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A NOTE ON THE MEANING OF STORAGE COEFFICIENT

By

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ABSTRACT

The term storage coefficient (the related terms of specific storage, storativity and others) is frequently used in hydrogeology as well as petroleum engineering. These terms seek to express the ability of a porous material to release water from storage in response to changes in hydraulic head. For convenience, the aforesaid terms are invariably defined in a volume normalized form. However, since more than one choice is available for the normalizing volume element, the expression for the storage term may apparently take different forms depending on the particular choice exercised. Losing sight of the importance of the normalizing volume can often lead to confusion in understanding the storage term. Furthermore, the concept of storage coefficient (or its related expressions) is specifically valid only for a system undergoing drainage (drained loading). There exist natural situations in which the fluid potential field in a groundwater system may change due to changes in external loads (pore pressure changes due to earthquakes, earth tides) which are characterized by responses without fluid drainage (undrained loading). The concept of storativity does not have direct relevance to undrained loading conditions. A recognition of the conventions used in defining the normalizing volume as well as the distinction between drained and undrained phenomena of loading can greatly help in a proper appreciation of the concept of storage coefficient.

INTRODUCTION

The term storage coefficient, S , and the synonymous terms, coefficient of storage and storativity, have been used extensively in the groundwater literature. It has been defined (Ferris et al., 1962) as "the volume of water which an aquifer releases from or takes into storage per unit surface area of aquifer per unit change in the component of head normal to the surface." Historically, this somewhat restricted definition appears to have been introduced to satisfy the need for a coefficient occurring in the differential equation describing nonsteady groundwater flow. The solution of that equation first led to the Theis equation followed by a host of other useful results.

While the aforementioned definition of storage coefficient is convenient and adequate for applications related to well testing, it is not adequate for the consideration of all nonsteady flow in arbitrary, heterogeneous, three-dimensional systems. A more concisely-defined, and somewhat more general, coefficient is specific storage, S_s , also referred to as specific storativity. As will be seen later, it is defined directly in terms of the compressibility of the skeleton of the aquifer and the compressibility of water. In using the storage coefficient concept, one invariably assumes that the coefficient describing change in storage is a

constant and not a function of hydraulic head which is changing in time. This assumption seems to be made primarily because it facilitates the analytic solution of the differential equation of diffusion.

In attempting to treat the coefficient describing changes in groundwater storage due to changes in hydraulic head one is lead to the consideration of defining a reference volume element. Based primarily on convenience, different choices for the definition of the volume element have been made in the fields of hydrogeology, petroleum engineering, soil mechanics, and soil physics. Each of these choices leads to an apparently different expression for the coefficient describing the phenomena of change in storage. This has led to considerable confusion between and even within the various fields. The purpose of this paper is to examine the meaning of the term specific storage in light of the various expressions that are used by different investigators.

THE PHENOMENON OF CHANGE IN STORAGE

The mass of fluid stored in a given volume of a porous medium may be expressed as (Narasimhan and Witherspoon, 1977):

$$M_f = V_v \rho_f S_f \quad (1)$$

where

M_f = the mass of fluid

V_v = the volume of the voids

ρ_f = the density of the fluid

S_f = the saturation of the fluid.

It is known empirically that V_v , ρ_f , and S_f are all functions of the average fluid pressure in a given volume of the porous medium. Hence, it is possible to define a "fluid mass capacity," M_c^* , which expresses the rate of change of the mass of fluid with respect to the change in average fluid pressure within the volume.

$$M_c^* = \frac{dM_f}{dp_f}$$

$$= V_v S_f \frac{d\rho_f}{dp_f} + \rho_f S_f \frac{dV_v}{dp_f} + V_v \rho_f \frac{dS_f}{dp_f} \quad (2)$$

The first term on the right-hand side of equation (2) represents the change in volume of the pore fluid. The second term represents the deformation of the skeleton of the porous medium, and the third term represents the desaturation of the medium.

The three terms on the right-hand side of equation (2) may now be considered individually. It is customary to evaluate the derivative of fluid density with respect to fluid pressure in terms of the compressibility of the fluid c_w , (reciprocal of bulk modulus). The compressibility of the fluid is defined as:

$$c_w = - \left(\frac{\frac{dV_f}{V_f}}{\frac{dp_f}{dp_f}} \right), \text{ at constant mass of fluid} \quad (3)$$

where

V_f = the volume of fluid.

