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Point Patterns In Urban Regions: Two Complementary Tests

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POINT PATTERNS IN URBAN REGIONS:
TWO COMPLEMENTARY TESTS

by

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Working Paper 91

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CONTENTS

	<u>Page</u>
Preface	i
Introduction.	1
I. THE SPATIAL STRUCTURE OF POINT LOCATIONS.	3
Clustering Properties of Urban Activities.	3
The Construction of Dendrites	5
Contour Maps of Clusters.	8
Four Illustrative Examples of Point Patterns	9
II. MEASURING CONCENTRATION AND DISPERSAL.	16
A Statistical Definition of the Problem	17
A Confidence Interval for the Stochastically Random Pattern	18
A Note on Further Research	21

PREFACE

This paper is the second working paper of a group that have as their common concern the development of statistical methods for the analysis of the spatial structure of population and employment in urban areas and regions. In it Mr. Angel reports on the development of two complementary statistical tests which allow one to describe the geometry of a scatter of points on a plane in a way that has not been thought of before.

This work is being conducted under the sponsorship of a grant from the Economic Development Administration of the U. S. Department of Commerce. A complete listing of the Working Papers under this grant appears on the back page of this paper. They deal with regional patterns, development, and policies.

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Working Papers on Spatial Pattern Analysis

#86 Andrei Rogers and Norbert G. Gomar, "Statistical Inference in Quadrat Analysis," October, 1968, to be published in Geographical Analysis.

#91 Shlomo Angel, "Point Patterns in Urban Regions: Two Complementary Tests," January 1969.

Andrei Rogers, "Quadrat Analysis of Urban Dispersion," forthcoming.

Andrei Rogers and Juan Martin, "Bivariate Quadrat Analysis of the Dispersion of Employment and Population in Urban Areas," forthcoming.

POINT PATTERNS IN URBAN REGIONS:
TWO COMPLEMENTARY TESTS

Introduction

This analysis is motivated by the observation that a large class of spatially distributed urban activities can be represented as points on a plane for purposes of description, analysis and evaluation. Retail activities, public and private services, specific industrial activities, residences of particular users of an urban system, location of criminal activities, warehouses and street intersections can all be viewed as patterns of points distributed in the two dimensional space identified as the study area. A language for identifying and describing such point patterns does not exist, however, and a method for identifying some overall properties of patterns which could match geometrically distinguishable patterns to intuitively perceived patterns is not readily available. Consequently, our ability to analyze point patterns is seriously limited.

This paper attempts to add to the limited literature on pattern analysis two interrelated tests. One test examines the spatial structure of point locations: the formation of clusters and the spatial interrelations among clusters. A method is described by which we can associate with any point pattern a pattern profile that describes the basic spatial structure of the pattern. The existence of such unique profiles enables us to classify patterns by their profile properties. Consequently, those patterns

which originates from a given profile resemble one another in their intuitively perceived properties, particularly when the number of points in the patterns is relatively large. By the use of profiles, each individual pattern can be associated with a contour map that summarizes the relationships among points in the pattern.

The second test attempts to establish a relation between a pattern of points and the region in which it lies. We attempt to give a more precise definition to the notions of 'Concentration' and 'Dispersal' of a collection of point locations within a given region by constructing a statistical test which discriminates between (i) a point pattern that is distributed randomly throughout the region, (ii) a pattern of point locations that displays a significant tendency to concentrate toward its center of gravity, and (iii) a pattern that displays a tendency to disperse away from its center of gravity.

Both tests are in the first stages of development. Further progress lies in developing more rigorous statistical tools for the first test, possibly by examining the one-dimensional case (point distributed along a line segment), and in testing various classes of patterns to determine the descriptive power of these proposed methods.

I. THE SPATIAL STRUCTURE OF POINT LOCATIONS

In order to obtain a detailed description of the structure of point patterns we proceed to construct a test. This test, which we shall call the Dendrite test, has the inherent advantage of being independent of the area or the shape of the region in which the point locations are embedded, and of the precise number of points in the region.

Clustering Properties of Urban Activities

Before introducing the test we would like to make an observation concerning the specific clustering properties of many urban activities that can be abstracted into point locations. Due to the interaction of transportation and land development in urban areas, clusters of activities tend to be in the form of chains rather than in the form of conglomerations (Figure 1):

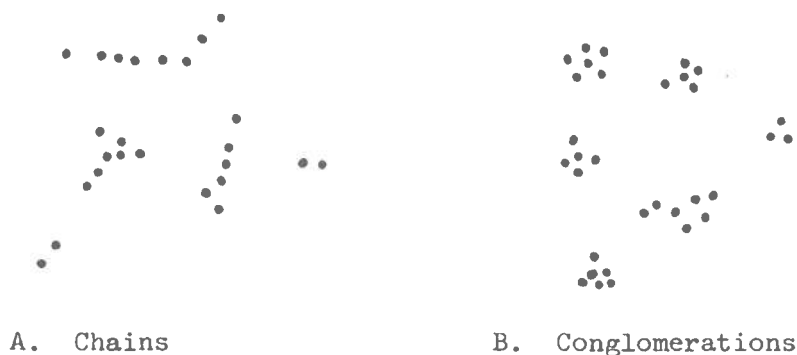


Figure 1: The Clustering of Point Locations

An interesting illustration of this phenomenon is the cluster of automobile dealers along Van Ness Avenue in San Francisco, California. All automobile lots and showrooms are adjacent to each other and face the avenue or streets directly branching off the avenue. There are

almost no automobile dealers one block away from the avenue.

We measure the clusteredness of point locations by measuring the maximal length of the link in a chain of line segments that connects all the points in a given cluster. This measure produces results which are different from those produced by other measures of clusteredness, such as average distance to the center of gravity or maximal distance to all cluster members. We illustrate this with an example (Figure 2):

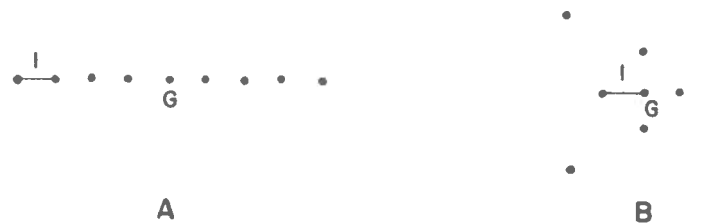


Figure 2: The Comparative Clusteredness of Patterns

- a. Under a measure of clusteredness which measures average distance to the center of gravity G:
 - A: Average distance to the center of gravity G is
 $(0+1+1+2+2+3+3+4+4)/9 = 20/9 = 2.25$ unit distances.
 - B: Average distance to the center of gravity G is
 $(0+4 \cdot 1 + 4 \cdot 2\sqrt{2})/9 = 1.7$ unit distances.
- b. Under a measure of clusteredness which measures maximal distance from center of gravity G:
 - A: Maximal distance from G is 4 unit distances.
 - B: Maximal distance from G is $2\sqrt{2} = 2.82$ unit distances.
- c. Under a measure of clusteredness which measures maximal distance to cluster members:

A: Every point is within 8 unit distances from every other point.

B: Every point is within $4\sqrt{2} = 5.66$ unit distances from every other point.

d. Under a measure of clusteredness which measures the longest link in a chain connecting all points in the cluster:

A: Maximal length of link is 1 unit distance.

B: Maximal length of link is $\sqrt{5} = 2.24$ unit distances.

Under the first three measures we would conclude that pattern B is more tightly clustered than pattern A. However, under the fourth measure we would conclude that pattern A is more tightly clustered than pattern B. This measure corresponds to our previous observation on the clustering of urban activities, and shall therefore be used in the following analysis.

The Construction of Dendrites

Given a collection of n pairs of coordinates of points in the plane, we can construct a dendrite, which can be uniquely identified with this collection, as follows:

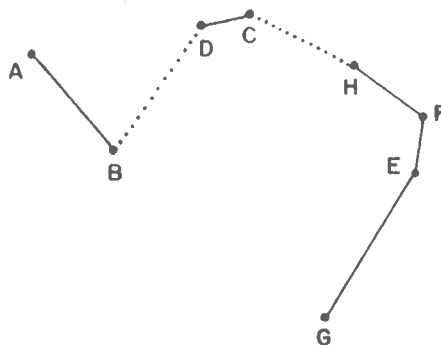


Figure 3: The Construction of a Dendrite

Start from any point, say A in the above figure, and connect it with a straight line segment to its nearest neighbor. A connects to B. Next connect B to its nearest neighbor, which in this particular case is A. Proceed to any other point, say C, and connect it to its nearest neighbor, D. Connect D to its nearest neighbor, C. Proceed in this fashion to connect E to F, F to G to E and H to F. After all points have been connected to their nearest neighbors, take any cluster, say CD, and connect it by a straight line segment to its nearest cluster, i.e., connect CD to that point outside the cluster which is closer to some point in the cluster than any other point outside the cluster. In our case connect H to C. In a similar fashion connect D to B.

