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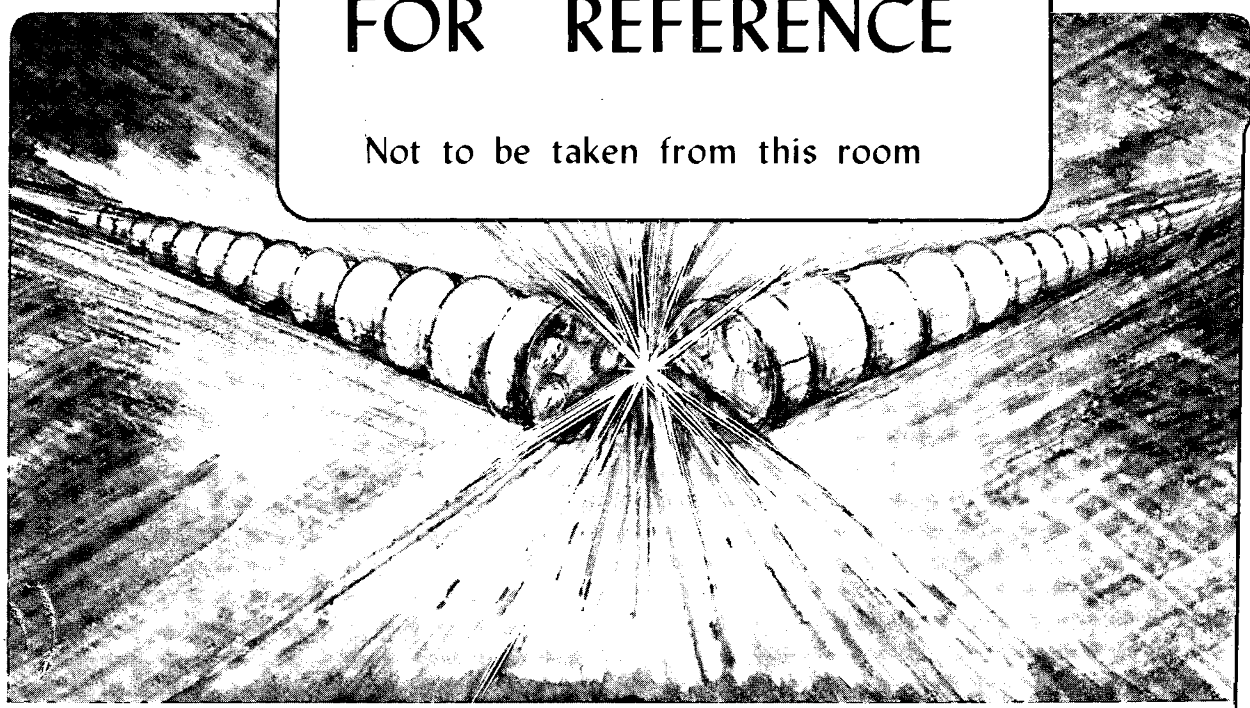

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Nonlinear, relativistic return current sheath for an ion-focused beam

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The plasma equilibrium is analyzed for propagation in the ion-focused regime, of a high-current relativistic electron beam through a radially infinite, collisionless plasma. Beam current variation is assumed adiabatic over an electron plasma period and ion-motion is neglected. The magnetohydrodynamic equations are reduced to a nonlinear ordinary differential equation describing the plasma return current sheath and solutions are characterized by the Budker parameter, with length scale set by the plasma skin depth.

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The transport of high-current, relativistic electron beams in the "ion-focused" regime (IFR)[1,2] is of great interest in accelerator research,[3,4] and has been proposed in a variety of novel forms for beam handling[5,6,7,8,9,10,11] and the generation of coherent radiation.[12,13,14,15] Typically the IFR refers to the limit in which the plasma charge per unit length is less than that of the beam, so that all plasma electrons are ejected radially to large distances. In this case, no plasma electrons remain to interact with the beam. For many novel applications, however, a plasma of large radial extent will exist initially or will be created due to beam and secondary ionization.[16] While the plasma equilibrium for *magnetically self-focussed* propagation has been studied in detail,[17,18,19] no simliar analysis exists for the return current sheath of an *ion-focussed* beam in a broad plasma.

In this work the equilibrium return current sheath of an ion-focussed beam in a broad plasma is described for arbitrary current. Results presented here are the starting point for studies of beam instabilities at high-current in a broad plasma, in particular, the "electron-hose" instability[20,21] and the "ion-channel laser" instability.[13,14] Related applications are dielectrically guided optical modes and IFR wakefield acceleration.

