requiring $a_\mu^{(\text{SUSY})} = 25.1 \times 10^{-10}$; here, we take $m_R/m_L = 1$. As the lightest smuon becomes heavier, S_{eff} becomes smaller and the constraint given in (5.2) may not be satisfied. Thus, the stability of the EW vacuum gives an upper bound on the smuon mass assuming that the SUSY contribution is responsible for the muon $g-2$ anomaly.

In order to see the impact of the one-loop calculation of the prefactor $\mathcal{A}$, we compare our result with a tree-level one. For this purpose, because the typical energy scale of the bounce for the decay of the EW vacuum is often taken to be around the EW scale, we define

$$S_{\text{eff}}^{(\text{tree})} \equiv \mathcal{B} - \ln\left(\frac{v^4}{1\,\text{GeV}^4}\right). \tag{5.3}$$

In Fig. 4, we show the contours of constant S_{eff} and $S_{\text{eff}}^{(\text{tree})}$, taking $a_\mu^{(\text{SUSY})} = 25.1 \times 10^{-10}$, $m_L/m_R = 1$ and $\tan\beta = 10$. The contours of constant S_{eff} and $S_{\text{eff}}^{(\text{tree})}$ show significant deviation. We find that S_{eff} and $S_{\text{eff}}^{(\text{tree})}$ differ by ~ 100, which results in the $O(10)$ GeV difference in the estimation of the upper bound on the smuon masses.

Now, we discuss the constraint on the lightest smuon mass. In Figs. 5, 6, and 7, we show the contours of $S_{\text{eff}} = 386$ for $m_R/m_L = 0.5$, 1, and 2, for $a_\mu^{(\text{SUSY})} = 25.1 \times 10^{-10}$ (0σ), 19.2×10^{-10} (1σ), and 13.3×10^{-10} (2σ). Requiring that $a_\mu^{(\text{SUSY})}$ is comparable to Δa_μ, we can see that the lightest smuon mass becomes maximally large when the Bino mass is $\sim 0.5-1$ TeV. In addition, as expected, the upper bound on the smuon mass becomes larger as $a_\mu^{(\text{SUSY})}$ becomes smaller. Notice that our smuon mass bound for the case of $m_R/m_L = 1$ is close to the one given in Ref. [29], which is based on the tree-level estimation of the decay rate.

Varying the Bino mass, we determined the maximal possible value of the lightest neutralino mass for fixed values of $\tan\beta$ and $a_\mu^{(\text{SUSY})}$. The result is shown in Fig. 8, in which the upper bound on the lightest neutralino mass is given as a function of the ratio m_R/m_L. We can see that the upper bound becomes the largest when $m_R \simeq m_L$. Requiring $a_\mu^{(\text{SUSY})} = 25.1 \times 10^{-10}$ (0σ), 19.2×10^{-10} (1σ), and 13.3×10^{-10} (2σ) with $\tan\beta = 10$ (50) and $m_R = m_L$, the lightest smuon mass is required to be smaller than 1.20, 1.38 and 1.68 TeV (1.18, 1.37 and 1.66 TeV), respectively. The bound is insensitive to the choice of $\tan\beta$. The muon $g-2$ anomaly can be hardly explained by the MSSM contribution if the lightest smuon is heavier than this bound.

