UCRL 8885

# UNIVERSITY OF CALIFORNIA

# Ernest O. Lawrence Radiation Laboratory

## EXPERIMENTAL STUDY OF PLASMA ALFVÉN-WAVE PROPERTIES

UCRL-8885
Controlled Thermonuclear Processes
TID-4500(15th Ed.)

UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

# EXPERIMENTAL STUDY OF PLASMA ALFVÉN-WAVE PROPERTIES

John M. Wilcox, Forrest I. Boley, and Alan W. De Silva

August 27, 1959

Printed for the U.S. Atomic Energy Commission

Printed in USA. Price 75 cents. Available from the
Office of Technical Services
U. S. Department of Commerce
Washington 25, D.C.

# EXPERIMENTAL STUDY OF PLASMA ALFVÉN-WAVE PROPERTIES

John M. Wilcox, Forrest I. Boley, and Alan W. De Silva

Lawrence Radiation Laboratory
University of California
Berkeley, California

August 27, 1959

ABSTRACT

Alfvén hydromagnetic waves are propagated through a cylindrical plasma. The wave velocity, attenuation, impedance, and energy transfer are studied. The theoretical equations predict correctly the functional dependence of the velocity and attenuation, and from these quantities accurate measurements of plasma density and temperature can be obtained. A qualitative agreement between theory and experiment is obtained for the hydromagnetic coaxial waveguide impedance, and the energy transferred from an oscillating circuit to the hydromagnetic wave is measured to be $43 \pm 10\%$.

# EXPERIMENTAL STUDY OF PLASMA ALFVÉN-WAVE PROPERTIES*

John M. Wilcox, Forrest I. Boley,† and Alan W. De Silva

Lawrence Radiation Laboratory
University of California
Berkeley, California

August 27, 1959

## INTRODUCTION

The generation and propagation of Alfvén waves in a gaseous discharge plasma have been reported recently at Berkeley[1] and Harwell.[2] Such hydromagnetic waves were first postulated by Alfvén[3] to account for certain properties of sun spots. Hydromagnetic waves were first generated in the laboratory by Lundquist[4] and by Lehnert[5] using liquid metals. The present experiments are similar except that the use of a gaseous plasma allows a more detailed study of the phenomena.

The purpose of this experiment is to determine various properties of Alfvén-wave propagation in a plasma. Among the properties investigated were propagation velocity, attenuation, energy transfer, and dielectric constant.

In a conducting fluid an imbedded magnetic field will be constrained to move approximately with the fluid. Such "frozen-in" magnetic field lines propagate Alfvén waves in a manner analogous to transmission of waves by a string. For Alfvén waves the tension of the magnetic lines is given by $B_0{}^2/4\pi$,[6] and the density $\rho$ by that of the plasma carried along with the lines. Here $B_0$ is the static magnetic-field intensity produced by external coils. Thus the propagation velocity is given by

$$v_A = B_0/\sqrt{4\pi\rho} \qquad (1)$$

*Work done under the auspices of the U. S. Atomic Energy Commission.
†Summer visitor from Wesleyan University, Middletown, Connecticut.

in agreement with the result derived from Maxwell's and the hydrodynamic equations for infinite conductivity. A more general result will be stated below.

The dielectric properties of a plasma may often be used to discuss plasma behavior. In particular, the propagation velocity is related to the velocity of light c by

$$v = v_A = c/\sqrt{K} \qquad (2)$$

and the impedance of a plasma-filled wave guide by

$$Z = Z_0/\sqrt{K} \qquad (3)$$

where K is the dielectric constant of the plasma and $Z_0$ the vacuum impedance of the guide.

When an electric field E is imposed upon a plasma in a direction perpendicular to an externally applied static magnetic field $B_0$, the dielectric constant may be calculated directly. The energy added by imposition of a field E is

$$\frac{KE^2}{8\pi} = \frac{E^2}{8\pi} + \frac{1}{2}\rho v^2. \qquad (4)$$

The magnitude of v is $Ec/B_0$, and thus we have

$$K = 1 + \frac{4\pi\rho c^2}{B_0^2} \qquad (5)$$

which correctly yields Eq. (1) for $4\pi\rho c^2/B_0^2 \gg 1$, as in these experiments.

## THEORETICAL DISCUSSION

Theoretical discussions of Alfvén-wave propagation under conditions similar to those imposed in the present experiments have been given by Newcomb[7] and by Lehnert.[8] Their results for propagation velocity and attenuation in the presence of noninfinite conductivity and plasma inertial effects will be stated here.

