

LBNL-
UCB-PTH0-11/07
September 2011

BRST Invariant PV Regularization of SUSY Yang-Mills and SUGRA*[†]

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Abstract

Pauli-Villars regularization of Yang-Mills theories and of supergravity theories is outlined, with an emphasis on BRST invariance. Applications to phenomenology and the anomaly structure of supergravity are discussed.

*This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC02-05CH11231, in part by the National Science Foundation under grant PHY-0457315.

[†]To be published in the proceedings of “Raymond’s 80th Birthday Party”, LAPP, Annecy-le-Vieux, July 8, 2011.

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Since I never collaborated on a paper with Raymond, I chose a topic that at least allowed me to put his initial in the title. I have been working for a number of years on Pauli-Villars (PV) regularization of supersymmetric theories and its applications, and I often get the question “Aren’t you breaking BRST [1] invariance?” In the following I will explain how the miraculous cancellations among boson and fermion loops in supersymmetry (SUSY) allow for the complete elimination of ultraviolet (UV) divergences by the introduction of only chiral supermultiplets and, in the case of local supersymmetry (supergravity or SUGRA) Abelian gauge supermultiplets. I will also describe some applications to phenomenology, and will discuss conformal and chiral anomalies in supergravity, and their cancellation in the context of effective theories from compactification of the weakly coupled heterotic string (WCHS).

1 SUSY Yang-Mills with chiral matter

A renormalizable, globally supersymmetric, theory is defined by two types of chiral superfields: Z^i for matter, with components $(z^i, \chi_\alpha^i, F^i)$, and the Yang-Mills (YM) superfield strengths W_α^a with components $(\lambda_\alpha^a, F_{\mu\nu}^a, D^a)$, where α is a Dirac index, i, a denote internal quantum numbers, and F^i, D^a are auxiliary fields. The theory is further defined by the superpotential:

$$W(Z) = \frac{1}{2}\mu_{ij}Z^iZ^j + \frac{1}{6}c_{ijk}Z^iZ^jZ^k, \quad (1.1)$$

and gauge transformation properties of the matter supermultiplets:

$$\delta_a Z^i = i(T_a Z)^i, \quad \delta_a Z^{\bar{m}} = -i(T_a^T Z)^{\bar{m}}. \quad (1.2)$$

There are no quadratic UV divergences; these are determined by the supertrace of the (field-dependent) mass matrix¹ which vanishes identically in this theory:

$$\text{STr}M^2(z^i, \bar{z}^{\bar{m}}, F_{\mu\nu}^a) = \sum_{S=0, \frac{1}{2}, 1} (-1)^{2S} (2S+1) M_S^2 = 0 \quad (1.3)$$

¹Throughout I use background field techniques and set fermions to zero in the background; the one-loop effective fermionic Lagrangian can be inferred from the bosonic result by supersymmetry.

The only logarithmic UV divergences are in wave function renormalizations. In particular the β -function is proportional to the parameter

$$b_a = -\frac{1}{16\pi^2}(3C_a - C_a^M) = g^{-3}(\mu)\frac{\partial g_a(\mu)}{\partial \ln \mu} = g_a^{-2}(\mu)\beta_a(\mu), \quad (1.4)$$

where C_a and C_a^M are quadratic Casimirs in the adjoint and matter representations, respectively. The superpotential (1.1) is not renormalized; with a “supersymmetric” choice [2] of gauge fixing (to be made explicit in the next section), the UV divergent contribution to the scalar potential is

$$\Delta V = \frac{1}{64\pi^2} \text{STr} M^4 \ln \Lambda^2 = -\frac{1}{2} \sum_a b_a D_a^2 \ln \Lambda^2, \quad (1.5)$$

which is just the supersymmetric completion of the vacuum polarization. In this gauge the anomalous dimension (matrix) for chiral superfields Z^i is given by

$$32\pi^2 \gamma_i^j = -4g^2 \sum_a C_2^a(r^i) \delta_i^j + \sum_{kl} c_{kli} c^{klj}, \quad C_2^a(r) = \frac{\dim a}{\dim r} C_a^r. \quad (1.6)$$

The logarithmic divergences of this theory can be canceled [3] by adding chiral PV supermultiplets Z^I, Y_I, φ^a with gauge transformation properties

$$\delta_a Z^I = i(T_a^I Z)^I, \quad \delta_a Y_I = -i(Y T_a)_I, \quad \delta_a \varphi^b = f^{abc} \varphi_c, \quad (+ \text{ other reps}), \quad (1.7)$$

where f^{abc} is a structure constant of the gauge group, and superpotential couplings

$$W_{\text{PV}} = \frac{1}{2} (\mu_{ij} + c_{ijk} Z^k) Z^I Z^J + \sqrt{2} g \varphi^a (T_a Z)^i Y_I, \quad (1.8)$$

leaving BRST unbroken. Cancellation of (one-loop) UV divergences is assured *provided*

$$C_a^M = \left(\text{Tr} T_a^2 \right)_{\text{matter}} = \text{Tr} (T_a^R)^2 \quad (1.9)$$

for some real representation R because one has to give gauge invariant masses to all the PV fields, and therefore they must form an overall real (reducible) representation which cancels the matter contribution to (1.4) – hence the “other reps” in (1.7). These additional chiral multiplets do not have any superpotential couplings to the light chiral supermultiplets Z^i . The condition (1.9) is satisfied in the minimal supersymmetric extension of the Standard Model (MSSM) and its extensions, as well as in the hidden sectors [4] of such extensions from all Z_3 orbifold compactifications of the heterotic string for which the full spectrum is known.