First, a number of simplifying assumptions. Current variation is assumed *adiabatic, i.e.* $\omega_p T_r >> 1$, where T_r is the current rise time, and ω_p is the plasma frequency, $\omega_p{}^2 = 4\pi n_p e^2/m$. The electron mass is m, the electron charge is $-e$, and the initial plasma density is n_p. The beam density $n_b > n_p$, is a function of $\zeta = V_b t - z$, where t is time, and z is axial displacement. The beam axial velocity is $V_b \sim c$, with c the speed of light. In addition, ion-neutralization of the beam will be neglected in the limit $\omega_i T << 1$, where T is the pulse length and ω_i is the ion plasma frequency, $\omega_i^2 = 4\pi n_p e^2/m_i$, with m_i the ion mass. Moreover collisions are neglected in the limit $\nu_e^{-1} >> T$, where ν_e is the collision rate of plasma electrons.[22] Finally to insure effective focussing, the Budker condition[1] is imposed, $n_p >> n_b/\gamma_b^2$, where γ_b is the Lorentz factor for the beam.

With these assumptions, the equilibrium, depicted in Fig. 1, consists of an ensemble of beam electrons streaming axially as they execute incoherent simple harmonic oscillations at the betatron frequency $\omega_\beta \sim \omega_p/(2\gamma_b)^{1/2}$. For definiteness, a step-profile beam density is assumed $\rho_b = -en_b(\zeta)H(a-r)$, where H is the step function and a is the beam radius. Ions are at rest, and plasma electrons have been adiabatically[23] expelled beyond some radius b, characterized by a density and velocity profile yet to be determined. A simple estimate of b is obtained from radial force balance assuming a *stationary* plasma (no axial drift). This gives $b \sim b_0$, where $b_0 = a(n_b/n_p)^{1/2}$ is the "neutralization" radius, *i.e.*, the radius of the cylindrical volume containing ion-charge sufficient to neutralize the beam electrostatic field. Since collisionless plasmas characteristically neutralize magnetic fields on the scale of a skin depth, $k_p^{-1} = c/\omega_p$, it is evident that neglect of the plasma drift is valid only for $k_p b \sim 2\nu^{1/2} << 1$, *i.e.*, $\nu << 1$, where $\nu = I/I_0$ is Budker's parameter, with I the beam current and $I_0 = mc^3/e \sim 17$ kA. For larger ν, the channel is widened due to the "*VxB*" force as will be seen.

The problem is simplified considerably with the observation that the radial drift $c\beta_r$ of plasma electrons is negligible compared to the axial drift $c\beta_z$; this is seen through a simple estimate. Equating the beam current and the total plasma current contained within a skin-depth of the channel wall gives $\beta_z \sim O(V^2)$, for $V<<1$, and $\beta_z \sim O(V)$ for $V \sim 1$, where $V=k_p b$ characterizes magnetic shielding. The radial drift may be estimated by assuming that any increase in beam charge is balanced by plasma electron charge flowing outward through the channel wall. This overestimate gives $\beta_r \approx b/cT_r \approx V/\omega_p T_r$. Thus in the limit of adiabatic current variation, $\omega_p T_r >> 1$, $\beta_r << \beta_z$.

With these estimates the equilibrium potentials and plasma electron flow are calculated using a cold fluid model. It is convenient to define dimensionless quantities, $a_z = eA_z/mc^2$, $\phi = e\varphi/mc^2$, $N_e = -\rho_e/\rho_i$, $N_b = -\rho_b/\rho_i$. Here φ is the scalar potential and A_z is the axial vector potential. The radial vector potential is neglected in the adiabatic approximation. The beam, electron and ion charge densities are ρ_b, ρ_e, and ρ_i. Maxwell's equations simplify with the "frozen-field" approximation, replacing the D'Alembertian operators with the Laplacian in the transverse coordinates. This amounts to the approximation $b/cT_r\gamma << 1$. Maxwell's equations then take the form

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial a_z}{\partial r} \approx k_p^2 (N_b + N_e \beta), \tag{1}$$

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \phi}{\partial r} \approx k_p^2 (N_e + N_b - 1), \tag{2}$$

where $\beta = \beta(r)$ is the axial velocity of the flow, normalized by c. Since $N_e = 0$ for $r<b$, Eqs. (1) and (2) may be integrated directly within the channel,

$$a_z = \begin{cases} \frac{1}{4} k_b^2 r^2 + a_z(0) & ; r < a \\ \frac{1}{4} k_b^2 a^2 (1 + 2 \ln(r/a)) + a_z(0) & ; a < r < b \end{cases}, \tag{3}$$

and $\phi = a_z - \psi$, where $\psi(r) = \psi(0) + (k_p r)^2/4$ is the "pinch" potential, and $k_b^2 = 4\pi n_b e^2/mc^2$. Equations (1), and (2), then need only be solved for $r > b$, subject to continuity at $r \to b^+$, with potentials vanishing for $r \to \infty$.