The resulting graph is called a Dendrite. It is a unique chain of straight line segments which connect all points in the pattern, and which is of minimal total length in cases where links in the space outside the points are not allowed.¹ The dendrite is also the solution to the problem of connecting a collection of towns on a flat terrain with railroad tracks, such that the total length of track is minimized and junctures can occur only inside towns (Figure 4).

¹For an elementary exposition of dendrites see H. Steinhaus, Mathematical Snapshots, Oxford University Press, New York, 1960. The author mentions the application of dendrite structures in Botany and Linguistics.

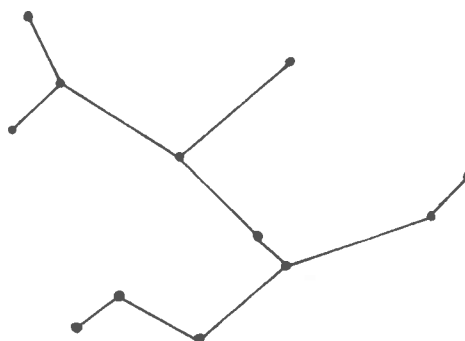


Figure 4: A Minimal Rail System Connecting 12 Towns
With Junctions Occurring Only Within Towns

A dendrite constructed on a given pattern of point locations can be considered unique although in case a point is equidistant from two or more other points, or in case a pattern is equidistant from two other patterns there can be two or more distinct dendrites (Figure 5).



Figure 5: Symmetric Dendrites

We shall call these dendrites symmetric dendrites and treat symmetric dendrites as identical. This is possible since our test is only concerned with the distribution of link lengths of a dendrite constructed on a given pattern, and symmetric dendrites have identical distribution of link lengths.

The Construction of Pattern Profiles

The first test proceeds as follows. Given a collection of n point locations in the plane, construct a dendrite through these points.² Next, measure the length of links of the dendrite and graph them in increasing order, such that the distance between each link is $\frac{1}{n-1}$ (in some unit of measurement), and the length of the longest link is unity (in the same units of measurement). (Figure 6):

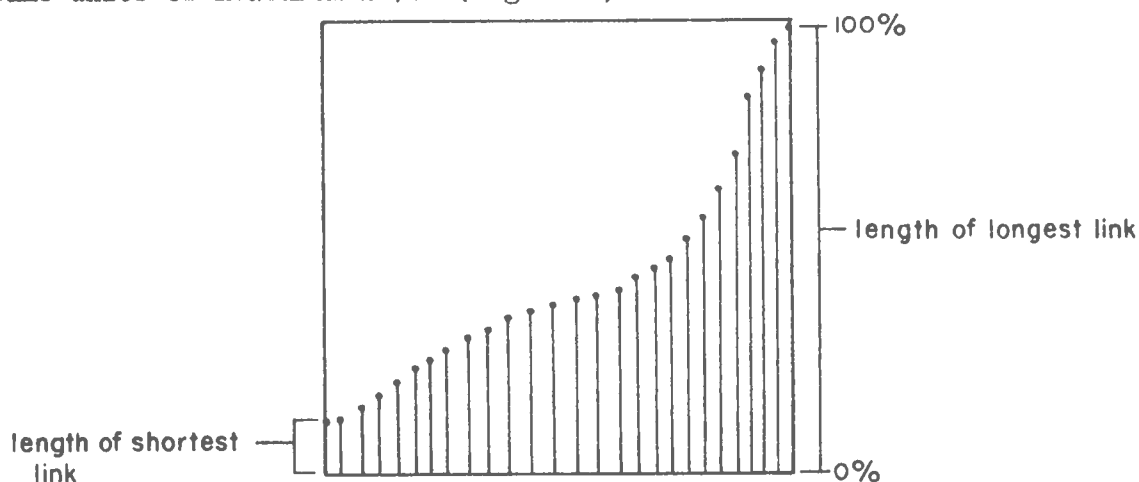


Figure 6: The Construction of Pattern Profiles

The envelope of the points thus identified provides a profile which characterizes the general clustering properties of the point pattern. Examples of different point patterns and their associated pattern profiles are given below.

Contour Maps of Clusters

On a point pattern map construct contour lines around all clusters

²Although the graphic construction described in Figure 4 is simple, I have not been able to find an algorithm for the construction of dendrites on large sets of points using a computer with common storage size.

for which the maximal length of link is 10%, 20%, 30%, 50%, etc., of the length of the longest in the dendrite. These clusters can be easily identified by examining the pattern profile and dividing the vertical axis into percentage points. The resulting contour map provides a description of the spatial structure of the point pattern. It describes what points form clusters of various degrees of compactness, and how clusters relate to one another in space.

The profile, on the other hand, provides us with a tool for comparing different point patterns using different scales of measurements. For example, we can compare the clusteredness of n stores within a Central Business District with the clusteredness of m small towns in Iowa, say, as long as a common unit of measurement is used in the construction of the profile, i.e., the length of the longest link in the dendrite should always be unity in some common system of measurement.

Four Illustrative Examples of Point Patterns

We now present, in Figures 7-10, four examples of point patterns, their related dendrites, pattern profiles, and contour maps. In Figure 11 the four pattern profiles are displayed together for purposes of comparison. The patterns are only representative of very large classes of point patterns, and we are still unable to classify all possible patterns in a meaningful way, i.e., in a way which discriminates between patterns geometrically in correspondence with their perceived variations. Preliminary examination of pattern profiles for various individual patterns, however, reveals significant difference in profiles of patterns which are perceived differently on an intuitive basis, and we therefore hope

