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An Upper Bound for the Capacity of Distributed MIMO Mobile Ad Hoc Networks

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Abstract— We compute an upper bound for the capacity of mobile wireless ad hoc networks (MANETs) when all the nodes in the network are endowed with multiple antennas. The derivation is based on a new communication scheme for wireless ad hoc networks utilizing the concept of cooperative many-to-many communication, which we show outperforms the conventional one-to-one communication approach. We assume that nodes transmit with low power, such that the total interference from other cells is much smaller than noise. Consequently, the results for the upper bound capacity is only valid for the low interference power regime.

Index Terms— Ad hoc networks, MIMO systems.

I. INTRODUCTION

The capacity of multiple-input multiple-output (MIMO) systems has received considerable attention [1], [2], [3], [4]; however, all these studies concentrate on the concept of one-to-one communication among nodes. Even the work by Jovičić et al [4] studies the capacity of wireless ad hoc networks by considering the whole network as a single MIMO system in which some nodes are part of the transmitter and the remaining nodes in the network are part of the receiver, with each node having a single antenna. A random line is used to cut the network into two parts consisting of senders and receivers. While this work was the first attempt to compute the capacity of networks based on MIMO systems, the results assume all the transmit and the receiving nodes in the network are capable of cooperating with each other.

In addition, Chen and Gans [5] and Blum [6] addressed the problem of capacity for MIMO ad hoc networks assuming fading for the wireless channel. However, both works only consider small-scale fluctuations of the fading channels.

In this paper, we compute upper bound capacity of nodes in MANETs considering both small-scale (fading) and large-scale (channel path loss) channel variations. We also propose that a more effective communication strategy among MIMO nodes in wireless ad hoc networks with a shared channel consists of enabling cooperative many-to-many communication. The new scheme is based on a collaboration-driven approach,

which we call opportunistic cooperation [7]. In this new approach, multiple nodes simultaneously communicate with each other within a pre-defined area that we call a *cell* of the network (as described later). Sender and receiver nodes collaborate rather than compete with each other to access the channel. Each sender node either relays a message to all receiver nodes or delivers a packet to its destination. Within the context of a cell in the network, multiple sender nodes either relay or transmit data to a destination simultaneously. The multiple access scheme that is proposed in this letter is based on a distributed MIMO system that does not require any bandwidth expansion. We explore the capacity behavior of wireless ad hoc networks when every single node has M antennas. The level of cooperation among nodes in a cell varies and depends on the number of nodes at each communication session. Consequently, the cooperation among nodes occurs opportunistically.

In this new paradigm, multiple nodes that are close to each other attempt to communicate concurrently. The nodes are divided into sender and receiver nodes. At each communication session, a node is selected either as a sender or a receiver node. If the node is a sender node, it sends packets from some of its antennas while the receiver nodes receive and decode packets from multiple nodes simultaneously using all of their antennas. Therefore, each MIMO system in this scheme consists of multiple transmitting nodes and a single receiver node in a cell, i.e., a distributed MIMO system. Hence, this approach does not require any cooperation among receiving nodes for decoding the received packets. In this paper, we only explore the capacity of such systems under the condition that the interference is much weaker than noise, by assuming low transmit power for the nodes. We also assume that the channel paths between transmitter and receiver antennas are statistically independent.

The remaining of the paper is organized as follows. Section II summarizes the network model. Section III introduces the principles of opportunistic cooperation. The communication model employed is explained in Section IV. In Section V, the design of parameters in the network is discussed. Section VI presents the capacity analysis. Sections VII and VIII provide the numerical results and the conclusion respectively.