A plasma of cylindrical geometry is assumed. A uniform magnetic field $B_0$ is in the $z$ direction along the axis of the plasma cylinder. Torsional waves are induced by an electric field acting between two concentric electrodes at one end of the cylinder. The radial dependence of the azimuthal magnetic field component $b_\theta$ of the resulting torsional wave is given by

$$\frac{\partial^2 b_\theta}{\partial r^2} + \frac{1}{r}\left(\frac{\partial b_\theta}{\partial r}\right) + \left(k_c^2 - \frac{1}{r^2}\right) b_\theta = 0 \qquad (6)$$

when the wave frequency is small compared with the ion cyclotron frequency. The solutions to this equation are first-order Bessel functions, and $k_c$ is determined by appropriate boundary conditions on those solutions. If the wave frequency is not small compared with the ion cyclotron frequency, the $b_\theta$ solutions are not easily separated from those for $b_r$ and $b_z$, but can be written as the sum over a set of "principal mode" solutions which are characterized by being divergence-free. Newcomb has shown that the higher principal modes suffer progressively larger damping. For this reason only the first principal mode is considered in this discussion, although future measurements may yield more information about this approximation.

The magnetic field components for the lowest-order principal mode have the following form:

$$b_\theta = b^0_\theta J_1(k_c r)e^{i(kz-\omega t)} \tag{7}$$

$$b_r = b^0_r J_1(k_c r)e^{i(kz-\omega t)} \tag{8}$$

$$b_z = b^0_z J_0(k_c r)e^{i(kz-\omega t)}, \tag{9}$$

where $J_0$ and $J_1$ are the zero- and first-order Bessel functions; $b^0_\vartheta$, $b^0_r$, $b^0_z$ are the amplitudes of the $\theta$, r, and z components, respectively; k is the complex propagation constant; $\omega$ is the wave angular frequency; and $k_c$ is determined by the boundary condition that at the outer radius we have

$$b_r = J_1(k_c a) = 0 \tag{10}$$

for a surrounding conducting cylinder of radius a.

The propagation velocity is contained in the real part of k. When collisions with neutrals are assumed negligible, the propagation velocity is given by[8]

$$U = v_A \left[ \frac{1 + \left( \frac{\omega}{\mu_0 \sigma v_A{}^2} \right)^2}{1 - \frac{k_c{}^2}{\mu_0{}^2 \sigma^2 v_A{}^2}} \right]^{1/2}, \tag{11}$$

where $\sigma$ is the conductivity. Similarly, the attenuation is contained in the imaginary part of k and is given by[7]

$$\epsilon_1 = \frac{(k_c{}^2 + k^2)}{2\mu_0 \sigma v_A} = \frac{(k_c{}^2 + k^2)(4\pi\rho)^{1/2}}{2\mu_0 \sigma B_0}. \tag{12}$$

The attenuation caused by neutral damping may be included with an additional term[9] in the total attenuation factor $\epsilon = \epsilon_1 + \epsilon_2$ involving the ion-neutral collision frequency $\nu_{in}$ of the form

$$\epsilon_2 = \frac{\nu_{in}}{2v_A} = \frac{\nu_{in}(4\pi\rho)^{1/2}}{2B_0} \tag{13}$$

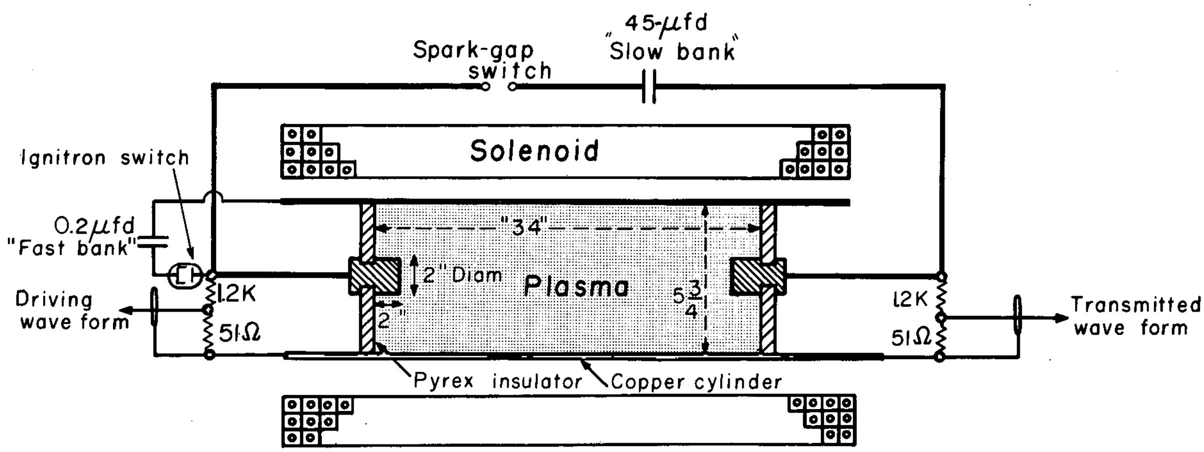
Both $\epsilon_1$ and $\epsilon_2$ have the same dependence on $B_0$. Since this added term assumes the loss at each collision of correlation between the colliding ion and the ordered wave motion, Eq. (13) provides an upper limit to the effects of neutral damping. For the conditions imposed in this experiment we have $k_c^2 \gg k^2$ and $\epsilon \approx 1.3\ m^{-1}$ at $B_0 = 12$ kgauss.

