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On Competition and Product Differentiation In Urban Transport:
The San Francisco Bay Area

Institute of Urban & Regional Development
University of California, Berkeley

Reprint No. 191

Reprinted from: *The Bell Journal of Economics,*
Volume 12, No. 2, Autumn 1981, pp. 362-379.

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This article considers Cournot-like competition among two public transit oligopolists (a rapid-rail system modeled on the Bay Area's BART, and a parallel bus system) in the presence of a competitively supplied third alternative. The oligopolists compete in both service quality and price, and do so myopically. With demand and cost data from the Bay Area numerically estimated equilibria are obtained and studied. The results indicate that: (1) with fares and product characteristics freely variable, neither mode need operate at a loss; (2) the rapid-rail mode can cover costs from the farebox even if the bus mode offers money-losing service; (3) the equilibria exhibit product differentiation, in sharp contrast to the presently observed situation.

1. Introduction

■ It was clear from the start that BART, the San Francisco Bay Area's new rapid-rail system, would face substantial competition from other modes. Chief among the competing modes was the private car. Bay Area auto-ownership rates are among the nation's highest, and reliance on the private car was fostered by an early full-freeway system (Webber, 1976, pp. 1–2). With a distribution of incomes skewed towards the upper tail, there was every reason to expect that substantial numbers of commuters would elect to remain in their cars.

But BART did not face competition only from automobiles. In the East Bay, as became clear during the period of construction, service provided by the Alameda-Contra Costa Transit District (AC Transit) would also be a major source of competition. (See Adler (1978) for a survey of this history.) Since 1960 AC Transit has provided what has been widely regarded as extremely effective transit service. The system provides extensive coverage of the East Bay, with express buses running to the principal East Bay Central Business District (CBD)—Oakland—as well as both local and express service to San

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Support from Research and Training Grant No. CA-11-0001 from the Urban Mass Transportation Administration of the U.S. Department of Transportation, and a grant from the UPS Foundation are gratefully acknowledged, but no concurrence with the views expressed here is implied. Discussions with M.M. Webber, M. Landau, G.S. Cluff, J. Bendor, and A. Klevorick were extremely helpful. The comments of an anonymous reviewer are gratefully acknowledged.

Francisco. In an era of increasing reliance on the private car, AC Transit managed to retain its ridership and even increased patronage by about 8% over the period 1962–1963 to 1976–1977 (*AC Transit Annual Reports*, 1962–1977). Thus, BART could expect substantial competition from the bus system as well as from the car.

When BART began operations, it was apparent that the rail system set its quality of service (including the fare) to parallel closely the competing bus system's. For example, for a (1975) transbay trip from Berkeley to San Francisco, where there is direct competition between the two modes, service characteristics are quite similar: BART is slightly faster and costs slightly more than regular bus service, while for express buses, the quality of bus service is virtually identical to that of BART. The service operated by AC Transit has failed to cover its operating costs in any year since its founding in 1960. BART was expected to rely for its construction costs on general obligation bonds secured by the property tax. On the other hand, it was expected to pay for all rolling stock and operating expenses out of the farebox.[1] The question may be raised whether this was a reasonable expectation, in light of the fact that BART elected to provide a level of service comparable to that provided by a deficit-ridden bus system.[2]

BART has already been built. There is no immediate reason to reexamine the question of the total resource costs expended in providing trips by different modes (Webber, 1976; Keeler *et al.*, 1975). Rather, the question raised by the considerations of the preceding paragraphs is: how, in the short run, could BART adjust its variable factors of production to minimize (if not cover) its operating loss? In posing this question it is important to realize that from the point of view of Bay Area transit supply, there are three main "industries": BART; the often competing bus systems run by AC Transit, San Mateo County's SAMTRANS, Golden Gate Transit, and San Francisco's MUNI; and the private car. Thus, to approach the question of loss-minimizing service, one needs to model the interdependencies among the three sets of carriers. Further, it is important to know just how the users of the system will react to short-run changes in service quality. What is needed, in short, is a full multimodal demand and supply model of Bay Area transit.

The remainder of this article formulates and then solves numerically such a model to provide answers to these questions. The supply-side decisions of the firms in the transit market are the heart of the model. I assume two firms, a bus company and the BART (or BART-like) rapid rail operator. Both firms are assumed to know the residual demand curves they face, and they act as myopic Cournot-like firms. Unlike the usual Cournot (or Bertrand) firm, these firms set not only the price, but also the characteristics of the service they offer.[3]

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[1] BARTD Organic Act. Stat 1957 c. 1056 p. 2309 ¶29038. *West's California Codes* ¶29038.

[2] One does observe instances in the three major transit modes—bus, rapid rail, and commuter rail—of services run to at least cover operating costs from the farebox. In general, this is accomplished by differentiating these services from those offered by competing "public service" modes. See Morlok and Viton (1980) for details.

[3] For a simple model of two duopolists providing substitutable products, see Dixit (1979). Dixit's analysis is for the linear case; while, as will be apparent, the problem to be discussed here is severely nonlinear. For an analysis of duopolists and monopolists subject to rate-of-return regulation, similar in spirit to Dixit's model, see Viton (1980).

Of course, the assumption of firm myopia is wrong except at the equilibrium iteration of the game. For this reason, attention has been given to models incorporating some sort of learning behavior, of which the von Stackelberg variant of duopoly is but one example. The model to be developed here does not explore this area, for two reasons. First, as will become apparent, closed-form solutions to the firm's profit-maximization problem are unobtainable. Thus, in solving the problem numerically we obtain points on the reaction functions and not the functions themselves as von Stackelberg analysis requires. Second, and more heuristically, in the model to be formalized here, convergence is rapid—generally within three to five iterations. In this case one may argue that the scope for learning is limited; and hence that myopic behavior is reasonable.