II. NETWORK PARAMETERS AND ASSUMPTIONS

We assume that time is slotted to facilitate the analysis. Define $A_T(n) := \frac{\theta n}{\rho}$ as the total area of the network. ρ is the average density of sender nodes and $0 < \theta < 1$ is the fraction of sender nodes at each time slot. Therefore, in this

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model, the total area of the network grows linearly with the number of nodes n . We define the term *cell* as the set of nodes located inside a square area. This assumption requires that each node knows its position using GPS [8] or some other technique. Consequently, a_{cell} denotes the area of every cell in the network, which does not depend on n . Also, the cell is the area where a group of nodes communicate. Accordingly, the total number of cells in the network is given by

$$\# \text{ of cells} = \frac{A_T(n)}{a_{cell}} = \frac{\theta n}{\rho a_{cell}}. \quad (1)$$

The movement of the nodes is modeled with a uniform mobility model [9] in which the steady-state distribution of the mobile nodes is uniform. Accordingly, the distribution of the number of nodes in a cell is binomial [10]. Let the random variable Z denote the number of nodes in a cell. Given (1), the probability of $Z = \ell$ is

$$\mathbb{P}\{Z = \ell\} = \binom{n}{\ell} \left(\frac{\rho a_{cell}}{\theta n}\right)^\ell \left(1 - \frac{\rho a_{cell}}{\theta n}\right)^{n-\ell}. \quad (2)$$

III. PRINCIPLES OF OPPORTUNISTIC COOPERATION

Conventionally, as illustrated in Fig. 1, the communication mechanisms and performance models of wireless ad hoc networks are based on the assumption that reliable communication occurs between a single node attempting to send packets to a destination, possibly via relaying [11] [12], [9], and the rest of the source-destination pairs sharing the channel concurrently are treated as destructive interference. This scenario is also described as an one-to-one communication framework. The works of Ghez et al [13] and Tong et al [14] explain a framework for many-to-one communication where multiple nodes transmit their packets to a single receiving node.

Fig. 1. Interference due to transmission at the same channel can cause collision and prevent packet delivery in the conventional one-to-one communication framework. Dashed lines indicate paths travelled by relay nodes.

In a competition-driven paradigm for MANETs, when two or more nodes become close enough to each other, they can transmit information to one another without any delay. With opportunistic cooperation, many nodes transmit concurrently to many other nodes that are close enough, and all such transmissions are decoded [7], as shown in Fig. 2. The communication between senders and receivers is time-slotted

and fully synchronized, for example, by assuming the nodes utilize GPS [8]. The nodes travel and eventually find the destination for a relayed packet (for example, nodes 4 and 5). The delivery is done similarly, where we assume in Fig. 2 that relays 4 and 5 approach the destination nodes simultaneously and deliver the relayed packets using a distributed MIMO system. The receiver nodes are assumed to employ multiuser detection in order to decode the simultaneous transmissions. Note that the packets are delivered using one-time relaying in this model, perhaps utilizing a multi-copy forwarding scheme [15] if there is enough receiver nodes in the cell. One of the benefits of using opportunistic cooperation comes from the increased probability of a relay node to deliver a packet to its destination in a short time, because a given packet is now carried by possibly many relay nodes (multi-copy forwarding [15]).

Fig. 2. Opportunistic Cooperation: nodes transmit simultaneously to different neighbors using a distributed MIMO approach. The nodes use multi-user detection to separate the signals from different senders.

In our specific implementation of opportunistic cooperation, we use two types of channels [7]. *Control channels* are used by nodes to obtain such information as the identifier (ID) of their neighbors and the state of data channels (CSI). *Data channels* are used to transmit data taking advantage of multiuser detection at the receivers. A detailed description for channel access can be found in [7], and Fig. 3 provides its time series representation. The nodes in each cell use the “neighbor discovery” t_{disc} time to find their neighbors (i.e., to obtain their IDs) and the period for transmission of data t_{data} to send information. t_{disc} and t_{data} are constant and independent of the number of nodes in the network (n). Together, they compose a “communication session.” Nodes are assumed to be synchronous [13]. The time t_{disc} is subdivided into N equal slots. Each time the discovery period is about to begin, each node randomly chooses one of the N mini-slots and transmits its control packet. If there is no collision, i.e., if the other nodes in the same cell choose different mini-slots to transmit, then all the other nodes in the cell will receive this packet. A collision happens every time two or more nodes in the same cell choose to transmit in the same mini-slot. When this happens, the nodes involved in the collision do not participate in the session. It is shown in [7] that the probability of collision can be reduced to a small value.