2 Supergravity

Supergravity is defined by the superpotential $W(Z^i)$, which is now an arbitrary function of Z , the real Kähler potential $K(Z^i, \bar{Z}^{\bar{m}})$ and the gauge kinetic function $f_{ab}(Z^i)$. Here I will assume that f is diagonal:²

$$f_{ab}(Z^i) = f(Z^i)\delta_{ab} \quad f(Z^i)| = f(z^i) = x + iy, \quad (2.1)$$

which is the case for supergravity from the heterotic string. To obtain the one-loop effective (bosonic) Lagrangian [5], we expand the action (covariantly) around a bosonic background, and integrate over quantum fluctuations $h_{\mu\nu}, \hat{A}_\mu^a, \hat{z}^i$ in the graviton, Yang-Mills and scalar fields, as well as fermions, ghosts and an auxiliary field α that is used to implement the gravitino gauge fixing. For the bose sector we use smeared gauges, defined by:

$$\mathcal{L}_{gf} = -\frac{\sqrt{g}}{2} (G_a G^a - G_\mu G^\mu) \quad G_a = \frac{1}{\sqrt{x}} \left[\mathcal{D}_\mu (x \hat{A}_a^\mu) + i K_{i\bar{m}} (T_a^{\bar{m}} \hat{z}^i - T_a^i \hat{z}^{\bar{m}}) \right] \quad (2.2)$$

$$G_\mu = \frac{1}{\sqrt{2}} \left[\nabla^\nu h_{\mu\nu} - \frac{1}{2} \nabla_\mu h_\nu^\nu - 2 \left(\mathcal{D}_\mu \bar{z}^{\bar{m}} K_{i\bar{m}} \hat{z}^i + \text{h.c.} \right) + 2x F_{\mu\nu}^a \hat{A}_a^\nu \right], \quad (2.3)$$

while for the gravitino ψ_α^μ we use an unsmeared gauge

$$G = -\gamma^\mu (i \mathcal{D} - \bar{M}) \psi_\mu - 2K_{i\bar{m}} \left[(\mathcal{D} \bar{z}^{\bar{m}} + iF^{\bar{m}}) \chi^i + (\mathcal{D} z^i + iF^i) \chi^{\bar{m}} \right] + \left(\frac{x}{2} \sigma^{\mu\nu} F_{\mu\nu}^a + \frac{1}{x} \gamma_5 D^a \right) \lambda_a = 0,$$

$$\delta(G) = \int d\alpha \exp(i\alpha G). \quad (2.4)$$

The choice (2.2) is the generalization to SUGRA of the ‘‘supersymmetric gauge’’ mentioned in Section 1. We drop terms that vanish by virtue of the tree equations of motion; much of this can be done *a priori* by adding a judicious choice of such terms to the inverse propagators. With the above gauge fixing procedures, the one-loop action takes the form

$$S_1 = \frac{1}{2} i \text{STr} [D^\mu D_\mu + H(g_{\mu\nu}, F_{\mu\nu}, z)] + T_-(g_{\mu\nu}, F_{\mu\nu}, z), \quad (2.5)$$

where $D_\mu(g_{\mu\nu}, F_{\mu\nu}, z)$ is a generalized covariant derivative, H a generalized squared mass matrix, and T_- is the helicity-odd fermion contribution. The explicit expression for H is invariant under all the symmetries of the SUGRA theory.

²The notation $X|$ stands for the lowest component (with the superspace coordinate $\theta = 0$) of the superfield X , with all fermion fields set to 0.

2.1 SUGRA with chiral matter

In evaluating the one-loop quadratic divergences for supergravity coupled to chiral matter [5], we use the trace of the graviton equation of motion:

$$g_{\mu\nu} \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} \Big|_{\text{tree}} = \frac{r}{2} - 2V + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} = 0, \quad (2.6)$$

which is equivalent to a metric redefinition that restores the Einstein term to canonical form in this order. Here V is the scalar potential

$$V = K_{i\bar{m}} F^i \bar{F}^{\bar{m}} - 3M\bar{M}, \quad M = e^{K/2} W(z) = M_\psi, \quad K_{i\bar{m}} = \frac{\partial^2 K}{\partial z^i \partial \bar{z}^{\bar{m}}}, \quad (2.7)$$

and M is an auxiliary field³ of the supergravity supermultiplet; its *vev* is the gravitino mass M_ψ . The one-loop quadratic divergences are determined by the sum of the supertraces from the gravity sector and the chiral matter sector, which, using (2.6), are given by

$$\text{STr} H_{\text{grav}} = -14|M|^2 + K_{i\bar{m}} \left(4F^i \bar{F}^{\bar{m}} - 3\mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} \right), \quad (2.8)$$

$$\text{STr} H_\chi = N_\chi \left(2|M|^2 + K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} \right) - 2R_{i\bar{m}} \left(F^i \bar{F}^{\bar{m}} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} \right), \quad (2.9)$$

where $R_{i\bar{m}}$ is the Ricci tensor derived from the Kähler metric $K_{i\bar{m}}$, and N_χ is the number of chiral supermultiplets. To cancel the quadratic divergences we add the following PV superfields [8]:

- chiral superfields Z^I with Kähler metric $K_{I\bar{M}} = K_{i\bar{m}}$ and signature $\eta_I = -1$,
- chiral superfields ϕ^α with Kähler metric $K_{\alpha\bar{\beta}} = \delta_{\alpha\bar{\beta}} e^{\alpha K}$,
- Abelian $U(1)$ vector fields W_α^n and $U(1)$ -charged chiral fields e^{θ^n} which together form massive vector fields by virtue of the superhiggs mechanism.