As a final introductory remark, it is worthwhile to place the present model in the spectrum of recent studies of price and quality behavior. The model to be formulated in subsequent sections is closest to that recently discussed by Lane (1980). The major difference between the two lies in the treatment of changing product characteristics (locations in product space). Lane assumes that fixed costs are high enough to prevent firms from altering the nonprice characteristics of their products. The present model makes no such assumption. Since the principal differentiations of service in urban transportation appear to lie in scheduling decisions of various kinds, it is entirely appropriate to allow for changes in product characteristics, and indeed we observe frequent transit schedule changes. Lane also makes an assumption of perfect foresight and hindsight, which is not made here. In the model discussed here, the transit industry consists of at most two firms, and the possibility of entry-deterring behavior either by BART or by the bus company is not investigated.

The following sections develop formally the model sketched in this introduction. In Section 2 I briefly discuss a model of the demand for the different modes of urban transportation and present an empirical estimate of the mode-choice model. Section 3 develops the Cournot-Nash duopoly framework. Section 4 indicates empirical estimates of the supply and demographic parameters. Finally, Section 5 presents the results of the model for a variety of potential urban transportation scenarios.

2. Demand

■ Consider, following the notation of Hausman and Wise (1978), a population of consumers, each facing a choice among J transportation modes. The consumers maximize utility, and the utility of mode j to the ith consumer is taken to be a linear function of the perceived characteristics of the mode (x_i^j) and the attributes of the consumer (a_i), plus a stochastic term reflecting unobserved (or unobservable) characteristics of the mode or the individual. Introduction of the stochastic element means that we can analyze only the probability that a given choice is made: each distribution of the stochastic term yields a set of mode-choice probabilities. For computational reasons, surveyed by Domencich and McFadden (1975), the stochastic terms for different individuals are often assumed to have independent Weibull distributions. Under this assumption, the probability of choosing a particular mode is given by the well-known conditional logit form: the probability that the ith consumer selects the jth mode, P_i^j, is

$$P_i^j = \frac{\exp[\beta' Z_i^j]}{\sum_k \exp[\beta' Z_i^k]}, \tag{1}$$

where $Z_i^j = (a_i, x_i)$, and β is an unknown weighting vector, which can be estimated, for example, by maximum likelihood methods (Domencich and McFadden, 1975, chapter 5).

McFadden, Talvitie *et al.* (1977) have recently completed a major study of mode choice in the San Francisco Bay Area, and I draw on their results for a set of empirical estimates of the mode-choice probabilities. They are shown in Table 1. These estimates are used in the sequel to represent the preferences of Bay Area commuters.[4]

3. Supply

■ This section presents the theoretical model of urban transportation supply sketched in the introduction. The model is completed in Section 4 with empirical estimates of the supply-side parameters. The plan of the present section is as follows. First, I discuss the situation of the traveler (consumer) and argue that he is both a price- and service-quality taker. I turn then to the question of competition. In the model formulated here, there are two suppliers, a bus company and the BART organization. Competition takes place in an extended Cournot-Nash framework, where the competitors select both price and quality of service. I then situate this competition in the context of an abstract market for transportation and present a model in which the decision variables of the transit agencies translate into precisely those features of travel that are revealed as important by the demand model of the previous section.

It is clear that we may model the user of the public transit modes as a price- and service-quality taker. However, the traveler who uses his car is in a unique position. He alone has immediate access to his means of transportation; and the consequent ease of travel planning has no doubt contributed significantly to the popularity of this means of locomotion. The economics of his actions may be characterized by two conditions.

First, the automobile user is a price-taker. That is to say, he perceives the cost of travel as given and independent of his actions. Whether or not he decides to use his car, the prices he faces will not change. In some important respects this is quite clearly true. For example, an individual's consumption of fuel and oil is likely to be miniscule when compared with the total consumption. Under these circumstances, the effect on the prices of these products of a change in individual travel demand behavior will be negligible, and the assumption of perfect elasticity is wholly reasonable. The assumption is less reasonable when applied to costs like auto insurance or depreciation; however, mileage-dependent variations in these costs are likely to be small (Keeler *et al.*, 1975, chapter 3, section 3.1). Second, an auto user is also a service-quality taker. Taking auto-ownership decisions as given, auto travel times are the only relevant measure of service quality. To be a quality-taker means that the auto user believes that attainable speeds on roadways are unaffected by his mode choice. This again is clearly plausible.[5]

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[4] McFadden, Talvitie *et al.* (1977, p. 165 *et seq.*) present an extensive discussion of specification tests on this and other models.

[5] I am indebted for this simple formulation to Professor A. Klevorick.

TABLE 1

WORK-TRIP MODE CHOICE MODEL, ESTIMATED POST-BART WITH NONGENERIC AUTO AND TRANSIT ON-VEHICLE TIME

(MODEL 1—AUTO ALONE, MODE 2—BUS, WALK ACCESS; MODE 3—BUS, AUTO ACCESS; MODE 4—BART, WALK ACCESS, MODE 5—BART, BUS ACCESS, MODE 6—BART, AUTO ACCESS; MODE 7—CARPOOL)