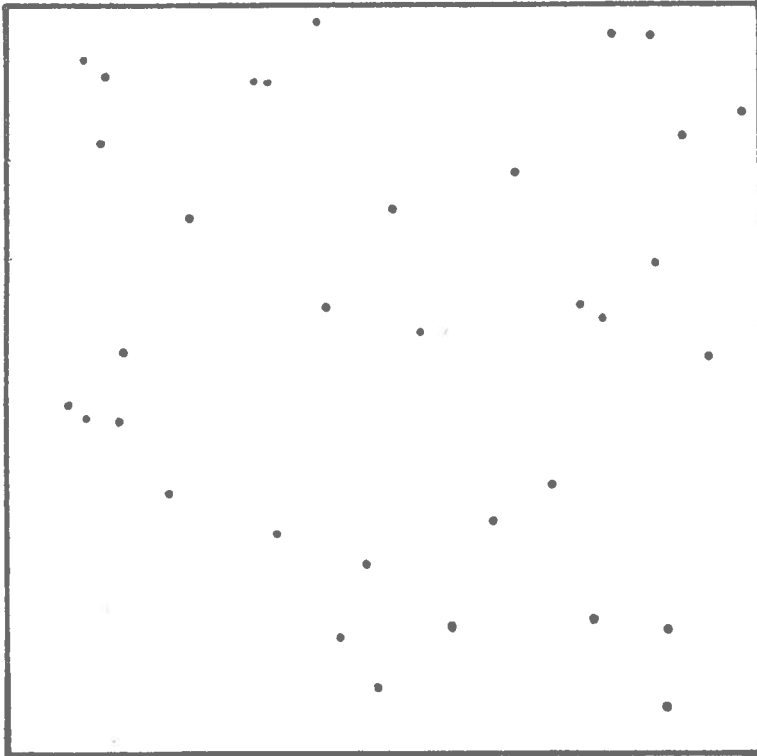


FIG. 7A
N PAIRS of COORDINATES of POINTS

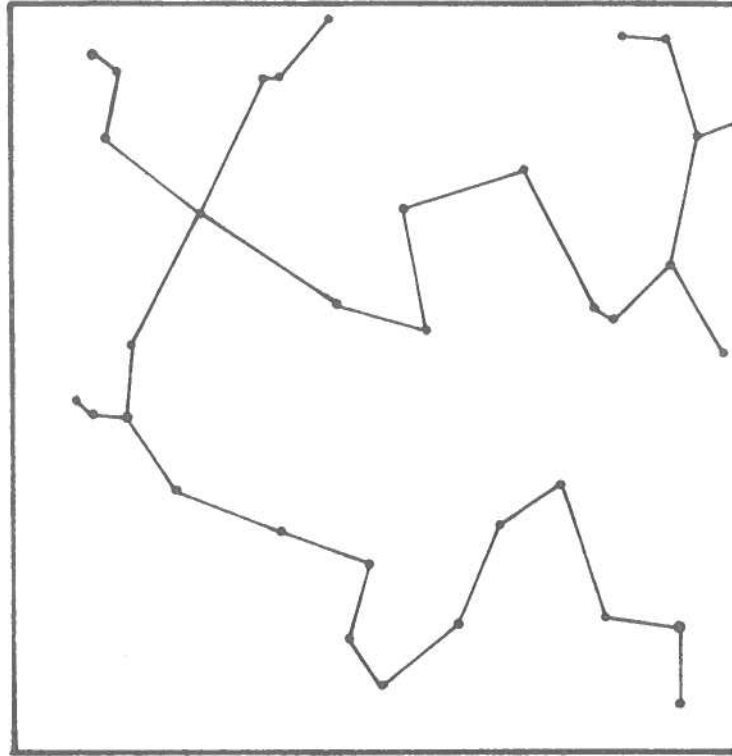


FIG. 7B
DENDRITE

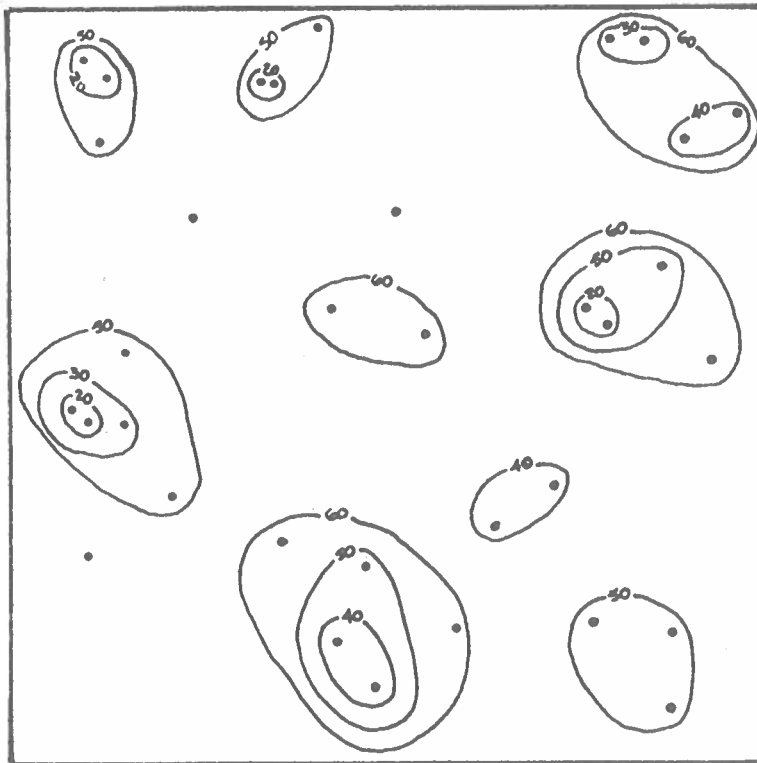


FIG. 7C
CONTOUR MAP of CLUSTERS

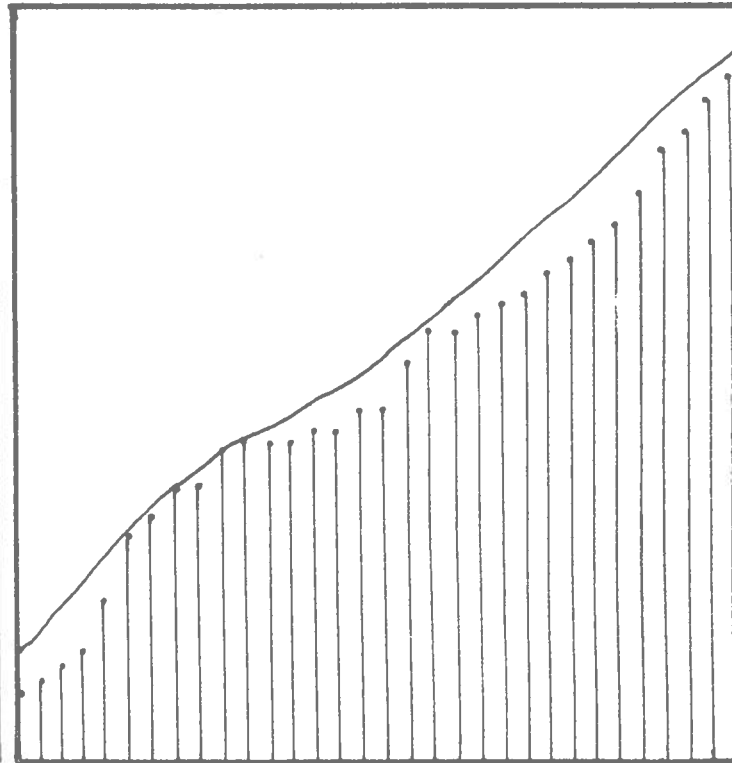


FIG. 7D
PATTERN PROFILE

FIGURE 7 : A "Random" Pattern