These give contributions

$$\text{STr} H_\chi^{PV} = N'_\chi \left(2|M|^2 + K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} \right) + 2(R_{i\bar{m}} - \alpha K_{i\bar{m}}) \left(F^i \bar{F}^{\bar{m}} + \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} \right), \quad (2.10)$$

$$\text{STr} H_{W_\alpha}^{PV} = N'_G \left[K_{i\bar{m}} \left(2F^i \bar{F}^{\bar{m}} - \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} \right) - 6|M|^2 \right], \quad (2.11)$$

where

$$N'_\chi = \sum_C \eta_{\phi_\chi^C}, \quad \phi_\chi^C = Z^I, \phi^\alpha, \theta_n, \quad N'_G = \sum_n \eta_{W_\alpha^n}, \quad \alpha = \sum_\beta \eta_{\phi^\beta} \alpha_\beta. \quad (2.12)$$

Cancellation of quadratic divergences is achieved with

$$N'_\chi = 3\alpha + 1 - N_\chi, \quad N'_G = \alpha - 2. \quad (2.13)$$

³The normalization for M used here differs by a factor $-\frac{1}{3}$ from the usual one [6, 7].

Full cancellation of logarithmic divergences imposes an additional constraint, giving [9]

$$N'_\chi = -29 - N_\chi, \quad N'_G = -12, \quad \alpha = -10. \quad (2.14)$$

It also requires the introduction of additional PV chiral superfields Y_I with Kähler metric $K_Y^{I\bar{M}} = K^{i\bar{m}}$, $K^{i\bar{m}}K_{j\bar{m}} = \delta_j^i$, as well as several copies of the Z^I with alternating signatures. All of these are included in the definition of N'_χ in (2.12).

2.2 SUGRA with YM and chiral matter

Now we add to the theory Yang-Mills superfields with canonical kinetic energy terms:

$$f(z^i) = x + iy = g^{-2} - i\frac{\theta}{8\pi} = \text{constant}. \quad (2.15)$$

If the chiral multiplets have gauge couplings as in (1.2), the potential acquires a D-term

$$\Delta V = \frac{1}{2}x D^a D_a, \quad D_a = K_i(T_\alpha z)^i = K_{\bar{m}}(T_\alpha^T \bar{z})^{\bar{m}}, \quad K_i = \frac{\partial K}{\partial z_i}, \quad K_{\bar{m}} = \frac{\partial K}{\partial z_{\bar{m}}}. \quad (2.16)$$

The PV superfields Z^I now transform as in (1.7), and the supertraces (2.8), (2.10) and (2.11) get the additional contributions [10]

$$\Delta \text{STr} H_\chi = -N_\chi x D^a D_a + 2D_a \left[\Gamma_{ij}^i (T^a z)^j + (T^a)_i^i \right], \quad (2.17)$$

$$\Delta \text{STr} H_\chi^{PV} = (2\alpha - N'_\chi) x D^a D_a - 2D_a \left[\Gamma_{ij}^i (T^a z)^j + (T^a)_i^i \right], \quad (2.18)$$

$$\Delta \text{STr} H_{W_\alpha}^{PV} = N'_G x D_a D^a, \quad (2.19)$$

where Γ_{jk}^i is the ‘‘affine connection’’ associated with the Kähler metric. In addition, there is an off-diagonal mass term connecting the gaugino to the auxiliary field α introduced in (2.4):

$$M_{\alpha\lambda^a} = -\sqrt{x} \left(D_a + \frac{1}{2} F_a^{\mu\nu} \sigma_{\mu\nu} \right) = -\sqrt{x} \left[D_a + \frac{1}{2} \left(\beta F_a^{\mu\nu} + i\gamma\gamma_5 \tilde{F}_a^{\mu\nu} \right) \sigma_{\mu\nu} \right], \quad \beta + \gamma = 1. \quad (2.20)$$

The second equality in (2.20) follows from the properties of Dirac matrices; it illustrates the ambiguity in defining γ_5 that is present in any regularization procedure. The ‘‘supersymmetric’’ choice is

$$\beta = 1, \quad \gamma = 0. \quad (2.21)$$

With this choice (2.20) matches off-diagonal squared masses that couple $h_{\mu\nu}$ to \hat{A}_μ and the graviton ghost c_μ to the YM ghosts c_a , and it allows for BRST invariant PV regularization. The Yang-Mills sector gives a contribution

$$\begin{aligned} \text{STr} H_{\text{YM}} &= (1 + N_G) x D_a D^a + \frac{1}{2} x F_{\mu\nu} F^{\mu\nu} \\ &\quad + N_G \left[K_{i\bar{m}} \left(2F^i \bar{F}^{\bar{m}} - \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} \right) - 6|M|^2 \right], \end{aligned} \quad (2.22)$$

and (2.9) gets an additional contribution

$$\Delta \text{STr} H_{\text{grav}} = 2x D_a D^a - \frac{1}{2} x F_{\mu\nu} F^{\mu\nu}. \quad (2.23)$$

The terms containing the YM field strength cancel, and all UV divergences can be canceled [8] provided N'_G in (2.13) and (2.14) is shifted by the amount

$$\Delta N'_G = -N_G. \quad (2.24)$$

Full cancellation of logarithmic divergences [9] requires including the chiral superfields Y_I , with Kähler metric as in Section 2.1 and gauge charges as in (1.7), the chiral superfields φ^a in the adjoint representation of the gauge group that were introduced in Section 1, as well as additional copies of these and other chiral superfields, such that, in particular, (1.9) is satisfied.