MODEL: MULTINOMIAL LOGIT, FITTED BY THE MAXIMUM LIKELIHOOD METHOD

INDEPENDENT VARIABLE (THE VARIABLE TAKES THE DESCRIBED VALUE IN THE ALTERNATIVES LISTED IN PARENTHESES AND ZERO IN NONLISTED ALTERNATIVES.)	ESTIMATED COEFFICIENT	t-STATISTIC
COST DIVIDED BY POSTTAX WAGE, IN CENTS DIVIDED BY CENTS PER MINUTE (1–7)	-.0266	3.92
AUTO ON-VEHICLE TIME, IN MINUTES (1,3,6,7)	-.0473	3.48
TRANSIT ON-VEHICLE TIME, IN MINUTES (2–6)	-.0197	2.03
WALK TIME, IN MINUTES (2–6)	-.0900	3.36
TRANSFER-WAIT TIME, IN MINUTES (2–6)	-.0438	1.81
NUMBER OF TRANSFERS (2–6)	-.120	0.856
HEADWAY OF FIRST TRANSIT CARRIER, IN MINUTES (2–6)	-.0290	2.60
FAMILY INCOME WITH CEILING OF $7,500, IN $ PER YEAR (1)	-.000289	1.78
FAMILY INCOME MINUS $7,500 WITH FLOOR OF $0 AND CEILING OF $3,000, IN $ PER YEAR (1)	.0000522	0.364
FAMILY INCOME MINUS $10,500 WITH FLOOR OF $0 AND CEILING OF $5,000, IN $ PER YEAR (1)	-.0000419	0.738
NUMBER OF PERSONS IN HOUSEHOLD WHO CAN DRIVE (1)	1.48	5.26
NUMBER OF PERSONS IN HOUSEHOLD WHO CAN DRIVE (3,6)	1.65	5.16
NUMBER OF PERSONS IN HOUSEHOLD WHO CAN DRIVE (7)	1.28	4.85
DUMMY IF PERSON IS HEAD OF HOUSEHOLD (1)	.668	3.19
EMPLOYMENT DENSITY AT WORK LOCATION (1)	-.00164	3.45
HOME LOCATION IN OR NEAR CBD (2=IN CBD, 1=NEAR CBD, 0 = OTHERWISE) (1)	.1546	0.835
AUTOS PER DRIVER WITH A CEILING OF ONE (1)	4.79	3.70
AUTOS PER DRIVER WITH A CEILING OF ONE (3,6)	3.63	4.81
AUTOS PER DRIVER WITH A CEILING OF ONE (7)	3.26	3.19
AUTO ALONE ALTERNATIVE DUMMY (1)	-4.18	2.82
BUS-WITH-AUTO-ACCESS DUMMY (3)	-8.24	6.67
BART-WITH-WALK-ACCESS DUMMY (4)	-2.28	3.36
BART-WITH-BUS-ACCESS DUMMY (5)	-.473	0.708
BART-WITH-AUTO-ACCESS DUMMY (6)	-7.30	5.93
CARPOOL ALTERNATIVE DUMMY (7)	-5.31	5.56

LIKELIHOOD RATIO INDEX	.4599	
LOG LIKELIHOOD AT ZERO	-964.4	
LOG LIKELIHOOD AT CONVERGENCE	-520.9	
PERCENT CORRECTLY PREDICTED	67.24	

VALUES OF TIME SAVED AS A PERCENT OF WAGE (t-STATISTICS IN PARENTHESES):

AUTO ON-VEHICLE TIME	178%	(2.53)
TRANSIT ON-VEHICLE TIME	74%	(1.84)
WALK TIME	338%	(2.46)
TRANSFER-WAIT TIME	165%	(1.65)
VALUE OF INITIAL HEADWAYS AS A PERCENT OF WAGE:	109%	(2.13)

ALL COST AND TIME VARIABLES ARE CALCULATED ROUND-TRIP. DEPENDENT VARIABLE IS ALTERNATIVE CHOICE (ONE FOR CHOSEN ALTERNATIVE, ZERO OTHERWISE).

NUMBER OF PEOPLE IN SAMPLE WHO CHOSE

AUTO-ALONE	378
BUS-WITH-WALK-ACCESS	68
BUS-WITH-AUTO-ACCESS	9
BART-WITH-WALK-ACCESS	4
BART-WITH-BUS-ACCESS	6
BART-WITH-AUTO-ACCESS	33
CARPOOL	137
TOTAL SAMPLE SIZE	635

SOURCE: McFADDEN, TALVITIE ET AL. (1977, PP. 149–150).

I now discuss the reaction-function approach to competition between two transit agencies. For simplicity I suppose that to each agency (bus or BART) there corresponds a single travel demand. In the mode-choice model presented in the last section this is not so: for example, for the BART mode there are three separate classes of demand to be considered, each corresponding to a different access mode. Inclusion of these classes in the model adds nothing but notational complexity.

It will be convenient to index the two modes by 1 and 2. Denote by x^1 and x^2 the characteristics of the two modes, including the fares charged. Suppose there is a given traveling population of size Q, and that it may be segmented into N homogeneous groups, where the homogeneity is reflected in the socioeconomic parameter vector a_i. If d_n is the proportion of group n in the travel population, then total demand for the ith mode is given by

$$D^i(x^1, x^2) = Q \sum_{n=1}^{N} d_n p^i_n, \qquad (i = 1, 2). \tag{2}$$

We can now form and study the profit expressions of the two modes. If p^i (which is an element of the characteristics vector x^i) is the fare on mode i and if $C^i(x^i)$ is the total cost of providing service at level x^i, then the profit of mode i is

$$\Pi^i(x^1, x^2) = p^i D^i(x^1, x^2) - C^i(x^i), \qquad (i = 1, 2). \tag{3}$$

We model each transit agency as behaving in a Cournot-like way with zero conjectural variation. Each takes the service-quality vector of the other as given and maximizes profit. This leads to reaction functions, $R^1(x^2)$ and $R^2(x^1)$, one for each agency, where the reaction function indicates the best choice of one's own level-of-service vector conditional on that of the other agency. A reaction-function equilibrium is a pair of vectors $\hat{x}^1$ and $\hat{x}^2$ that satisfy

$$\begin{aligned} \hat{x}_1 &= R^1(\hat{x}_2), \\ \hat{x}_2 &= R^2(\hat{x}_1). \end{aligned} \tag{4}$$

This reaction-function equilibrium will be the object of our empirical study in Section 5. It represents a stable point in the sequential reactions of one agency to the actions of the other.

I next place reaction-function competition in the more detailed context of an abstract transportation market. The market is shown schematically in Figure 1.[6] The structure of the market is one of transportation between a set of origins and a given destination area, such as a CBD or major shopping center. There is a circular residential area of radius r miles. We suppose that origins are uniformly distributed over the radii of the residential circle so that the "average" resident lives $r/2$ miles from the center. We assume too that the relevant socioeconomic characteristics (to be described in Section 4) are also uniformly distributed.[7] Streets lie along the radii of the residential circle.