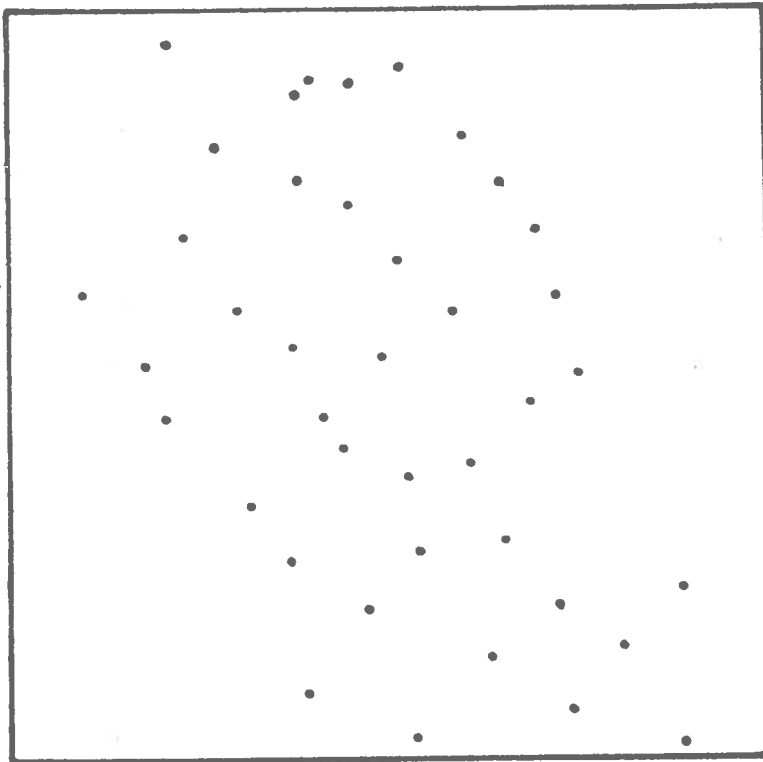


FIG. 8A
N PAIR of COORDINATES of POINTS

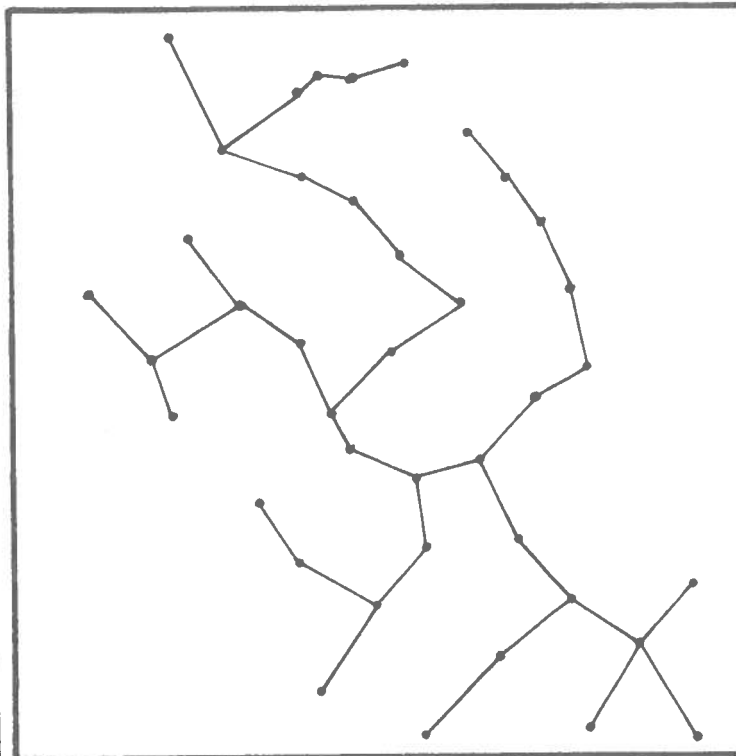


FIG. 8B
DENDRITE

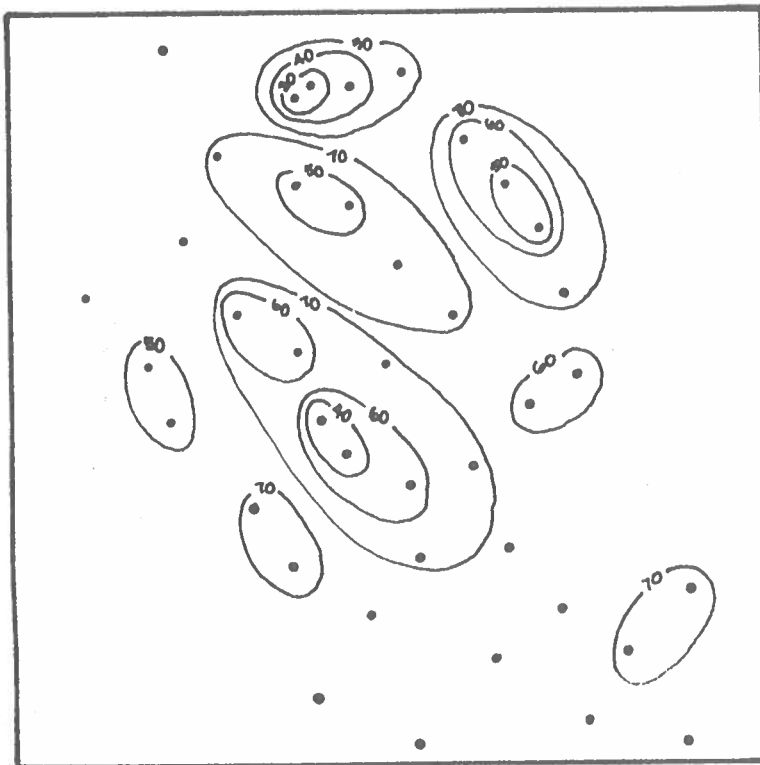


FIG. 8C
CONTOUR MAP of CLUSTERS

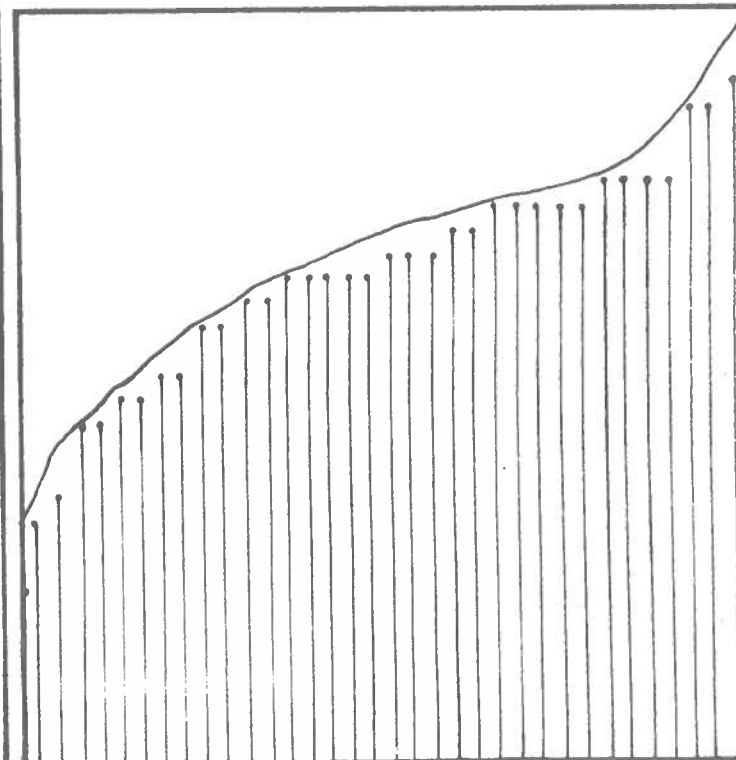


FIG. 8D
PATTERN PROFILE

FIGURE 8: A Semi-Regular Pattern

