

# Lawrence Berkeley National Laboratory

## Recent Work

### Title

MODELING OF PROXIMITY COUPLING BETWEEN SUPERCONDUCTING FILAMENTS IN SUPERCONDUCTING DIPOLE MAGNETS

### Permalink

<https://escholarship.org/uc/item/86h6s9m6>

### Author

Green, M.A.

### Publication Date

1987-06-01

2



# Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

## Engineering Division

RECEIVED  
LAWRENCE  
BERKELEY LABORATORY

SEP 18 1987

LIBRARY AND  
DOCUMENTS SECTION

Presented at the Cryogenic Engineering Conference,  
St. Charles, IL, June 15-18, 1987, and to be published  
in Advances in Cryogenic Engineering, Vol. 33

**Modeling of Proximity Coupling between  
Superconducting Filaments in Superconducting  
Dipole Magnets**

M.A. Green

June 1987

### TWO-WEEK LOAN COPY

*This is a Library Circulating Copy  
which may be borrowed for two weeks.*



LBL-23823  
c.2

## **DISCLAIMER**

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

## MODELING OF PROXIMITY COUPLING BETWEEN SUPERCONDUCTING FILAMENTS IN SUPERCONDUCTING DIPOLE MAGNETS

M. A. Green

Lawrence Berkely Laboratory  
University of California  
Berkeley, California 94720

There is strong experimental evidence for superconducting current coupling between filaments in fine filament multifilamentary Nb-Ti which has been proposed for the SSC. The coupling between the fine filaments appears to be a Type II superconducting coupling. As a result, a.c. loss and magnetization can increase as one reduces the filament size in a fine filamentary conductor. This paper presents a computer model which fits experimental data for proximity superconducting coupling between filaments in a superconductor matrix. Using this model, one can predict the effect of proximity coupling on the field uniformity of a dipole magnet for the SSC.

### BACKGROUND

The field generated by circulating currents flowing within the superconductor of a superconducting dipole was observed as early as 1970.<sup>1</sup> This magnetic field was observed to be rich in higher multipoles even though the coil configuration in which the circulating currents flowed did not generate higher multipoles when it was powered by a transport current. It was recognized from the very beginning that these higher multipoles would adversely affect the operation of a superconducting storage ring at injection if the field at injection was low enough.

The SSC requires very uniform magnetic fields in its dipole magnets at injection, better than 1 part in 10,000 (1 unit), over its useful aperture of 10 mm in radius.<sup>2</sup> Cost considerations dictate that the injection induction for the SSC dipoles be low (about 0.33 T when the full field of the SSC magnet is 6.6 T). At an injection induction of 0.33 T, the circulating currents in the superconducting filaments generate a field error of 17 to 20 units when the average superconductor filament diameter is 20 microns.<sup>3</sup> The magnitude of the field generated by circulating currents goes down as the filament diameter goes down. It was thought that reducing the filament diameter would go a long way toward reducing the field generated by the circulating currents.

Two effects have made the reduction of filament diameter as a means of reducing the circulating current field less attractive. The first is the "so-called" surface current which can be attributed to an extension of

the lower critical field  $H_{c1}$ . The  $H_{c1}$  effect places a lower limit on the field generated by magnetization or circulating currents. The second effect which has been observed is coupling due to a form of Type II tunneling between filaments within the strands. As spacing between filaments is reduced, the magnetization observed increases as the filaments carry the circulating currents which are coupled across the filaments by superconducting currents. Unlike a.c. loss coupling between filaments, the proximity effect coupled currents do not decay with time.

This paper explains a computer modeling method which has been used to predict how proximity coupling will affect the magnetic field in an SSC dipole at injection. Computer results generated using the LBL-SCMAG04 computer code are presented.

## BASIC THEORY, MAGNETIZATION AND DOUBLET STRENGTH

The field generated by circulating currents in a single filament of superconductor can be represented by the classical hydrodynamic doublet equation<sup>4</sup> which takes the following form in the complex plane:<sup>5</sup>

$$H^{**}(Z) = \frac{\Gamma e^{i\alpha}}{2\pi i (Z - Z_c)^2} \quad (1)$$

where  $H^{**}(Z)$  is the complex conjugate of the field  $H^*(Z)$  at a point  $Z$ . This field is generated by a current doublet with a strength  $\Gamma$  and a doublet angle  $\alpha$  at a location  $Z_c$ .  $\Gamma$  is the product of the circulating current  $I$  and the average distance between the current  $d$ . The doublet angle  $\alpha$  is the angle of the magnetic flux line through the conductor (mostly due to the magnetic field generated by the transport current) minus  $\pi/2$ . Both  $\Gamma$  and  $\alpha$  are functions of the previous flux history of the superconducting filament.