Trip-making between origin and destination takes place as follows. Suppose

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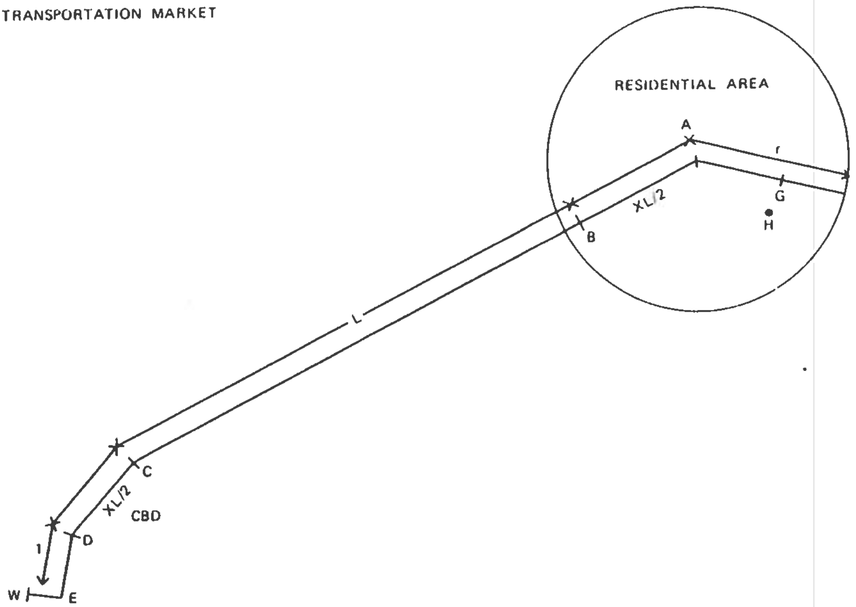
[6] This model first appeared in Pozdena (1975). It has received extensive use in the analysis of Bay Area transportation. See Webber (1976) and Keeler *et al.* (1975).

[7] Thus we rule out the possibility of (say), the left side of the residential area's being poor and the right side's being rich. In this formulation rich and poor live together. (The model is easily adapted to the analysis of other residential patterns.)

FIGURE 1

TRANSPORTATION MARKET



the trip-maker lives at H. If the selected mode is the automobile, the consumer drives (after a possible "carpool delay") from H to A, interpreted as a freeway entrance. He drives at freeway speeds from A to C, the freeway exit, and then proceeds along arterial streets from C to D to E, where he parks. From E he walks to work at W, assumed to be .25 mile away from the parking lot.

The description of bus trips is slightly more complicated. From home at H the commuter either walks or drives to the nearest bus stop, assumed to be located at G. When the next bus arrives, the commuter boards it; and the bus proceeds along arterial streets to B, which is also a freeway entrance. Note that this formulation introduces a perceived disadvantage for the bus, equivalent to the fact that, in reality a bus generally takes a more circuitous route to the freeway. The present structure, instead of increasing bus distance traveled, introduces a circuity factor by having the bus travel from A to B (a distance of $XL/2$ miles) at the slower arterial speed. Note that this adequately captures the circuity-induced disadvantage since, from Table 1, modal choices depend only on travel time and not distance. The bus proceeds on a freeway from B to C, proceeds on arterial roads from C to D to E, and at E the commuter alights and walks to work at W.

Trips by BART are again slightly different. Here we have three possible access modes. Assume that the BART station is located at A.[8] If the access

[8] That the freeway and BART tracks are essentially coextensive in Figure 1 reflects the observed Bay Area layout.

mode is by automobile or by walking, the consumer proceeds directly from H to A. If the access mode is automobile, it is assumed that free parking is available at or immediately adjacent to the BART station. If BART is accessed by bus, then as before he walks to the nearest bus stop, waits for the bus, and rides to A. No matter what the access mode, at the station he waits for the next train, which takes him from A to E. Note that if the BART speed is high enough, this represents an advantage in travel time over the competing modes which travel at arterial speeds for $XL/2 + 1$ miles (auto) or $XL + 1$ miles (bus). The BART station at the destination is assumed to be located at E from which one has the usual walk to work of W.

I turn now to a discussion of the concept of transit service quality. From the demand model of Table 1, the relevant measures are cost, on-vehicle time, walk time, transfer wait time between transit modes, number of transfers, and headway (the interval between successive vehicles) of the first transit carrier. I assume that each type of road is characterized by a (possibly mode-dependent) attainable speed, and that this speed is invariant, in any given period of the day, to traffic volume.[9] I make the similar assumption that BART speeds are given. Then on-vehicle travel times are given, both to consumers and to transit-providing agencies. The structure of the market determines the number and duration of transfers between transit modes. Hence, the measures that are subject to agency determination are fare, walk time, transfer wait time, and headway. The detailed model below incorporates the last three in the form of operating characteristics.

Since we are interested in describing the outcome of a bus access system that is run to feed efficiently to the BART system, we suppose that there is a feeder bus service distinct from the regular bus service and that it is run by BART only to feed into the rapid rail system. This means that along some of the residential streets there will be in effect two bus lines: one destined for the CBD and one for the nearest BART station. We allow the bus systems to be entirely separate, running at different levels of service and charging different fares.[10]

Suppose that a bus system runs R equally spaced bus routes over the residential area. From previous assumptions, the average commuter lives $r/2$ miles from the freeway entrance/BART station at A in Figure 1. At this level of bus service, the average commuter, who is assumed to walk at three miles per hour, will take $\pi r/6R$ hours to walk to the nearest bus stop. Thus, from our assumption about the level at which the residential area is "covered" with bus routes we can immediately deduce the average access walk time a commuter faces.

Similarly, denote by ψ the headway of transit vehicles along their routes. If, as is conventional, arrivals at stops follow a Poisson distribution, then the relation between average wait at a stop and vehicle headway is simply "wait

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[9] The following may make this assumption plausible. For expressways, the traffic between a given origin and destination is likely to be a small proportion of total traffic on the road, and hence will have only a small influence on speeds. For residential and city streets, attainable speeds depend as much on geometric layout (frequency of intersections, traffic lights, etc.) as on traffic.