With the knowledge by each node of who their neighbors

Fig. 3. Time series representation of control and data packets. t_{disc} is the neighbor discovery phase. t_{data} is the time period for transmission of data. t_{disc} plus t_{data} form a communication session.

are in the same cell, a distributed protocol can be used to allow the nodes to coordinate access to the channel [7].

IV. COMMUNICATION MODEL

We assume every node has M antennas, where M is a constant that is greater than or equal to 2. In our analysis, we consider that the channel state information (CSI) is known only at the receiver. The assumption of using low transmit power for each node allows us to simplify the analysis such that the interference from adjacent cells is negligible compared to the thermal noise, i.e., the additive white Gaussian noise (AWGN) power is much stronger than the interference. Hence, nodes only communicate with other nodes in the same cell and there is no significant interference from other cells in the network. Fig. 4 illustrates the MIMO distributed communication setup in a cell for S sender nodes and a receiver node j .

Fig. 4. Distributed MIMO communication in a cell.

Each transmit node uses T of its M antennas to transmit a packet using total power P_t . According to Blum [6], in the lower power interference regime and considering no CSI at the sender side, transmitting with equal power in each antenna maximizes capacity. Thus, each transmit antenna uses power P_t/T . The distance between a sender s and a receiver j is denoted as r_{sj} . Assuming no small-scale fading, the received signal power at node j from antenna k in node s is

$$P_{k,s,j} = \frac{P_t/T}{(1+r_{sj})^\alpha}, \quad (3)$$

where α is the path loss parameter and assumed to be greater than or equal to 2. In (3), the distance between receiver node

j and the sender node s (r_{sj}) is not a function of transmit and receive antennas, i.e., k and m . This is because the distances between any transmit antenna in sender node s and any receive antenna in node j is assumed to be equal for practical considerations and to simplify the analysis.

We use the channel model, described in [4], in which the received signal power is proportional to $1/(1+r_{sj})^\alpha$, as opposed to the more common approach of $1/r_{sj}^\alpha$ [11], [12] in order to ensure that the received power is never greater than the transmitted power.

We use boldface capital letters to represent matrices and boldface lower case letters to denote vectors. Accordingly, the received signal vector for one receiver node is defined as $\mathbf{y} = [y_1, y_2, \dots, y_M]^t$, where the operator t is the transpose of a vector. The transmission signal is $\mathbf{x} = [x_1, x_2, \dots, x_{TS}]^t$, where S is the number of senders in the cell, i.e., $S = \theta\ell$. This assumption implies that transmit nodes use T antennas while receiving nodes utilize all their M antennas for communication. Thus, the total transmitted power for the cell is SP_t . The received signal for a receiver node is defined as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$, where $\mathbf{z} = [z_1, z_2, \dots, z_M]^t$ is a zero-mean complex AWGN. We assume $E[\mathbf{z}\mathbf{z}^\dagger] = \sigma_z^2 \mathbf{I}_M$, where the operator $\dagger$ is defined as the conjugate transpose of a matrix, $\mathbf{I}_M$ is the $M \times M$ identity matrix, and σ_z^2 is the noise variance. $\mathbf{H}$ is the $M \times TS$ channel matrix with its elements defined as

$$h_{m,s} = \frac{\eta_{m,s}}{(1+r_{sj})^\alpha}, \quad (4)$$

where $1 \leq m \leq M$, $1 \leq s \leq TS$, and $\eta_{m,s}$ is a stationary and ergodic stochastic fading process that is independent for each sender antenna and receiver antenna pair. The power of fading coefficients is normalized to one, i.e., $E[|\eta_{m,s}|^2] = 1$. Consequently, we have that

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,TS} \\ h_{2,1} & h_{2,2} & \dots & h_{2,TS} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,1} & h_{M,2} & \dots & h_{M,TS} \end{bmatrix}. \quad (5)$$

In every cell, each MIMO system consists of multiple transmitting nodes (the S senders, each with T active transmitting antennas) and a single receiver node (with M receiving antennas). As a cell may have more than one receiver node, the communication is S -to-many (opportunistic cooperation). In practice, it is possible that each node may have a packet for different nodes in a cell. One can then assume that each transmit node utilizes different frequencies to transmit packets to all other nodes [7]. The details of such protocols are beyond the scope of this paper.