## GEOMETRY

The geometry of the apparatus used in these experiments is shown in Fig. 1. A 5-3/4-in. i.d., 34-in.-long copper cylinder is placed in a uniform axial magnetic field of the order of 10 kgauss. At each end of the cylinder is a pyrex insulator in which is mounted a coaxial electrode 2 in. in diameter and 2 in. long. After evacuation of the cylinder to approximately 0.6 micron of Hg, hydrogen gas is allowed to flow through the cylinder at a pressure of 100 microns Hg ($7.1 \times 10^{15}$ protons/cm$^3$). The equilibrium pressure is monitored with a Pirani gauge which is periodically calibrated by use of a McLeod gauge. The voltage on each electrode is measured with a resistive divider, and the results are presented on a dual-trace oscilloscope.

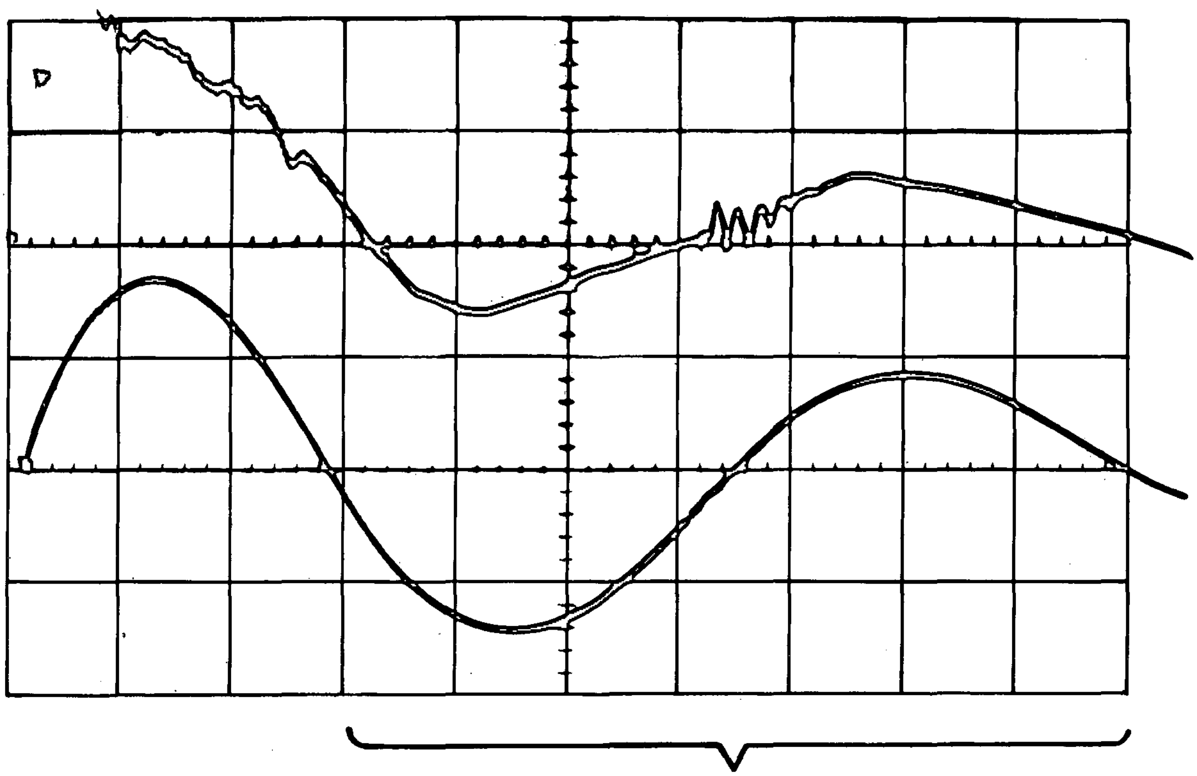
## PLASMA PREPARATION

The gas is ionized by discharging a 45-μf "slow" condenser bank between the two center electrodes. The resulting current and voltage waveforms are displayed in Fig. 2. The (inductive) impedance of the external circuit is




MU-18257

Fig. 1. Experimental geometry.