2.3 Including the dilaton

Finally, we include a nontrivial gauge kinetic function:

$$f(z^i) = f(Z^i) \Big| = x(z^i) + iy(z^i), \quad \langle x(z^i) + iy(z^i) \rangle = g^{-2} - i \frac{\theta}{8\pi}, \quad f_i = \partial_i f \neq 0. \quad (2.25)$$

This introduces [10] an additional off-diagonal mass term that mixes gauginos with the fermion superpartner of the dilaton $f(z^i)$:

$$\Delta M_{\chi^i \lambda^a} = -i \frac{f_i}{2\sqrt{x}} \left[D_a + \left(\beta F_a^{\mu\nu} + i\gamma \gamma_5 \tilde{F}_a^{\mu\nu} \right) \sigma_{\mu\nu} \right], \quad \beta + \gamma = 1. \quad (2.26)$$

In this case the ‘‘supersymmetric’’ choice is

$$\beta = \gamma = \frac{1}{2}, \quad (2.27)$$

which matches a squared mass term that couples the dilaton to the Yang-Mills fields, and BRST invariant PV regularization is again possible. The YM field strength terms vanish identically in the squared masses, *e.g.*,

$$\left| \Delta M_{\chi^i \lambda^a} \right|^2 = \frac{f_i \bar{f}^i}{4x} D_a D^a, \quad \bar{f}^i = K^{i\bar{m}} \bar{f}_{\bar{m}}, \quad (2.28)$$

and the new contribution to the chiral multiplet supertrace is

$$\Delta \text{STr} H_\chi = \frac{f_i \bar{f}^i}{4x} D^a D_a. \quad (2.29)$$

There is also an additional term in the gaugino connection:

$$\Delta A_\lambda^\mu = -\frac{\partial^\mu y}{2x} \left(i\delta\gamma_5 - \epsilon \frac{\epsilon^{\tau\nu\rho\sigma}}{24} \gamma_\tau \gamma_\nu \gamma_\rho \gamma_\sigma \right), \quad \delta + \epsilon = 1. \quad (2.30)$$

We choose:

$$\delta = 0, \quad \epsilon = 1, \quad \Delta A_\mu^\lambda = 2x h^{\nu\rho\sigma} \gamma_{[\mu} \gamma_\nu \gamma_\rho \gamma_{\sigma]}, \quad (2.31)$$

where $h_{\mu\nu\rho}$ is the three-form, dual to the axion y , that comes from compactification of the ten dimensional supergravity limit of the heterotic string. With the choice (2.31) the connection (2.30) is a vector current. There is no associated anomaly, the QCD vacuum angle θ is not renormalized, in agreement with earlier results [11], and the modified linearity condition is respected [12] at one-loop order in the dual linear multiplet formulation for the dilaton supermultiplet. Specifically, in the effective supergravity theory from the WCHS, the dilaton supermultiplet $f(Z) = S(s, \chi_\alpha^s, F^s)$ is dual to a (modified) linear supermultiplet $L(\ell, \chi_\alpha^\ell, b_{\mu\nu})$, where $b_{\mu\nu}$ is a two-form whose curl is the three-form $h_{\mu\nu\rho}$ in (2.31). With the choices (2.27) and (2.31), the new contributions to the YM supertrace are

$$\Delta \text{STr} H_{\text{YM}} = -\frac{f_i \bar{f}^i}{4x} D_a D^a - \frac{N_G}{2x^2} \left[f_i \bar{f}_{\bar{m}} F^i \bar{F}^{\bar{m}} + (\partial_\mu x \partial^\mu x + \partial_\mu y \partial^\mu y) \right]. \quad (2.32)$$

The D-terms in (2.29) and (2.32) cancel, and we obtain an overall contribution

$$\Delta \text{STr} (H_\chi + H_{\text{YM}}) = -\frac{N_G}{2x^2} \left[f_i \bar{f}_{\bar{m}} F^i \bar{F}^{\bar{m}} + (\partial_\mu x \partial^\mu x + \partial_\mu y \partial^\mu y) \right]. \quad (2.33)$$

This contribution can be canceled by adding [8] chiral PV multiplets π^α with Kähler metric $K(\pi, \bar{\pi}) = (f + \bar{f})|\pi|^2$ and/or by coupling [13] some Abelian gauge PV multiplets to the dilaton, that is, by setting $f_{W_\alpha^n} = e_n f(Z)$. Cancellation of (2.33) requires

$$N_\pi - e = N_G \quad N_\pi = \sum_\alpha \eta_{\pi^\alpha} \quad e = \sum_n \eta^n e_n. \quad (2.34)$$

Cancellation of logarithmic divergences requires [13] the second mechanism:

$$N_\pi = 0 \quad e = -N_G. \quad (2.35)$$

Large PV masses for the chiral superfields $Z^I, Y_I, \varphi^a, \phi^a$, as well as those needed to assure that the condition (1.9) is satisfied, are generated by including gauge invariant bilinears of these superfields in the superpotential, and large PV masses for the (W_α^n, θ^n) arise from the Abelian superhiggs mechanism. The squared cut-off in the UV divergent terms are replaced by the relevant squared PV masses, and one obtains an expression of the form

$$\mathcal{L}_{\text{tree} + 1\text{-loop}} = \mathcal{L}_{\text{tree}}(g_{\mu\nu}^R, K^R, g_a^R) + \text{operators dim} \geq 6. \quad (2.36)$$

All of the higher dimension terms that cannot be absorbed into renormalizations (denoted by the superscript R) are associated with UV logarithmic divergences.

3 Two Applications

In this section I will describe applications to particle phenomenology of the regularization procedure described above. Both cases illustrate the sensitivity of the scalar potential to the choice of PV masses, which cannot be completely fixed by the requirement of UV finiteness.