Equation 1 can be used as it is or it can be expanded in a Taylor series about the origin  $Z = 0$ . This series will take the following general form (with no iron):

$$H^{**}(Z) = \sum_{N=1}^{\infty} a_N^{**} Z^{N-1} \quad (2)$$

the general form of the multipole coefficient  $a_N^{**}$  is as follows:

$$a_N^{**} = \frac{-\Gamma e^{i\alpha}}{2\pi i} N Z_c^{-(N+1)} \quad (3)$$

which applies for all multipoles  $N$  ( $N = 1$  is dipole,  $N = 2$  is quadrupole and so on). The radius of convergence is  $|Z_c|$ .

One can extend the theory to the case where there is an infinitely permeable circular iron shell which has its center at  $Z = 0$ . One can use the method of images to do this calculation. The method is described further in Reference 5. The power series expansion of the image currents doublets is included in the SCMAG04 code.

The doublet strength factor is proportional to the superconductor magnetization. On the filamentary level, the relationship between the doublet strength factor  $\Gamma$  and magnetization takes the following form:

$$\Gamma_f = \frac{\pi D_f^2 M_f}{4} \quad (4a)$$

where  $D_f$  is the filament diameter (m);  $M_f$  is the superconductor filament magnetization ( $\text{Am}^{-1}$ ); and  $\Gamma_f$  is the product of the doublet current  $I$  and the doublet distance  $d$ . On the strand level, the relationship between magnetization and  $\Gamma$  is as follows:

$$\Gamma_s = \frac{\pi D_s^2 M_s}{4} \quad (4b)$$

where  $D_s$  is the strand diameter (m);  $M_s$  is the superconducting strand magnetization ( $\text{Am}^{-1}$ ); and  $\Gamma_s$  is the strand doublet strength factor (Am).

The magnetization of the superconductor in an SSC dipole has several sources: 1) There is magnetization due to bulk current circulation within the filaments. These currents are responsible for a.c. losses at the filamentary level when the field in the filament changes. These a.c. losses per cycle are frequency independent. 2) There is magnetization due to  $H_{c1}$ .<sup>6</sup> This magnetization is maximum at  $H_{c1}$ , and it is zero when  $H = 0$  or as  $H$  approaches  $H_{c2}$ . This magnetization generates no a.c. loss. 3) There is magnetization due to coupled eddy currents across the filaments. These currents are proportional to  $dB/dt$ , and they are inversely proportional to the square of the twist pitch of the strand.<sup>7</sup> These currents generate an a.c. loss per cycle which is frequency dependent. 4) There is magnetization due to superconducting tunneling between filaments. This magnetization is  $dB/dt$  independent, and it does generate a.c. losses per cycle which is frequency independent. 5) There is magnetization due to eddy current coupling due to coupling between strands of a cable. This coupled eddy current magnetization behaves like coupled eddy current magnetization between filaments. The SCMAG04 code calculates the magnetization and field which are generated by the first four items.

The first two items on the magnetization list occur at the filamentary level so that  $M_f$  in Equation 4a is the sum of these two items. For a fully penetrated superconductor with a constant  $J_c$ , the bulk current magnetization  $M_{f1}$  will take the following approximate form:

$$M_{f1} \approx \frac{2}{3\pi} D_f J_c [1 - \delta] \quad (5)$$

where  $D_f$  is the filament diameter;  $J_c$  is the critical current density of the superconductor in filament (this should come from magnetization measurements rather than short sample measurements); and  $\delta$  is the fraction of the filament current carrying capacity which carries transport current or coupled currents. The second filamentary term is the  $H_{c1}$  effect magnetization  $M_{f2}$  which has the following form in the SCMAG04 code:

$$M_{f2} \approx H_{c1} - \frac{\ln[(H - H_{c1}/2)\phi]}{\ln[(H_{c2} - H_{c1}/2)\phi]} H_{c1} \quad \text{for } H > H_{c1} \quad (6)$$

for  $H \leq H_{c1}$  the magnetization  $M_{f2} = H$  where  $H_{c2}$  is the upper critical field;  $H_{c1}$  is the lower critical field; and  $\phi = \lambda/(2.07 \times 10^{-15})$  where  $\lambda$  is the penetration depth of the conductor.