6   Conclusions and discussion

In this letter, we have studied the stability of the EW vacuum in the MSSM, paying particular attention to the parameter region where the muon $g-2$ anomaly can be explained by the SUSY contribution. We consider the case where the Higgsino mass parameter μ is significantly large; in such a case, the SUSY contribution to the muon $g-2$ is enhanced
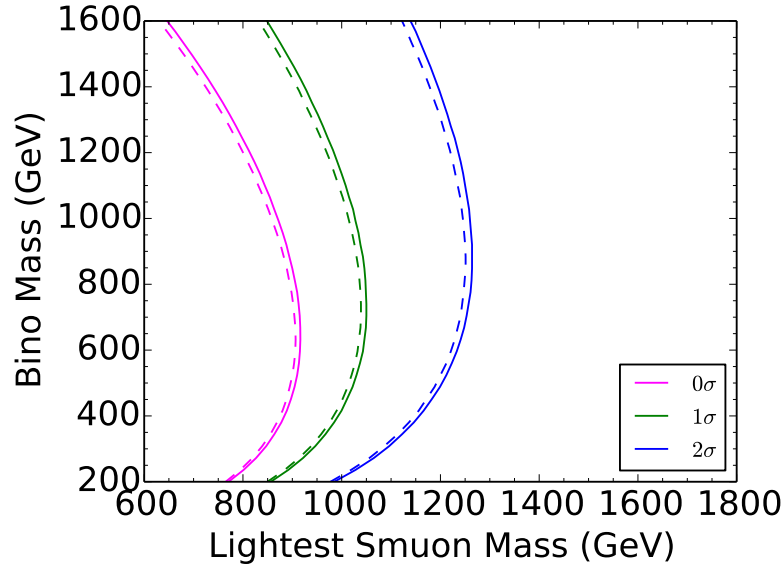
Figure 5: Contours of $S_{\text{eff}} = 387$ for $m_R/m_L = 0.5$. The magenta, green, and blue lines are for $a_\mu^{(\text{SUSY})} = 25.1 \times 10^{-10}$ (0σ), 19.2×10^{-10} (1σ), and 13.3×10^{-10} (2σ), respectively. The solid (dashed) lines are for $\tan\beta = 10$ (50).
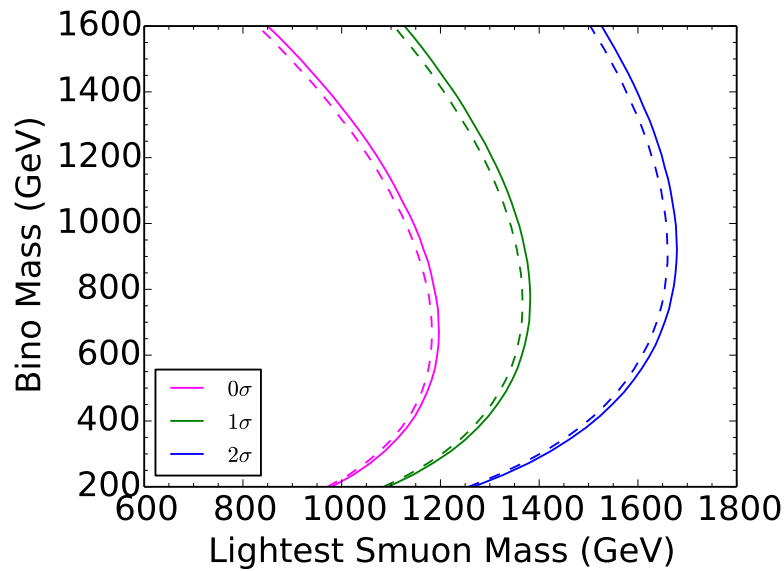

Figure 6: Same as Fig. 5, except $m_R/m_L = 1$.

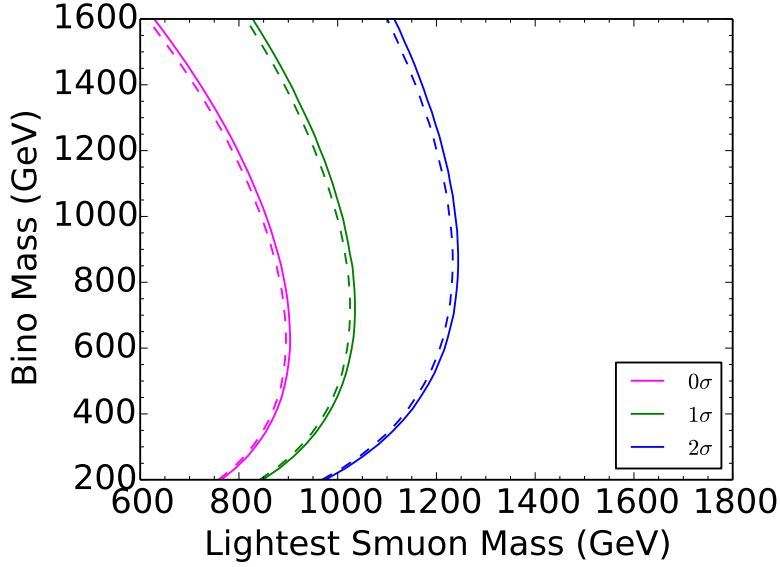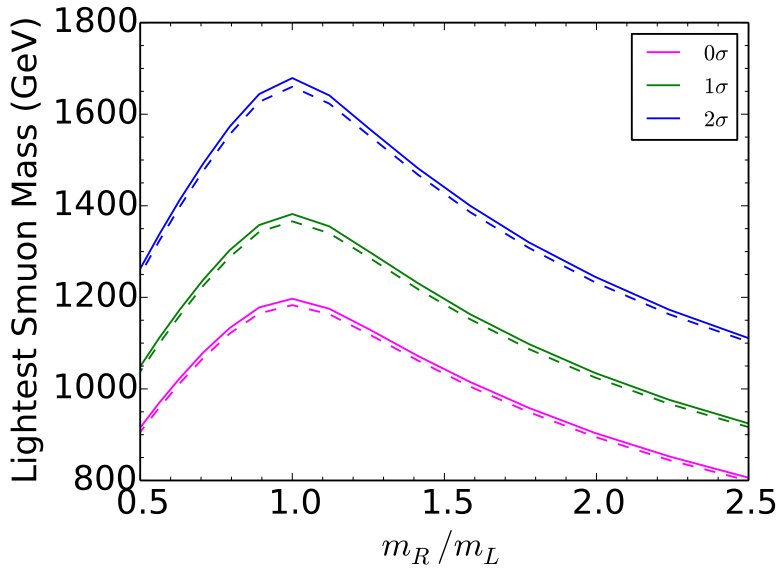Figure 7: Same as Fig. 5, except $m_R/m_L = 2$.

