Abstract

Edward Chamberlin conjectured that the number of trades in realistic trading systems is likely to exceed that predicted by competitive equilibrium theory. He supported this conjecture by data from a large number of classroom experiments and with a plausible argument based on a numerical example. This paper states and proves a theorem that supports and illuminates Chamberlin's intuition, supplies examples of trading processes that lead to excess trading, and presents some additional experimental evidence.

Experimental Markets and Chamberlin's Excess Trading Conjecture

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Edward Chamberlin [2] claimed that in real world trading systems where there is no recontracting, the number of transactions is likely to exceed the number predicted by competitive equilibrium theory. Chamberlin conducted a series of classroom market experiments in which he induced market demand and supply by assigning each student a role either as a demander with a specified buyer value or a supplier with a specified seller cost. Chamberlin reports that in forty-six classroom market experiments, the number of trades exceeded the competitive equilibrium quantity forty-two times and was never smaller.

Chamberlin showed a numerical example in which partially informed traders make some trades at non-equilibrium prices and where the number of trades is greater than that found in competitive equilibrium. On the basis of this example and his experimental results, Chamberlin stated that:

> "The conclusion seems unavoidable that 'price fluctuations render the volume of sales normally greater than the equilibrium amount which is indicated by the supply and demand curves.'"

Chamberlin does not offer a formal proof of his assertion, nor does he spell out precise conditions under which the assertion is true. This paper supplies a theorem that confirms Chamberlin's intuition. Stated informally, our theorem asserts that if traders are sufficiently aggressive to find mutually profitable trade possibilities so long as they exist, then the number of trades must be at least as large as the competitive equilibrium quantity.

Let us define a *simple trading economy* to be one in which there are suppliers and demanders. Each supplier i has a specified seller cost c_i at which she can supply at most one unit. Each demander j has a specified buyer value v_j. If supplier i sells a unit to demander j at price p, then supplier i has a profit of $p - c_i$ and demander j has a profit of $v_j - p$.

Let us define a *trading outcome* as a list of all buyer-seller pairs who trade and the price at which each pair trades. We say that a *trading outcome is exhaustive* if no demander who did not trade and no supplier who did not trade in this outcome could trade with each other at a price which is profitable for both.

Theorem 1 *A competitive equilibrium is an exhaustive trading outcome. If a simple trading economy has a unique competitive equilibrium quantity, the number of trades in any exhaustive trading outcome is at least as large as the number of trades in competitive equilibrium.*

On the way to proving Theorem 1, we establish the following lemma.

Lemma 1 *If competitive equilibrium has a unique trading quantity, then it must be that in competitive equilibrium the buyer value of every buyer who trades exceeds the seller cost of every seller who trades.*[1]

Proof of Lemma 1:

Consider a competitive equilibrium with price p. Since traders have the option of not trading, no buyer or seller makes a negative profit. Therefore every demander who trades has a buyer value at least as high as p and every supplier who trades has a seller cost no higher than p. It follows that if supplier i and buyer j trade in competitive equilibrium then $b_j \geq p \geq v_i$. Suppose that $b_j = p = c_i$. Then, since both i and j make zero profit in equilibrium, there exists another competitive equilibrium with the same price p but with a smaller number of trades. In this alternative equilibrium i and j do not trade and all other traders who traded in the original competitive equilibrium continue to trade. Therefore if the competitive equilibrium

[1]Notice that Lemma 1 does not state that if competitive equilibrium quantity is unique, all traders must make positive profits. It might be that some traders on one side of the market make zero profits, but if so then all traders on the other side of the market make positive profits.

quantity is unique, and supplier i and demander j both trade in competitive equilibrium, it must be that $b_i > v_j$. QED

With Lemma 1 in hand, the proof of Theorem 1 is easy.