V. NETWORK DESIGN

In a practical MIMO system, it is usually preferred that the number of transmit and receive antennas to be defined within a range. The number of nodes within a single cell is a random number due to the mobility of the nodes. Thus, it is desirable to set the network parameters such that the maximum number of nodes in a cell is bounded with a high probability as $n \rightarrow \infty$. For this reason, the number of nodes in a cell should fall into a range, e.g., $[2, G/\theta]$, where $G \geq 1$. In this example, the total

number of transmit nodes in each cell will not exceed G with high probability (*whp*).¹ The maximum number of nodes is a design parameter for each application, and for the rest of this paper we choose the value of G/θ .

If a cell has only one or no node, it is not possible to implement a distributed MIMO system in that cell. For this reason, we make the probability of $\mathbb{P}\{Z = 0\} + \mathbb{P}\{Z = 1\}$ less than an arbitrary small positive number ϵ_1 . That is, for large n , given that $\binom{n}{\ell} \leq \frac{n^\ell}{\ell!}$ for $n \gg \ell$ and the $\lim_{x \rightarrow \infty} (1 - 1/x)^x = e^{-1}$, we have

$$\begin{aligned} \mathbb{P}\{Z = 0\} + \mathbb{P}\{Z = 1\} &= \sum_{i=0}^1 \binom{n}{i} \left(\frac{\rho a_{cell}}{\theta n}\right)^i \left(1 - \frac{\rho a_{cell}}{\theta n}\right)^{n-i} \\ &\leq \exp\left(-\frac{\rho a_{cell}}{\theta}\right) \times \left(1 + \frac{\rho a_{cell}}{\theta}\right) \\ &< \epsilon_1. \end{aligned} \quad (6)$$

Also, from (2), observe that the limiting probability distribution for the nodes in a cell equals

$$\lim_{n \rightarrow \infty} \mathbb{P}\{Z = \ell\} = \frac{1}{\ell!} \left(\frac{\rho a_{cell}}{\theta}\right)^\ell \exp\left(-\frac{\rho a_{cell}}{\theta}\right), \quad (7)$$

that is, as $n \rightarrow \infty$, the nodes attain a Poisson distribution with parameter $\lambda = \frac{\rho a_{cell}}{\theta}$.

Now, let ℓ_{max} represent the maximum number of nodes in a cell, and $\lceil x \rceil$ denote the ceil function (i.e., the smallest integer greater than or equal to x). Thus, for the uniform mobility model, the maximum number of nodes in any cell can be shown [10] to be given *whp* by

$$\ell_{max} = \left\lceil \frac{3 \log(n)}{\log\left(\log\left(n^{\frac{\theta}{\rho a_{cell}}}\right)\right)} \right\rceil. \quad (8)$$

This result demonstrates that the maximum number of nodes in a cell increases very slowly as n increases in the network. Accordingly, we can compute the probability that the number of nodes in a cell is greater than G/θ and attempt to limit this probability. Therefore,

$$\begin{aligned} \sum_{\ell=\frac{G}{\theta}+1}^{\ell_{max}} \mathbb{P}\{Z = \ell\} &= \sum_{\ell=\frac{G}{\theta}+1}^{\ell_{max}} \binom{n}{\ell} \left(\frac{\rho a_{cell}}{\theta n}\right)^\ell \left(1 - \frac{\rho a_{cell}}{\theta n}\right)^{n-\ell} \\ &\leq \sum_{\ell=\frac{G}{\theta}+1}^{\ell_{max}} \frac{1}{\ell!} \left(\frac{\rho a_{cell}}{\theta}\right)^\ell \exp\left(-\frac{\rho a_{cell}}{\theta}\right) \\ &\leq \epsilon_2, \end{aligned} \quad (9)$$

where ϵ_2 is an arbitrarily small positive number.