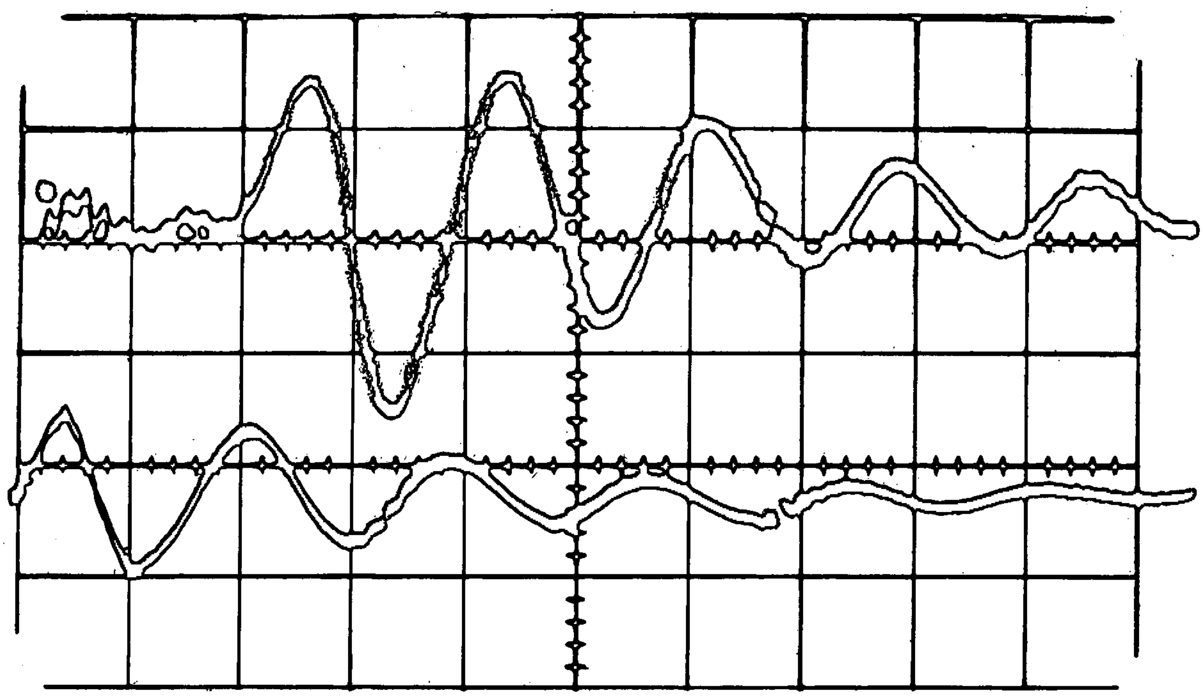


MU-18302

Fig. 2. Slow-bank voltage and current. Horizontal scale 10μ sec/large division. Top trace is voltage between the center electrodes at 1000 v/large division. Bottom trace is current at 40 ka/large division.

larger than the impedance of the tube, so that while the condenser bank is charged to 10 kv, only 2 kv appear across the tube. The ionization mechanism is not well understood, and has not been studied in this experiment. After the plasma has been formed by the slow bank, a 0.2-μf fast condenser bank is discharged between one electrode and the outer cylinder. This voltage creates a radial electric field at one end of the plasma which induces the hydromagnetic wave. The resulting voltage waveforms on the sending and receiving electrodes are displayed in Fig. 3.

The wave has been propagated at various times after the slow bank has been fired, as indicated by the bracket on Fig. 2. The fast bank was fired at approximately 65 μsec for this figure. At the earliest times (before the bracket) the traces are hashy and the Alfvén wave cannot be distinguished. Over a range of 70 μsec the measured wave velocity is essentially constant, as shown in Fig. 4, which indicates that the density $\rho$ is Eq. (1) is not changing during this time. The measured wave velocity is shown in Fig. 5 as a function of the fast-bank voltage. Since the velocity is constant over the range from 4 to 16 kv, the density $\rho$ in Eq. (1) is not changing due to ionization or other effects of the fast-bank voltage. As the slow-bank voltage is increased, the measured wave velocity decreases. This is apparently caused by evolution from the tube walls. Since this effect has not been measured quantitatively, and since the percentage ionization has not been independently determined, the density $\rho$ in Eq. (1) is computed from the observed wave velocity.



MU-18303

Fig. 3. Received and driving waveforms. Horizontal scale is 1 μsec/large division. Top trace is received voltage and bottom trace is driving voltage. A filter is used to give a horizontal base line. A delay of about 2 μsec in the received signal can be seen.