All that remains is to set down self-consistent equations for N_e and β. Before doing this it is instructive to consider the single-particle plasma electron equations of motion. They are derived from the Hamiltonian normalized by mc^2,

$$h = \sqrt{1 + (p_z + a_z)^2 + p_\perp^2} - \phi, \tag{4}$$

where $\vec{p}_\perp$, and p_z are the transverse and axial canonical momenta normalized by mc. In the frozen-field limit, $\partial/\partial t \approx -c\partial/\partial z$, and one may show from the equations of motion that the quantity $h - p_z$ is conserved along an electron orbit. For an initially cold plasma, this provides an integral of the fluid equations,

$$1 = \gamma(1 - \beta) + a_z - \phi = \left(\frac{1 - \beta}{1 + \beta}\right)^{1/2} + \psi, \tag{5}$$

where $\gamma = (1 - \beta^2)^{-1/2}$. The remaining fluid equations reduce simply to radial force balance (for $r > b$),

$$\frac{\partial\phi}{\partial r} - \beta\frac{\partial a_z}{\partial r} \approx 0 \quad . \tag{6}$$

The magnetohydrodynamic (MHD) Eqs. (1), (2), (5) and (6) completely describe the plasma equilibrium. In terms of the pseudopotential, χ, defined so that $\psi=1-e^{-\chi}$, these MHD equations, reduce for $r \geq b$ to a *single* nonlinear sheath equation

$$\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial\chi}{\partial R} = sinh(\ \chi) \quad , \tag{7}$$

where $R=k_p r$. Boundary values are $\chi \rightarrow 0$, for $R \rightarrow \infty$ and

$$\chi(V\ ) = \frac{1}{2}\ ln\left(2\frac{U^2}{V^2} - 1\right), \tag{8}$$

$$\frac{\partial\chi}{\partial R}(V\ ) = \frac{V}{2}\left(2\frac{U^2}{V^2} - 1\right)^{1/2} \quad , \tag{9}$$

where the current parameter $U = k_b a = 2\nu^{1/2}$, and $V=k_p b$. Other quantities are determined from χ according to $\beta = tanh(\ \chi)$, $\gamma = cosh(\ \chi)$, and

$$N_e = cosh^2(\ \chi) + cosh(\ \chi)\left(\frac{\partial\chi}{\partial R}\right)^2 \quad . \tag{10}$$

The potentials are given by $a_z \sim sinh(\chi)$, $\phi \sim cosh(\chi)$, up to additive terms independent of R (with an adiabatic dependence on ζ). The boundary value for β is $\beta(b)=1-V^2/U^2$.

To understand the form of the solution note that $\chi \to 0$ in the limit $R \to \infty$, while, from Lenz's law, the variation in β and hence χ should be monotone. Thus for $R \to \infty$, $\chi \to 0^+$, $\chi^{(1)} \to 0^-$ and $\chi^{(2)} \to 0^+$. For large R the solution of Eq. (7) will then be a superposition of modified Bessel functions, $K_0(R)$ and $I_0(R)$, with coefficients depending on V and U. The parameter V (and the physical solution) is thus determined from U by the condition that the coefficient of I_0 should vanish. Numerically, this is accomplished by solving Eq. (7), with an iteration over V in the interval $U<V<2^{1/2}U$. Since for a given U, there is a unique solution for V, it is evident that the pinch potential ψ, the axial drift β, and the plasma electron density N_e, are determined only by U (i.e., current) with the radial scale set by the plasma skin depth k_p^{-1}.

As an analytically tractable special case, consider the low-current limit $U<<1$. Approximating $sinh(\chi) \sim \chi$ in Eq. (7) the solution is just $\chi \sim -U^2 K_0(R)/2$ for $R>V$. The boundary conditions reduce to a transcendental equation for the skin depth parameter V, with the approximate solution,

$$V^2 \approx U^2 - \frac{1}{2} U^4 \, ln\left(\frac{Ue^C}{2}\right) + O(U^6) \, , \qquad (11)$$

where $C \sim 0.5772$ is Euler's constant. Other quantities for $R \geq V$ are determined according to $\beta \sim \chi$, $\gamma \sim 1+\chi^2/2$, and

$$N_e \approx 1 + \frac{U^4}{4}\left(K_0(R)^2 + K_1(R)^2\right) . \qquad (12)$$

The potentials are just $a_z \sim \chi$, and $\phi \sim \chi^2/2$, up to additive constants determined by the boundary values from Eq. (3).