The right term of the inequality in (9) is the right-tail function of Poisson distribution with parameter $\lambda = \rho a_{cell}/\theta$. This probability decreases with the increase in G/θ or decrease in λ . Using the four parameters G , θ , ρ , and a_{cell} , the inequalities in (6) and (9) can be satisfied.

In general, the network design becomes the task of carefully tuning the parameters G , θ , ρ , and a_{cell} (selected from a practical range) to make sure that there is a high probability that the number of nodes in the cell is in the range of 2

¹With high probability means with probability $\geq 1 - \frac{c_1}{n}$ [10], where c_1 is a positive constant.

TABLE I
POSSIBLE NETWORK PARAMETERS AND CORRESPONDING
PROBABILITIES.

θ	G	ρa_{cell}	Cell node range [$\min, G/\theta$]	$\mathbb{P}\{\min \leq Z \leq G/\theta\}$
1/2	4	2	[2,8]	0.8871
1/2	6	4	[2,12]	0.9332
1/3	3	2	[2,9]	0.8987
1/3	4	2	[2,12]	0.9738

and G/θ . Table V illustrates some examples for network parameters and their corresponding probabilities when the number of nodes in the cell is between 2 and G/θ .

VI. CAPACITY ANALYSIS

We compute the capacity for unit bandwidth also known as spectral efficiency. The communication bandwidth for the network is a constant that depends on the transmission bit rate of the nodes.

Theorem 1: The average capacity (in units of nats/s/Hz) of each node in the network is upper bounded by the following constant when $n \rightarrow \infty$

$$\begin{aligned} \frac{P_t M \theta}{\sigma_z^2} \left[\frac{(1 + \sqrt{2a_{cell}})^{2\alpha-1} - 1 - \sqrt{2a_{cell}}(2\alpha-1)}{a_{cell}(2\alpha-2)(2\alpha-1)(1 + \sqrt{2a_{cell}})^{2\alpha-1}} \right] &\left\{ \frac{\rho a_{cell}}{\theta} \left[\frac{\Gamma(\frac{G}{\theta}, \frac{\rho a_{cell}}{\theta})}{\Gamma(\frac{G}{\theta})} \right] \right. \\ &\left. - \exp\left(-\frac{\rho a_{cell}}{\theta}\right) \right\} + 1 - \frac{(\frac{G}{\theta}+1)\Gamma(\frac{G}{\theta}+1, \frac{\rho a_{cell}}{\theta})}{\Gamma(\frac{G}{\theta}+2)}, \end{aligned}$$

where $\Gamma(m+1) = m!$, and $\Gamma(m, x) = \int_x^\infty y^{m-1} e^{-y} dy$ is the incomplete Gamma function.

Proof: The conditional mutual information between input sequence, $\mathbf{x}$, and the output data sequence, $\mathbf{y}$, is given by

$$I(\mathbf{x}, \mathbf{y} | \mathbf{H}) = E_{\mathbf{H}}[I(\mathbf{x}, \mathbf{y} | \mathbf{H} = H)], \quad (10)$$

where H is a $M \times TS$ channel matrix realization of $\mathbf{H}$, in which $S = \theta\ell$ is the number of senders in the cell.

With CSI known only at the receiver, maximizing mutual information (in units of nats/s/Hz) is equivalent to maximize the following equation over $\mathbf{x}$ [1], [2], [3]²

$$C = \max_{\mathbf{K}_{\mathbf{x}} \geq 0, (\mathbf{K}_{\mathbf{x}})_{ii} \leq P_t/T} E_{\mathbf{H}} \left[\log \det \left(\mathbf{I}_M + \frac{1}{\sigma_z^2} \mathbf{H} \mathbf{K}_{\mathbf{x}} \mathbf{H}^\dagger \right) \right], \quad (11)$$

where $\mathbf{K}_{\mathbf{x}}$ is defined as $E(\mathbf{x} \mathbf{x}^\dagger)$.