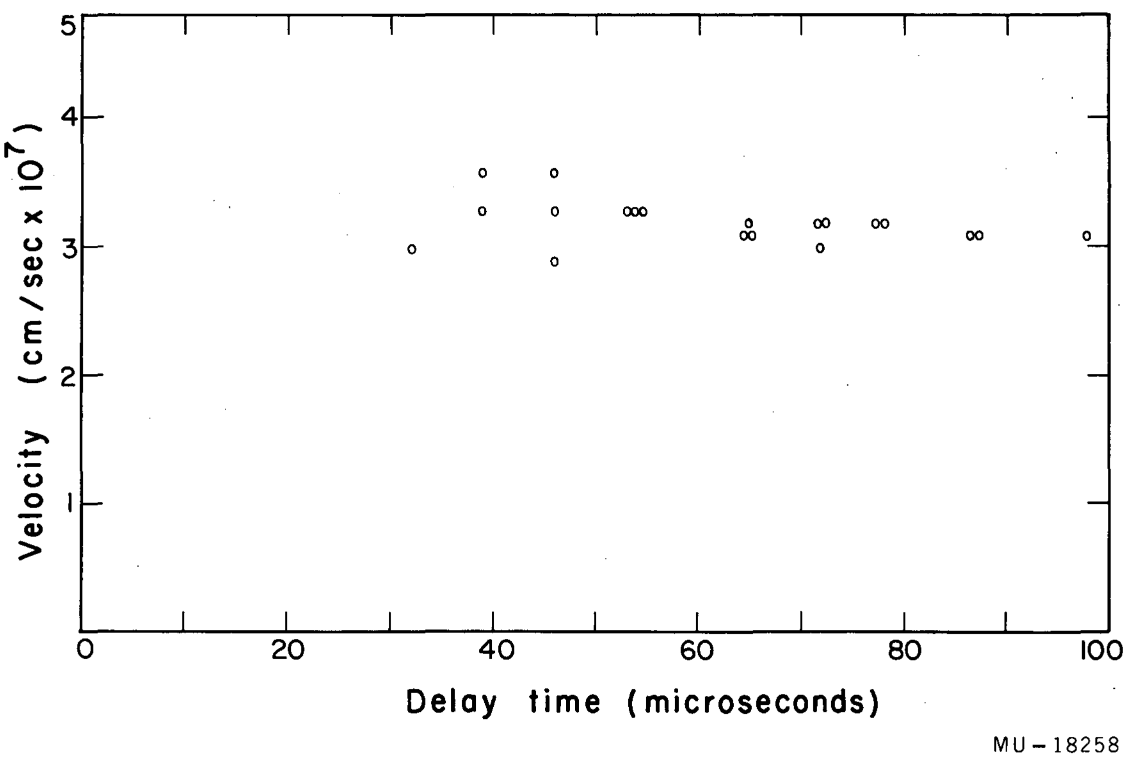



Fig. 4. Measured wave velocity vs. time delay between firing of slow bank and fast bank.