Figure 1 shows magnetization curves generated by the SCMAG04 code for a conductor with a copper to superconductor ratio of 1.4 and various filament sizes from 2 microns to 20 microns. There is no coupling at the strand level shown in Figure 1. One will note that the curves are skewed. This skewing is the same for all filament diameters. The skew is observed in magnetization loops which are generated when the field change goes through zero.<sup>8,9</sup> The skewed behavior can be attributed to  $H_{c1}$  currents and vortex currents.<sup>6</sup> This skew does not die out until the field on the filament reaches  $H_{c2}$ .

Items 3 and 4 on the list of contributing magnetizations occur at the strand level. To first order, these two magnetizations can be summed to get  $M_s$  in Equation 4b. The magnetization due to coupled eddy current takes the following form provided the conductor has some twist and the rate of flux change is moderate.<sup>7</sup>

$$M_{s1} \approx \frac{\dot{B}}{\rho_e} \left( \frac{L}{2\pi} \right)^2 \quad (7)$$

where  $L$  is the twist pitch of the superconductor;  $\dot{B}$  is the rate of flux change; and  $\rho_e$  is the effective transverse resistivity of the matrix which is defined as follows:

$$\frac{1}{\rho_e} = \frac{1}{\rho_t} + \frac{2W}{D_s \rho_M} + \frac{D_s W}{2\rho_M} \left( \frac{2\pi}{L} \right)^2 \quad (7a)$$

where  $\rho_t$  is the interfilament resistivity which is defined as:

$$\rho_t = \frac{6.56 \times 10^{-16}}{W} + \rho_M \quad (7b)$$

where  $W$  is the interfilamentary spacing;  $\rho_M$  is the matrix material bulk resistivity including magnetoresistivity; and  $D_s$  is the strand diameter. The proximity coupling magnetization takes the following approximate form:<sup>10</sup>

$$M_{s2} = \frac{2}{3\pi} \frac{D_s J_c}{r+1} e^{-\epsilon} \quad (8)$$

where  $D_s$  and  $J_c$  are previously defined;  $r$  is the copper to superconductor ratio; and  $\epsilon$  is a function of  $W$ ,  $H$  and  $\rho_t$ . The value of  $\epsilon$  derived for use in the SCMAG04 code was fit to the Brookhaven National Laboratory measured magnetization data.<sup>11</sup> A value  $\epsilon$  which fits the data is as follows:

$$\epsilon = 7.18 \times 10^{11} (B + 0.5)(W(\rho_t)^{-1/2} - 7.8 \times 10^{-12}) \quad (8a)$$

or zero, whichever is larger. The expression given in 8a does not predict the onset of proximity coupling very well, but it is useful in the region where  $M_{s2}$  becomes important to the total magnetization.<sup>12</sup>

Figure 2 shows magnetization curves generated by the SCMAG04 code for a conductor which exhibits proximity coupling between filaments. The proximity coupling is pronounced for filament spacings below 0.8 microns. The 0.8 micron spacing case exhibits a little coupling near zero induction. Compare this curve with the 5 micron diameter filament in Figure 1.

### MAGNETIZATION LOOPS

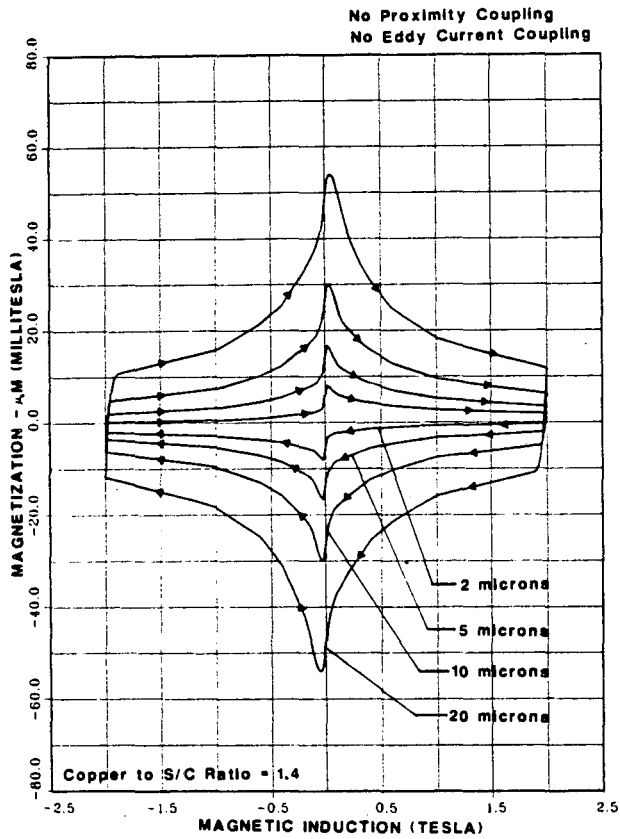


Figure 1. Magnetization Versus Magnetic Induction and Filament Diameter (no proximity coupling)