Physically, these results show that the small axial drift (β) of the plasma electrons results in a "VxB" force which tends to push electrons outward radially. These electrons then "pile up" near the shifted channel wall (N_e>1). Near $r=b$ the unneutralized ion-charge produces an attractive electrostatic potential ($\phi \sim O(U^4)$). The resulting radial electric field just balances the "VxB" force and maintains the equilibrium. Near the channel wall the magnetic field is approximately the vacuum result, while at large radii, it evanesces on the scale of a skin depth. In this low current limit, return current effects are quite small. For example, at $I \sim 1$ kA, the plasma electron velocity at the channel wall is $\beta(b) \approx \nu \ln(\nu e^{2C}) \sim -0.1$. The density at the channel wall is $N_e(b) \approx 1 + \nu \sim 1.06$ and the channel radius $b/b_0 \approx 1 - \beta/2 \sim 1.05$.

At higher currents the flow becomes relativistic and the linearization errs. As an example the numerical solution for $I \sim 9.4$ kA is depicted in Fig. 2. To depict the dependence of the various quantities on current the results of several numerical solutions have been collated in Fig. 3.

In deriving these results it was assumed that the beam is unperturbed by the return current sheath and this is easily checked. Considering a beam electron at position ζ with energy $mc^2\gamma_b$, it is straightforward to show that γ_b varies according to $d\gamma_b/dz = \partial W/\partial \zeta$, where W is the monopole wake,

$$W = \psi(r = 0) = 1 - \left(2\frac{U^2}{V^2} - 1\right)^{-1/2} - \frac{1}{4}V^2 \quad , \tag{13}$$

and is a function only of current as depicted in Fig. 3. This result reflects the fact that beam electrons at the head must do work to induce a plasma return current; plasma electrons then deposit this energy in the beam tail. As long as $W/\gamma_b \sim \nu/\gamma_b << 1$, the work done by the beam head is only a small fraction of the beam energy. Equation (13) also provides a simple estimate of the maximum IFR wakefield accelerating gradient, $E \sim -mc^2 W k_p / e$, corresponding to a current pulse truncated at the tail. It should also be noted that this longitudinal wake will amplify local maxima in the beam current; however the gain length is typically quite long. For example, for a gaussian current profile $I(\zeta) \propto exp(-\zeta^2 / 2c^2 T_r^2)$, the longitudinal wake produces bunch compression on a length scale,

$$L_B \approx 2\pi c T_r \gamma_b^{1/2} \left[-I\frac{\partial W}{\partial I}\right]^{-1/2} \quad , \tag{14}$$

where the expression in brackets is evaluated at the current peak. Since $L_B/\lambda_\beta \sim O(\gamma_b) x O(\omega_p T_r) >> 1$, compression is a small effect.

To summarize, a quantitative description has been given of the return current sheath surrounding an ion-focussed beam in a broad plasma. At high-current the sheath is nonlinear and relativistic, in marked constrast to the result for magnetically self-focussed propagation.

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FIG. 1. A relativistic electron beam of radius a and charge density ρ_b propagating in steady-state through an underdense plasma, expels plasma electrons from the beam volume and beyond to produce an "ion-channel." The ion charge density ρ_i is uniform, while plasma electrons have been expelled from a cylindrical volume of radius b, as indicated by the plasma electron density ρ_e. From Lenz's law, the beam current J_b, will induce a return current J_e in the plasma.

FIG. 2. The numerical solution for various quantities associated with the sheath, as a function of the radial coordinate $R=k_p r$, for I~9.4 kA (U~1.5). **(a)** The pinch potential ψ, the axial vector potential a_z, and the scalar potential ϕ, all normalized by e/mc^2. (Here b_0/a~6 is assumed in solving for ϕ and a_z.) **(b)** N_e is the plasma electron density normalized to its initial value and β is the axial drift velocity normalized by c. Due to the axial drift depicted in (b), electrons are "held off" the channel wall by the VxB force. This deficit of charge at the wall produces a potential trough as depicted in (a), just adequate to balance the VxB force for the remaining electrons.

FIG. 3. The results of several numerical solutions of Eq. (7) have been collated to depict **(a)** the variation of channel radius $b/b_0=V/U$ and wake potential W with current. Also depicted are **(b)** the plasma electron axial drift velocity and the plasma electron density, each evaluated at the channel edge.