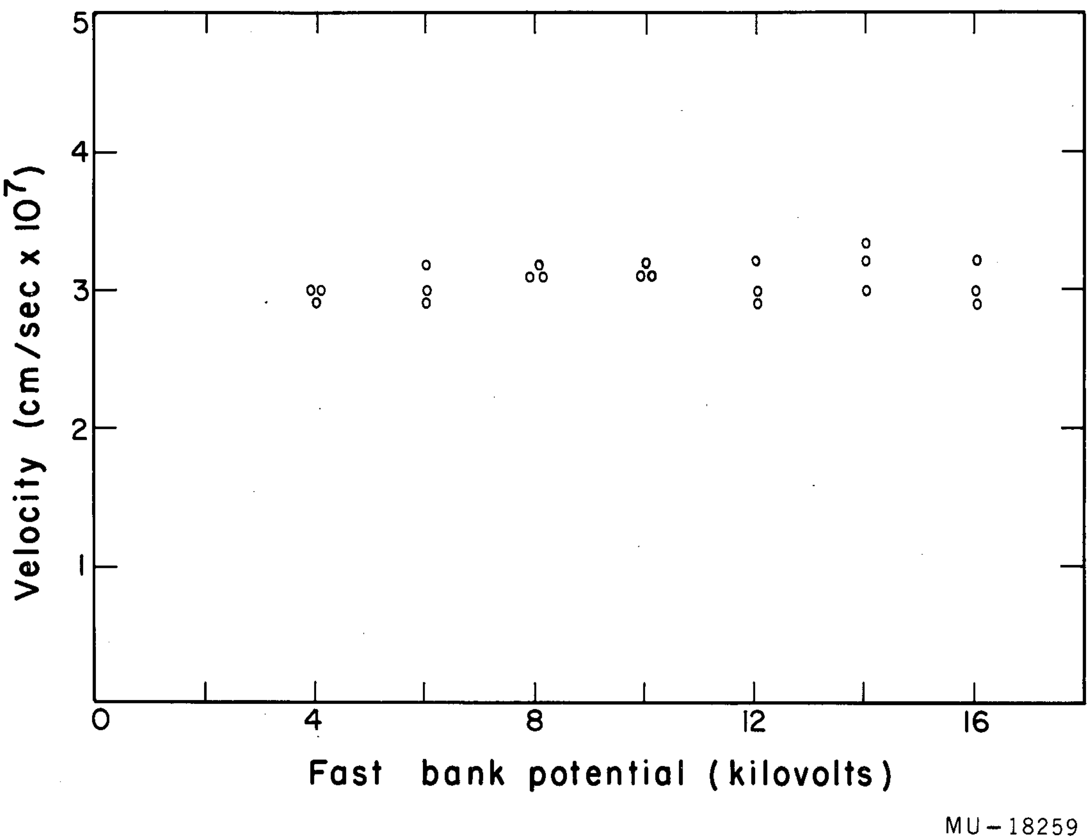



Fig. 5. Measured wave velocity vs. fast-bank potential.

## VELOCITY

The wave velocity was determined as a function of $B_0$ from dual-beam oscilloscope traces of driving and received voltage wave forms similar to those in Fig. 3. The resulting velocities are plotted in Fig. 6. The dependence of wave velocity on $B_0$ is very nearly that predicted from Eq. (1), although the dashed "best-fit" straight line extrapolates to a nonzero velocity intercept. The cause of this intercept is not known.

The plasma density can be evaluated from the measured wave velocity at a particular magnetic field. At $B_0 = 10$ kgauss, $v_A = 2.8 \times 10^7$ cm/sec yields $\rho = 1.02 \times 10^{-8}$ gm/cm$^3$ which is equivalent to $6.1 \times 10^{15}$ protons/cm$^3$.

## ATTENTUATION

The ratio of the received to driving wave voltage as measured between the center electrode and the outer cylinder is plotted as a function of magnetic field in Fig. 7. This ratio R is equal to

$$R = e^{-(\epsilon_1 + \epsilon_2) L} \qquad (14)$$

where $\epsilon_1$ is given by Eq. (12), $\epsilon_2$ is given by Eq. (13), and L is the length of the tube. The solid curve has been calculated by the use of Eq. (14) and is normalized to fit the experimental data at 12 kgauss. We may note that if the damping caused by ion-neutral collisions can be made negligibly small ($\epsilon_2 \ll \epsilon_1$), the transverse conductivity can be calculated from Eq. (12). Thus the electron temperature can be determined from a theoretical discussion of conductivity such as that given by Spitzer.[10] A small change in temperature results in a large change in the ratio of the received to driving wave voltage.