3.1 Taming large quadratic divergences

It has been pointed out [14, 15] that the loop suppression parameter

$$\epsilon = \frac{1}{16\pi^2} \quad (3.1)$$

may be compensated by large coefficients, leading to significant effects from loop corrections. Specifically, once (2.6) is imposed, the quadratically divergent correction to the scalar potential includes the terms:

$$V_Q = \frac{1}{2}\epsilon\Lambda^2\text{STr}H_{\text{nonderiv}} \ni \epsilon\Lambda^2 \left[|M|^2 (N_\chi - 3N_G - 7) - N_G M_\lambda^2 - R_{i\bar{m}} F^i \bar{F}^{\bar{m}} \right]. \quad (3.2)$$

Typical WCHS orbifold compactifications have many more chiral multiplets than gauge multiplets: $N_\chi \gtrsim 300$, $N_G \lesssim 65$, so since in many gravity mediated supersymmetry-breaking scenarios the gaugino mass M_λ is much smaller than the the gravitino mass:

$$M_\lambda^2 = \frac{1}{4} f_i \bar{f}^i M^2 \ll M^2, \quad (3.3)$$

the first term in (3.2) suggests the possibility of a significant *positive* contribution to the vacuum energy [14], perhaps curing the problems with classes of models that have negative vacuum energy at tree level. However, in the regulated theory (3.2) is replaced by

$$V_Q \rightarrow \epsilon \left[|M|^2 \left(N_\chi \Lambda_\chi^2 - 3N_G \Lambda_G^2 - 7\Lambda_{\text{grav}}^2 \right) - N_G M_\lambda^2 \Lambda_G'^2 - R_{i\bar{m}} F^i \bar{F}^{\bar{m}} \Lambda_\chi'^2 \right] + \dots, \quad (3.4)$$

where the ellipsis indicates finite terms proportional to M_{PV}^2 such that the one-loop quadratically divergent corrections are completely absorbed into renormalizations:

$$\mathcal{L}_Q = \mathcal{L}_{\text{tree}}(g_{\mu\nu}^R, K^R) - \mathcal{L}_{\text{tree}}(g_{\mu\nu}, K) + O(\epsilon^2), \quad K^R = K + \epsilon \sum_A \Lambda_A^2. \quad (3.5)$$

The effective squared cut-offs Λ_A^2 in (3.4) and (3.5) are determined by the PV masses:

$$\Lambda_A^2 = C_A \left(\eta^i M_i^2 \ln M_i^2 \right)_A, \quad \left(\sum_i \eta_i M_i^2 \right)_A = 0, \quad (3.6)$$

where C_A is a constant. Several remarks are in order [16].

- The sign of Λ_A^2 is indeterminate [17] if there are five or more terms in the sum, which is generally required to eliminate all the UV divergences of SUGRA.
- If $N_\chi \sim \epsilon^{-1}$ one has to sum the leading $(\epsilon\Lambda^2)^n$ terms [16].
- Supersymmetry dictates that the higher order terms complete the Lagrangian $\mathcal{L}_{\text{tree}}(g_{\mu\nu}^R, K^R)$ with K_R given by (3.5).

So, for example, if the M_i^2 are field independent constants, we just get

$$V_Q = e^{K+\Delta K} \left(W_i K^{i\bar{m}} \bar{W}_{\bar{m}} - |W|^2 \right) + \frac{1}{2} x D^a D_a, \quad W_i = \frac{\partial}{\partial z^i} W = -e^{-K/2} K_{i\bar{m}} \bar{F}^{\bar{m}}. \quad (3.7)$$

If, in addition, supersymmetry is broken only by F-terms, $\langle D_a \rangle = 0$, the vacuum energy is just multiplied by a positive constant.

It has also been pointed out [15] that the last term in (3.2) or (3.4) can be significant because it involves a sum over all the chiral supermultiplets. The Kähler potential for the untwisted sector from orbifold compactification of the heterotic string is not known beyond leading (quadratic) order, and could include terms that induce flavor changing neutral current (FCNC) effects in the observable sector. Experimental limits on these effects therefore imply restrictions on the Kähler potential. A sufficient condition [16] for a “safe” Kähler potential is the presence of isometries of the Kähler geometry. For example, the Kähler potential for an untwisted sector n from orbifold compactification takes the form

$$K^n = -\ln(T^n + \bar{T}^{\bar{n}} - \sum_{A=1}^{N_n} |\Phi_n^A|^2), \quad (3.8)$$

which has an $SU(N_n + 1, 1)$ symmetry that is necessarily also a symmetry of the Ricci tensor:

$$R_{i\bar{m}}^n = (N_n + 2) K_{i\bar{m}}^n. \quad (3.9)$$

Alternatively the suppression of FCNC effects can be achieved through a judicious choice of PV masses [16].

3.2 Anomaly mediated SUSY breaking

One-loop contributions to soft supersymmetry breaking parameters for the superpartners of the Standard Model particles can be important, particularly in models where they are suppressed at tree level. If they arise only through loop effects, the mechanism for supersymmetry is referred to as “anomaly mediation”.

The one-loop contribution to gaugino masses m_a is independent of Planck-scale physics, and is completely determined by the properties of the effective low energy (sub-Planck scale) theory. The result is [18, 19, 20]

$$\Delta m_a(\mu) = -3\beta_a(\mu)M - \frac{g^2(\mu)}{14\pi^2} F^j \left[(C_a - C_a^M) K_j + 2 \sum_i C_a^i \partial_j \ln K_{i\bar{i}} \right]. \quad (3.10)$$

The term proportional to the β -function (1.4) is related [21, 22] to the conformal anomaly, in that it arises from the running of the coupling constant from the Planck scale to the scale μ , and has been shown to be exact to all orders in perturbation theory.