XBL 876-2495

### MAGNETIZATION LOOPS

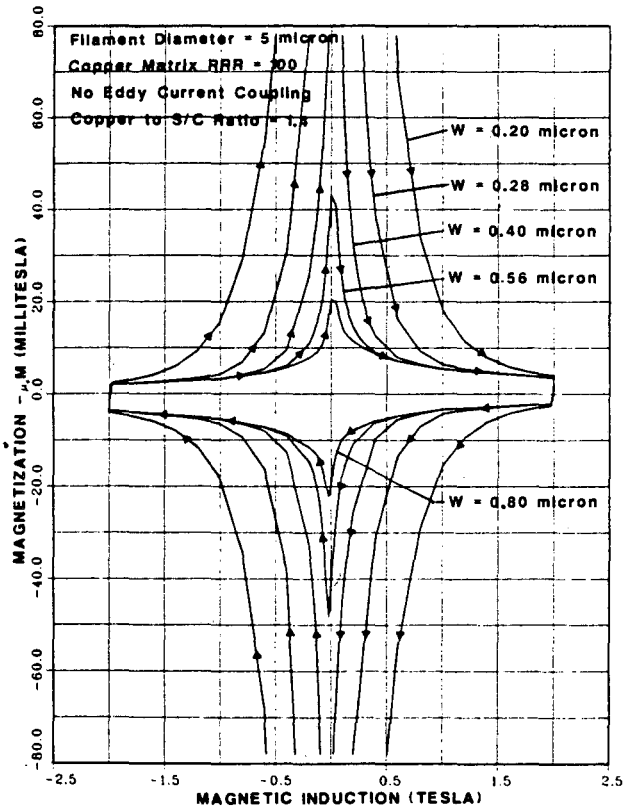


Figure 2. Magnetization Versus Magnetic Induction and Filament Spacing W with Proximity Coupling (copper matrix RRR=100)



## THE EFFECT OF PROXIMITY COUPLING ON THE SSC FIELD AT INJECTION

The SCMAG04 code was used to calculate the multipole components of the magnetization field generated within the LBL-NC-7 configuration of the SSC dipole shown in Figure 3. This figure shows the direction of the magnetic flux lines within the coil which determines the doublet angle  $\alpha$  in various parts of the coil. Figure 3 shows that the flux lines change direction in coil 5 of the magnet and that the field is lowest in coils 5 and 6 of the magnet. Since proximity coupling is strongest at low fields, it is these outside coil blocks which have the largest effect on the proximity coupling magnetization seen in the good field region of the magnet.