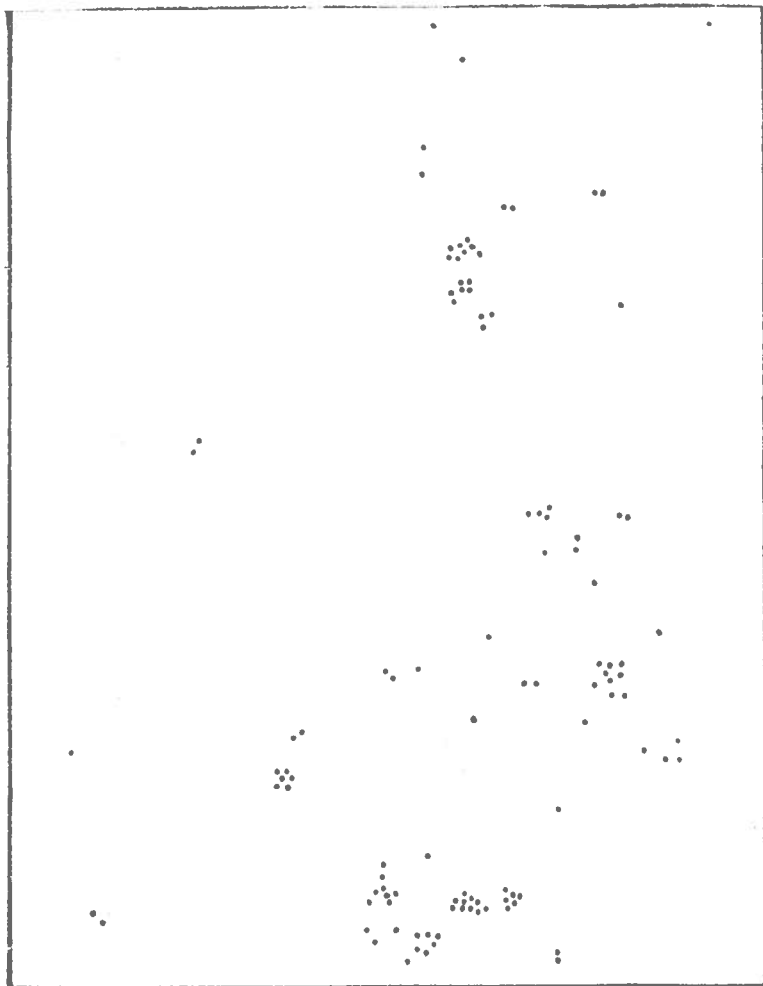


FIG. 9A N PAIRS of COORDINATE of POINTS

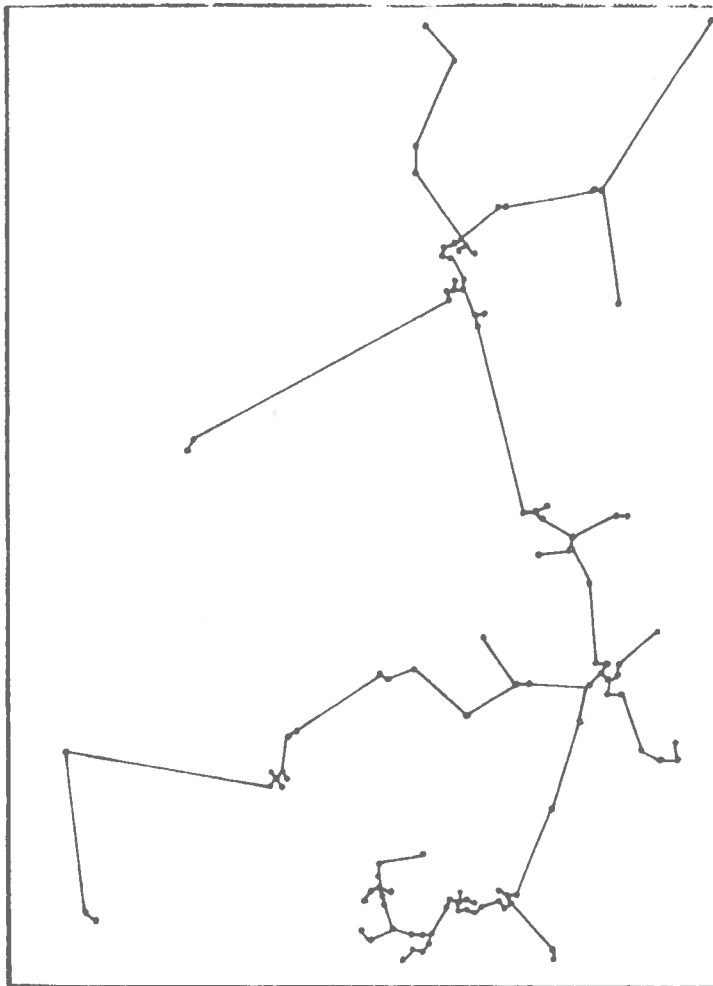


FIG. 9B DENDRITE

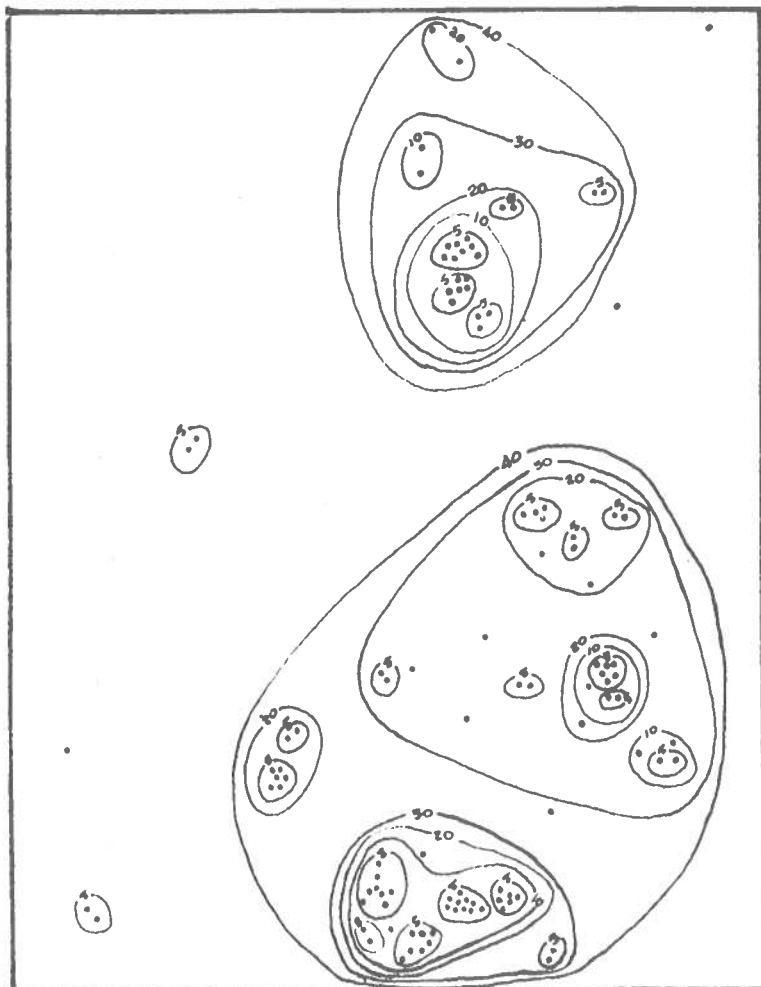


FIG. 9C CONTOUR MAP of CLUSTERS

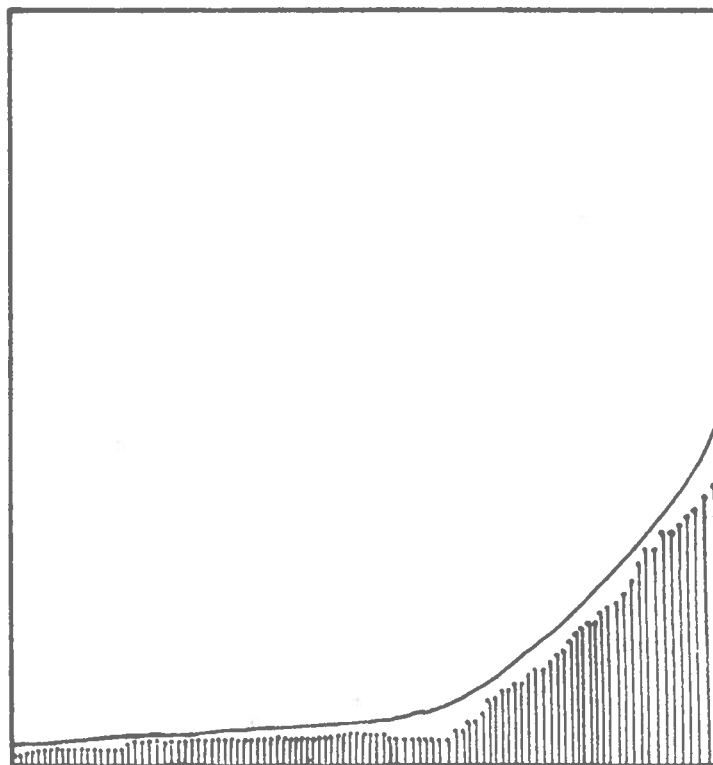


FIG. 9D PATTERN PROFILE

FIGURE 9: A Random Distribution of Small Clusters