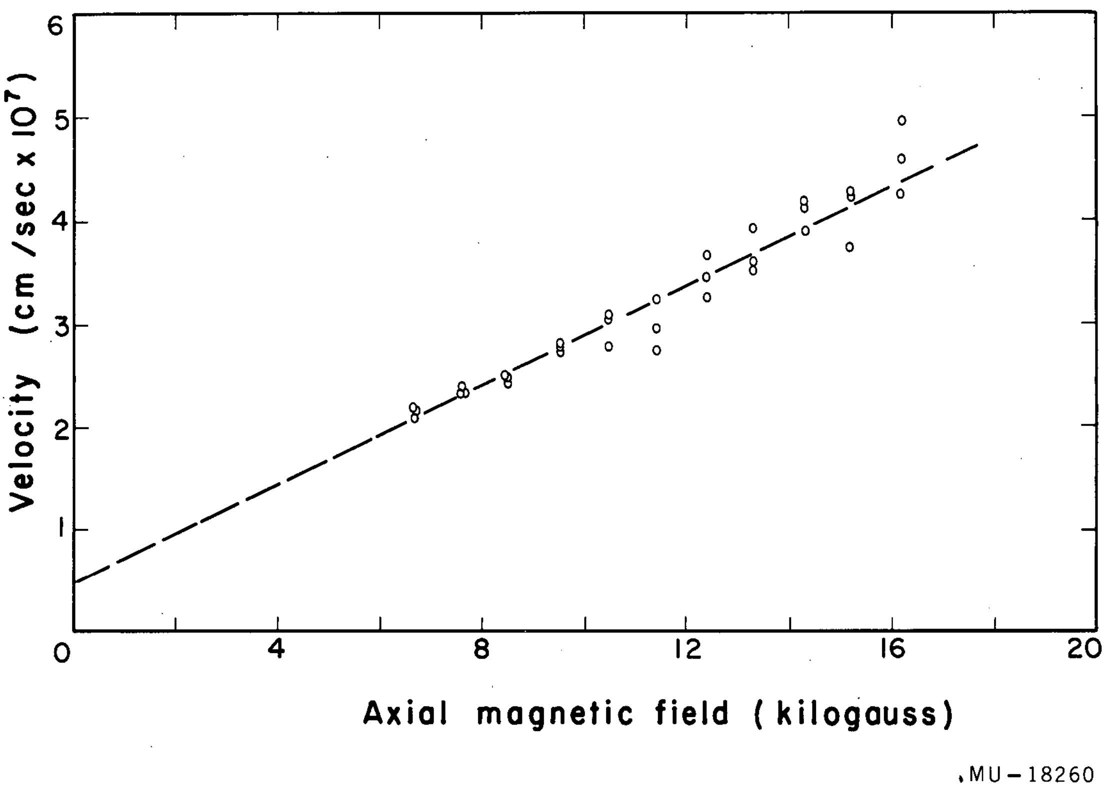



Fig. 6. Measured wave velocity vs. axial magnetic field. Dashed line indicates linear dependence predicted by theory.

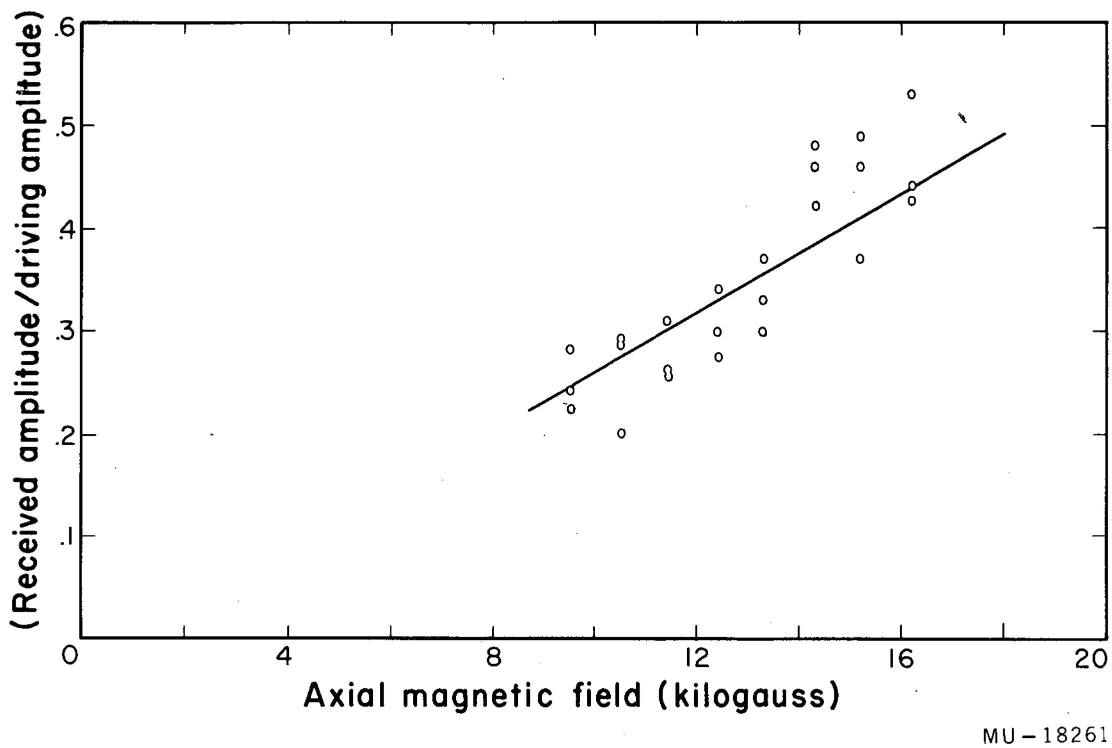



Fig. 7. Received amplitude/driving amplitude vs. axial magnetic field. Solid curve is plot of Eq. (14) normalized at 12 kgauss.

## IMPEDANCE

The impedance of the device was determined as a function of magnetic field from the waveforms of the driving current and voltage. At 13.3 kgauss this measured impedance was found to be 0.085 ohms and the theoretical impedance obtained from Eqs. (3) and (4) to be 0.080 ohms. However, the impedance was observed to be not as strongly dependent upon $B_0$ as the linear relationship predicted from Eqs. (3) and (4).