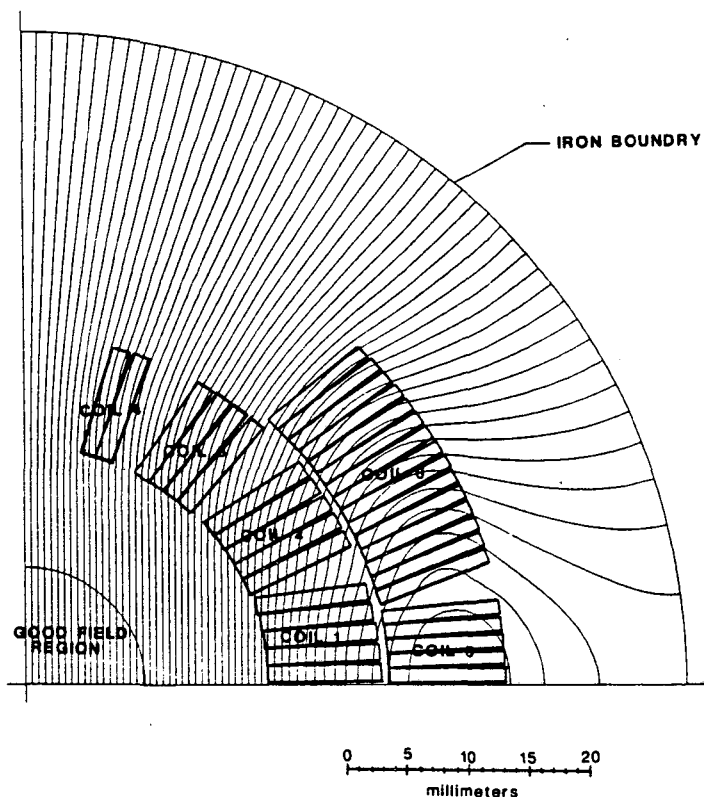


Figure 3. SSC Dipole Configuration LBL-NC-7 Showing Coils, Iron and Magnetic Flux Lines

Figure 4 shows the ratio of sextupole to dipole at a radius of 10 mm as a function of central induction and filament spacing. The multipole ratio is presented for the central field going down from 6.6 T to zero and back up again. In a superconductor without proximity coupling, most of the 5 micron filaments are fully penetrated by the flux change at a central field of 0.08 T going up. One can see in Figure 4 that when the filament spacing is 0.2 microns, flux penetration of the second flux change does not occur until the central induction change reaches about 0.5 T going up. The shape of the 0.2 micron spacing curve is very different when the field goes up as compared to when the field goes down. This difference reflects what is happening in the low field region of the magnet.

Figure 5 shows the sextupole component at SSC injection as a function of filament diameter  $D$  and the ratio of filament spacing  $W$  to diameter for a Residual Resistance Ratio (RRR) 100 copper matrix conductor. The dashed line indicates what the sextupole ratio would be with no proximity coupling. This curve does not approach zero even when the filament diameter approaches zero. This is due to the  $H_{c1}$  surface current effect.

**MAGNETIZATION SEXTUPOLE RATIO VERSUS  
TRANSPORT CURRENT INDUCTION  
AND FILAMENT SPACING**

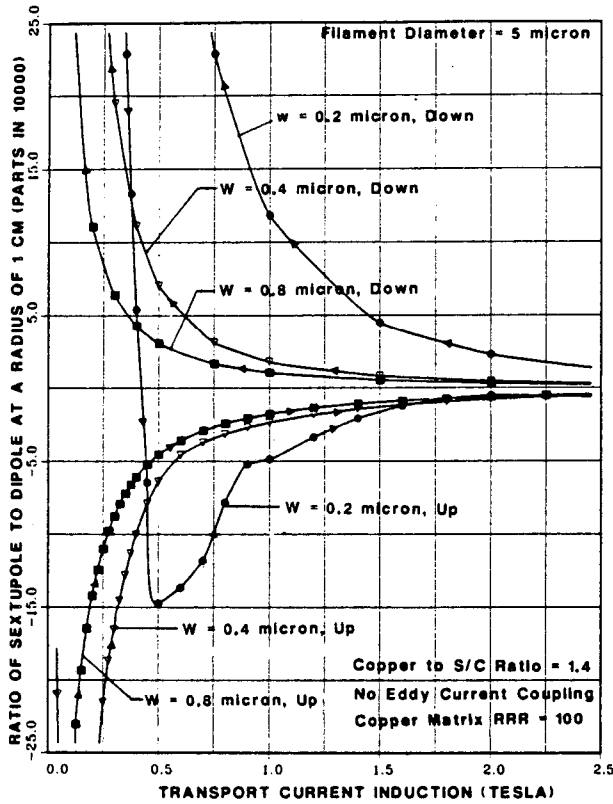


Figure 4. The Ratio of Magnetization Sextupole to Dipole as a Function of Transport Current Induction, Induction Change Direction and Filament Spacing  $W$