In view of the definition of fluid density, the volume of fluid may be expressed as:

$$V_f = \frac{M_f}{\rho_f} \quad (4)$$

and

$$dV_f = - \frac{M_f d\rho_f}{\rho_f^2} \quad (5)$$

since the mass of fluid, M_f , is constant. Substituting (4) and (5) into equation (3) yields:

$$c_w = + \frac{(d\rho_f/\rho_f)}{dp_f} \quad (6)$$

Hence, the change in density with respect to change in fluid pressure is:

$$\frac{d\rho_f}{dp_f} = c_w \rho_f \quad (7)$$

It might be noted that the solution to this equation is the conventional equation of state for a slightly compressible fluid like water,

$$\rho_f = \rho_0 e^{c_w(p_f - p_0)} \quad (8)$$

where ρ_0 = the density of the fluid at reference pressure, p_0 .

Since the derivative in the second term of equation (2) is related to the deformation of the solid skeleton of the porous medium, a compressibility reflecting the compressive properties of the skeleton is needed. It is in the introduction of coefficients describing this compressibility that many of the major differences between the definitions of change in storage in dealing with the flow of fluids through porous media arise in different disciplines. At least in the fields of hydrology, petroleum engineering, and soil mechanics, the deformation of the skeleton is generally related to "effective stress," σ' , that is defined as:

$$\sigma' = \sigma - \chi p_f,$$

$$\chi = \begin{cases} 1, & p_f \geq 0 \\ 0 \leq \chi \leq 1, & p_f < 0 \end{cases} \quad (9)$$

(Bishop, 1960; Narasimhan, and Witherspoon, 1977). Here σ is the total stress and χ is a coefficient that is a function of fluid saturation. χ varies from zero to one and is included to account for that portion of the pore fluid pressure (moisture suction, where fluid pressure is negative) that may be converted to effective stress (Narasimhan, 1978). Using the chain rule of differentiation, the derivative in the second term of (2) may be written as:

$$\frac{dv_v}{dp_f} = \frac{dv_v}{d\sigma'} \frac{d\sigma'}{dp_f} \quad (10)$$

If the total stress is assumed to be constant in (9), differentiating σ' with respect to p_f yields

$$\begin{aligned} \frac{d\sigma'}{dp_f} &= -(\chi + p_f \frac{d\chi}{dp_f}) \\ &= -\chi' \end{aligned} \quad (11)$$

where, in general, χ' is a coefficient that must be empirically determined. In light of (11), equation (10) simplifies to

$$\frac{dv_v}{dp_f} = - \frac{dv_v}{d\sigma'} \chi' \quad (12)$$

The derivative involving saturation in the last term on the right-hand side of (2) is generally of interest in fields where zones of partial saturation are commonly dealt with, such as soil physics. The relations for saturation are generally empirical in nature and the derivative, $\frac{ds_f}{dp_f}$, may be thought of as the slope of a plot of saturation versus fluid pressure.

From the above relations (2), (7), (12), it can be seen that

$$M_c^* = v_v s_f c_w \rho_f - \rho_f s_f \chi' \frac{dv_v}{d\sigma'} + v_v \rho_f \frac{ds_f}{dp_f} \quad (13)$$

SATURATED POROUS MEDIA

Equation (13) describes how, in an arbitrary volume element of the porous medium, water is released from or taken into storage as the average fluid pressure over the element changes. Note that M_c^* in (13) depends on the expansion of water, the deformation of the skeleton of the porous medium, and the desaturation of the pores. If the porous medium is assumed to be fully saturated, a number of simplifications are possible.

In the saturated case it is obvious that the third term of equation (13), defining M_c^* , goes to zero since $S_f = 1$. Also, it can be seen from (9), that for $p_f > 0$, $\chi' = 1$. Then, in view of (11) and (12), the derivative in the second term of equation (2) is:

$$\frac{dv_v}{dp_f} = - \frac{dv_v}{d\sigma'} \quad (14)$$

The fluid mass capacity for a fully-saturated porous medium then reduces to

$$(M_c^*)_{sat} = \rho_f \left[v_v c_w - \frac{dv_v}{d\sigma'} \right] \quad (15)$$