[10] There exists at least one such feeder service, from the University of California to the University Avenue (Berkeley) station. The difficulties of ensuring that such a service is used only for BART-bound travel are not discussed here. It is also assumed that the vehicle types run by the two bus operations are the same. For the costs of smaller buses see, e.g., Fisher and Viton (1974).

equals half the headway." Thus, from the system variables of vehicle headways, we may deduce the average wait times consumers incur.[11]

The results of the last paragraphs imply that by considering only prices, number of routes in the residential area provided by the two bus systems, and the headways of the transit modes, we may capture all the relevant measures of service quality revealed as important by the choice model of Table 1. We shall also restrict the mode-choice model to six modes by assuming that the extent of carpooling, represented by an average number of travelers per car, is fixed and not an object of choice by the auto-using segment of the market.

We now present the profit expressions of the bus and BART operators. We use the modal indices of Table 1. Thus, for example, P_n^4 is the probability that a member of the nth homogeneous group of travelers will choose to travel by BART after walking to the system. We have just shown how the independent variables in these probabilities are related to the system characteristics. Again, we assume that the total traveling population in the period under consideration is Q.

First, we give the profit expression for the bus system. If each rider is charged an identical fare p, then the bus operator chooses R, ψ, and p to maximize

$$\Pi = Qp \sum_n (P_n^2 + P_n^3)d_n - C(\psi, R), \tag{5}$$

where $C(\psi, R)$ is the cost of providing this level of service.

The BART operator provides in effect two modes: the BART rapid-rail system and the feeder bus system. Denote by p_B and p_f the BART and feeder fares, by R_f the number of feeder routes, and by ψ_B and ψ_f the headways of BART trains and feeder buses. Then the BART agency selects p_B, p_f, R_f, ψ_B, and ψ_f to maximize

$$\Pi^B = Qp_B \sum_n (P_n^4 + P_n^5 + P_n^6)d_n + Qp_f \sum_n (P_n^5 d_n) - C^B(R_f, \psi_B, \psi_f), \tag{6}$$

where C^B is the cost of providing the multiproduct level of service. This formulation is deceptively simple: recall that the probabilities P_n^j are given by equation (1), and each of them is in general a nonlinear function of all the choice variables. It is clear from inspection of the equations that a closed-form solution is not possible.

Finally, we impose a set of technological constraints on each of the profit-maximization problems. There is an upper limit to the number of persons who can enter a single bus; and we take this to be the seating capacity (fifty) of a bus. Since the number of trips made at headway ψ is $1/\psi$, we require that

$$\psi Q \sum_n (P_n^2 + P_n^3)d_n \leq 50,$$

$$\psi_f Q \sum_n (P_n^5)d_n \leq 50, \tag{7}$$

for the regular bus and feeder bus systems, respectively.

For BART there is also a capacity constraint, though it depends on the

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[11] See Fisher and Viton (1974, pp. 18-22), for simulation models of average waiting time under various arrival distributions.

number of cars in a train. If T is the number of cars and we require that each car hold not more than 100 commuters, the constraint is:[12]

$$\psi_H Q \sum_n (P_n^A + P_n^B + P_n^W) d_n \leq 100T. \tag{8}$$

There is also, in the case of BART, a safe-headway requirement. Because of the confluence of three lines at Oakland, trains there must be 120 seconds apart, or six minutes on each BART's three lines. Therefore, on the BART line we require that $\psi_H \geq .1$.

The cost terms are discussed in the next section. Though the model formulation is quite general, we shall make the following additional assumptions. The demand model of the last section is for work-trips; and hence attention is focused on the peak period alone. This is in line with the observation by Webber (1976) that BART is essentially a commuter rail operation. Second, the influence of minor-direction traffic is ignored. In light of the focus on the peak alone, this is a reasonable assumption, especially for CBD-bound travel. For other periods, this assumption is questionable; however, since offpeak travel volumes are low compared with peak volumes, the results in Section 5 indicate that the possibilities of covering variable costs in low-demand periods are quite limited.

4. Empirical parameters

■ Table 2 compiles the costs of transportation in the Bay Area for the three modes. Costs for an automobile are those for a compact car. It is possible to consider different sizes of cars and assign them to the different demographic groups, but this is not done here. To make auto costs comparable with the way in which McFadden's researchers calculated them (Reid, 1977), the auto costs include only direct out-of-pocket costs. No specific externality tolls are assumed, which is in keeping with current practice, even though taxes, which are included in auto costs, do cover most of the noncongestion external costs (Viton, 1978a).

It is assumed that both the regular bus system and BART's feeder bus system use the same technology and are subject to the same costs. In at least one instance we observe feeder buses that are substantially smaller than regular transit buses, but this innovation is not discussed.[13] All capital costs of buses are assigned to a four-hour daily peak, and they are included in the costs of Table 2.

BART costs include the operating costs of running the system plus a capital cost of the rolling stock. Thus it is assumed that in the short run rolling stock costs are variable: that is, the fleet could be easily sold. All other capital costs are excluded: it is assumed that the capital equipment (excluding rolling stock) is fixed and that its costs would continue to be incurred if zero output were produced.

The model implicitly assumes constant returns to density over the relevant portion of the BART cost curve and the bus cost function. The constant-cost assumption for buses is standard in the literature. But Pozdena and Merewitz present evidence that this is not so for rapid-rail. Nevertheless, even in their

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[12] In fact, the seating capacity of BART cars is approximately 72. The procedure adopted here, due to Keeler *et al.* corrects for different amounts of space provided by the two modes.

[13] See footnote 10.