Writing the superpotential in the form

$$W(Z) = \sum_{ijk} W_{ijk} Z^i Z^j Z^k + \sum_{ij} \mu_{ij} Z^i Z^j + O(Z^4), \quad (3.11)$$

supersymmetry breaking generates so-called A and B terms in the scalar potential that are, respectively, cubic and quadratic in the scalar fields z^i :

$$V \ni \frac{1}{6} \sum_{ijk} A_{ijk} W_{ijk} z^i z^j z^k + \frac{1}{2} \sum_{ij} B_{ij} \mu_{ij} z^i z^j + \text{h.c.} \quad (3.12)$$

Neglecting small flavor mixing in the anomalous dimension matrix (1.6), the one-loop contributions to the parameters A and B are [20, 23]

$$\Delta A_{ijk}(\mu) = (\gamma_i + \gamma_j + \gamma_k)_\mu M + a_{ijk} \ln(M_{PV}/\mu), \quad (3.13)$$

$$\Delta B_{ij}(\mu) = (\gamma_i + \gamma_j)_\mu M + b_{ij} \ln(M_{PV}/\mu). \quad (3.14)$$

The first term in each expression is the conformal anomaly contribution [22, 21], again valid to all orders in perturbation theory. The (field-dependent) parameters a and b vanish if there are no tree-level soft terms in the observable sector.

In contrast to the above, the supersymmetry-breaking (“soft”) scalar squared masses m_i^2 are strongly dependent on Planck scale physics [20]:

$$\Delta m_i^2 = 9\gamma_i |M|^2 + \nu_i (m_{\text{soft}}^{PV}) + \mu_i, \quad (3.15)$$

where the last term vanishes if there is no tree-level SUSY breaking in the observable sector. The first, “conformal anomaly”, term was not found in earlier analyses [21, 22]; they found instead a universal two-loop contribution proportional to the derivative of the anomalous dimension matrix γ . The second term vanishes only if the tree-level *Pauli-Villars* soft squared masses vanish, which is generally not the case. In the “sequestered sector” model of Randall and Sundrum [22] the

first term is exactly canceled by the second; this requires⁴ a very special form of the hidden sector scalar potential, as well as of PV masses. The spurion analysis [21] missed the second term in (3.15) because of an assumption of holomorphicity that is not borne out by the explicit PV calculation [20]. The sensitivity of soft scalar masses to Planck scale physics can easily be understood in the framework of PV regularization. Superpotential and gauge couplings of light chiral superfields are regulated by the PV fields $\Phi^A = Z^I, Y_I, \varphi^a$, which obtain large PV masses through gauge invariant superpotential couplings to other fields Φ'_A :

$$W_{PV} \ni \mu_A \Phi^A \Phi'_A, \quad m_A^2 = m_{A'}^2 = e^K K_{\Phi}^{A\bar{A}} K_{\Phi}^{\prime\bar{B}B} |\mu_A|^2. \quad (3.16)$$

The finiteness requirement constrains the Kähler metric for Φ^A , but not for Φ'_A , since they need not have any couplings to light sector fields, except for electromagnetic couplings if they carry gauge charges. Since *all* gauge-charged PV fields contribute to the β -functions (1.4), the PV loop contribution to the gaugino masses is uniquely fixed. On the other hand the fields Φ'_A need not have any superpotential couplings to the light fields, so the constraint that the UV divergence associated with the anomalous dimension matrix γ vanishes places no restriction on their Kähler metric, and no restriction on the corresponding PV masses, so Δm_i^2 is undetermined.

There is a parallel situation concerning the Kähler chiral and conformal anomalies associated, respectively, with linear and logarithmic UV divergences. Supergravity is classically invariant [25] under Kähler transformations

$$\begin{aligned} K &\rightarrow K + F(Z) + \bar{F}(\bar{Z}), & W &\rightarrow e^{-F} W, \\ \chi &\rightarrow e^{i\text{Im}F/2} \chi, & \lambda &\rightarrow e^{i\text{Im}F/2} \lambda, & \psi &\rightarrow e^{i\text{Im}F/2} \psi, \end{aligned} \quad (3.17)$$

which are anomalous at the quantum level. The chiral anomaly of the Yang-Mills Lagrangian associated with the phase transformation on the fermions in (3.17) forms an F-term superfield component together with the conformal anomaly associated with the β -functions; this operator is completely fixed by the requirements of UV finiteness and supersymmetry, and is independent of Planck scale physics or the regularization procedure. By contrast, the conformal anomaly associated with the γ -functions is a D-term superfield component, with no chiral counterpart, and depends on the details of the regularization procedure, which in turn should parametrize Planck scale physics. The WCHS is perturbatively invariant under all gauge transformations, as well as a group of transformations on the chiral superfields $Z = T^n, \Phi^i$,

$$T^n \rightarrow f(T^n), \quad \Phi^i \rightarrow g(q_n^i, T^n) \Phi^i, \quad (3.18)$$

⁴It was noted in [24] that this result rests on the assumption that $\langle F^i K_i \rangle$ is negligible.