Proof of Theorem 1:

First we show that a competitive equilibrium outcome is exhaustive. Let p be a competitive equilibrium price. In a simple trading economy, supplier i with will trade if $p > c_i$ and will not trade if $p < c_i$. Demander j will trade if $p < v_j$ and will not trade if $p > v_j$. If demander i does not trade in competitive equilibrium, then $v_i \leq p$, where v_i. Therefore if neither buyer i nor seller j makes a trade in competitive equilibrium, it must be that $b_i \leq c_j$. But this implies that i and j could not trade together at a price that is profitable for both. It follows that competitive equilibrium is an exhaustive trading outcome.

Consider a trading outcome with fewer trades than there are in competitive equilibrium. It must be that there is at least one demander i and at least one supplier j who does not trade in this trading outcome, but does trade in competitive equilibrium. From Lemma 1 it follows $b_i > c_j$. But if this is the case, the proposed trading outcome is not exhaustive. QED

Theorem 1 tells us that if traders understand their buyer values and seller costs and are sufficiently aggressive to find mutually profitable trade possibilities so long as such remain, then the number of trades will be at least as large as the competitive equilibrium quantity.

Exhaustive and Voluntary Trading Outcomes

It is not remarkable that some trading outcomes have more trades than the competitive outcome. Additional trades can always be forced if one party is coerced to trade. The interesting thing that Chamberlin observed is that there are trading outcomes where the number of trades is greater than the trading outcome and where all traders make non-negative profits.

Let us define a trade to be *voluntary* if both profits of both the buyer and the seller are non-negative. In a simple trading economy, competitive equilibrium is voluntary, since traders choose to trade only if they make a non-negative profit. The following remark shows given a very weak restriction, a simple trading economy will have voluntary trading outcomes with more trade than a competitive equilibrium.

Remark 1 *In a simple trading economy, there will exist a voluntary trading outcome with more trades than in competitive equilibrium unless either (i) the seller cost of all suppliers who do not trade exceeds the highest buyer value or (ii) the buyer value of all demanders who do not trade is smaller than the lowest seller cost.*

Proof of Remark 1 Suppose that Conditions (i) and (ii) of the remark are not satisfied and consider a competitive equilibrium at price p. Then there is a supplier i whose seller cost is less than the buyer value of some demander j^* who does not trade in competitive equilibrium, and there is a demander j whose buyer value is greater than the seller cost of some supplier i^* who does not trade. Consider the trading outcome in which all traders other than supplier i and demander j continue to trade at the price p, but supplier i trades with demander j^* at a price between c_i and b_{j^*} and demander j trades with demander i^* at a price between c_{i^*} and b_j. This trading outcome is voluntary and there is one more trade in this trading outcome than in competitive equilibrium. QED

An Example

To illustrate these ideas, it is helpful to consider an example. Here we consider a distribution of buyer values and seller costs taken from the experiments studied by Bergstrom and Kwok [1]. The market has 16 "low-cost" suppliers, each of whom can supply one unit at a cost of $10 and 8 "high-cost" suppliers, each of whom can supply one unit at a cost of $30. There are also 16 "low-value" demanders, each of whom has a buyer value of $20 for a single unit and 8 "high-value" demanders, each of whom has a buyer value of $40. The demand and supply curves for this economy are shown in Figure 1.

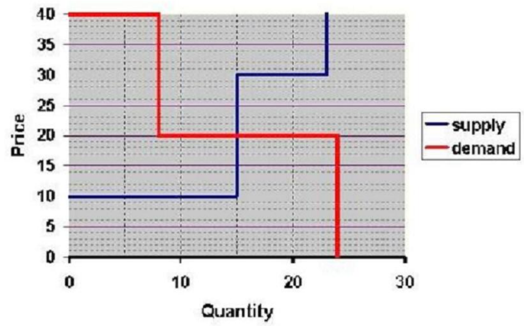

Figure 1: Supply and Demand in Session 1

We see from Figure 1 that there is a unique competitive equilibrium at the competitive price $20 and the competitive quantity 16 units.