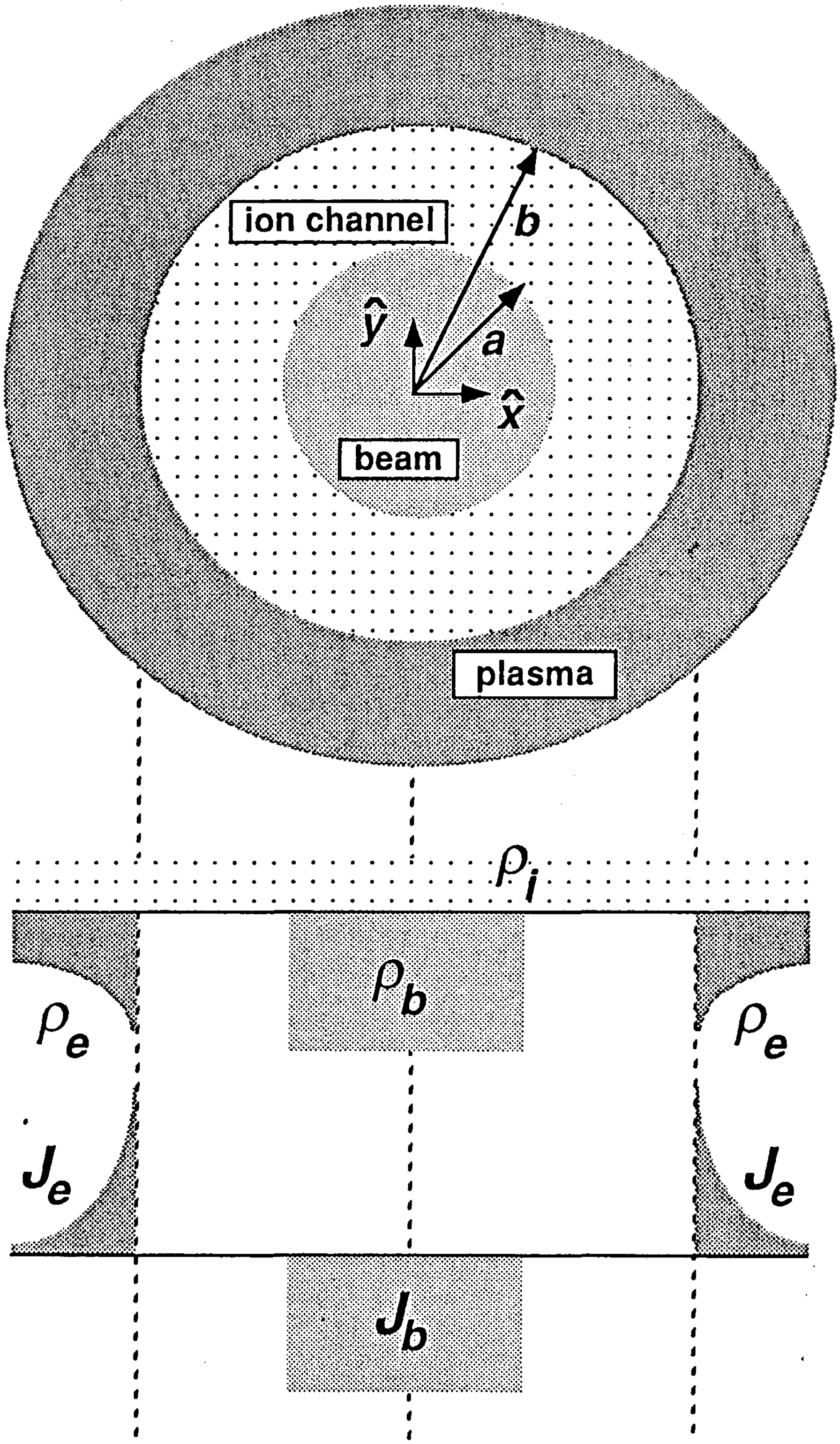

FIG. 1

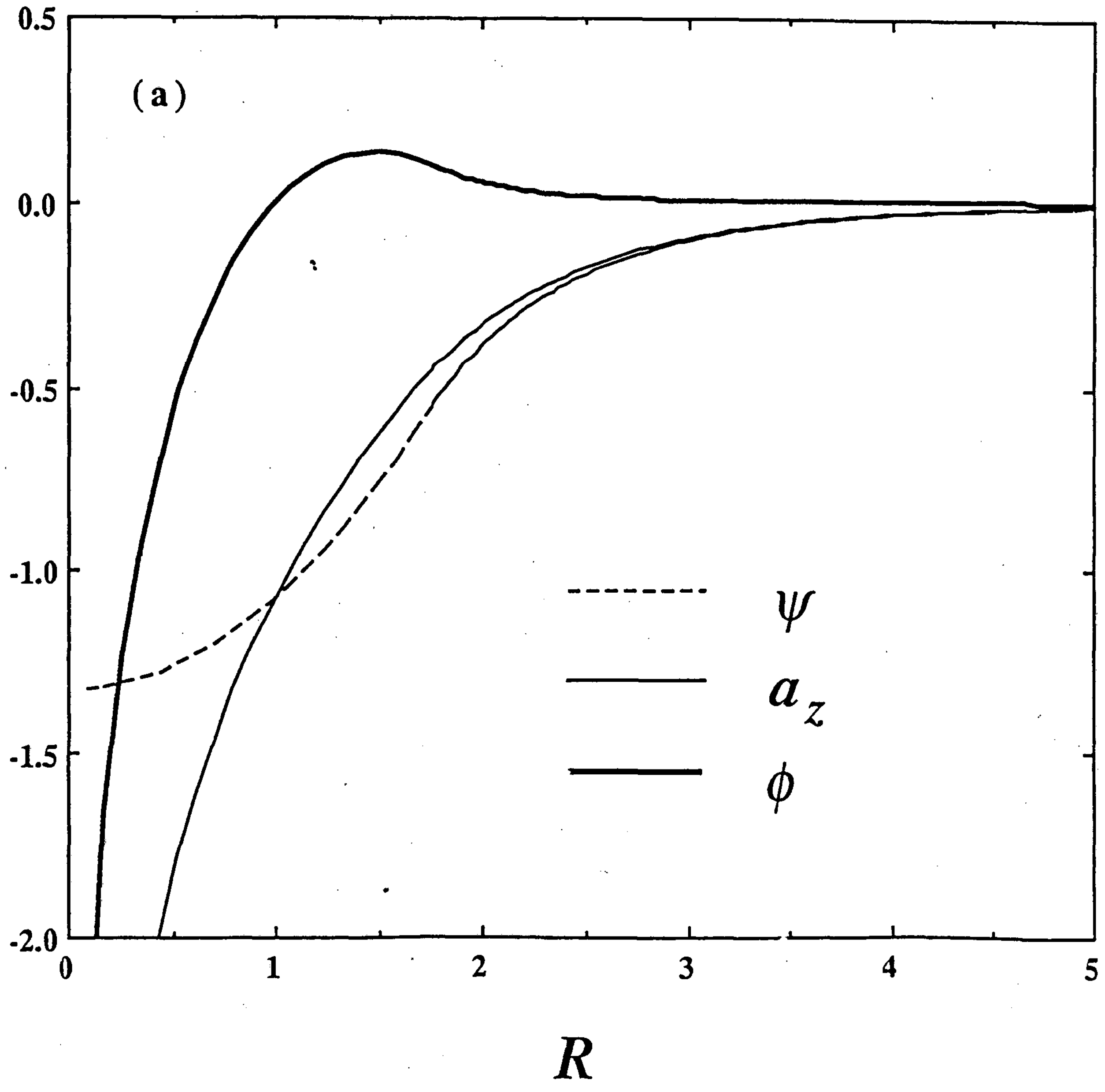

FIG. 2(a)