Recall, however, that in arriving at this equation, it was assumed that the total stress is constant. In light of equation (9), this implies that

$$d\sigma' = -dp_f \quad (16)$$

DRAINED AND UNDRAINED LOADING

It is appropriate at this juncture to digress a little and discuss the phenomena of "drained loading" and "undrained loading" which are commonly used in the soil mechanics literature (e.g., Lambe and Whitman, 1969). First, let us consider drained loading. If the total stress, σ , on the volume element remains constant, then, according to (16) any change in pore pressure is converted into an equivalent effective stress change with the opposite sign. Thus, if pore pressure falls, the effective stress increases and vice versa. At the same time, since water and the skeleton have different abilities to deform, the volume changes undergone by the water contained in the voids and the void themselves will not be the same. That is, for drained loading:

$$|\Delta\sigma'| = |\Delta p_f|, \sigma = \text{const.} \quad (17 \text{ a})$$

$$|\Delta V_v| \neq |\Delta V_f| \quad (17 \text{ b})$$

In fact, the quantity of water drained or taken into storage, ΔV_w , is equal to the sum of two different terms. Thus,

$$|\Delta V_w| = |\Delta V_v| + |\Delta V_f|. \quad (17 \text{ c})$$

It may be noted here that ΔV_w denotes the water released from storage to the outside as opposed to ΔV_f which denotes the volume change of the fluid contained in the voids.

The concept of storage coefficient specifically corresponds to the drained loading case. Physically this is easy to understand, since, in groundwater systems the tectonic stresses remain practically constant and the drainage of water (e.g., withdrawal of water from wells, regional groundwater movement) is the principal mechanism causing stress changes.

The concept of "undrained loading" is widely used in soil engineering although relatively little considered in hydrogeology. Undrained loading occurs when the total stress on a system is changed without the addition or removal of fluid, as in the case of a sealed system. The response of saturated materials to instantaneous or quickly applied loads with very little or no time available for fluid drainage is also characterized by undrained loading. As the pore volume decreases due to the external load, the volume of the water contained must decrease by an identical amount if the medium is to remain saturated. Since water and the skeleton have different compressibilities, it follows that the water and the skeleton will bear different portions of the change in total stress.

Typically, change in effective stress in undrained loading can only be caused by changing the total stress since drainage is absent. Physically, such conditions may result from construction of buildings or dams or soil masses; cyclic loading of soil mass due to earthquakes; loading of aquifers due to earth tides and so on. The undrained response of a saturated porous medium is characterized by:

$$\Delta V_v = \Delta V_f \quad (17 \text{ d})$$

and

$$|\Delta \sigma'| \neq |\Delta p_f|, \quad (17 \text{ e})$$

$$\Delta \sigma = \Delta \sigma' + \Delta p_f \quad (17 \text{ f})$$

Strictly speaking, the concept of specific storage is not applicable to the undrained case. This is because a lack of drainage implies that no water is released from storage. Or, equivalently, ΔM_f = mass of water released from storage = 0.

In comparison, therefore, the drained and the undrained conditions of loading have an interesting asymmetry. In the former, the changes in stress in the skeleton and in the fluid are equal, but the changes in the volumes of the two are not. In the latter, however, the changes in stress in the two phases are unequal while the corresponding volume changes are the same.

The undrained response of a porous medium is outside the scope of the transient groundwater flow equation (diffusion-type equation) normally used in hydrogeology or petroleum engineering. The generation of pore-fluid pressures in undrained systems (e.g., cyclic loading due to earthquakes or earth tides; gradual changes in external loads due to construction or engineering structures or due to tectonic causes) may be almost instantaneous in elastic materials and have to be computed through a stress-strain analysis of the system in conjunction with the constitutive laws governing effective stress. In fact, for analyzing groundwater flow in a system subject to changes in external loads, the appropriate initial conditions and the source terms have to be obtained from independent stress-strain calculations.

It is obvious from the above that some insight into the nature of loading existing in the field is necessary before one can set out to calculate the storage coefficient from field data. A comparison of drained and undrained flow phenomena in saturated porous media is presented in Table 1.