Suppose that a matchmaker controlled the pairings for this trading economy and only allowed the 8 high-value demanders to meet the 8 high-cost sellers and the 16 low-value demanders to meet the 16 low-cost sellers. The high-value, high-cost pairings could make mutually profitable trades at a price of $35 and the low-value, low-cost pairings could make mutually profitable trades at a price of $15. This trading outcome is voluntary, since every trader makes a positive profit, and the total number of trades is 24 as compared to only 16 trades in competitive equilibrium.

Another interesting trading process with a voluntary and exhaustive trading outcome is the following. In the first round of trading, demanders and suppliers are randomly paired. If the buyer value of the demander exceeds the seller cost of the supplier with whom he is paired, they transact at a price intermediate between the buyer value and seller cost. Those pairs who can not make a trade join a pool of unmatched individuals. This procedure is iterated on the pool of unmatched individuals until there are no two unmatched individuals who can make a mutually profitable exchange.

Let us apply this process to the Bergstrom-Kwok example described above. In the first round, all low-cost suppliers will trade with whoever they meet, since a seller cost of \$10 is lower than the buyer value of any demander. Likewise all high-value demanders will trade in the first round, since a buyer value of \$30 exceeds the seller cost of every supplier. Therefore the only traders left after the first round are high-cost suppliers and low-value demanders, who can not make a mutually profitable trade. Thus the trading process ends after a single round. The number of trades that result is a random variable, which is determined by the random matching of suppliers and demanders in the first round. The total number of trades must equal the total number of suppliers who trade. The 16 low-cost suppliers will certainly trade. Of the 8 high-cost suppliers, those who meet high-value demanders will trade. The number of high-cost suppliers who trade is therefore a random variable $\mathbf{X}$ with a hypergeometric distribution determined by the number of high-value buyers who are drawn in 8 draws, with replacement, from a population of 8 high-value and 16 low-value demanders. The number of trades will exceed the competitive number whenever at least one high-cost supplier is paired with a high-value demander. The probability that this happens is greater than 0.98.[2]

[2]The maximum likelihood outcome is 19 trades, which happens with a probability of about 1/3.

Extension to More General Economies

In a simple trading economy, as we have defined it, each trader can buy or sell at most one unit. The result of Theorem 1 extends, using essentially the same proof, to economies of the following kind.

Suppliers and demanders both seek to maximize profits as follows. For each supplier i there is a cost function $c_i(n)$ such that her profits if she sells n units and receives a total payment of $\$P$ are $P - c_i(n)$. For each buyer j there is a value function $b_j(n)$ such that his profits if he pays a total of $\$P$ for n units are $b_j(n) - P$.

This class of economies encompasses economies where suppliers satisfy the standard economic theory of the firm. The assumption about demanders is satisfied in a market where consumers seek to maximize quasi-linear utilities, but does not apply to commodities for which there are "income effects" on demand.

Empirical Confirmation

Bergstrom and Kwok [1] examined the results in 31 separate classrooms in which, two rounds of two separate classroom market experiments were conducted.[3] Figure 2 shows the frequency of excess trading across these classrooms.

Like Chamberlin, we found that the number of trades frequently exceeded that in competitive equilibrium. But we also found that in more than 1/3 of the sessions in which traders had some experience, the number of trades was

[3] In both of these experiments there were two types of buyers and two types of sellers. In the first of these experiments, the distributions of supplier and demanders were in the same proportions shown in Figure 1. In the second of these experiments, the types were the same, but the proportions of low and high-cost suppliers were reversed as were the proportions of low and high-value demanders.