Now, we define the following vector $\mathbf{r}$ with its elements representing the distance between different transmit antennas of a sender nodes and the receive antennas of a receiver node, i.e., $\mathbf{r} = [r_{11}, r_{21}, \dots, r_{M1}, \dots, r_{1TS}, r_{2TS}, \dots, r_{MTS}]$. Note that the elements of the vector $\mathbf{r}$ are random variables, in which every MT elements associated with the same transmit node and receive node pair are equal.

Let R_j denote the capacity for the receiver j when there is no interference from adjacent cells. It is already shown in [4] that, for S sender nodes in a cell,

$$\begin{aligned} R_j(S) &\leq \max_{\mathbf{K}_{\mathbf{x}} \geq 0, (\mathbf{K}_{\mathbf{x}})_{ii} \leq P_t/T} E_{\mathbf{H}} \left[\log \det \left(\mathbf{I}_M + \frac{1}{\sigma_z^2} \mathbf{H} \mathbf{K}_{\mathbf{x}} \mathbf{H}^\dagger \right) \right] \\ &\leq E_{h,r} \left[\sum_{m=1}^M \log \left(1 + \sum_{s=1}^{TS} \frac{P_t/T}{\sigma_z^2} |h_{ms}|^2 \right) \right] \\ &\leq \sum_{m=1}^M \sum_{s=1}^{TS} E_r \left[\frac{P_t/(T\sigma_z^2)}{(1+r_{ms})^{2\alpha}} \right]. \end{aligned} \quad (12)$$

²Note that $\sigma_z^2 = 1$ in [1], [2], and [3]. Also, $\log(\cdot)$ stands for the natural logarithm.

r_{ms} is a random variable that considers the uniform distribution of the nodes. The distance between the nodes are independent and identically distributed (iid) [9]. Each receiver location in the cell in polar coordinates is (r, γ) , where r and γ represent radius and angle from the center of the cell, respectively. Accordingly, averaging over distance in a cell results in

$$\begin{aligned} E_r \left[\frac{P_t / (T\sigma_z^2)}{(1 + r_{ms})^{2\alpha}} \right] &= E_r \left[\frac{P_t / (T\sigma_z^2)}{(1 + r)^{2\alpha}} \right] \\ &= \int \int_{area} \frac{P_t / (T\sigma_z^2)}{(1 + r)^{2\alpha}} \frac{1}{area} r dr d\gamma \\ &\leq \frac{P_t}{T\sigma_z^2} \int_0^{2\pi} \int_0^{r_{max}} \frac{1}{\pi r_{max}^2} \frac{r dr}{(1 + r)^{2\alpha}} d\gamma \\ &= \frac{P_t}{T\sigma_z^2} \int_0^{2\pi} \int_0^{\sqrt{2a_{cell}}} \frac{r dr}{(1 + r)^{2\alpha}} d\gamma \quad (13) \\ &:= \frac{P_t}{T\sigma_z^2} f(a_{cell}, \alpha), \quad (14) \end{aligned}$$

where $f(a_{cell}, \alpha) := \frac{(1 + \sqrt{2a_{cell}})^{2\alpha-1} - 1 - \sqrt{2a_{cell}}(2\alpha-1)}{a_{cell}(2\alpha-2)(2\alpha-1)(1 + \sqrt{2a_{cell}})^{2\alpha-1}}$. In (13), we use the fact that the maximum distance inside a square cell between two nodes is $r_{max} = \sqrt{2a_{cell}}$.