called T-duality, that effects a Kähler transformation (3.17) with $F = F(T^n)$, with the fields T^n known as “Kähler moduli”, and q_n^i the “modular weights” of Φ^i . However the effective quantum field theory is anomalous under T-duality. The regularized theory is anomaly free if PV mass terms respect the classical symmetries. This is not possible in the case of T-duality, or for an anomalous Abelian symmetry, $U(1)_X$, that is present in almost all realistic theories that break part of the gauge symmetry at the string scale by Wilson loops. For example, the PV superfield ϕ^β gives a contribution to the quadratically divergent one-loop Lagrangian

$$(\mathcal{L}_Q)_{\phi^\beta} \propto \text{STr} H_{\phi^\beta} \ni (1 - 2\alpha^\beta) \left(K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} - x D^a D_a \right) + 2D_a q_X^\beta. \quad (3.19)$$

To obtain an invariant mass, ϕ^β must have a superpotential coupling to another field ϕ^γ with

$$\alpha^\beta + \alpha^\gamma = 1, \quad q_X^\beta + q_X^\gamma = 0, \quad (3.20)$$

such that the contribution from ϕ^γ exactly cancels (3.19). One could instead restore T-duality by making the mass parameters in (3.16) field-dependent: $\mu \rightarrow \mu(T^n)$; this would be interpreted as a threshold correction [26]. However such corrections are known to be absent [27] in, for example, Z_3 and Z_7 orbifold compactifications.

4 Anomalies and anomaly cancellation in supergravity

It has long been known how to cancel the T-duality [28] and $U(1)_X$ [29] anomalies involving Yang-Mills field strength bilinears. The full anomaly structure of PV regulated supergravity has been determined only recently [30]; its detailed form, and therefore the possibility of anomaly cancellation, depends on the choice of PV couplings. It was recently shown [31] that for specific Z_3 and Z_7 compactifications, with no Wilson lines and therefore no anomalous $U(1)_X$, the *string theory* anomaly is completely canceled by the four dimensional version of the Green-Schwarz mechanism [32]. If PV regularization can be a faithful parameterization of the higher string and Kaluza-Klein modes that render the full theory finite, there should be a choice that realizes this result at the field theory level; determining this prescription could in turn restrict the loop corrections to the scalar potential discussed in Section 3.

4.1 Anomalous YM couplings, their cancellation and two applications to phenomenology

Under T-duality and $U(1)_X$, the shift in the YM Lagrangian is given, in the Kähler $U(1)$ superspace formulation [7] of SUGRA, by the expression

$$\Delta \mathcal{L}_{\text{YM loop}} = \frac{1}{8} \sum_a \int d^4\theta \frac{E}{R} \left[\sum_n c_a^n H(T^n) + c_a \Lambda_X \right] (W_a^\alpha W_\alpha^a)_a + \text{h.c.}, \quad (4.1)$$

where

$$c_a^n = \frac{1}{8\pi^2} \left[C_a - \sum_i C_a^i (1 - 2q_n^i) \right], \quad c_{a \neq X} = \frac{1}{4\pi^2} \text{Tr} T_a^2 T_X, \quad c_X = \frac{1}{12\pi^2} \text{Tr} T_X^3. \quad (4.2)$$

The anomaly is canceled by a 4-dimensional version of the Green-Schwartz (GS) mechanism; the dilaton is no longer invariant under these transformations:

$$\Delta S = -bH(T) - c\Lambda_X. \quad (4.3)$$

Then the variation (4.1) is canceled by the variation of the tree Lagrangian:

$$\mathcal{L}_{\text{YM tree}} = \frac{1}{8} \int d^4\theta \frac{E}{R} S \sum_a (W_a^\alpha W_\alpha^a)_a + \text{h.c.}, \quad \Delta \mathcal{L}_{\text{YM tree}} = -\Delta \mathcal{L}_{\text{YM loop}}. \quad (4.4)$$

To make the theory fully invariant, the dilaton Kähler potential $K(S + \bar{S})$ is replaced by

$$K[S + \bar{S} + V(T, \bar{T}) + cV_X],$$

where V_X is the $U(1)_X$ vector superfield:

$$\Delta V_X = \Lambda_X + \bar{\Lambda}_X, \quad (4.5)$$

and the function $V(T, \bar{T})$ satisfies

$$\Delta V = H + \bar{H}. \quad (4.6)$$

The full T -dependence of V is determined by matching [12] string and field theory calculations of the $\text{Im}tF \cdot \tilde{F}$ vertex:

$$V(T, \bar{T}) = - \sum_n \ln(T^n + \bar{T}^{\bar{n}}). \quad (4.7)$$

Anomaly cancellation requires

$$c_a = c = \frac{\text{Tr} T_X}{96\pi^2} \quad \forall a, \quad c_a^n = b \quad \forall a, n \quad (4.8)$$

for compactifications with no threshold corrections, such as Z_3 and Z_7 orbifolds. For those with string loop threshold corrections of the form

$$\mathcal{L}_{\text{th}} = \sum_n \frac{b_a^n}{8} \int d^4\theta \frac{E}{R} f(T^n) \sum_a (W_a^\alpha W_\alpha^a)_a + \text{h.c.}, \quad \Delta f(T^n) = H(T^n), \quad (4.9)$$

the second condition in (4.8) is replaced by

$$b_a^n = b - c_a^n. \quad (4.10)$$

Note that the one-loop calculation yields a supersymmetric anomaly; the higher loop corrections to the β -function are encoded in the PV cut-off demanded by supersymmetry [12, 33, 34]; for example

$$\Lambda_G^2 = e^{K/3} = \left[16(\text{Res}) \prod_n (\text{Ret}^n) \right]^{-\frac{1}{3}} = g^{-\frac{4}{3}} R_{\text{comp}}^{-2} = g^{-\frac{4}{3}} \Lambda_{\text{comp}}^2, \quad (4.11)$$

where the subscript ‘‘comp’’ refers to the compactification radius/scale.