XBL 876-2499

**MAGNETIZATION SEXTUPOLE RATIO VERSUS  
FILAMENT DIAMETER AND W/D RATIO  
AT SSC INJECTION,  $B = 0.33$  TESLA**

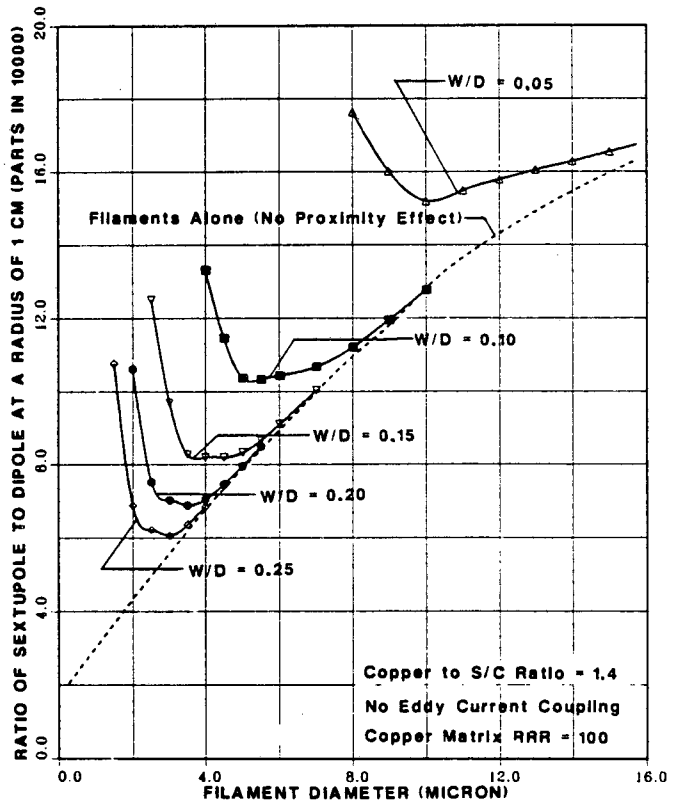


Figure 5. The Ratio of Magnetization Sextupole to Dipole at the SSC Injection Induction of 0.33 T (with the field rising) for Various  $W/D$  Ratios

From Figure 5, one can see that the proximity effect curves depart from the dashed line when the filament spacing  $W$  drops below 0.8 to 1 micron. This calculation is consistent with the Brookhaven measurements of a proximity coupled superconductor.<sup>11, 12</sup> Figure 5 presents the superconductor selection dilemma of the SSC. One wants to reduce magnetization by reducing filament diameter, but one wants to maintain a high-transport current  $J_c$  which means the superconducting filaments have to be continuous and smooth.<sup>9</sup> A good-quality, high  $J_c$  conductor has a relatively small spacing between filaments as compared to the filament diameter ( $W/D$  should be in the 0.1 to 0.2 range).

#### ACKNOWLEDGMENTS

The author wishes to acknowledge the fruitful discussions he has had with W. Gilbert, C. Taylor, J. Peterson and S. Caspi of LBL; and the author thanks A. Ghosh of the Brookhaven National Laboratory for the magnetization data which he shared. This work was supported by the Office of High Energy Physics of the United States Department of Energy under Contract Number DE-AC03-76SF00098.

#### REFERENCES

1. M. A. Green, IEEE Transactions on Nuclear Science NS-18 (1971), p.664.
2. Superconducting Super Collider Conceptual Design Report SSC-SR-2020 (1986).
3. M. A. Green, "Control of the SSC Fields Due to Superconductor Magnetization in the SSC Magnets", to be published in IEEE Transactions on Magnetics MAG-23 (2) (1987).
4. L. M. Milne-Thomson, Theoretical Hydrodynamics, McMillan Company, N.Y. (1960), p. 204.
5. M. A. Green, "Residual Fields in Superconducting Magnets", published in the Proceedings of the MT-4 Conference at Brookhaven National Laboratory (1972), p. 339.
6. W. J. Carr, IEEE Transactions on Magnetics MAG-21 (2), p. 335.
7. M. N. Wilson, Superconducting Magnets, Clarendon Oxford Press, Oxford, UK (1983), pp. 174-181.
8. W. J. Carr and C. R. Wagner, Advances in Cryogenic Engineering Vol. 30, Plenum Press (1984), p. 923.
9. A. K. Ghosh and W. B. Sampson, Advances in Cryogenic Engineering Vol. 32, Plenum Press (1986), p. 809.
10. E. W. Collings, "Stabilizer Design Considerations in Ultrafine Filamentary Cu/Ti-Nb Composites", Proceedings of the 6th Workshop on Niobium-Titanium Superconductors, University of Wisconsin, Madison, WI (1986).
11. A. K. Ghosh et al, "Anomalous Low Field Magnetization in Fine Filament Nb-Ti Superconductor", to be published in IEEE Transactions on Magnetics MAG-23 (2) (1987).

*LAWRENCE BERKELEY LABORATORY  
TECHNICAL INFORMATION DEPARTMENT  
UNIVERSITY OF CALIFORNIA  
BERKELEY, CALIFORNIA 94720*