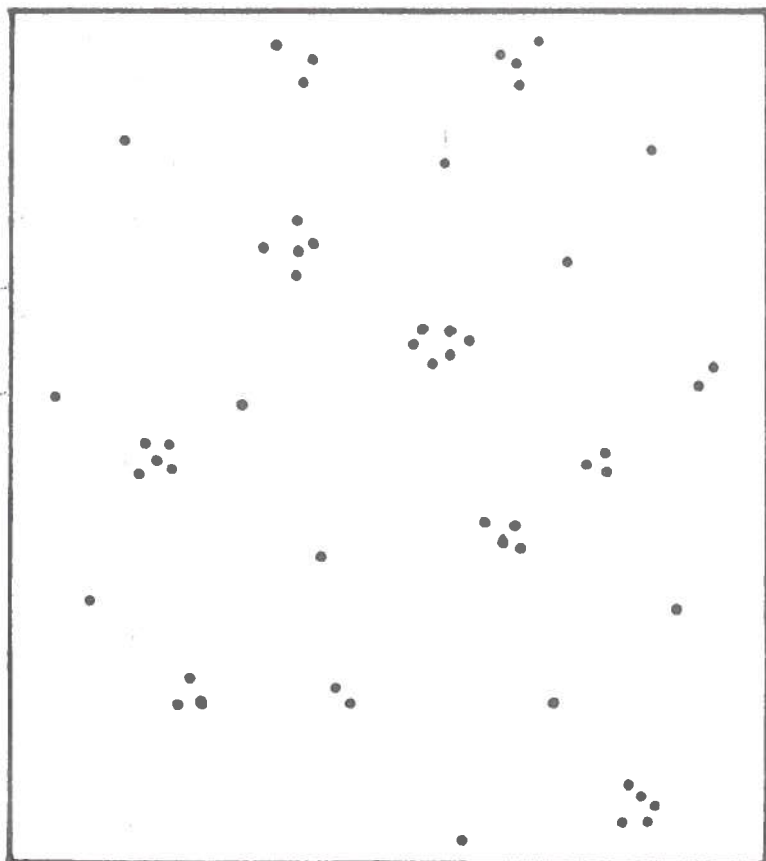


FIG. 10A

N PAIRS of COORDINATES of POINTS

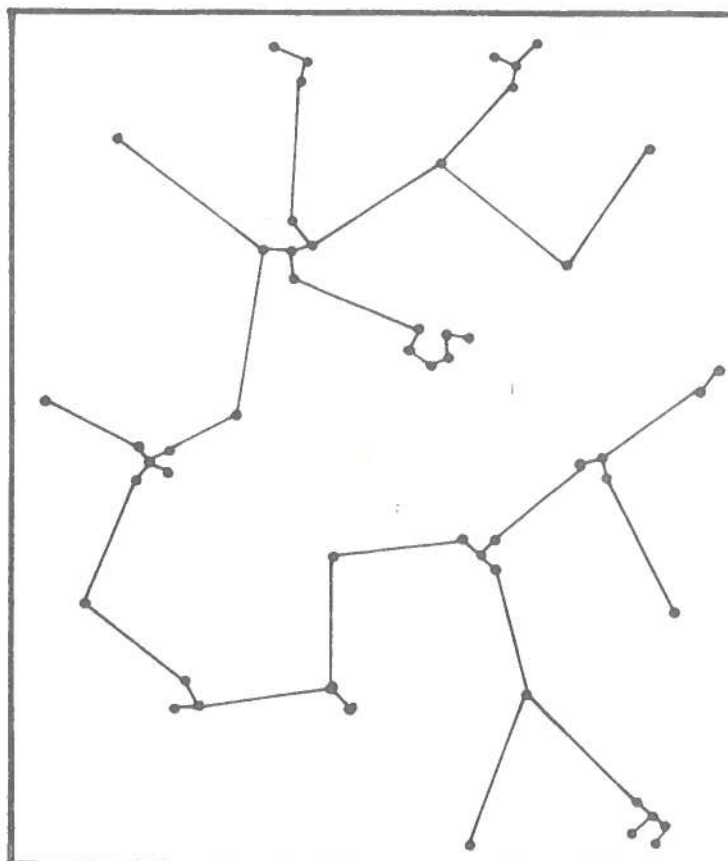


FIG. 10B

DENDRITE

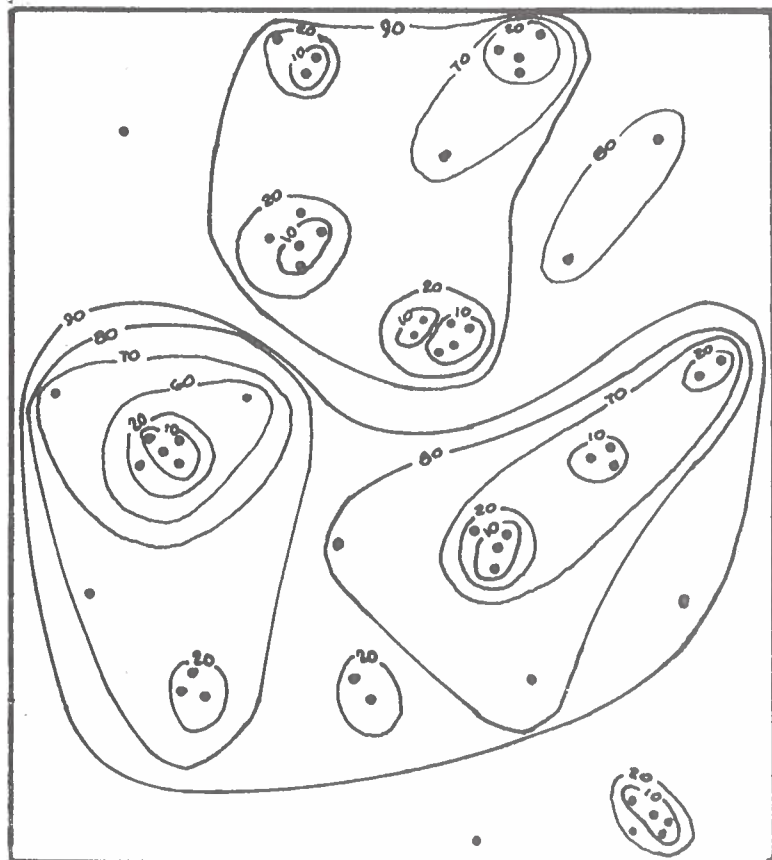


FIG. 10C

CONTOUR MAP of CLUSTERS

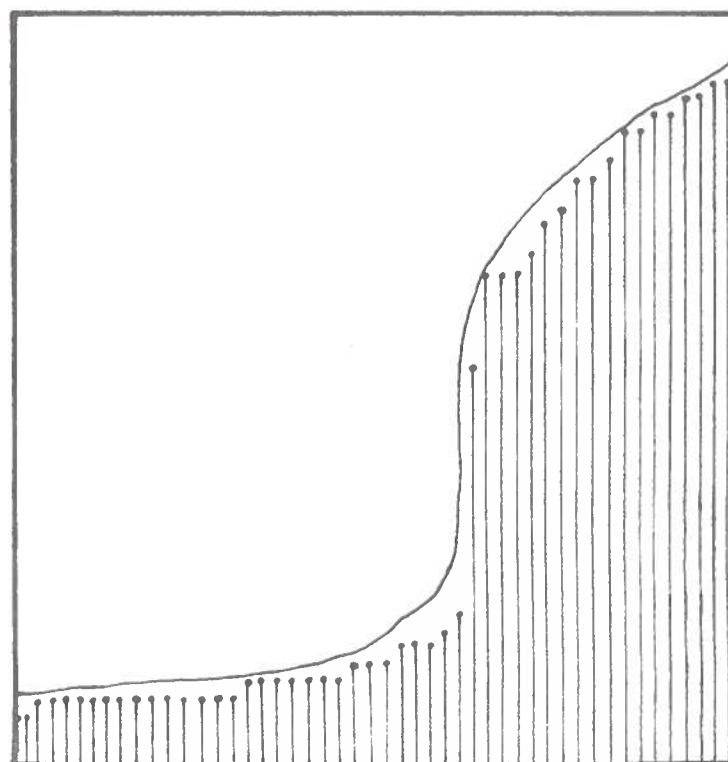
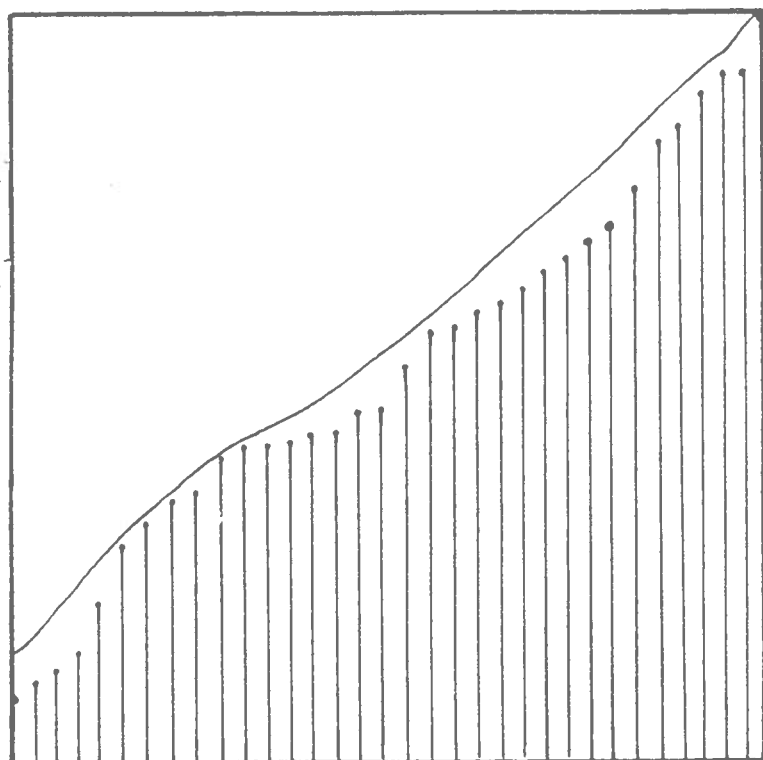