From (12) and (14), and noting that $S = \theta\ell$, the upper bound unconditional capacity for the receiver node j is given by

$$\begin{aligned} R_j(\ell) &\leq \sum_{m=1}^M \sum_{s=1}^{T \min(G, \theta\ell)} E_r \left[\frac{P_t / (T\sigma_z^2)}{(1 + r_{ms})^{2\alpha}} \right] \\ &\leq \frac{P_t M \min(G, \theta\ell)}{\sigma_z^2} f(a_{cell}, \alpha). \quad (15) \end{aligned}$$

Observe that the maximum number of senders in a cell is limited to G . Furthermore, (15) is the total capacity of the cell, because all receiver nodes in a cell receive all the transmitted packets in that cell (opportunistic cooperation) as either destination, relay, or simply drops that packet because the destination is one of the receivers in the cell.

The number of nodes in the cell is a random variable with its probability given by (2). Hence, with $n \rightarrow \infty$, from (7), and G/θ being the maximum number of nodes allowed to communicate per cell, the average node capacity is upper bounded by

$$\begin{aligned} R_j &= \sum_{\ell=2}^{\infty} \mathbb{P}\{Z = \ell\} R_j(\ell) \\ &\leq \frac{P_t M \theta}{\sigma_z^2} f(a_{cell}, \alpha) \left(\sum_{\ell=2}^{G/\theta} \mathbb{P}\{Z = \ell\} \times \ell \right. \\ &\quad \left. + \frac{G}{\theta} \sum_{\ell=G/\theta+1}^{\infty} \mathbb{P}\{Z = \ell\} \right) \\ &= \frac{P_t M \theta f(a_{cell}, \alpha)}{\sigma_z^2} \left\{ \frac{\rho a_{cell}}{\theta} \left[\frac{\Gamma(\frac{G}{\theta}, \frac{\rho a_{cell}}{\theta})}{\Gamma(\frac{G}{\theta})} - \exp(-\frac{\rho a_{cell}}{\theta}) \right] \right. \\ &\quad \left. + 1 - \frac{(\frac{G}{\theta} + 1) \Gamma(\frac{G}{\theta} + 1, \frac{\rho a_{cell}}{\theta})}{\Gamma(\frac{G}{\theta} + 2)} \right\}. \quad (16) \end{aligned}$$

These results show that the upper bound capacity increases linearly as the number of receive antennas or transmit power for each node increases. This result is only valid for values of P_t such that the total interference experienced by each cell

is much smaller than the thermal noise. Note that the upper bound capacity is not a function of n , neither a function of T .

VII. NUMERICAL RESULTS

To have a better understanding of the capacity behavior, we plot the upper bound obtained in Theorem 1 as a function of the size of the area for some network parameters.

Fig. 5 shows that the capacity upper bound increases from lower values of a_{cell} up to a maximum value, and after that it begins to decrease with the increase of the cell area. This is a consequence of the fact that for high values of a_{cell} the average number of nodes in each cell increases. However, the number of nodes per cell allowed to communicate is limited to G/θ , as explained in Section V. Therefore, beyond a certain point, the average upper bound decreases with the increase in a_{cell} . Another interesting observation is that the capacity upper bound decreases with the increase in α . This is explained by the fact that no interference is considered; thus, the increase of the path loss parameter reduces the received signal strength of each receiver node. Note that, if interference is considered, we would expect the opposite effect for the upper bound as a function of α , which is the subject of our future research.

Fig. 5. Average node capacity upper bound as a function of the cell area (a_{cell}) for $G = M = 4$, $P_t = 0.1$ Watt, $\rho = 0.5$ node/m², $\theta = 0.5$, $\sigma_z^2 = 1$, for different values of α .

Fig. 6 presents the comparison for our opportunistic cooperation MIMO scheme with a MIMO one-to-one communication approach. In the latter case, we model each node using M antennas for communication, i.e., either a node behaves as a sender transmitting with M antennas (using P_t in each one such that the total transmitted power is MP_t) or it behaves like a receiver using M antennas for reception. Also, in the one-to-one technique, only the cells containing exactly one sender and at least one receiver are able to communicate successfully [12]. We observe that our many-to-many scheme outperforms the one-to-one case. Moreover, the opportunistic cooperation is a distributed approach which allows the MIMO system to support more than M transmit antennas increasing even further the capacity.