## ENERGY TRANSFER

The efficiency of the transfer of energy from the external oscillating circuit to the hydromagnetic wave has been measured. The energy delivered to the driving electrodes by the oscillating circuit has been obtained by numerical multiplication of the measured current and voltage waveforms. The second and third half cycles have been chosen for this comparison. The energy content of the wave in this same interval has been measured with magnetic probes. For a wave traveling in the positive z direction Cowling[11] gives

$$\mathbf{b} = -\frac{B_0}{v_A}\mathbf{V},$$

where $\mathbf{b}$ is the magnetic field associated with the wave, $\mathbf{V}$ is the (transverse) plasma velocity, $B_0$ is the static axial magnetic field, and $v_A$ is the Alfvén velocity. Then the magnetic energy, $b^2/8\pi$ per unit volume, equals the kinetic energy, $1/2\ \rho V^2$ per unit volume. The magnetic field of the wave is measured with a probe located 1-3/4 in. beyond the end of the driving electrode at a radial position $r = r_p$ midway between the coaxial electrodes. The largest magnetic-field component associated with the wave is $b_\theta$. We also observe the presence of $b_r$ and $b_z$, but these are small compared with $b_\theta$ so that $b_r^2$ and $b_z^2$ can be neglected in comparison with $b_\theta^2$ for a first-order calculation.[12] Thus from the energy density $b_\theta^2/8\pi$ we calculate the energy that flows past the probe during the second and third half-cycles of the wave, add an equal amount of energy to represent the kinetic energy $1/2\ \rho V^2$, and compare the result with the energy input from the external oscillating circuit to the driving electrodes.

We measure with the magnetic probe the value of $b_\theta$ at $r = r_p$. Since our analysis includes only the lowest-order mode, we have shown above that

$$b_\theta = b^0_\theta J_1(k_c r)e^{i(kz-\omega t)} \qquad (7)$$

where $b^0_\theta$ is determined from the measured value of $b_\theta$ at $r = r_p$. The magnetic energy of the wave is calculated by integrating $b^2_\theta/8\pi$ over $r$ and $t$ at $z = 0$. The time integral is done as a summation employing the output wave forms of the magnetic probe. Energy input during the two half-cycles is found to be $9.5\times10^6$ ergs, while that in the wave for the same interval is $4.1\times10^6$ ergs, indicating the transfer of $(43 \pm 10)\%$ of the input energy to the wave.

The fate of the remaining energy is not presently known, but may be in insulator or sheath losses, or possibly in a rapidly attenuated wave propagated in the negative $z$ direction. In any event, the transfer of an appreciable fraction of the input energy into Alfvén-wave energy is possible.[13]

The magnetic probe measurement indicates that the field associated with the wave is about 100 gauss. Since the static field is 10,000 gauss, the wave field is about 1%, and therefore a small-amplitude theoretical treatment is valid.

## ACKNOWLEDGMENTS

We wish to thank Mr. William R. Baker for his interest in this work. Mr. Richard O'Sullivan made valuable contributions to the experimental work. Illuminating theoretical discussions were held with Drs. Lloyd Smith and William Newcomb, and we wish to thank Dr. Bo Lehnert for sending us a copy of his paper prior to publication. We wish to thank Gerald V. Wilson, Louis Biagi, and Pierre Pellisier for able assistance.

# FOOTNOTES

1. Allen, Baker, Pyle, and Wilcox, Phys. Rev. Lett. 2, 383 (1959).
2. D. F. Jephcott, Nature 183, 1652 (1959).
3. H. Alfvén, Arkiv Mat. Astron. Fysik 29B, No. 2 (1943).
4. S. Lundquist, Phys. Rev. 76, 1805 (1949).
5. Bo Lehnert, Phys. Rev. 94, 815 (1954).
6. H. Alfvén, Cosmical Electrodynamics (Oxford University Press, London, 1950), p. 82.
7. W. A. Newcomb in Magnetohydrodynamics (Stanford University Press, 1957), p. 109.
8. Bo Lehnert, Plasma Physics on Cosmical and Laboratory Scale, (To to published in Supplement to Nuovo cimento).
9. W. A. Newcomb, Lawrence Radiation Laboratory, Livermore, California, private communication.
10. L. Spitzer, Jr., Physics of Fully Ionized Gases (Interscience Publishers, Inc., New York, 1956).
11. T. G. Cowling, Magnetohydrodynamics (Interscience Publishers, Inc., New York, 1957), p. 36.
12. Since $b_z$ is in the same direction as the static axial field $B_0$, the energy density will be of the form $(B_0 + b_z)^2/8\pi = B_0^2/8\pi + b_z^2/8\pi + B_0 b_z/4\pi$. The first term is constant, the second is negligibly small, but the third term could be appreciable since $B_0$ is large. However, it can be shown that the spatial integration of this term over the tube radius, at any value of z, is identically zero.

13. This efficiency is of particular importance for the "magnetic beach" plasma heating proposal of T. H. Stix in Generation and Thermalization of Plasma Waves, Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy (United Nations, New York, 1958), Paper No. 361. In this scheme a hydromagnetic wave is induced at one end of a magnetic-mirror geometry, and conditions are arranged so that the local ion cyclotron frequency at the center of the mirror geometry is approximately equal to the wave frequency. Under these conditions Stix predicts that the wave energy will be absorbed and thermalized by the ions of the plasma, thus providing an attractive heating mechanism. The proposed plasma density is considerably smaller than in the present experiment.