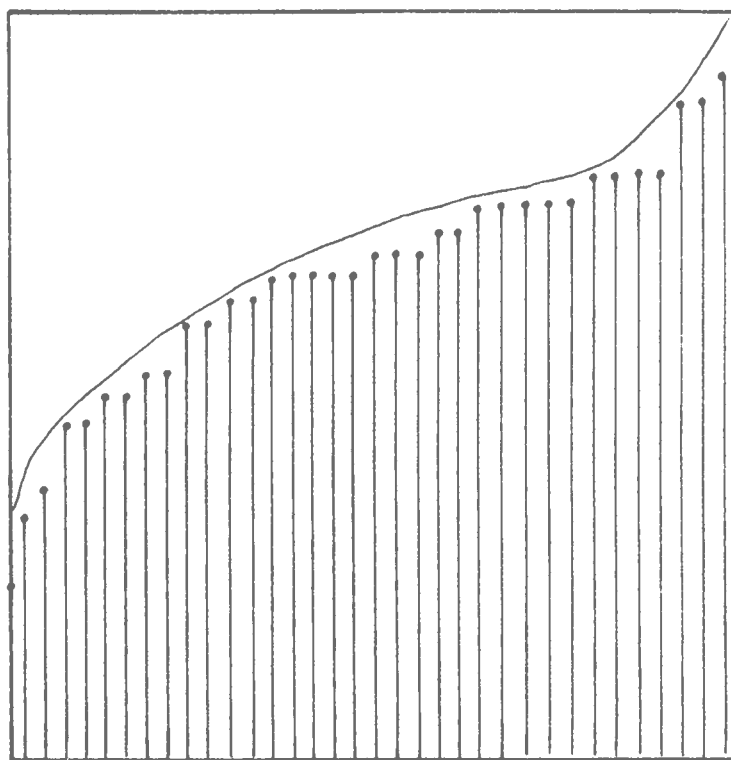
FIG. 10D

PATTERN PROFILE

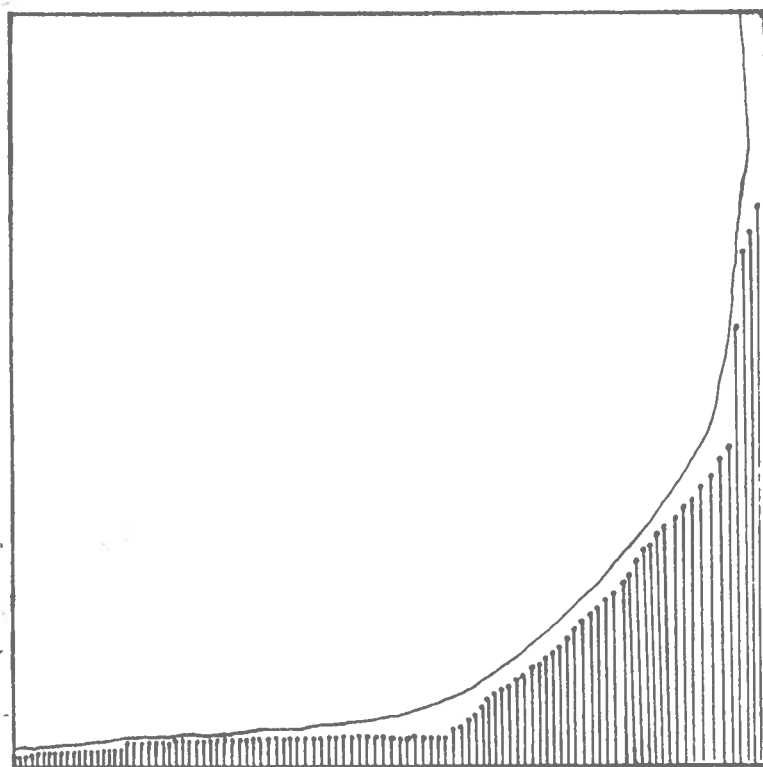
FIGURE 10: Semi-Regular Distribution of Small Clusters



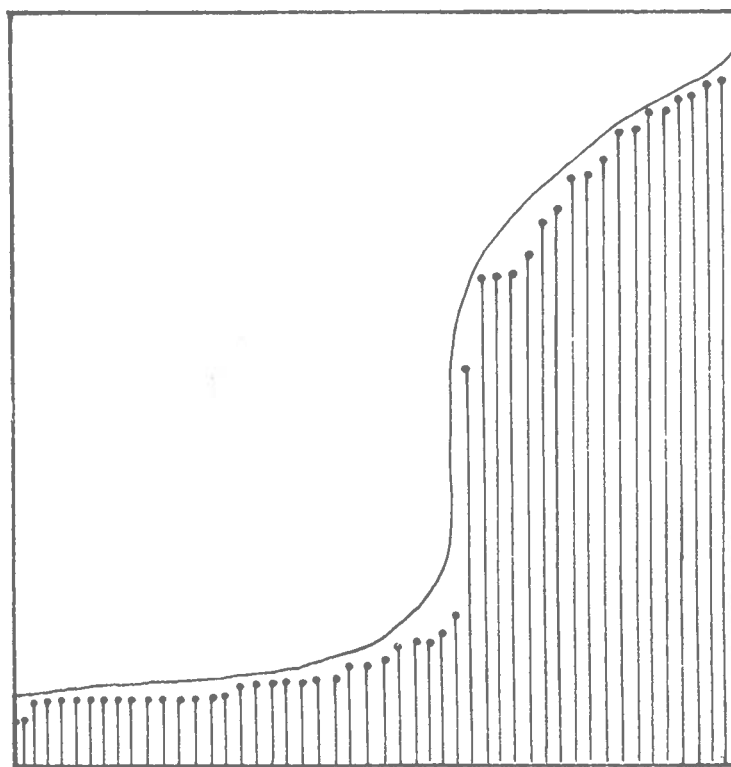
"RANDOM"



SEMI-REGULAR



"RANDOM" DISTRIBUTION of SMALL CLUSTERS



SEMI-REGULAR DISTRIBUTION of SMALL CLUSTERS

FIGURE II: Pattern Profiles

that such profiles ultimately can be used to discriminate between classes of patterns.

The "random" pattern profile (upper left in Figure 11) is distinguished by the observation in that it tends to exhibit a fairly regularly increasing slope.³ There are no significant bends or sharp rises. This can possibly be explained rigorously by statistically analyzing the distribution of intervals in a random distribution of points on a line segment. This analysis has not yet been attempted and provides an interesting problem for further research.

The "semi-regular" pattern (upper right in Figure 11) is characterized by its convexity and by the observation that most link lengths are of a similar size, and are not significantly shorter than the maximal link length.

The "random" distribution of small clusters (lower left in Figure 11) is characterized by the significant concavity of the pattern profile. It can also be interpreted, in this case, as a combination of semi-regular clusters distributed randomly in the plane. The shallow segment of the profile then corresponds to the semi-regular pattern profile, and the uniformly increasing segment to the "random" pattern profile. We must note here, however, that the "random" distribution of small clusters presented here was not generated by a computer. It has been obtained empirically from the examination of the distribution of insurance firms in Oakland, California. The observation that it

³The distribution of points was generated by a computer using a Monte Carlo technique.

might correspond to a "random" distribution of small clusters was arrived at a posteriori through the examination of the similarities between the profiles.

The semi-regular distribution of small clusters (lower right) can be characterized by the steep rise that divides the profile into two parts. This profile can also be seen as a combination: semi-regular clusters distributed semi-regularly in space. In contrast with the semi-regular pattern above, most distances are either small or large.

Another property of pattern profiles concerns the number of link lengths which are within a given percentage (10%, 20%, etc.) of the maximal link length. This may be a misleading property in case the pattern analyzed happens to contain some points which are relatively far away from the majority of points. Meaningful analysis of this property, therefore, calls for another measure, or another method for the construction of profiles that is not so dependent on individual point locations. This is another interesting problem for further research. One should note, however, that the most important properties of the profile are its slopes and bends, and not its position in the square, which is heavily dependent on the length of the longest link.

In conclusion, we see that this method provides us with a means for identifying patterns of observed distributions of point locations in urban regions. Patterns can be classified according to their pattern properties, and each individual pattern can be associated with a contour map that summarizes the spatial relationships of points, small clusters and large clusters within the region. Relatively high peaks in the landscape correspond to a high degree of clustering, while a shallow landscape corresponds to relative regularity.

II. MEASURING CONCENTRATION AND DISPERSAL

The second test is designed to discriminate between patterns that are likely to have been generated by a "random" mechanism and those that were generated by a process which leads to concentration or dispersal within the region.

We can intuitively identify concentrated and dispersed patterns visually as, for example, in Figure 12 below.

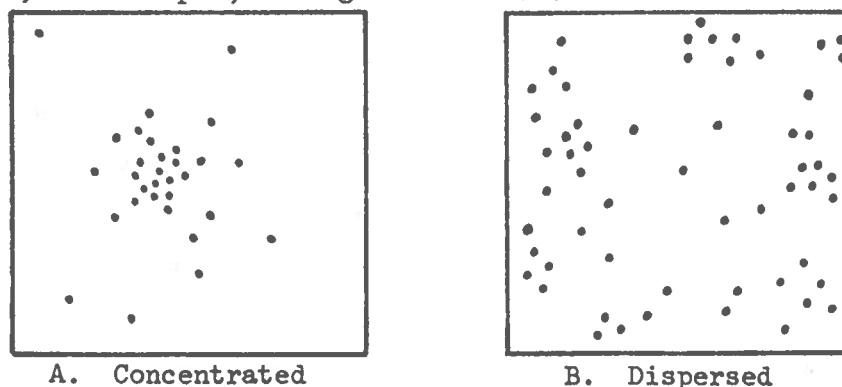


Figure 12: Concentration And Dispersal Within A Region

We are using the terms "Concentrated" and "Dispersed" to stress that we are dealing here with the location of points within a region with a given finite area. Thus, for example, the dispersed pattern in the above figure would be classified as a concentrated one if the entire page were the study region.