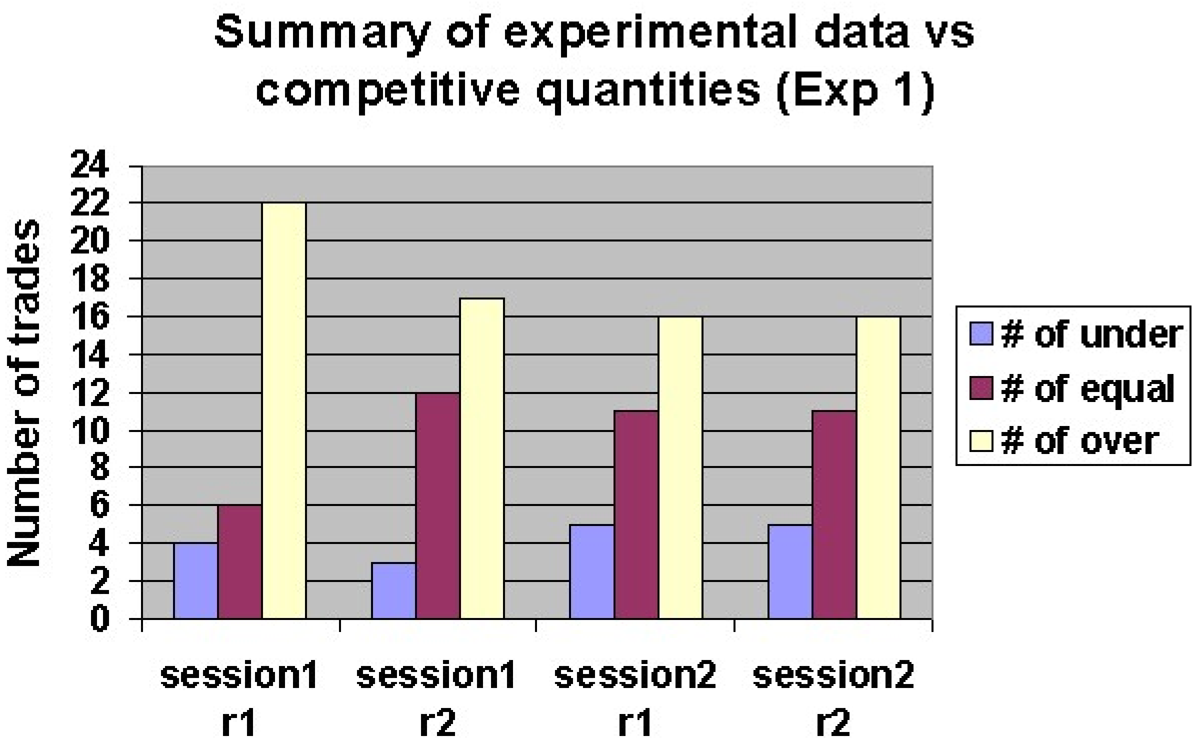

Figure 2: **Excess Trading in the Classroom**

equal to the competitive prediction. Unlike Chamberlin, we observed a few classrooms in which the number of trades fell short of that in competitive equilibrium. From Theorem 1, we conclude that in these classrooms, there were some suppliers and some demanders who did not trade, but could have made a profit if they had found each other before trading ended.

In his paper, Chamberlin reveals a detail of his procedure that appears to explain why he never observed fewer than the competitive number of trade. In a footnote, Chamberlin explains that he forced the execution of any mutually profitable trades that remained unconsummated during regular trading.[4] Chamberlin's procedure guarantees that the trading process is in our terms, "exhaustive." Theorem 1 tells us that in a simple trading economy the number of trades resulting from an exhaustive trading process can not be smaller than the number of trades in competitive equilibrium.

[4]Chamberlin's footnote reads as follows:

> In perhaps four or five cases out of the forty-six it was discovered . . . that a single transaction which could have been made had not been made. In other words, the highest remaining buyer's ticket was higher than the lowest remaining seller's ticket. In each of these cases the bargain was ruled as having been made at the midpoint between the two figures. This procedure was justified on the ground that, since there was pressure for time, the buyer and seller would, in fact, have found each other if the market had lasted longer.

References

[1] Theodore Bergstrom and Eugene Kwok. Extracting valuable data from classroom trading pits. Technical report, University of California Santa Barbara, Santa Barbara, California, June 2004.

[2] E.H. Chamberlin. An experimental imperfect market. *Journal of Political Economy*, April:95–108, 1948.