TABLE 1.
COMPARISON OF DRAINED AND UNDRAINED LOADING PHENOMENA
IN SATURATED POROUS MEDIA

	Drained Loading	Undrained Loading
1.	External load assumed constant	External load changing
2.	Effective stress changes caused from within, due to fluid pressure changes induced by addition or withdrawal of water.	Effective stress changes caused from the outside due to changes in boundary loads.
3.	Water is released from storage from the system to the outside: $V_w \neq 0$.	Water is not released from storage to the outside: $V_w = 0$.
4.	$ \Delta\sigma' = \Delta p_f $ (assuming $\chi = 1$)	$ \Delta\sigma' \neq \Delta p_f $ (assuming $\chi = 1$)
5.	$ \Delta V_v \neq \Delta V_f $	$ \Delta V_v = \Delta V_f $
6.	$ \Delta V_w = \Delta V_v + \Delta V_f $	$ \Delta V_w = \Delta V_v - \Delta V_f = 0$
7.	S_s can be directly computed from a drained experiment (e.g., conventional aquifer test). By definition: $S_s = \frac{1}{V_b} \frac{\Delta V_w}{\Delta \Psi}$	S_s cannot be directly determined from an undrained experiment.
8.	Increase (decrease) in pore pressure goes hand in hand with increase (decrease) in void volume.	Increase in pore pressure goes with decrease in void volume and vice versa.

REPRESENTATIONS OF SPECIFIC STORAGE

The formulation of M_c^* , in particular, equation (15), relates to an arbitrary volume element. In predictive mathematical models, M_c^* is a required input parameter that is invariably generated from laboratory experiments and related tests, based on the deformation of the porous medium. Since laboratory tests are conducted on samples of different sizes, it is convenient to report the results on porous medium deformation in some volume-normalized form. In general, there are three choices available for the normalizing volume. These are (a) the bulk volume, V_b ; (b) the solid volume, V_s ; and (c) the pore volume, V_v .

Case a: Use of bulk volume for normalization

If the right-hand side of (15) is multiplied and divided by V_b

$$\begin{aligned} (M_c^*)_{\text{sat}} &= V_b \rho_f \left[c_w \frac{V_v}{V_b} - \left(\frac{dV_v}{V_b d\sigma'} \right) \right] \\ &= \rho_f V_b \left[n c_w - \frac{dn}{d\sigma'} \right] \end{aligned} \quad (18)$$

where n is porosity.

In the soil mechanics literature it is customary to treat $-\frac{dn}{d\sigma'} \approx m_v$ the volumetric coefficient of compressibility, where

$$m_v = \frac{-1}{\Delta\sigma'} \frac{\Delta V_v}{V_{b0}}$$

and V_{b_0} is the bulk volume of the sample before the incremented load, $\Delta\sigma'$, is applied. Hence, (18) may be written as:

$$(M_c^*)_{sat} = \rho_f V_b [nc_w + m_v]. \quad (19)$$

The fluid mass capacity may then be normalized with respect to the bulk volume, V_b , of the porous medium to give a coefficient that may be called the "specific fluid mass capacity." The more conventional coefficient, specific storage, S_s , however, is expressed as the volume of fluid released or taken into storage per unit volume per unit change in hydraulic head, ϕ , rather than the mass of fluid released per unit change in pressure. Change in mass of fluid may be converted to change in fluid volume if the change in density is assumed to be small.

Also, change in fluid pressure may be written as

$$dp_f = \rho_f g d\Psi \quad (21)$$

where g is the acceleration due to gravity (assumed constant) and Ψ is the pressure head. By definition the hydraulic head is

$$\phi = z + \Psi \quad (22)$$

where z is the elevation. Hence,

$$d\phi = dz + d\Psi. \quad (23)$$

Since M_f is not a function of elevation, $\frac{dM_f}{d\phi} = \frac{dM_f}{d\Psi}$, however,

and, as a consequence of (21),

$$\frac{dM_f}{dp_f} = \frac{1}{\rho_f g} \frac{dM_f}{d\Psi} \quad (24)$$

Further,

$$\frac{dM_f}{dp_f} = \frac{\rho_f}{\rho_f g} \frac{dV_w}{d\Psi} = \frac{1}{g} \frac{dV_w}{d\Psi} = (M_c^*)_{sat} \quad (25)$$