A measure of concentration or dispersal cannot discriminate between different spatial structures of points such as, for example, the tendency to form small clusters or the tendency of clusters to gravitate toward one another (or to repel each other). Hence, we shall only concern ourselves with the aggregate point structure within the region.

A. Statistical Definition of the Problem

Suppose that n particles are distributed in a stochastically random process over a rectangular region of area AB and coordinates $(0,0)$, $(A,0)$, (A,B) and $(0,B)$, such that the probability density function of X_i (the X coordinate of the i 'th point) is:

$$(1) \quad f(X_i) = \begin{cases} \frac{1}{A} & \text{on } [0,A] \\ 0 & \text{elsewhere} \end{cases} \quad i = 1, 2, \dots, n$$

and, similarly, the p.d.f. of Y_i is:

$$(2) \quad f(Y_i) = \begin{cases} \frac{1}{B} & \text{on } [0,B] \\ 0 & \text{elsewhere} \end{cases} \quad i = 1, 2, \dots, n$$

Assume that X_i , Y_i are independent.

We are given an observation of n particles with coordinates (x_1, y_1) , $(x_2, y_2), \dots, (x_n, y_n)$, respectively, where the (x_i, y_i) are in the rectangle AB . We would like to construct a confidence interval for testing whether it is likely that our observed pattern is the result of a random process.

First, we construct a random variable T which is the sum of squares of distances from the observed points to their center of gravity $(\bar{x}, \bar{y})$.

$$(3) \quad T = \left[(x_1 - \bar{x})^2 + (y_1 - \bar{y})^2 \right] + \left[(x_2 - \bar{x})^2 + (y_2 - \bar{y})^2 \right] + \dots \\ + \left[(x_n - \bar{x})^2 + (y_n - \bar{y})^2 \right],$$

where

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad \text{and} \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i.$$

Intuitively, we note that when the observed value of T is much smaller than its expected value under the above distribution, the pattern is likely to have been the result of a process which tends to produce concentration. Conversely, when the observed value of T is large, we suspect that the pattern is the result of a process which tends to produce dispersal.

A Confidence Interval For The Stochastically Random Pattern

We now establish that when n is large, a $(1-\alpha)$ confidence interval for T is:

$$\left(\frac{n(A^2 + B^2)}{12} + u_{\frac{\alpha}{2}} \sqrt{\frac{n(A^4 + B^4)}{180}}, \frac{n(A^2 + B^2)}{12} + u_{1-\frac{\alpha}{2}} \sqrt{\frac{n(A^4 + B^4)}{180}} \right),$$

where the u 's are the $(1-\alpha)$ confidence intervals of the Normal Distribution.

Proof:

$$(4) \quad T = \sum_{i=1}^n (x_i - \bar{x})^2 + \sum_{i=1}^n (y_i - \bar{y})^2 = nS_1^2 + nS_2^2,$$

where S_1^2 is the sample variance of $x_1, x_2, \dots, x_n$, and S_2^2 is the sample variance of $y_1, y_2, \dots, y_n$.

The expectation and variance of these sample variances can be obtained directly from the first four moments of the x_i 's and y_i 's. To develop the expressions for $E(S_i^2)$, $\text{Var}(S_i^2)$ we follow the discussion in Kendall.⁴

⁴ See Maurice George Kendall and Alan Stuart, The Advanced Theory of Statistics, Vol. 1, 5th edition, C. Griffin, London, 1958.

$$(5) \quad \mu_1' = \int_0^A x_i \cdot f(x_i) dx_i = \int_0^A x_i \frac{1}{A} dx_i = \frac{A}{2}$$

$$(6) \quad \mu_2' = \int_0^A x_i^2 f(x_i) dx_i = \frac{A^2}{3}$$

$$(7) \quad \mu_3' = \int_0^A x_i^3 f(x_i) dx_i = \frac{A^3}{4}$$

$$(8) \quad \mu_4' = \int_0^A x_i^4 f(x_i) dx_i = \frac{A^4}{5}$$

Following Kendall, we obtain expressions for μ_1 , μ_2 , μ_3 , and μ_4 .⁵

$$(9) \quad \mu_1 = \mu_1' = \frac{A}{2} [= E(x_i)]$$

$$(10) \quad \mu_2 = \mu_2' - \mu_1'^2 = \frac{A^2}{3} - \frac{A^2}{4} = \frac{A^2}{12} [= \text{Var}(x_i)]$$

$$(11) \quad \mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3 = \frac{A}{4} - 3 \cdot \frac{A}{2} \cdot \frac{A^2}{3} + 2 \frac{A^3}{8} = 0$$

$$(12) \quad \mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4 = \frac{A^4}{80}$$

We now can obtain expressions for $E(S_1^2)$ and $\text{Var}(S_1^2)$:

$$(13) \quad E(S_1^2) = \frac{n-1}{n} \mu_2 = \frac{(n-1)A^2}{12n}$$

$$(14) \quad \text{Var}(S_1^2) = \frac{(n-1)^2}{n^3} \mu_4 - \frac{(n-1)(n-3)}{n^3} \mu_2^2,$$

and for a large n the above expressions reduce to:

$$(15) \quad E(S_1^2) = \frac{A^2}{12}$$

$$(16) \quad \text{Var}(S_1^2) = \frac{\mu_4 - \mu_2^2}{n} = \frac{1}{n} \left(\frac{A^4}{80} - \frac{A^4}{144} \right) = \frac{A^4}{180n}$$

⁵See Kendall, op. cit., pp. 56 and 277.

Similar expressions are obtained in the same manner for $E(S_2^2)$ and $\text{Var}(S_2^2)$. For a large n , T as the sum of two independent and approximately normally distributed random variables is itself approximately normally distributed with:

$$(17) \quad \begin{aligned} E(T) &= E(nS_1^2 + nS_2^2) = nE(S_1^2) + nE(S_2^2) \\ &= \frac{nA^2}{12} + \frac{nB^2}{12} = \frac{n(A^2 + B^2)}{12}, \end{aligned}$$

and

$$(18) \quad \begin{aligned} \text{Var}(T) &= \text{Var}(nS_1^2 + nS_2^2) = n^2 \text{var}S_1^2 + n^2 \text{var}S_2^2 \\ &= \frac{nA^4}{180} + \frac{nB^4}{180} = \frac{n}{180} (A^4 + B^4). \end{aligned}$$

This completes the proof.

As a special case, consider the distribution of n restaurants in San Francisco, California, taken to be a square region of area A^2 . Given n and A we can compute a 95% confidence interval for T from the above result:

$$(19) \quad \left(\frac{nA^2}{6} - 1.96 \sqrt{\frac{nA^4}{90}}, \frac{nA^2}{6} + 1.96 \sqrt{\frac{nA^4}{90}} \right).$$

We can also compute the observed value of T , given the coordinates (x_i, y_i) of the n restaurants:

$$(20) \quad \hat{T} = (x_1 - \bar{x})^2 + (y_1 - \bar{y})^2 + \dots + (x_n - \bar{x})^2 + (y_n - \bar{y})^2$$

$$\text{where } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad \text{and} \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i.$$

If $\hat{T}$ falls within the confidence interval we cannot reject our original hypothesis that the overall locational concentration or dispersal of restaurants in San Francisco is the result of a random process. However, if the $\hat{T}$ falls to the left of our interval, it would be reasonable to argue that restaurants in San Francisco tend to concentrate and gravitate toward their center of gravity $(\bar{x}, \bar{y})$. If $\hat{T}$ falls to the right of the interval we conclude that they tend to disperse away from their center of gravity $(\bar{x}, \bar{y})$.

A Note On Further Research

One can see why the two tests we have presented in this paper are complementary: one measures the position of points within a point pattern, and the other measures the position of the pattern within a given region. The discriminatory power of these tests has not yet been determined conclusively and awaits operational development of programs for simulating and testing different point patterns. Moreover, this preliminary excursion into the realms of point patterns also has raised the following other interesting problems: (i) What is the relationship of travel behavior to the formations of chains of urban activities? (ii) Is it possible to define statistical forms for the distribution of link lengths on one dimensional, and possibly two dimensional, spaces? (iii) Is it possible to construct a computer algorithm for the construction of dendrites, pattern profiles and contour maps of point locations? (iv) How do the results obtained in these tests correspond to results obtained by other pattern analysis methods? These and other related questions are currently under study.

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