Fig. 6. Average node capacity upper bound comparison for opportunistic cooperation (many-to-many) and the one-to-one MIMO schemes as a function of the cell area (a_{cell}) for $P_t = 0.1$ Watt, $\rho = 0.5$ node/m², $\theta = 0.5$, $\sigma_z^2 = 1$, and $\alpha = 2$.

VIII. CONCLUSIONS

In this letter, we have computed an upper bound for the capacity of MIMO nodes in ad hoc networks. This upper bound does not decrease with the increase in the number of nodes in the network. This result is rather optimistic assuming that the interference is no longer the dominant factor. Another important aspect of these results is the use of a new communication scheme (called opportunistic cooperation) that allows multiple nodes in a cell to communicate concurrently with other nodes in the same cell. This approach allows strong interfering nodes that are adjacent to each other to provide useful information by cooperation rather than compete for the channel. Our results show that the cooperative many-to-many approach outperforms the conventional one-to-one communication scheme.

Future work should consider the effect of interference and finding protocols to mitigate the destructive effect of interference using opportunistic cooperation.

REFERENCES

- [1] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Comm., Kluwer Academic Press*, no. 6, pp. 311–355, 1998.
- [2] E. Telatar, "Capacity of multi-antenna gaussian channels," *European Trans. on Telecomm.*, vol. 10, no. 6, pp. 585–595, November 1999.
- [3] A. J. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of mimo channels," *IEEE Journal on Selected Areas in Comm.*, vol. 21, no. 5, pp. 684–702, June 2003.
- [4] A. Jovičić, P. Viswanath, and S. R. Kulkarni, "Upper bounds to transport capacity of wireless networks," *IEEE Trans. on Information Theory*, vol. 50, no. 11, pp. 2555–2565, November 2004.
- [5] B. Chen and M. J. Gans, "Limiting throughput of MIMO ad hoc networks," in *Proc. of IEEE International Conference on Acoustics, Speech, and Signal Processing*, Philadelphia, PA, March 2005.
- [6] R. Blum, "MIMO capacity with interference," *IEEE Journal on Selected Areas in Comm.*, vol. 21, no. 5, pp. 793–801, June 2003.
- [7] R. M. de Moraes, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "A new communication scheme for MANETs," in *Proc. of IEEE WirelessCom*, Maui, Hawaii, June 2005.
- [8] B. W. Parkinson and J. J. Spilker, *Global Positioning System: Theory and Applications Vol. I*. American Institute of Aeronautics and Astronautics, Inc., 1996.
- [9] N. Bansal and Z. Liu, "Capacity, delay and mobility in wireless ad-hoc networks," in *Proc. of IEEE Infocom*, San Francisco, California, March 2003.
- [10] R. Motwani and P. Raghavan, *Randomized Algorithms*. Cambridge Univ. Press, 1995.
- [11] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. on Information Theory*, vol. 46, no. 2, pp. 388–404, March 2000.
- [12] M. Grossglauser and D. Tse, "Mobility increases the capacity of wireless ad-hoc networks," in *Proc. of IEEE Infocom*, Anchorage, Alaska, March 2001.
- [13] S. Ghez, S. Verdú, and S. Schwartz, "Optimal decentralized control in the random access multipacket channel," *IEEE Trans. on Automatic Control*, vol. 34, no. 11, pp. 1153–1163, November 1989.
- [14] L. Tong, Q. Zhao, and G. Mergen, "Multipacket reception in random access wireless networks: From signal processing to optimal medium access control," *IEEE Comm. Magazine*, pp. 108–112, November 2001.
- [15] R. M. de Moraes, H. R. Sadjadpour, and J. J. Garcia-Luna-Aceves, "Throughput-delay analysis of mobile ad-hoc networks with a multi-copy relaying strategy," in *Proc. of IEEE SECON*, Santa Clara, California, October 2004.