If normalized with respect to the bulk volume

$$\frac{(M_c^*)_{sat}}{V_b} = \frac{1}{gV_b} \frac{dV_w}{d\Psi} \quad (26)$$

Hence, from the definition of S_s (see Table I)

$$S_s = \left(\frac{dV_w}{V_b} \right) \frac{d\Psi}{d\Psi} = \frac{g(M_c^*)_{sat}}{V_b} \quad (27)$$

or, from (19),

$$S_s = \rho_f g (nc_w + m_v). \quad (28)$$

It might be noted in passing that specific storage with respect to pressure, S_s^* , is occasionally used by some. It is given as the volume of fluid released or taken into storage per unit volume per unit change in fluid pressure and expressed as:

$$S_s^* = (nc_w + m_v) \quad (29)$$

Case b: Use of solid volume for normalization

Since Darcy's law applies to the velocity of fluid relative to that of the grains, it is sometimes more convenient to assume that the volume element contains a constant volume of incompressible solids (Smiles and Rosenthal, 1968; Narasimhan and Witherspoon, 1977). For this Lagrangian volume element, V_b changes with time, while V_s remains constant and is used as the normalization volume. Thus, multiplying and dividing the right-hand side of (15) by V_s we get

$$\begin{aligned} (M_c^*)_{sat} = \rho_f V_s & \left[\frac{V_b}{V_s} c_w - \left(\frac{dV_b}{V_s d\sigma'} \right) \right] \\ & - \rho_f V_s \left[e c_w - \frac{de}{d\sigma'} \right] \end{aligned} \quad (30)$$

In the soil mechanics literature, e is void ratio and $\frac{-de}{d\sigma'}$ is often designated as the coefficient of compressibility, a_v , that is, $a_v = -\frac{1}{\Delta\sigma'} \frac{\Delta V_b}{V_s}$. Hence, (30) can be rewritten as:

(17)

$$(M_c^*)_{\text{sat}} = \rho_f V_s \left[e c_w + a_v \right]. \quad (31)$$

If the definition of S_s is expanded to allow normalization with respect to the volume of solids, then

$$S_s = \frac{(M_c^*)_{\text{sat}}}{V_s} g = \rho_f g [e c_w + a_v]. \quad (32)$$

Case c: Pore volume for normalization

In the petroleum engineering literature (e.g., Earlougher, 1977), the pore volume, V_v , is sometimes used for normalization instead of V_b or V_s . If V_v is used for normalization in conjunction with an Eulerian volume element, the following interesting result is obtained in two steps. First, the term $\frac{dv_v}{d\sigma'}$ on the right-hand side of (15) is multiplied and divided by V_v .

$$(M_c^*)_{\text{sat}} = \rho_f V_v (c_w + c_p) \quad (33)$$

where

$$c_p = - \left(\frac{\frac{dv_v}{V_v}}{\frac{d\sigma'}{V_v}} \right) \text{ is the "pore volume compressibility"}$$

$$\approx - \left(\frac{\frac{dv_v}{V_{v0}}}{\frac{d\sigma'}{V_{v0}}} \right)$$

and where V_{v_0} is the void volume prior to applying σ' . Note, incidentally, that

$$m_v = - \left(\frac{dv_v}{v_b} \right) = - \left(\frac{dv_v}{v_v} \right) \cdot \frac{v_v}{v_b} = c_p n. \quad (34)$$

Second, the right-hand side of (33) is multiplied and divided by V_b as in (19),

$$\begin{aligned} (M_c^*)_{sat} &= \rho_f V_b \frac{v_v}{v_b} (c_w + c_p) \\ &= \rho_f V_b n (c_w + c_p) \end{aligned} \quad (35)$$

In the petroleum literature (Earlougher, 1977), the total compressibility of the reservoir is defined as

$$c_t = c_w + c_p \quad (36)$$

and the quantity, nc_t , is often used as a fundamental reservoir parameter, " ϕc ", (in petroleum literature the symbol ϕ is used for porosity instead of n). Thus, in the light of (35) and (36),

$$(M_c^*)_{sat} = \rho_f V_b nc_t \quad (37)$$

and

$$\begin{aligned} S_s &= \frac{(M_c^*)_{sat} g}{V_b} = \rho_f g n (c_w + c_p) \\ &= \rho_f g nc_t \end{aligned} \quad (38)$$

It is of interest to note that the total compressibility, c_t , is the sum of compressibilities of two different material phases and

$$c_t \neq \left(\frac{dv_b}{v_b} \right) \frac{d\sigma}{d\sigma}$$

as may be sometimes thought.