TABLE 2

TRANSPORTATION COSTS[1]

1. AUTOMOBILE – COMPACT CAR		2. BUS		3. BART	
ARTERIAL COST/ VEHICLE MILE[2]	$ .0827	COST/MAJOR DIRECTION PEAK VEHICLE-HOUR[4]	$16.34	COST/PEAK CAR-MILE[6]	$7.27
LINEHAUL COST/ VEHICLE MILE[2]	.0597	COST/MAJOR DIRECTION PEAK VEHICLE-MILE[5]	.3274	CURRENT COST/ CAR MILE	2.85
PARKING COST/DAY[3]	1.71			VEHICLE CAPITAL COST/CAR-MILE	4.42

[1]THE REFERENCES BELOW GIVE COSTS IN 1972 DOLLARS. THE FIGURES IN THE TABLE MULTIPLY THOSE COSTS BY 1.3516, THE 1972–1976 INCREASE IN THE SAN FRANCISCO–OAKLAND CONSUMER PRICE INDEX (STATISTICAL ABSTRACT OF THE U.S., 1978, TABLE 772).

[2]KEELER ET AL. (1975, PP. 84–86). INCLUDES ONLY COSTS OF AUTO OPERATION AND MAINTENANCE.

[3]ROUGH AVERAGE OF ALL-DAY RATES IN 1978 FOR SEVEN DOWNTOWN OAKLAND LOTS.

[4]VITON (1977, P. 50).

[5]IBID. INCLUDES $.0737 ADMINISTRATIVE EXPENSES.

[6]KEELER ET AL. (1975, P. 122). SIX PERCENT INTEREST RATE ASSUMED.

preferred Model 4, one cannot reject at the 15% level of significance the hypothesis of constant returns to density.[14] Thus, in what follows, constant returns in the short run are assumed, and hence average and marginal costs are the same.

Table 3 summarizes the physical parameters to be used in the sequel. Note that the average trip length in this formulation is 17 miles, which is slightly longer than the average Bay Area trip length of 16 miles (*Urban Transportation Factbook*, 1974, I-19). Recall that we assume that speeds on any road segment are invariant to the volume of travel on that road. (See Viton (1977) for a simple model in which speed does depend on traffic.)

Table 4 lists the demographic variables used in estimating the mode-choice probabilities, and the distribution of Bay Area incomes. Pretax income, household size, and autos per household, and their distributions, are based on the 1975 *Survey of Housing and Transportation*. The posttax wage was constructed using average Federal and California tax rates, and assuming a 40-hour work week and a 50-week work year.

A rough estimate of the number of drivers per household may be obtained by assuming that the household contains two adults and then estimating the

TABLE 3

TRANSPORTATION MARKET PARAMETERS

SYMBOL	SEGMENT	REFERENCE IN FIG. 1	DISTANCE (MI)	SPEEDS			TRANSFER TIMES	MINUTES
				AUTO	BUS	BART		
L	LINEHAUL	BC	12	40	40	--	BUS TO BART (150 YDS., 3 MPH.)	1.7
r	RESIDENTIAL	AI	2	25	12	--		
XL	CIRCUITY	AB,CD	4	25	12	--	AUTO TO BART: (100 YDS., 3 MPH.)	1.1
	BART LINE	AE	17	--	--	45		

[14] Pozdena and Merewitz (1978); *t*-statistics calculated from Table 1, p. 76.

TABLE 4

DEMOGRAPHIC DATA

(1) GROUP	(2) PERCENTAGE OF POPULATION d_i[1]	(3) PRETAX INCOME ($/YEAR)[2]	(4) HOUSEHOLD SIZE[1]	(5) AUTOS/ HOUSEHOLD[2]	(6) POSTTAX WAGE ($/MINUTE)[3]	(7) DRIVERS/ HOUSEHOLD[4]	(8) AUTOS/ DRIVERS[5]
1	.29	5,000	2.39	.58	.040	2.04	.28
2	.29	10,000	2.92	.60	.077	2.02	.30
3	.15	17,500	3.09	.91	.129	2.06	.44
4	.11	22,500	3.33	1.00	.163	2.12	.47
5	.16	35,000	3.50	1.88	.222	2.16	.87

[1] SOURCE: U.S. CENSUS, CURRENT HOUSING REPORTS (H-170-75-39), 1975.

[2] ASSUMED.

[3] BASED ON AVERAGE UNITED STATES (STATISTICAL ABSTRACT OF THE UNITED STATES, 1978, TABLE 427) AND CALIFORNIA (HOACHLANDER, 1979) TAX RATES, ASSUMING ALL WORK 40 HOURS/WEEK, 50 WEEKS/YEAR. CALCULATED FROM COLUMN (3).

[4] SEE TEXT.

[5] RATIO OF COLUMNS (5) AND (7).

probability that the remaining members are over sixteen (and hence eligible for a driver's license). Based on 1970 Census of Population data, 72% of children were over sixteen in 1970, and based on data for California, we take the probability of having a driver's license, given that one is of age, to be 90% (U.S. Federal Highway Administration, 1972).[15]

5. Competition

■ I now present some results of the intermodal competition model for several cases of interest.[16] The approach taken here is to make the size of the market (the number of trips from origin to destination in a given peak hour) a parameter, and then to analyze the competitive possibilities dependent on that market size. Recall that the model is a *route* model: with three East Bay routes, a given market size is equivalent, assuming equal demands on each line, to a system-wide market three times that size. Also note that although the term "competition" is used, in fact there can be a substantial exercise of market power. Each of the two transit modes perceives a downward sloping demand curve and, with price- and quality-taking consumers, sets fare and level-of-service to maximize profit. To the extent that this results in large profits, some taxing or regulatory authority would be needed to balance social and private benefits. However, this question is not discussed here. I focus only on the nature of the competition between the two operators and leave open the knotty question of the distribution of gains to market power. Note too that we focus on peak

[15] The actual probabilities ranged from .81 to 1.0 for the population 18 years of age and older (the working population of interest here).

[16] In this section, results on waiting and access times are computed, using the data in the tables presented and the following relationships: waiting time = headway/2; access time = (feeder radius) π/(6 number of routes). The numerical analysis used the ACDPAK algorithm, written by M.J. Best of the University of Waterloo. See Best (1975) and Best and Ritter (1974) for properties. Work was performed on the DEC-10 computer at the Wharton School, University of Pennsylvania.

travel: to the extent that offpeak service is required, peak-period profits might be used to subsidize it.