These results have two important applications to phenomenology:

- Matching the coefficient of $F_{\mu\nu}^a \cdot F_a^{\mu\nu}$ to the two-loop RGE invariant [11] of supersymmetric Yang-Mills theories fixes [12, 35] the gauge unification scale; this gives in the \overline{MS} scheme

$$\mu_{\text{unif}}^2 = \frac{m_{\text{string}}^2}{2e} = \frac{g^2 m_{\text{Planck}}^2}{2e} \sim 2 \times 10^{17} \text{GeV}. \quad (4.12)$$

This is an order of magnitude lower than what is obtained by extrapolating from low energy data in the context of the minimal supersymmetric extension of the Standard Model, but in effective theories from superstrings one expects heavy states that are vector-like under the Standard Model gauge group, as well as corrections to the dilaton Kähler potential from string nonperturbative effects and/or field theory loop effects.

- The effective Lagrangian $\mathcal{L}_{\text{eff}}(U_a, \Pi^i)$ for gaugino condensates $U_a \simeq (W_a^\alpha W_\alpha^a)_{\text{hid}}$ and matter condensates $\Pi^i \simeq \prod_A (\phi^A)_{\text{hid}}^{n_A^i}$ in a strongly coupled hidden sector can be constructed by matching [36] the anomalies of \mathcal{L}_{eff} to (4.1), thus providing a mechanism for supersymmetry breaking.

4.2 Full anomaly cancellation?

The linear divergences of supergravity can be canceled by the PV fields introduced in Section 2, except for some from nonrenormalizable terms in the ψ, λ connections. The residual chiral anomalies associated with the latter terms form supersymmetric (F-term) anomalies together with residual conformal anomalies proportional to total divergences, *provided* the cut-off is field-dependent:

$$\Lambda(Z) = e^{K/2} \Lambda_0, \quad \Lambda_0 \rightarrow \infty. \quad (4.13)$$

The resulting effective theory is fully finite, with the remainder of the anomalies arising from a subset of chiral PV superfields with noninvariant masses (3.16), that can be chosen to have a simple Kähler metric. The total anomaly in the regulated theory is then given by [30, 37]

$$\begin{aligned} \Delta\mathcal{L} &= \frac{1}{8\pi^2} \left[\text{Tr}\eta\Phi(T^n, \Lambda_X) \left(\frac{1}{3}\Omega_W + \Omega_{\text{YM}} + \dots \right) + H(T) (\Omega_W + \dots) \right] + \text{h.c.} \\ &= \Delta\mathcal{L}_{\text{YM loop}} + \frac{1}{8} \left(\int d^4\theta \frac{E}{R} \left[\sum_n c_n H(T^n) + c\Lambda_X \right] W^{\alpha\beta\gamma} W_{\alpha\beta\gamma} + \text{h.c.} \right) + \dots \end{aligned} \quad (4.14)$$

The first term in the first line of (4.14) comes from the noninvariant PV masses, and the second from the variation of the effective cut-off:

$$\Delta \ln M_{PV} = \Phi, \quad \Delta K = H + \bar{H}. \quad (4.15)$$

Ω_{YM} and Ω_W are Chern-Simons superfields whose chiral projections are, respectively, the Yang-Mills and curvature superfield strengths:

$$(\bar{D}^2 - 8R)\Omega_{YM} = \sum_{a \neq X} W_a^\alpha W_\alpha^a, \quad (\bar{D}^2 - 8R)\Omega_W = W^{\alpha\beta\gamma} W_{\alpha\beta\gamma}. \quad (4.16)$$

The terms proportional to Ω_{YM} are just those found in (4.1), and can be canceled as in Section 4.1. In the case where threshold corrections are present, these can be included by an appropriate T -dependence in the PV masses. The constants c_n and c are determined by the requirement that on-shell quadratic UV divergences vanish; c is given by (4.8), and

$$c_n = \frac{1}{192\pi^2} \left(2 \sum_A q_n^A - N_\chi + N_G - 21 \right). \quad (4.17)$$

We have checked [37] for specific Z_3 and Z_7 orbifolds, with [38] and without [31] Wilson lines, that $c_n = b$, so the term proportional to Ω_W can also be canceled by the GS mechanism, provided the tree Lagrangian contains a term

$$L_{\text{tree}} \ni - \int d^4\theta E(S + \bar{S})\Omega_W, \quad (4.18)$$

which is indeed present in effective supergravity from the heterotic string. The ellipsis in the second parentheses in (4.14) represents ‘‘D-term’’ anomalies from additional logarithmic divergences of the form

$$\mathcal{L}_{1 \text{ loop}} \ni \partial_\mu O^\mu \ln \Lambda^2; \quad (4.19)$$

these have not yet been completely determined. The ellipsis in the first parenthesis in (4.14) represents terms nonlinear in the parameters of anomalous transformations. Their coefficients depend on the detailed choice of the PV Kähler potential, and therefore of the PV masses. The challenge is to find a choice that mimics the string result [31]. It may also be the case that full cancellation of the anomalies requires constraints on the Kähler potential for the twisted sector, analogous to the constraint (1.9) on gauge charges that is required for cancellation of UV divergences. Resolving these questions would have important implications for the phenomenological issues discussed in Section 3.

5 Afterword

Although Raymond and I have never written a paper together, we did have one very successful collaboration. Of the 51 students at the 1981 Les Houches summer school that we co-directed, at least 38 (some at this meeting) are still active in particle physics, and many are leaders in the field, not just in terms of scientific productivity, but also in terms of service to the scientific community.

Happy Birthday Raymond!



Acknowledgment

This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract

DE-AC02-05CH11231, in part by the National Science Foundation under grant PHY-0457315.

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