Case d: Combined use of V_s and V_b for normalization

In addition to the three different conventions described above, a fourth convention is sometimes followed in hydrogeology. In this convention V_s is used for normalizing the porous medium deformation parameter, while V_b is used in defining the volume element over which S_s is defined. This can be done in two steps as in case C. First, the term $\frac{dv_v}{d\sigma'}$ on the right-hand side of (15) is multiplied and divided by V_s .

$$\begin{aligned} (M_c^*)_{sat} &= \rho_f \left[V_v c_w - V_s \frac{de}{d\sigma'} \right] \\ &= \rho_f [V_v c_w + V_s a_v] \end{aligned} \quad (39)$$

Second, the right-hand side of (39) can be multiplied and divided by V_b

$$(M_c^*)_{sat} = \rho_f V_b [nc_w + (1-n) a_v] \quad (40)$$

since $\frac{V_s}{V_b} \frac{V_b - V_v}{V_b} = (1-n).$

From (40) it is seen that

$$S_s = g \frac{(M_c^*)_{sat}}{V_b} = \rho_f g [nc_w + (1-n) a_v] \quad (41)$$

SUMMARY

In summary, then, the coefficient describing change in storage can be expressed in four different ways depending on the choice of (a) the normalization volume used for describing porous medium deformation observed in the laboratory and (b) the normalization volume used in defining the volume element over which specific storage is defined.

The four cases may be summarized as:

- (i) Bulk volume, V_b , used in both cases:

$$S_s = \rho_f g [nc_w + m_v]$$

- (ii) Solid volume, V_s , used in both cases

$$S_s = \rho_f g [ec_w + a_v]$$

- (iii) Pore volume, V_p , used for deformation and V_b used for volume over which specific storage is defined.

$$S_s = \rho_f g n [c_w + c_p]$$

and $nc_t = n (c_w + c_p).$

- (iv) V_s used for deformation and V_b used for volume over which specific storage is defined.

$$S_s = \rho_f g [nc_w + (1-n) a_v].$$

Finally, in view of the definition of the storage coefficient, S , it can be seen that specific storage is simply the storage coefficient normalized with respect to the thickness of the aquifer, that is,

$$S = S_s b$$

where b is the thickness of the aquifer. Also, since nc_t , is change in storage with respect to fluid pressure,

$$S_s = \rho_f g n c_t,$$

and the commonly used petroleum engineering term " ϕ_{ch} " ($= nc_t b$ in the present notation) can be written as:

$$"\phi_{ch}" = nc_t b = \frac{S_s}{\rho_f g} b = \frac{S}{\rho_f g}$$

It is interesting to note that Jacob's original definition of specific storage is given in terms of aquifer thickness and change in aquifer thickness rather than volume. It is likely that he was primarily concerned with the one-dimensional problem of flow to a fully penetrating well. In this restricted case, the volume element can be conveniently assumed to be of unit cross sectional area and spanning the thickness of the aquifer. If uniform thickness is assumed, as commonly done in hydrology, normalization with respect to aquifer thickness becomes of lesser importance, hence the use of the storage coefficient, which is the non-normalized form of specific storage. For the more general problem, however, the storage coefficient is not a suitable parameter (Freeze and Witherspoon, 1966). While the concept of specific storage may be employed, it is perhaps best to consider each of the terms on the right-hand side of equation (13) separately.

Before discussing the relative merits of the above representations, perhaps the assumptions and suitable applicability of the expressions should be restated. First, and most important, it is assumed that the total stress remains constant. This implies that the above only applies directly to problems where fluid is being injected or drained. Second, in going from mass to volume and from fluid pressure to hydraulic head it is assumed that change in fluid density is small.

From a general modeling point of view, it appears that cases (i) and (ii) are preferable to (iii) and (iv) for the reason that both are consistent in choosing the same normalization volume for the laboratory experiments and the field problem. Although (iii) and (iv) may yield practically as accurate results as (i) and (ii), from a conceptual point of view they lack clarity in that they employ one convention for interpreting laboratory data and another for the field problems.

From a theoretical point of view, if one wishes to use (i) and define S_g over a volume element with V_b fixed in time, then Darcy's law has to be formulated as the difference in the velocity between water and grains across a surface of interest. Furthermore, as the boundary of the flow region changes due to deformation, one has to visualize addition or subtraction of porous material so that the surface of the flow region will remain fixed as required by the Eulerian formulation.