Finally, a technical comment is required. Recall that we shall discuss only equilibria and not the interim points on the reaction functions. Roberts and Sonnenschein (1977) have pointed out the possibility of multiple equilibria in complex models; and indeed, no proof of uniqueness is attempted here. However, in light of the findings below, it is the *existence* of profitable equilibria that is important.

☐ **BART with free choice of train size.** The competitive possibilities are shown in Table 5. When there are 1,000 commuters per hour, neither transit mode covers its variable costs, as indicated by the blank entries in the table. But the striking fact is that when the BART system can adjust the size of its trains, it can compete at any travel volume exceeding 2,500 commuters per hour.[17] The BART-plus-feeder system always chooses to provide a free feeder service to the trains; and this is provided at headways less than those of the competitive bus system. The BART system, too, provides significantly greater coverage of the residential area than does the bus system. For example, in the case of 10,000 hourly commuters, 25 routes are provided by the feeder system, as opposed to 16 by buses.

There is product differentiation since the combined BART plus feeder system is perceived to provide better service at a higher price. Comparing the two modes at a travel volume of 10,000 commuters per peak hour, access time to the bus system is 25 minutes, while to the feeder system it is 4.9 minutes. Average waiting times are 5.5 minutes for the express bus option, for BART three minutes when access is by car, and seven minutes when the feeder bus is used.

TABLE 5

FREE CHOICE OF BART TRAIN SIZE[1]

CORRIDOR TRAVELLERS PER PEAK HOUR	INTEGRATED BUS				BART + FEEDER BUS						
					FEEDER SYSTEM			BART			MARKET SHARE
	HEADWAY (MINS.)	ROUTES	FARE ($)	MARKET SHARE	HEADWAY (MINS.)	ROUTES	FARE ($)	HEADWAY (MINS.)	FARE ($)	CARS	
1,000	--	--	--	--	--	--	--	--	--	--	--
2,500	21	8	1.71	25%	12	16	0.0	15	2.23	2	32%
5,000	15	12	2.00	33%	10	20	0.0	8	2.81	2	31%
7,500	13	14	2.19	38%	9	23	0.0	6	3.23	2	27%
10,000	11	16	2.30	40%	8	25	0.0	6	3.39	2	25%

[1] MINIMUM FEASIBLE TRAIN SIZE IS TWO CARS.

[17] In all the results presented here, answers are rounded as follows: headways to the nearest minute; number of routes to the nearest integer; fares to the nearest cent; mode share to the nearest 1%. A single mode share is presented, averaged over all users, though the analysis is done in terms of the individual groups of travelers shown in Table 4. An equilibrium was attained when the mode choices of the individual groups were stationary to an error of 1%. In every case, equilibrium was attained within 3 complete iterations. The initial point for all problems was: buses per hour, 40; bus routes, 10; BART fare, $1.50; BART train size (when not constrained), 2 cars; bus fare, $1.50.

What this means is reflected in the distribution of demands over the various income groups. If we consider the lowest and highest income classes (groups 1 and 5, respectively) the following pattern emerges.[18] First, auto use differs considerably: 39% of group 1 use the car, while 62% of group 5 do so. Second, substantial differences emerge in public transit mode use as well. The low-income commuters overwhelmingly use the bus—78% of their transit travel takes place on this mode with virtually all (96%) walking to the system. By contrast, 65% transit travel by the high-income commuters is on BART, with only 30% of that population electing to use the feeder service. With high values of time, the members of group 5 tend to use their cars to get to the BART system.

These differences in travel patterns are also reflected in the fares charged. With 10,000 commuters per peak hour, the bus fare is $2.30, while the BART system charges $3.39. Clearly, the wealthy consumers are willing to pay for the superior service option BART offers. This is in accord with the observations discussed by Morlok and Viton (1980).

And one conclusion stands out clearly. If it is desired to operate BART in a way that covers variable costs (and even makes a contribution, through the exploitation of market power, to capital costs), then it appears best to regard the station size as sunk investment and run small trains frequently. If this is done, cost-covering operations are feasible even at quite low corridor travel volumes.

□ **Fixed ten-car BART trains.** BART's original plan for full operation called for the scheduling of ten-car trains in the peak period and five-car trains in the offpeak period (Merewitz, 1975, p. 23). By 1975 this level of operation had not been reached. As Merewitz (1975) reports, the average peak train size ranged from seven cars on the Concord-Daly City line to four in Richmond-Fremont service. The eventual aim, however, is still to run ten-car peak trains, and therefore this case is worth analyzing.

The outcome is easily described. If the aim of BART were to minimize loss with no constraints on the level of service (so that zero variable costs are incurred at zero output), then only at high levels of corridor travel—approximately 10,000 or more per peak hour—should service be provided at all. In cases where travel volumes range from 2,500 to 7,500 per peak hour, only the integrated (express) bus system covers its costs.[19] In these situations, the best strategy is to close down BART operations. It is true that the capital costs of the system will still be incurred; but at least no operating losses will accrue. When travel volumes are low—1,000 per peak hour or less—no public mode is able to cover its variable costs.

In the single case where BART is competitive with the bus service and the private car, there is not much product differentiation between the public modes. Both public modes provide expensive but high-quality service at roughly comparable travel times and fares. The BART system charges a fare of $2.03 and provides free feeder bus service which covers the residential area more completely than the comparable "integrated bus" service. The latter charges more for service ($2.17), a reversal made possible by the more flexible level of service it provides. The result is that BART and the bus system roughly

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[18] This discussion draws on group-specific data not shown in Table 5. See footnote 17.

[19] Of course, this assumes that AC Transit behaves in Cournot fashion. See Section 5 for BART possibilities when AC behaves otherwise.

split the transit market (carrying 32% and 37%, respectively) at approximately equal service levels. In this respect the calculated equilibrium is not unlike the situation now observed. But, of course, to cover costs, both modes exploit their market power in the equilibrium presented here.