Figure 8: Upper bound on the lightest smuon mass as a function of m_R/m_L. The magenta, green, and blue lines are for $a_\mu^{(\text{SUSY})} = 25.1 \times 10^{-10}$ (0σ), 19.2×10^{-10} (1σ), and 13.3×10^{-10} (2σ), respectively. The solid (dashed) lines are for $\tan\beta = 10$ (50).

so that the muon $g-2$ anomaly can be explained with relatively large values of the smuon masses. With μ being large, however, the smuon-smuon-Higgs trilinear coupling is enhanced, and there may show up a charge breaking minimum of the potential, resulting in the meta-stability of the EW vacuum. With the size of the SUSY contribution to the muon $g-2$ being fixed to alleviate the muon $g-2$ anomaly, the trilinear coupling is more enhanced with a larger value of the smuon mass. Thus, if the smuon mass is too large, the muon $g-2$ anomaly cannot be solved in the MSSM even if we consider a very large value of μ because the longevity of the EW vacuum cannot be realized.

We have performed a detailed calculation of the decay rate of the EW vacuum, assuming that the SUSY contribution to the muon anomalous magnetic moment is large enough to alleviate the muon $g-2$ anomaly. Our calculation is based on the state-of-the-art method to calculate the decay rate of the false vacuum, which includes the one-loop effects due to the field coupled to the bounce. The most important advantage of the inclusion of the one-loop effect is to determine the mass scale of the prefactor $\mathcal{A}$, which is mass-dimension 4. Another advantage is that the scale dependence of the bounce action $\mathcal{B}$ can be canceled by that of $\mathcal{A}$ at the leading-log level. Requiring $a_\mu^{(\mathrm{SUSY})} = 25.1 \times 10^{-10}$ (0σ), 19.2×10^{-10} (1σ), and 13.3×10^{-10} (2σ), we found that the lightest smuon should be lighter than 1.20, 1.38 and 1.68 TeV (1.18, 1.37 and 1.66 TeV) for $\tan\beta = 10$ (50), respectively. It is challenging to find such a heavy smuon with collider experiments. A very high energy collider, like muon colliders [92], the FCC [93–95], or the CLIC [96] may be able to perform a conclusive test of the SUSY interpretation of the muon $g-2$ anomaly.

In this letter, we assumed that the superparticles other than the smuons and the Bino are so heavy that they are irrelevant for the muon $g-2$ as well as for the stability of the EW vacuum. If some of the superparticles are as light as the smuons and the Bino, the upper bound on the smuon mass we obtained may become more stringent. For example, if the stau is relatively light, then the decay rate of the EW vacuum may become larger because the large μ also enhances the stau-stau-Higgs trilinear coupling which is orders of magnitude larger than the smuon-smuon-Higgs coupling. In such a case, the upper bound on the slepton mass becomes more stringent compared to the case only with the smuons. More detailed discussion on such a case will be given elsewhere [91].

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