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Analytical expressions for fusion spectra produced in "beam-target" fusion reactions

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For "beam-target" fusion reactions, collimated measurements of the energy spectrum of one of the reaction products can provide information on the degree of anisotropy of the reacting beam ions. Analytical expressions relating the fusion spectrum to the velocity distribution of the beam ions are given. Application of the expressions to measurements of the spectrum of 15-MeV protons produced by reactions between energetic ^3He ions and relatively cold deuterons during ICRF heating in the PLT tokamak indicate that the velocity distribution of fast ^3He ions is peaked perpendicular to the tokamak magnetic field.

Several authors^{1,2} have related the energy distribution of neutrons produced by thermonuclear reactions to the temperature of the reactants, but no analytical treatments³ and only one set of numerical calculations⁴ relating the distribution function of nonthermal reactants to the energy spectrum of fusion reaction products has been published. In actual controlled fusion experiments, however, the largest fusion reaction rates⁵ and the most easily measured fusion spectra are achieved in plasmas with nonthermal populations. Measurements of these spectra can provide information about the "tail" (nonthermal part) of the ion distribution function that may be difficult to obtain using other diagnostic techniques. In the past, analysis of fusion spectra has relied on comparison of measured spectra with a set of spectra calculated numerically for various reactant distributions. This approach, though accurate, is tedious and provides little physical insight into the factors affecting the fusion product energy distribution. This paper presents analytical expressions for the fusion energy distribution produced in reactions between cold target ions and a monoenergetic, possibly anisotropic, beam in a strong magnetic field for the case of a fusion reaction that is isotropic in the center-of-mass (cm) frame. The utility of these expressions is then illustrated with a 15-MeV proton spectrum produced in the $d(^3\text{He}, p)\alpha$ fusion reaction during ion cyclotron range of frequency (ICRF) heating on the PLT tokamak.

For an isotropic monoenergetic beam (particle 1) that

reacts with cold target ions (particle 2) in a magnetized plasma, the energy distribution $\hat{F}(E)$ of the daughter product (particle 3) is⁶

$$\int_{E_l}^{E_u} \hat{F}(E) dE = \ln \left(\frac{1 + 2k_0 \frac{(E_u - E_0^i)}{\Delta E}}{1 + 2k_0 \frac{(E_l - E_0^i)}{\Delta E}} \right) \ln \left(\frac{1 + 2k_0}{1 - 2k_0} \right), \quad (1)$$

for $|E - E_0^i| < \Delta E$,

where

$$E_0^i = \frac{m_4}{m_3 + m_4} (Q + K) + \frac{1}{2} m_3 V^2,$$

$$\Delta E = V \sqrt{\frac{2m_3 m_4}{m_3 + m_4} (Q + K)},$$

and

$$k_0 = V \sqrt{\frac{m_3(m_3 + m_4)}{2m_4(Q + K)}}.$$

Here Q denotes the fusion energy, $K = \frac{1}{2} m_1 m_2 v_1^2 / (m_1 + m_2)$ is the relative kinetic energy, $V = m_1 v_1 / (m_1 + m_2)$ is the cm velocity, and v_i and m_i are the (laboratory) velocity and mass of particle i . The energies E_l and E_u represent the lower and upper limits of an energy bin, respectively. The corresponding expression for an anisotropic monoenergetic beam is⁶

$$\int_{E_l}^{E_u} \hat{F}(E) dE = \frac{1}{\pi} \left[\sin^{-1} \left(\frac{E - E_0^a}{k_1 \Delta E} \right) + k_2 \sqrt{1 - \left(\frac{E - E_0^a}{k_1 \Delta E} \right)^2} \right]_{E=E_l}^{E=E_u}, \quad (2)$$

for $|E - E_0^a| < k_1 \Delta E$,

where

$$E_0^a = E_0^i + \Delta E [\cos \chi_1 \cos \chi_3 - k_0 (\cos^2 \chi_1 \sin^2 \chi_3 + \sin^2 \chi_1 \cos^2 \chi_3)]$$

$$k_1 = \sin \chi_1 \sin \chi_3 (1 + 2k_0 \cos \chi_1 \cos \chi_3),$$

and

$$k_2 = k_0 \sin \chi_1 \sin \chi_3 / (1 + 2k_0 \cos \chi_1 \cos \chi_3).$$

The pitch angle χ_i is the angle between the velocity vector $\mathbf{v}_i$ and the magnetic field $\mathbf{B}$.

In actual fusion plasmas, the beam distribution is seldom monoenergetic. For a beam with energy distribution $f_1(E_1)$, the fusion-product energy distribution function is given

FIG. 1. Physical picture of the proton spectral measurement. The ^3He ions gain perpendicular energy at the "resonance layer", where the ^3He cyclotron frequency equals the wave frequency. As the ions gain perpendicular energy, their pitch angle changes until, at high energy, most of the ^3He ions execute banana orbits with turning points near the resonance layer. The fast ^3He ions produce 15-MeV protons in beam-target $d(^3\text{He}, p)\alpha$ fusion reactions,⁷ some of which are detected by a proton spectrometer⁸ mounted on the bottom of the vacuum vessel ($R = 132$ cm).

en by the weighted average of monoenergetic distribution functions $\hat{F}(E_3, E_1)$:

$$F(E_3) = \int dE_1 \sigma(E_1) E_1 f_1(E_1) \hat{F}(E_3, E_1), \quad (3)$$

where σ is the fusion cross section. In many cases, the weighting factor $\sigma E_1 f_1$ peaks strongly for some energy $E_1 = E_{\text{peak}}$ and can be treated approximately as a δ function. When this is the case, the fusion-product energy distribution function is given approximately by the monoenergetic expressions, Eqs. (1) and (2).

As an illustration of the application of Eqs. (1) and (2), consider the 15-MeV protons produced in $d(^3\text{He}, p)\alpha$ fusion reactions during ICRF heating in the PLT tokamak (Fig. 1). The minority ^3He ions are accelerated to high energies by the rf waves⁷ and the high-energy ^3He distribution is expected to be strongly anisotropic in velocity space.⁹ Proton spectra measured with a collimated surface barrier detector¹⁰ are characterized by a broad, twin-lobed structure (Fig. 2). Comparison of the spectra with the distribution produced by an isotropic ^3He distribution [Eq. (1)] or a fully perpendicular ^3He velocity distribution [Eq. (2)] indicates that most ($\approx 90\%$) of the reacting ^3He ions have much larger velocities

FIG. 2. Proton spectrum during ^3He minority ICRF heating (180 kW) in a discharge with $I_p = 520$ kA, $n_e = 2.6 \times 10^{13} \text{ cm}^{-3}$, $B_\phi = 30.5$ kG, and $\bar{n}_{^3\text{He}} \approx 0.5 \times 10^{12} \text{ cm}^{-3}$. The curve is the spectrum produced by an anisotropic perpendicular ^3He beam with maximum energy of 400 keV and temperature of 50 keV. The absolute accuracy of the energy measurement is $\pm 2.5\%$; the resolution of the spectrometer is 0.5 MeV (full linewidth).

perpendicular to the magnetic field than parallel to the field.⁹ Using Eq. (2), the spacing of the peaks in Fig. 2 implies that most of the reactions were produced by ^3He ions with about 300 keV of energy, in agreement with estimates based on the magnitude and time evolution of the proton emission.^{7,9} Measurements of the $d(^3\text{He}, p)\alpha$ reaction rate are much more sensitive to changes in the ^3He energy distribution than are measurements of the spectral width, however.⁹ The principal value of the collimated spectral measurements is the information they provide on the degree of anisotropy of the reacting beam ions.

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