If one wishes to use (ii), then V_b (and hence the surface configuration) of the volume element over which S_g is defined changes with time. In this scheme, therefore, Darcy's law is strictly applied in the sense of fluid velocity relative to solid grains. A major consideration in this formulation is that the flow region geometry is a function of time.

An important limitation of the storage coefficient concept is that it relates the amount of water released to change in hydraulic head. However, the skeletal deformation component of storage coefficient is controlled by change in effective stress rather than in change in pressure head. In such field situations where skeletal deformation depends on the magnitude of effective stress, it is obvious that the coefficient quantifying change in storage will vary as a function of stress. Such dependence of S_g or S on σ' cannot be easily incorporated into a differential equation in which hydraulic head is the dependent variable. The conventional transient groundwater flow equation is thus reasonable only when hydraulic heads are small and the storage coefficient is essentially constant and independent of effective stress.

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NOTATION

a_v	coefficient of compressibility $[LT^2/M]$
b	thickness of the aquifer $[L]$
c_p	pore volume compressibility $[LT^2/M]$
c_w	compressibility of water $[LT^2/M]$
c_t	total compressibility $(c_w + c_p)$ $[LT^2/M]$
e	void ratio $[1]$
g	acceleration due to gravity $[L/T^2]$
m_v	volumetric coefficient of compressibility $[LT^2/M]$
M_c^*	fluid mass capacity of a volume element $[LT^2]$
$(M_c^*)_{sat}$	fluid mass capacity for a saturated volume element $[LT^2]$
M_f	mass of water contained in an arbitrary volume element $[M]$
n	porosity $[1]$
p_f	fluid pressure $[M/LT^2]$
p_o	reference fluid pressure $[M/LT^2]$
S	storage coefficient $[1]$
S_f	fluid saturation $[1]$
S_s	specific storage $[1/L]$
V_b	bulk volume of a volume element $[L^3]$
V_{b0}	V_b at a reference state $[L^3]$
V_f	volume of fluid contained in the voids $[L^3]$
V_s	volume of solids $[L^3]$
V_v	volume of voids $[L^3]$
Z	Elevation above datum $[L]$

ΔV_w volume of water removed (added) from system $[L^3]$

γ_f specific weight of fluid $[M/L^2T^2]$

ρ_f density of fluid $[M/L^3]$

ρ_o reference density of fluid at p_o $[M/L^3]$

σ total stress $[M/LT^2]$

σ' effective stress $[M/LT^2]$

ϕ hydraulic head or fluid potential $[L]$
(in petroleum engineering, ϕ is porosity)

χ a dimensionless factor $[1]$

χ' a dimensionless factor $[1]$

Ψ pressure head $[L]$

BIBLIOGRAPHY

- Bishop, A. W., "The Principle of Effective Stress, Publ. 32, pp. 1-5, Norwegian Geotechnical Inst., Oslo, Norway, 1960.
- Earlougher, T. C., Jr., Advances in Well Test Analysis, Soc. of Petroleum Engineers of AIME, New York, 1977.
- Ferris, J. G., D. B. Knowles, R. H. Brown, and R. W. Stallman, Theory of Aquifer Tests, Geological Survey Water-Supply Paper 1536-E, U. S. Gov. Printing Office, Washington, 1962.
- Freeze, R. A., and P. A. Witherspoon, "Theoretical Analysis of Regional Groundwater Flow, I: Analytical and Numerical Solutions to the Mathematical Model," Water Resour. Res. 2. 64 (1966).
- Lambe, T. W. and R. V. Whitman, Soil Mechanics, John Wiley & Sons, Inc., New York, 1969.
- Narasimhan, T. N., The Significance of the Storage Parameter in Saturated-Unsaturated Groundwater Flow, Preprint No. LBL-7041, Lawrence Berkeley Laboratory, 1978, Water Resources Res. (in press)
- Narasimhan, T. N. and P. A. Witherspoon, "Numerical Model for Saturated-Unsaturated Flow in Deformable Porous Media, Part I Theory," Water Resources Res., 13 (3), 657-666, 1977.
- Smiles, D. E., and M. J. Rosenthal, "The Movement of Water in Swelling Materials," Australian Jour. Soil Res., 6, 237-248, 1968.

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