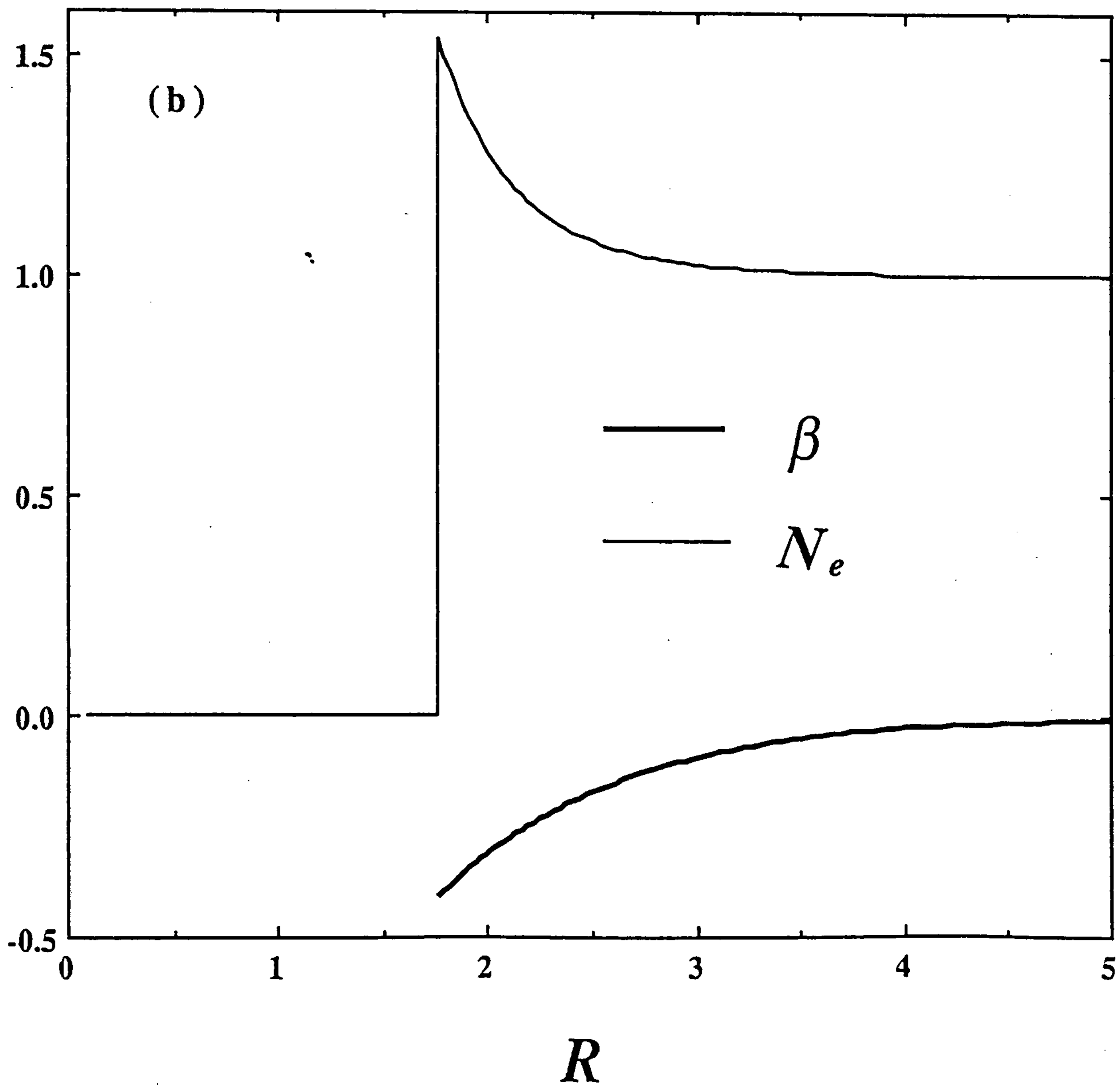

FIG. 2(b)

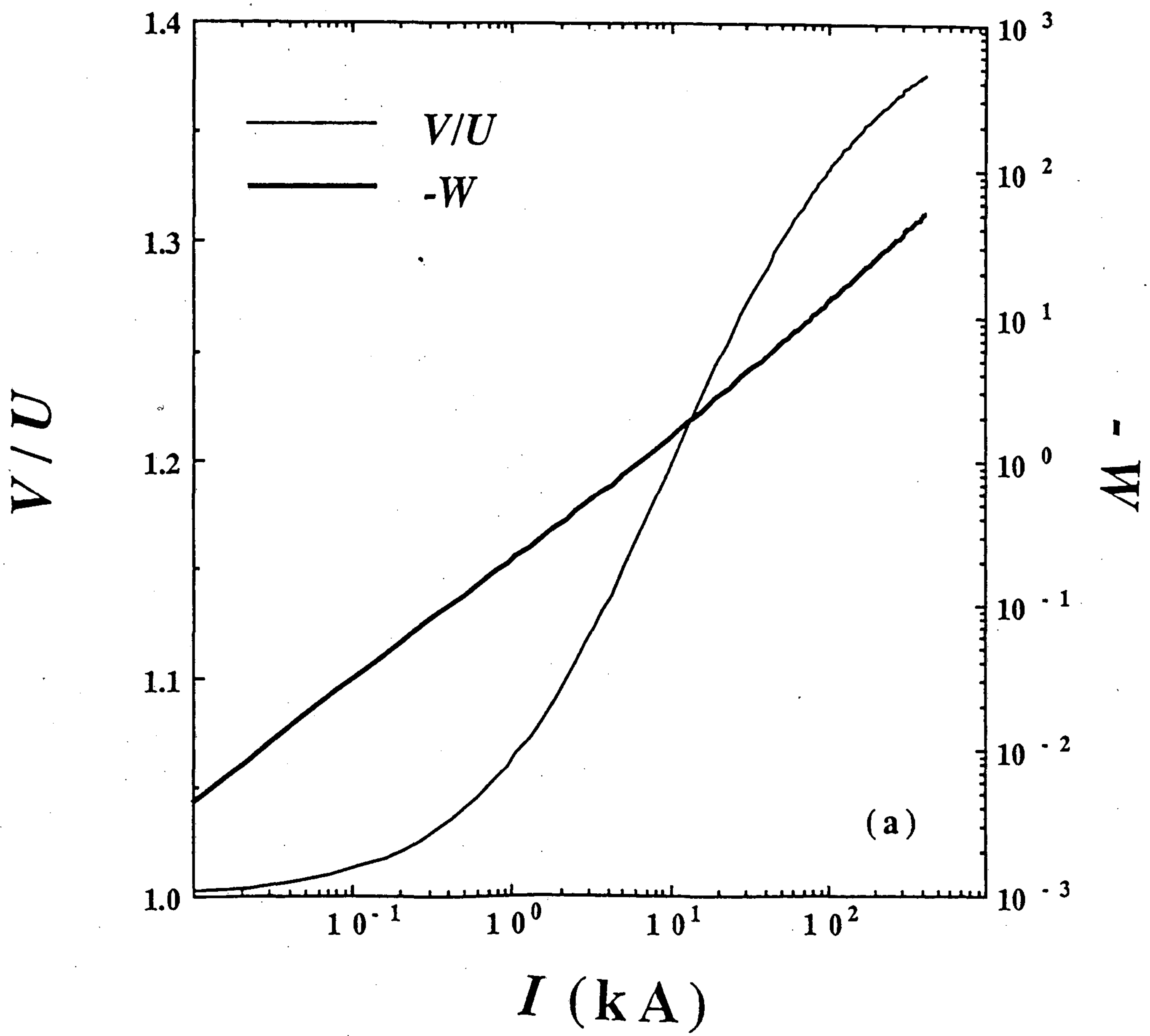
V / U
- W
V/U
-W
(a)
1.4
1.3
1.2
1.1
1.0
10 3
10 2
10 1
10 0
10 -1
10 -2
10 -3
10 -1
10 0
10 1
10 2
I (kA)