□ **A public-utility bus system.** It can be argued that the results presented thus far do not fairly indicate the potential for BART, for two reasons. First, the model discussed depicts the integrated bus system as a profit-maximizing duopolist, and this does not adequately capture the motivations of AC Transit. The second reason is derived from the first: when BART was being planned, it was reasonable to view the possibilities of covering variable costs in the light of the prevailing system of bus transit in which good but money-losing service was the rule. As pointed out in Section 1, however, BART has not in general moved to exploit any opportunities here, since service levels are quite similar to those of the competing linehaul bus. The question of what BART could do in the face of a linehaul bus system with no cost-covering constraint is nonetheless interesting.

With free choice of fares and service levels, the bus system's best action is to shut down when variable costs cannot be covered. This does not hold when the choice variables are constrained, because satisfaction of the constraints may be possible only by losing money. For this case, the model gives the loss-minimizing level of service, subject to satisfaction of the constraints.

To discuss a bus system operating as a subsidized public utility, we consider two sets of constraints on the bus system. The first is the fare. To the extent that BART planners took the competing bus system into account, it is likely that they assumed that the basic low-fare service would continue. The second set of constraints directly affects the quality of bus service offered. Rather than put constraints directly on the two remaining choice variables (bus headway and route coverage), we impose a user-perceived service constraint: average wait-plus-access time must be less than some specified amount. This leaves the bus system free to adjust headways and route coverage to lower costs as much as possible, just as long as this basic service is provided.

We therefore make two assumptions about the nature of the bus system. The first is that the fare be set at 1975–1976 levels, that is, \$.65 per rider. We also assume that the average wait-plus-access time for bus users must be less than 20 minutes. The question of interest is whether, given this bus competition, BART can still cover variable costs; and we assume that BART can freely select its train size.

Table 6 shows the result. Two general features are of interest. First, of course, the bus system always provides service, though it never covers its costs. Second, as far as mode shares are concerned, the bus system carries a significantly larger share of the traffic.

Turning to the details of the service, note that the bus system is in fact operating under three constraints, the third being the service constraint common to all the situations studied, that all passengers be seated. This, in conjunction with the other two constraints (\$.65 fare, wait-plus-access time less than 20 minutes), accounts for the rather different service characteristics at different travel volumes. As the number of travelers increases, it becomes progressively easier to run full buses and maintain the required level of service. At large travel volumes the number of people who want to ride the \$.65 express

TABLE 6

CONSTRAINED BUS SYSTEM FARE = $.65, WAIT AND ACCESS TIME ≤ 20 MINUTES

CORRIDOR TRAVELLERS PER PEAK HOUR	INTEGRATED BUS				BART + FEEDER BUS						
					FEEDER SYSTEM			BART			
	HEADWAY (MINS.)	ROUTES	FARE ($)	MARKET SHARE	HEADWAY (MINS.)	ROUTES	FARE ($)	HEADWAY (MINS.)	FARE ($)	CARS	MARKET SHARE
1,000	21	7	.65	49%	--	--	--	--	--	--	--
2,500	22	7	.65	37%	13	15	0.0	18	2.03	2	26%
5,000	35	25	.65	43%	10	20	0.0	9	2.79	2	25%
7,500	37	48	.65	52%	--	--	--	12	2.29	2	12%
10,000	3	3	.65	39%	--	--	--	8	2.62	2	15%

[1] MINIMUM FEASIBLE TRAIN SIZE IS TWO CARS.

bus is large: to accommodate them seated, many buses must be run. But one can reduce the number of routes to make service less attractive, and this is what happens.[20]

The BART and feeder service is unremarkable. At all travel volumes over 2,500 per hour BART is able to distinguish its high-fare service from the low-fare, relatively time-costly bus service, and cover its costs. Again the feeder service is free; BART's share is between 12 and 26%. But the central result still holds: BART can still adjust its service to cover variable costs, even when competing with a subsidized public transit mode. It does not appear that BART's failure to cover its operating costs from the farebox can be ascribed solely to the nature of the competition it faces.

6. Conclusions

■ This article has examined the price and product-space choices of Bay Area transit duopolists. The duopolists, who are also competing against an outside good supplied competitively (the private car), set both price and service characteristics according to the classical rule of zero conjectural variation. Two distinct regimes were examined. First, both transit suppliers were constrained to cover all operating costs from the farebox. This induced product differentiation, but at the same time fares considerably higher than those currently observed. Since this result might be unacceptable as a matter of policy, the article also examined the possibility of BART's continuing to cover operating costs from the farebox while the competing bus system was under no constraint to cover its costs. In this case, the bus system provided a money-losing but popular service, much the way it now does. This second case is of particular interest, since the legislation establishing BART required it to cover operating costs; but AC Transit was under no such statutory requirement.

The results of the model indicate, surprisingly, that neither bus nor rapid-rail transit need always be deficit systems. Furthermore, by appropriately differentiating their products, both services are viable even when directly com-

[20] Why between 7,500 and 10,000 commuters per peak hour the bus service provided to satisfy the wait-plus-walk constraint changes so suddenly is not clear.

peting. The principal operations change in BART would appear to be that train size be tailored more closely to the size of the traveling population: with shorter trains, optimal scheduling considerations call for smaller headways which have the added advantage of making the mode more attractive to consumers. In addition, the model solutions indicate that there are benefits to BART in rethinking the concept of feeder service to the relatively widely spaced train stations.

Even if a bus system not constrained to cover costs is retained, some will no doubt object to the segmented nature of the transit service the model predicts, with the new BART system attracting the wealthy and leaving those less well off to ride the buses. This is to a large extent a judgment of applied social welfare, but it may be worth pointing out that such product differentiation is not necessarily revealed as offensive to our notions of public policy. For example, on its Northeast Corridor service Amtrak provides both coach and club-car service. This is an example of what may be thought of as a quasi-public utility providing precisely the sort of high cost/high level of service and lower cost/poorer service product differentiation suggested here as viable in urban transportation.

Transit deficits in the United States increased by over 700% between 1970 and 1978. The anticipated reluctance of governmental officials to see this trend continue suggests that the possibilities of even limited competition and product differentiation explored in this article should at least be studied. Transit does not have to be a drain everywhere on the public purse.

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