FIG. 3(a)

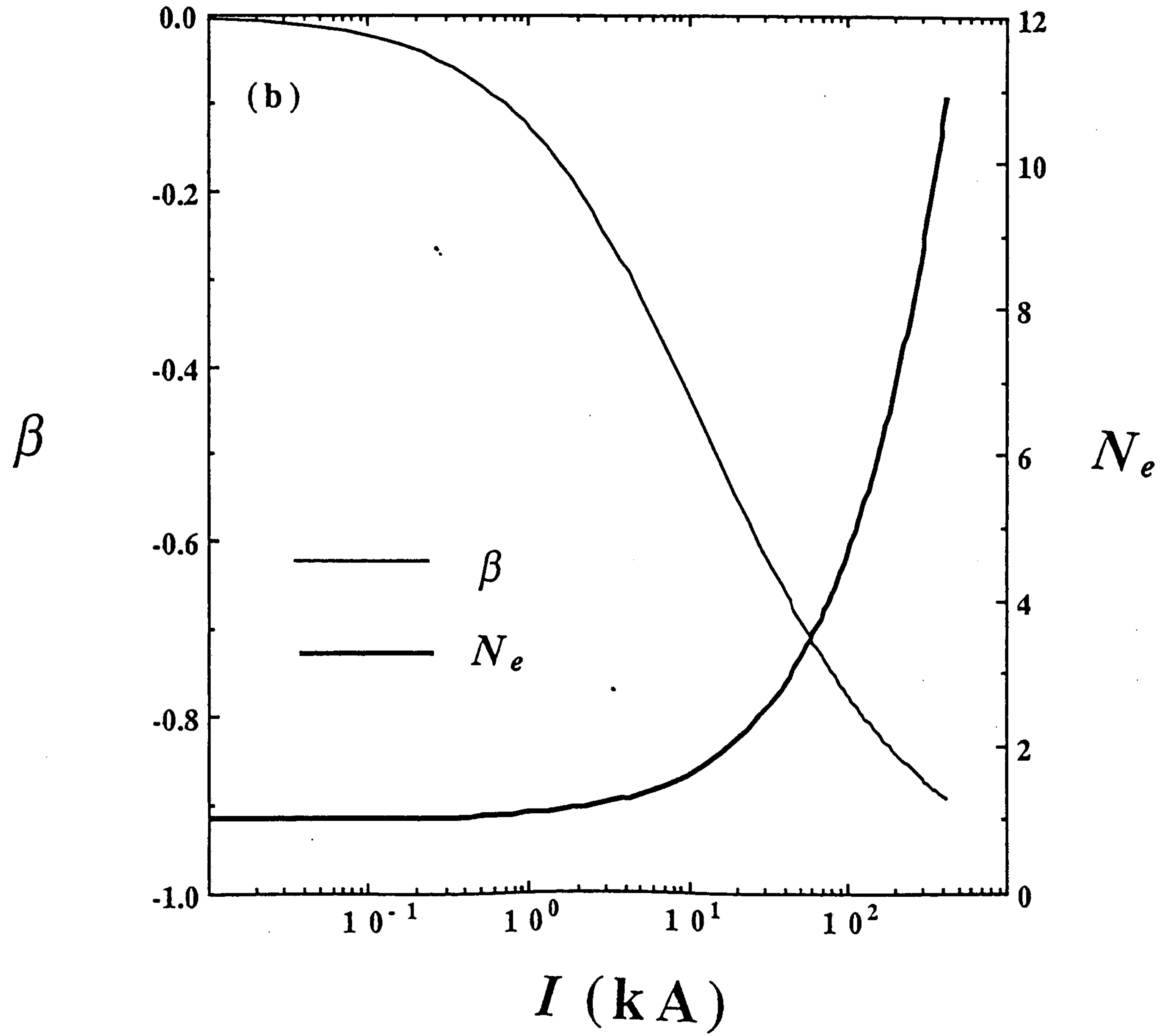

FIG. 3(b)

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